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7. Se considera la ecuación diferencial $x' = \cos(t + x)$.

a) Buscar soluciones de la forma $x(t) = at + b$.

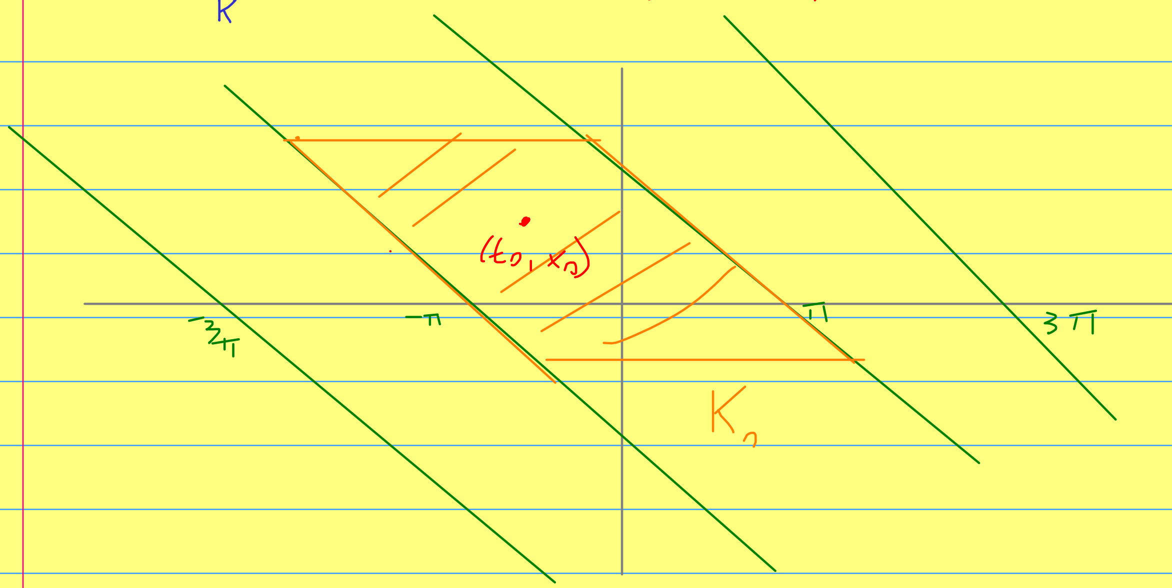
b) Probar, usando salida de compactos, que todas las soluciones maximales están definidas en $\mathbb{R}$.

c) Hallar el lugar geométrico de los máximos, mínimos y puntos de inflexión de las soluciones.

d) A partir de las partes anteriores, realice un bosquejo del gráfico de las soluciones maximales para distintas condiciones iniciales.

a) $x_K(t) = -t + \pi(1 + 2K)$ con $K \in \mathbb{Z}$

b)



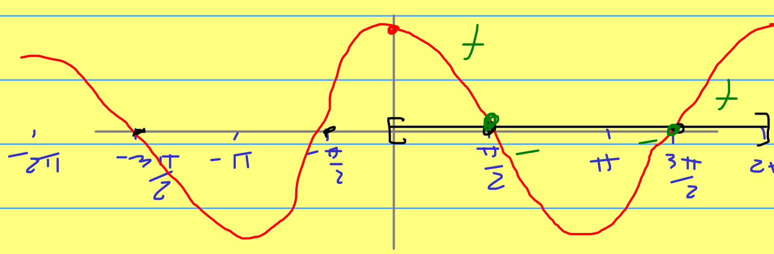
c) Hallar el lugar geométrico de los máximos, mínimos y puntos de inflexión de las soluciones.

Máx y mín: $\boxed{\begin{array}{l} \dot{x} = 0 \\ \ddot{x} \neq 0 \end{array}}$

Puntos de inflexión $\boxed{\ddot{x} = 0}$

$\dot{x} = \cos(t + x) \overset{\text{queremos}}{=} 0$

$t + x = \frac{\pi}{2} + 2K\pi$



$t + x = \frac{3\pi}{2} + 2K\pi$

$$\left.\begin{aligned} x(t) &= -t + \pi\left(\frac{1}{2} + 2K\right) \\ x(t) &= -t + \pi\left(\frac{3}{2} + 2K\right) \end{aligned}\right\} \text{Candidatos}$$

$\dot{x} = \cos(t+x)$

$\ddot{x} = -\operatorname{sen}(t+x)(1+\dot{x})$

Como solo nos importa $\ddot{x}$ en los puntos críticos, podemos tomar $\dot{x} = 0$

$\ddot{x} = -\operatorname{sen}(t+x)$

Obs: $\underbrace{\cos^2(t+x)}_{0} + \operatorname{sen}^2(t+x) = 1 \to \operatorname{sen}(t+x) = \pm 1$

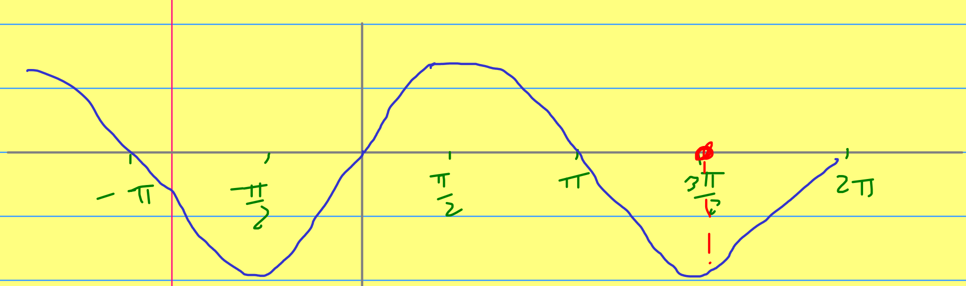
Máximos: $\ddot{x} < 0$ sii $-\operatorname{sen}(t+x) = -1$

sii $\operatorname{sen}(t+x) = 1$

$\boxed{\bullet\ x = -t + \frac{\pi}{2} + 2K\pi} \to \operatorname{sen}(t+x) = 1$

sii $t + x = \frac{\pi}{2} + 2K\pi$

• $x = -t + \frac{3\pi}{2} + 2K\pi$

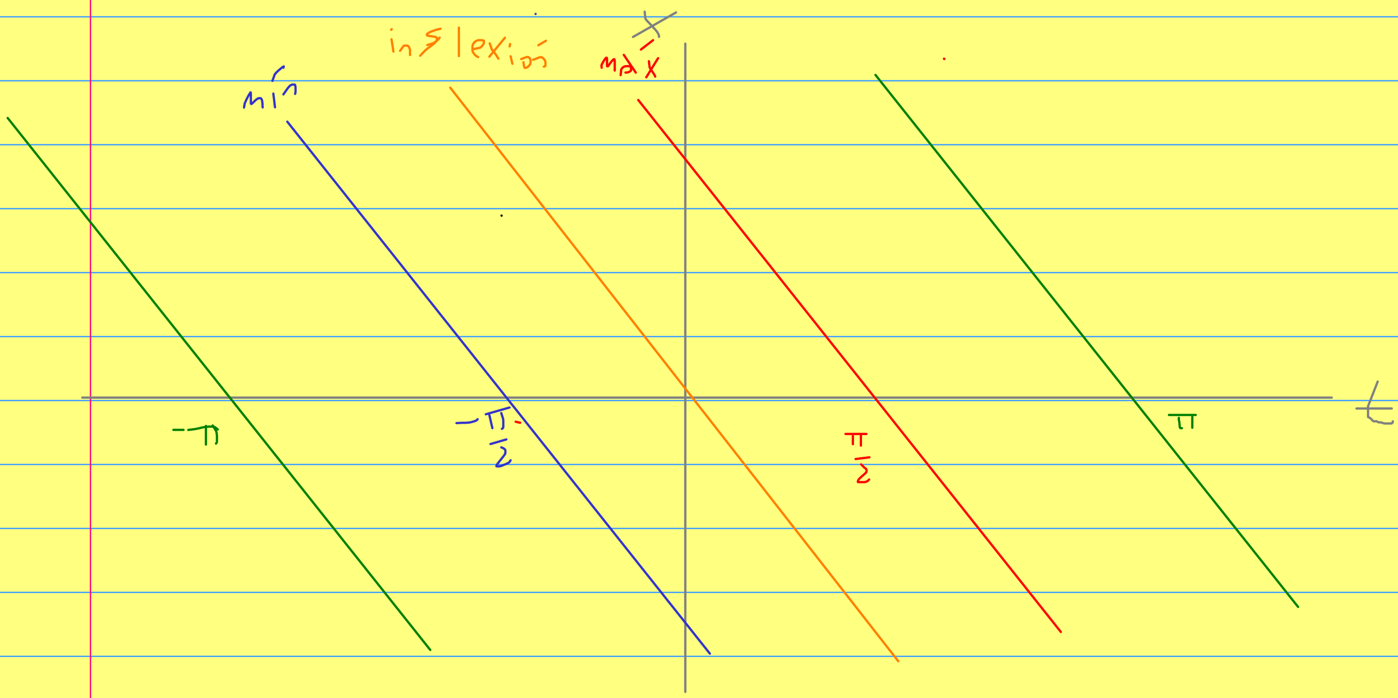


Mínimos: $\ddot{x} > 0$

sii $-\operatorname{sen}(t+x) = 1$

sii $\operatorname{sen}(t+x) = -1$

sii $t+x = \frac{3\pi}{2} + 2k\pi$



$\dot{x} = \cos(t+x)$

$\ddot{x} = -\operatorname{sen}(t+x)(1+\dot{x})$

$= -\operatorname{sen}(t+x)\cdot(1+\cos(t+x)) = 0$

Queremos

sii $\operatorname{sen}(t+x) = 0$ sii $t+x = k\pi$

y/o

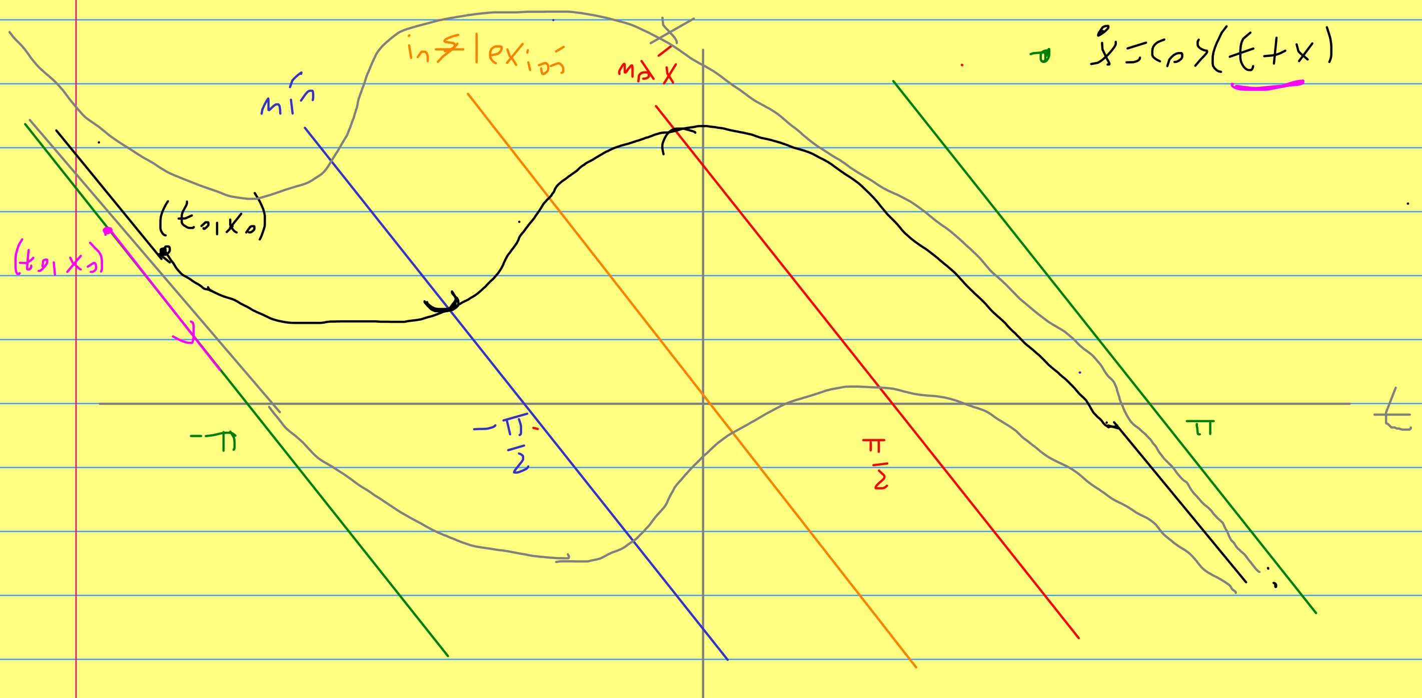
$1+\cos(t+x) = 0$ sii $\cos(t+x) = -1$

Sii $t+x=\pi+2K\pi$
$=\pi(1+2K)$

Puntos de inflexión están en las rectas

$x(t)=-t+K\pi$ con $K\in\mathbb{Z}$

• $\dot{x}=\cos(t+x)$

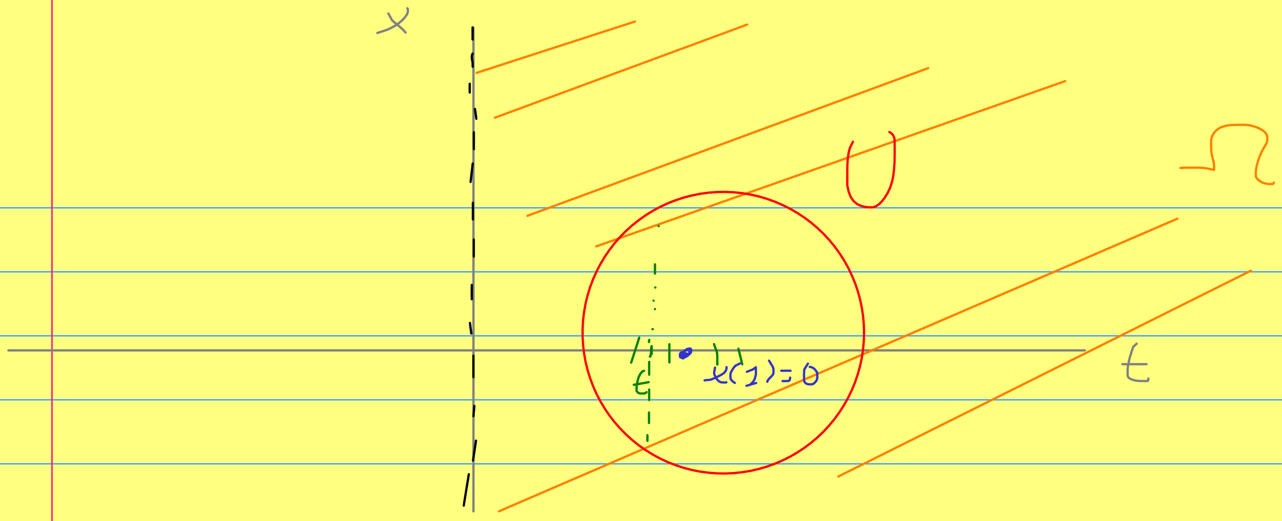


• $\dot{x}=\cos(t+x)$

Consultas

4. $\dot{x}=f(t,x)$, $\quad x(1)=0$ con $f:(0,+\infty)\times\mathbb{R}\to\mathbb{R}$ dada por $f(t,x)=\min\left\{\frac{x}{t},t\right\}$

$$\begin{cases}\dot{x}=\min\left\{\frac{x}{t},t\right\}\\ x(1)=0\end{cases}$$

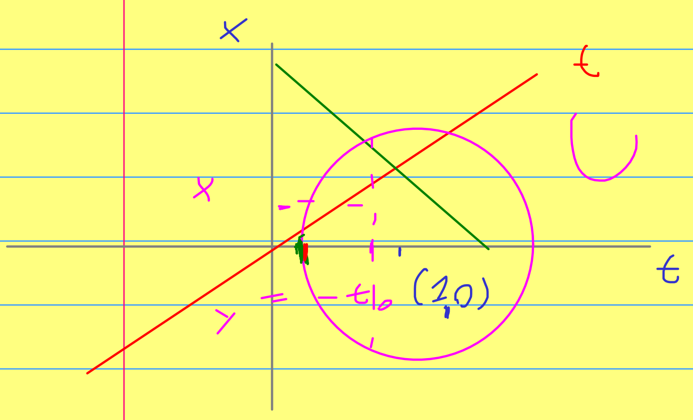


Sea t_0 fijo "cerca" de $t=1$

$$\exists K>0 \text{ tq } \frac{|f(t_0,x)-f(t_0,y)|}{|x-y|} \le K$$

$$|f(t_0,x)-f(t_0,y)| = \left|\min\left\{\frac{x}{t_0}, t_0\right\} - \min\left\{\frac{y}{t_0}, t_0\right\}\right|$$

$\{ |t_0 - t_0| = 0$



$$\left|\frac{x}{t_0} - t_0\right|$$

$$\left|\frac{x}{t_0} - \frac{y}{t_0}\right|$$

$$\Rightarrow \frac{1}{|t_0|}\frac{|x-y|}{|x-y|} \le K$$

1

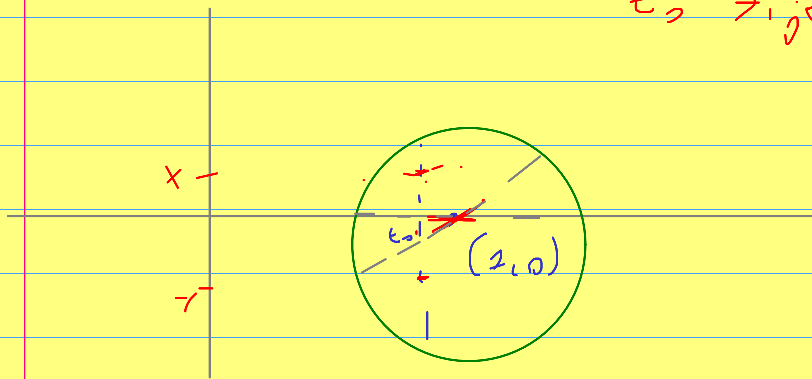
$$K = \min\{t : (t,x)\in U\}$$

$K > 0$

$$\left|\frac{x}{t_0} - t_0\right| \le K\,|x-1|$$

t_0 fijo $\quad f(t_0, x) = \frac{x}{t_0}$

$f(t_0, 1) = t_0$



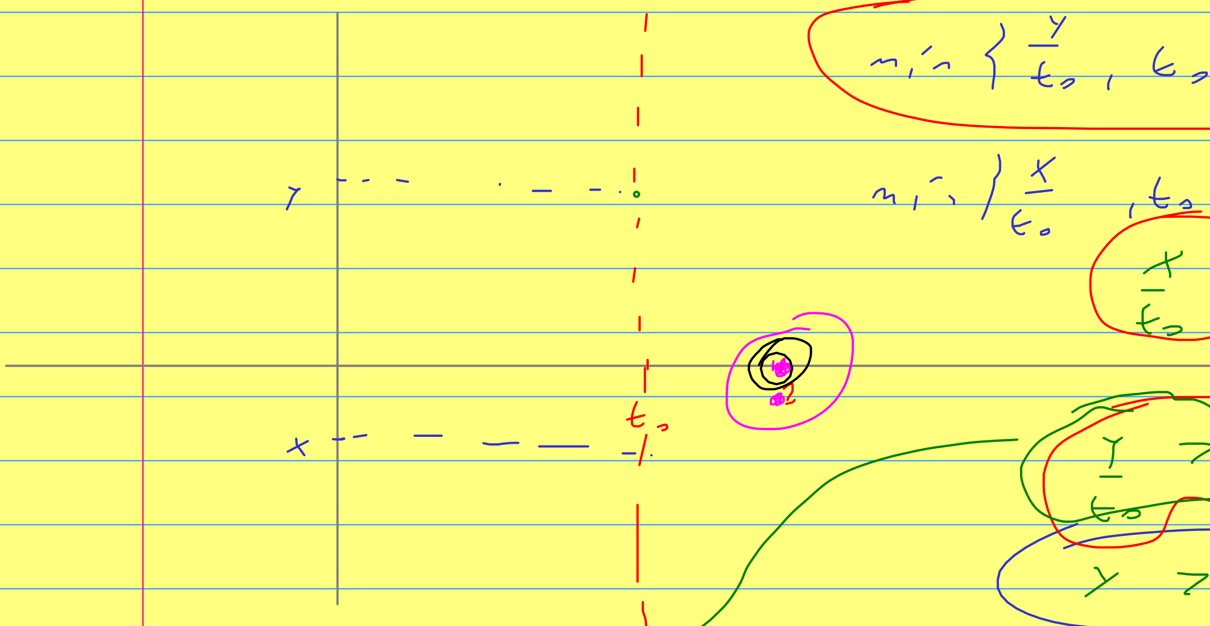
$\min\left\{\frac{1}{t_0}, t_0\right\} = t_0$

$\min\left\{\frac{x}{t_0}, t_0\right\} = \frac{x}{t_0}$

$\frac{x}{t_0} < t_0$, $x < t_0^2$

$\frac{1}{t_0} > t_0$

$1 > t_0^2$



$$\left|\frac{x}{t_0} - t_0\right| \le K\,|x-1|$$

MAL

$$\boxed{\frac{x}{t_0} - t_0 \;\cancel{<}\; \frac{x}{t_0} - \frac{1}{t_0}} \quad = \frac{1}{t_0}(x-1)$$

$>$

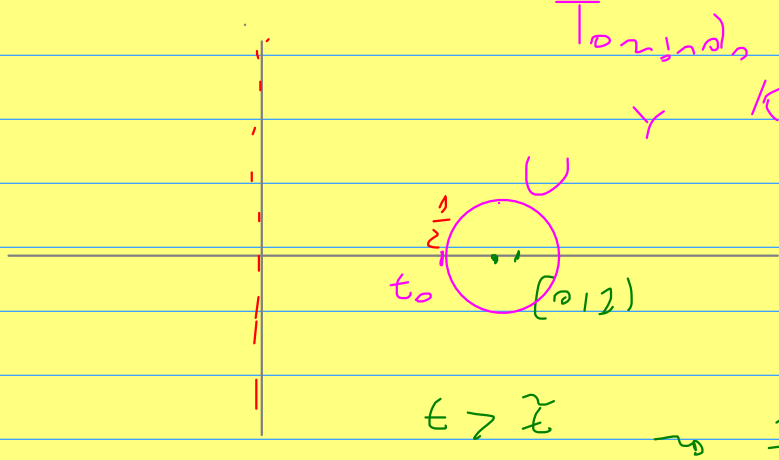
$$\frac{\left|\frac{x}{t_0} - t_0\right|}{|x-1|} < \frac{\left|\frac{x}{t_0} - \frac{1}{t_0}\right|}{|x-1|} = \left|\frac{1}{t_0}\right| \frac{|x-1|}{|x-1|}$$

1

$$\frac{\left|\frac{x}{t_0} - t_0\right|}{|x-1|} < \left|\frac{1}{t_0}\right| < K$$

Tomando $\tilde{t} = \min\{t_0 : (t_0, x) \in U\}$

y $K = \left|\frac{1}{\tilde{t}}\right|$.



$t > \tilde{t} \to \frac{1}{t} < \frac{1}{\tilde{t}} = K$

$\frac{y}{t_0} > t_0 \to \frac{x}{t_0} - \frac{1}{t_0} < \frac{x}{t_0} - t_0$