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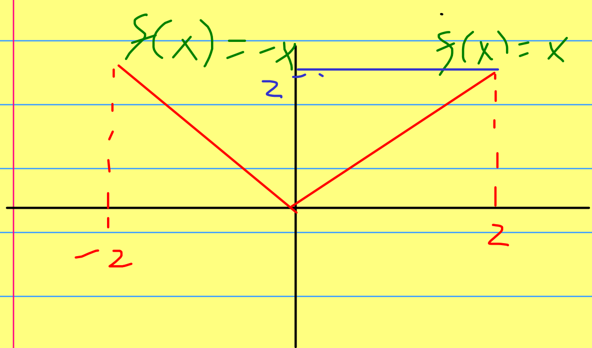
14. Hallar las series de Fourier de las funciones definidas por:

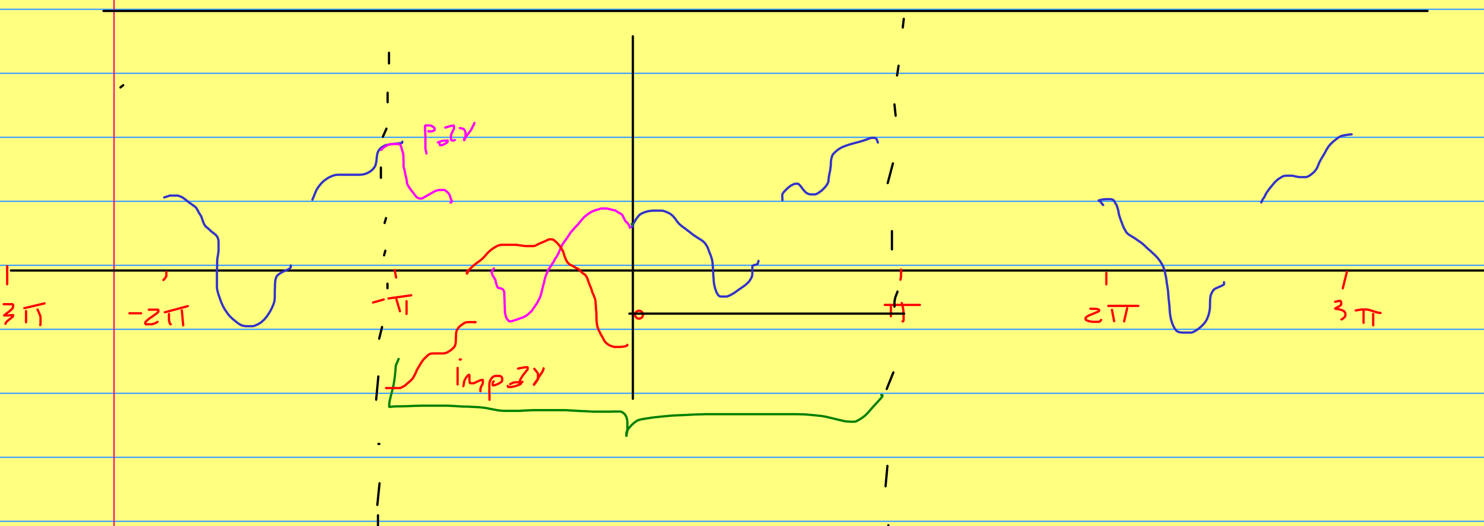
a) $f(x) = |x|$, si $-2 \leqslant x \leqslant 2$, 4-periódica .

$$a_0 = \frac{1}{2}\int_{-2}^{2} |x|\,dx = \frac{2}{2}\int_0^2 |x|\,dx = \int_0^2 x\,dx$$



$$|x| = \begin{cases} x & si\ x \geq 0 \\ -x & si\ x < 0 \end{cases}$$

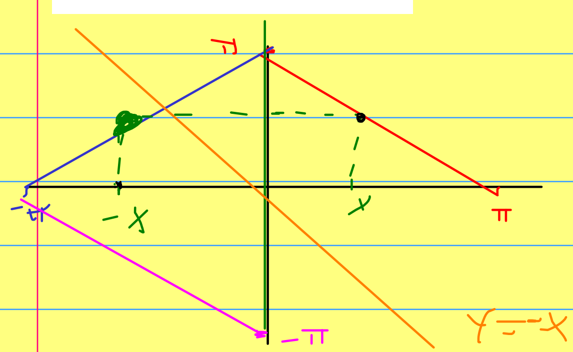
$$\int_{-2}^{2} |x|\,dx = \int_{-2}^{0} -x\,dx + \int_0^2 x\,dx$$



Ej. b) $f(x) = \pi - x$, $x \in (0, \pi)$

$g(x) = g(-x)$

$g(-x) = -g(x)$



$f:(0,\pi) \to \mathbb{R}$

Extensión par:

$\boxed{b_K = 0}$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_K = \frac{2}{\pi}\int_0^{\pi} f(x)\cos Kx\,dx$$

Serie de Fourier de cosenos.

Extensión impar

Serie de Fourier de senos.

$\boxed{a_K = 0}$

$$b_K = \frac{2}{\pi}\int_0^{\pi} f(x)\,\text{sen}\,Kx\,dx$$

b) $f(x) = \pi - x$

Serie de $\underbrace{\text{Fourier de cosenos}}_{b_K = 0\ \forall K}$:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} \pi - x\,dx = -\frac{2}{\pi}\int_{\pi}^{0} u\,du$$

$u = \pi - x$, $u(\pi) = 0$

$du = -dx$, $u(0) = \pi$

$$= -\frac{2}{\pi}\left.\frac{u^2}{2}\right|_{\pi}^{0}$$

$$= -\frac{2}{\pi}\left[0 - \frac{\pi^2}{2}\right]$$

+

$\boxed{a_0 = \pi}$

$$= \frac{\cancel{2}\pi^{\cancel{2}}}{\cancel{2}\cancel{\pi}} = \pi$$

$$a_K = \frac{2}{\pi}\int_0^{\pi} f(x)\cos Kx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} (\pi - x)\cos Kx\,dx$$

$$= \frac{2}{\pi}\left[\pi \underbrace{\int_0^{\pi} \cos Kx\,dx}_{=0} - \int_0^{\pi} x\cos Kx\,dx\right]$$

$$\frac{\operatorname{sen} Kx}{K}\Bigg|_0^{\pi}$$

partes

$$\bullet \int_0^{\pi} \overset{f}{x}\,\overset{g'}{\cos Kx}\,dx = \cancel{\frac{x \operatorname{sen} Kx}{K}\Bigg|_0^{\pi}}^{\,0} - \int_0^{\pi} \frac{\operatorname{sen} Kx}{K}\,dx$$

$$= -\frac{1}{K}\int_0^{\pi} \operatorname{sen} Kx\,dx$$

$$= -\frac{1}{K}\left(-\frac{\cos Kx}{K}\Bigg|_0^{\pi}\right)$$

$$= \frac{1}{K^2}\Big(\underbrace{\cos K\pi}_{(-1)^K} - \underbrace{\cos 0}_{1}\Big)$$

$$\bullet \int_0^{\pi} x\cos Kx\,dx = \frac{(-1)^K - 1}{K^2}$$

$$a_K = -\frac{2}{\pi}\int_0^{\pi} x\cos Kx\,dx = \frac{2}{K^2\pi}\cdot\left(1-(-1)^K\right)$$

$$\boxed{a_K = \frac{2}{K^2\pi}\left(1-(-1)^K\right)}$$

La serie de Fourier de cosenos es:

$$S_\infty(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{K=1}^{\infty}\frac{1-(-1)^K}{K^2}\cos Kx = g(x)$$

Cuando $x \in (0,\pi) \Rightarrow g(x) = f(x)$

Cuando $x \in (-\pi,0) \Rightarrow \boxed{g(x) = f(|x|)}$

pdv.

Práctico 7

1. Hallar la solución de la ecuación de ondas,

$$\boxed{u_{tt}(x,t) - c^2 u_{xx}(x,t) = 0} \quad (x,t) \in (0,L) \times (0,\infty)$$

con las condiciones de contorno:

Posición inicial

Velocidad inicial

$$\begin{cases} \boxed{u(x,0) = x(L-x)} & x \in [0,L] \\ \boxed{u_t(x,0) = 0} & x \in [0,L] \\ \boxed{u(0,t) = u(L,t) = 0} & t \in [0,\infty) \end{cases}$$

utilizando el método de separación de variables.

Extremos de la onda están fijos.

$u(x,0) = 0 \quad \forall t$

Solución trivial

Suponer:

• Variables separables: $u(x,t) = X(x)\, T(t)$

$u_{tt}(x,t) = X(x)\, T''(t)$

$u_{xx}(x,t) = X''(x)\, T(t)$

Sustituyo en la ecuación

$X(x)\, T''(t) - c^2 X''(x)\, T(t) = 0$

$$\underbrace{\frac{1}{c^2} \frac{T''}{T}(t) = \frac{X''}{X}(x)}$$

Son constantes K

$$\begin{cases} \dfrac{1}{c^2}\dfrac{T''(t)}{T} = K \;\to\; T''(t) - Kc^2 T(t) = 0 \\[2ex] \dfrac{X''(x)}{X} = K \;\to\; \boxed{X''(x) - K X(x) = 0} \end{cases}$$

Ecuaciones diferenciales ordinarias e independientes.

Necesitamos condiciones iniciales para X y T

$$\begin{cases} u(x,0) = x(L-x) & x \in [0,L] \\ u_t(x,0) = 0 & x \in [0,L] \\ u(0,t) = u(L,t) = 0 & t \in [0,\infty) \end{cases}$$

$u(x,t) = X(x)\,T(t)$

- $u(0,t) = X(0)\,T(t) = 0 \quad \forall t \in [0,\infty)$

$$\boxed{X(0) = 0}$$

$$\underline{T(t) = 0} \quad \forall t \in [0,\infty)$$

Si $T(t) = 0 \quad \forall t \in [0,\infty)$

$\Rightarrow u(x,t) = X(x)\,T(t) = 0 \quad \forall t$

$$\mathcal{U}(L,t) = X(L)\,T(t) \overset{\text{hipótesis}}{=} 0 \qquad \forall t \in [0, +\infty)$$

$$\boxed{X(L) = 0}$$

$$T(t) = 0 \quad \forall t$$

$$\begin{cases} X'' - KX = 0 \\ X(0) = 0 \\ X(L) = 0 \end{cases}$$

Polinomio característico: $\lambda^2 - K = 0$

• $K=0$: $X'' = 0 \Rightarrow X(x) = ax + b$

$0 = X(0) = b \Rightarrow \boxed{b=0}$

$0 = X(L) = aL \underset{L>0}{\Rightarrow} \boxed{a=0}$

$X(x) = 0 \ \forall x$

$u(x,t) = 0$.

• $K>0$: $\lambda^2 = K \rightarrow \boxed{\lambda = \pm\sqrt{K}}$

$$X(x) = C_1 e^{\sqrt{K}x} + C_2 e^{-\sqrt{K}x} \Rightarrow \boxed{X(x) = 0}$$

$$0 = X(0) = C_1 + C_2 \rightarrow C_2 = -C_1$$

$$0 = X(L) = C_1 e^{\sqrt{K}L} + C_2 e^{-\sqrt{K}L} \qquad \boxed{C_2 = 0}$$

$$0 = C_1 e^{\sqrt{K}L} - C_1 e^{-\sqrt{K}L}$$

$$= C_1 [\underbrace{e^{\sqrt{K}L} - e^{-\sqrt{K}L}}_{>0}] \rightarrow \boxed{C_1 = 0}$$