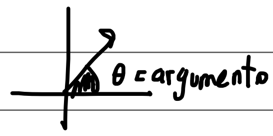
Práctica 1 : (complejos) $\mathbb{C}$

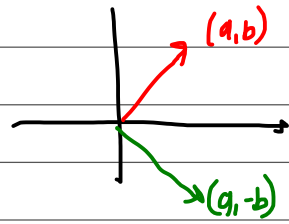
Repaso teórico: Los complejos son $\mathbb{R}^2$ con más estructura : Son un cuerpo completo.

- módulo : $|(a,b)| = \sqrt{a^2+b^2}$
- argumento : ángulo que forma el vector con eje x



- Parte real e imaginaria : (a,b), a → real, b → imag.

- Conjugado : $\overline{(a,b)} = (a,-b)$



- Definimos $(a,b)\overset{\text{prod. complejo}}{\cdot}(c,d) = (ac-bd,\ ad+bc)$

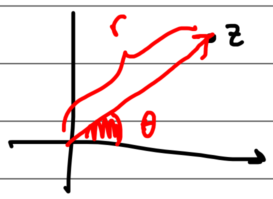
- Notaciones :

  - Binómica : $(a,b) = a\underbrace{(1,0)}_{1} + b\underbrace{(0,1)}_{i} = a+bi$

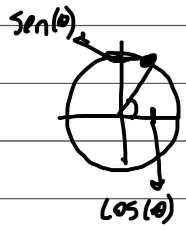
    ↳ el prod en esta notación es "más fácil" $(i^2=-1)$

$$(a+bi)(c+di) = ac + i.i\,bd + i.bc + i\,ad$$
$$= ac - bd + i(bc+ad)$$

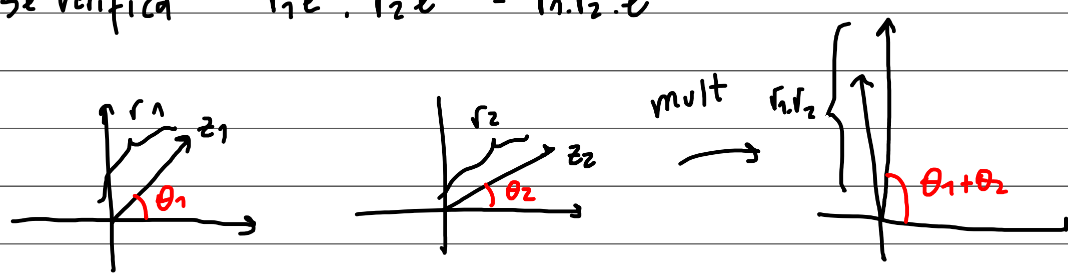
  - Polar : Un pto de $\mathbb{R}^2$ se determina por la dist. al cero y el ángulo (argumento)



$$z = re^{i\theta} = r.(\cos(\theta), \operatorname{sen}(\theta))$$
$$= r(\cos(\theta) + i\operatorname{sen}(\theta))$$



Obs : se verifica $r_1e^{i\theta_1} . r_2e^{i\theta_2} = r_1.r_2.e^{i(\theta_1+\theta_2)}$

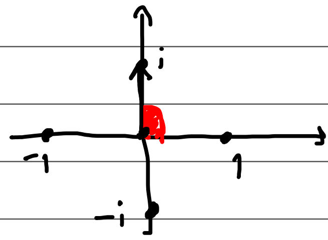


- Exponencial compleja : $e^{a+bi} := e^a . e^{bi} = e^a(\cos(b) + i\operatorname{sen}(b))$

  ↓ complejo de radio e^a y ángulo b

Obs : se verifica $e^{z_1}.e^{z_2} = e^{z_1+z_2}$.

1)



$i = r.e^{i\theta} = 1.e^{i\pi/2}$.

$i^1 = i$

$i^2 = -1 = e^{i\pi}$

$i^3 = -i$

$i^4 = 1$

$i^n.\underbrace{i^{4k}}_{1} = i^{n+4k} = i^n$

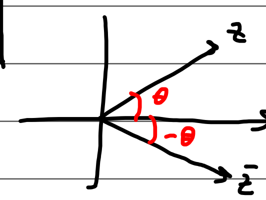
2) Antes :

- Prop conjugado :
  - $z.\bar{z} = |z|^2$
  - $z+\bar{z} = 2Re(z)$
  - $z-\bar{z} = 2i.Im(z)$

- inverso de un num complejo : $\frac{1}{z} = z^{-1}$, por esto se deduce que

$$\boxed{z^{-1} = \frac{\bar{z}}{|z|^2}}$$

- Conjugado en polares : $\overline{re^{i\theta}} = re^{-i\theta}$



c) $\frac{1}{1+i} = (1+i)^{-1} = \frac{1-i}{1^2+1^2} = \frac{1}{2} - \frac{i}{2} \rightarrow$ binómica.

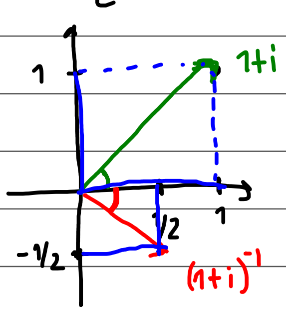
$1-i = (1,-1)$

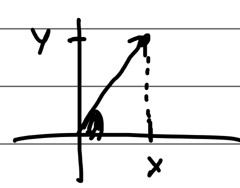
$|1-i|^2 = 1^2+(-1)^2 = 2$.

¿Y en polares?

- radio : $|(\frac{1}{2},-\frac{1}{2})| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{1/2} = 1/\sqrt{2}$
- argumento : $-\pi/4$

$\frac{1}{1+i} = \frac{1}{\sqrt{2}}.e^{-i\pi/4}$





$tg(\theta) = \frac{y}{x}$

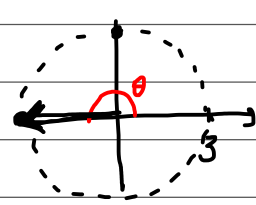
$\theta = arctg(y/x)$

e) $\frac{1}{(1+i)^2} = ((1+i)^{-1})^2 = ((1+i)^2)^{-1} = (2i)^{-1} = -\frac{i}{2}$

3) b) $3e^{\pi i}$.

radio = 3

ángulo = π



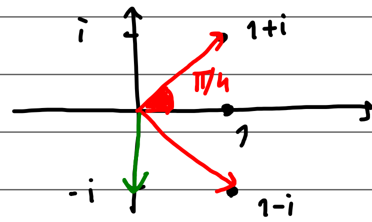
$3e^{\pi i} = 3(-1) = -3$.

$$\left[e^{i\pi} + 1 = 0 \right]$$

$3e^{\pi i} = 3.(\cos(\pi) + i\,sen(\pi)) = 3(-1+1.0) = -3$.

c) $\dfrac{1-e^{i\pi/2}}{1+e^{i\pi/2}} = \dfrac{1-i}{1+i} = \dfrac{1-i}{(1+i)}\dfrac{(1-i)}{(1-i)} = \dfrac{\cancel{1+i^2}-2i}{1^2-i^2} = \dfrac{-2i}{2} = -i$

$1e^{i\pi/2} = i$ — radio$(i) = 1$, ángulo$(i) = \pi/2$



d) $(1+i)^{100}$

$(1+i) = \sqrt{2}\,.\,e^{i\pi/4}$

$$(1+i)^{100} = \left(\sqrt{2}\,.\,e^{i\pi/4}\right)^{100} = \sqrt{2}^{100}.\left(e^{i\pi/4}\right)^{100}$$

$$= \left((\sqrt{2})^2\right)^{50}.\,e^{i\frac{100\pi}{4}} = 2^{50}.\,e^{i25\pi}$$

$$= 2^{50}.\,e^{i\pi} = -2^{50}$$

$1.e^{i\pi}$ = complejo de radio 1 y ángulo π