Seguimos con práct 4.

1). Recuerden clase pasada hablamos de $\sum_{k}^{\infty} r^n$ (converge si $r \in [0,1)$).

e). $\sum_{n=1}^{\infty} \log\left(\frac{n^2+2n+1}{n^2}\right) = \sum_{n=1}^{\infty} \log\left(\left(\frac{n+1}{n}\right)^2\right) = \sum_{n=1}^{\infty} 2\log\left(\frac{n+1}{n}\right)$

$= 2\sum_{1}^{\infty}\left(\log(n+1) - \log(n)\right)$ $\qquad \frac{(n+1)^2}{n^2} = \left(\frac{n+1}{n}\right)^2$

En gral una serie telescópica es de la forma $\sum_{1}^{\infty}(a_{n+1}-a_n)$

$\underbrace{(\cancel{a_2}-a_1)}_{1} + \underbrace{(\cancel{a_3}-\cancel{a_2})}_{2} + \underbrace{(\cancel{a_4}-\cancel{a_3})}_{3} + \underbrace{(a_5-\cancel{a_4})}_{4} = a_5 - a_1.$

$S_K = \sum_{1}^{K}(a_{n+1}-a_n) = a_{K+1} - a_1.$

$= 2\lim_{K\to\infty} \sum_{1}^{K} \log(n+1) - \log(n) = 2\lim_{K\to\infty} \log(K+1) - \log(1)$

$= 2\lim_{K\to\infty} \log(K+1)$

$= \infty.$

$\sum_{1}^{\infty} \log\left(\frac{n^2+2n+1}{n^2}\right) = \infty$

* Recordar $\sum_{1}^{\infty} a_n = \lim_{K\to\infty} \sum_{1}^{K} a_n$

d) $\sum_{1}^{\infty} \frac{3}{n(n+3)} = \sum_{1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+3}\right)$ (es como la telescópica pero sobreviven más términos)

f). $\sum_{1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)} = \sum_{1}^{\infty} \frac{a}{n+1} + \frac{b}{n+2} + \frac{c}{n+3}$ $\rightarrow \begin{cases} a = -1/2 \\ b = 2 \\ c = -3/2. \end{cases}$

$= \sum_{1}^{\infty} \frac{-1}{2(n+1)} + \frac{2}{(n+2)} - \frac{3}{2(n+3)}$

$= \sum_{1}^{\infty} \frac{-1}{2(n+1)} + \frac{4}{2(n+2)} - \frac{3}{2(n+3)}$

$$= \sum_{1}^{\infty} \underbrace{\frac{-1}{2(n+1)} + \frac{1}{2(n+2)}} + \underbrace{\frac{3}{2(n+2)} - \frac{3}{2(n+3)}} =$$

$$\downarrow \quad -a_{n+1} \quad a_{n+2} \quad 3a_{n+2} \quad -3a_{n+3}$$

Llamamos $\boxed{a_n = \frac{1}{2n}}$

$$3a_{n+2} = 3 \cdot \frac{1}{2(n+2)}$$

$$= \lim_{K\to\infty} \sum_{1}^{K} (a_{n+2} - a_{n+1}) + 3(a_{n+2} - a_{n+3}).$$

$$\lim_{K\to\infty} [\cancel{a_3} - a_2 + 3(a_3 - \cancel{a_4})] + [a_4 - \cancel{a_3} + 3(\cancel{a_4} - a_5)]$$

$$+ \dots [a_{K+2} - a_{K+1} + 3(a_{K+2} - a_{K+3})].$$

$$\lim_{K\to\infty} (-a_2 + a_{K+2}) + 3(a_3 - a_{K+3}).$$

$$= -a_2 + 3a_3. = \frac{-1}{2(2)} + 3 \cdot \frac{1}{2\,3} = \frac{-1}{4} + \frac{1}{2}.$$

g) $$\sum_{n=1}^{\infty} \frac{n\,\text{arctg}(n+1) - (n+1)\,\text{arctg}(n)}{n.(n+1)} = \sum_{1}^{\infty} \underbrace{\frac{\text{arctg}(n+1)}{n+1}}_{a_{n+1}} - \underbrace{\frac{\text{arctg}(n)}{n}}_{a_n}.$$

$$= \lim_{K\to\infty} \sum_{1}^{K} a_{n+1} - a_n = \lim_{K\to\infty} a_{K+1} - a_1$$

$$= -a_1 = -\text{arctg}(1).$$

$$= -\pi/4.$$

$$a(n) = a_n = \frac{\text{arctg}(n)}{n} \xrightarrow{n\to\infty} 0.$$

acotado.

$$\left(\lim_{K\to\infty} \frac{\text{arctg}(K+1)}{K} = 0 \right)$$

$$\lim_{K\to\infty} (\cancel{a_2} - a_1) + (\cancel{a_3} - \cancel{a_2}) + \dots (a_{K+1} - \cancel{a_K})$$

Los sig. criterios (cj 2,3,4,5) son para series de términos positivos

2) Si $\sum_{1}^{\infty} a_n < \infty$ y b_n tq $b_n < a_n . \forall n \geq K. \Rightarrow \sum_{1}^{\infty} b_n < \infty.$

Recordar que $\sum \frac{1}{n} = \infty$, $\sum \frac{1}{n^2} < \infty$.

a). $\sum_{1}^{\infty} \frac{1}{n^n}$ a partir de $n=2$, $n^n > n^2 \Rightarrow \frac{1}{n^n} < \frac{1}{n^2}$

Como $\sum \frac{1}{n^2} < \infty \Rightarrow \sum \frac{1}{n^n} < \infty$. ✓

b). $\sum_{1}^{\infty} e^{-\sqrt{n+1}}$ Veamos $e^{-\sqrt{n+1}} = \frac{1}{e^{\sqrt{n+1}}} < \frac{1}{(n+1)^2}$

Con un cambio de var. $\left(u^2 = (n+1) \right)$ es equiv a ver $\frac{1}{e^u} < \frac{1}{u^4}$

equiv a ver $u^4 < e^u$

(a partir de un momento).

(Hay varias formas de ver esto.
Una de ellas es mostrar que $e^u - u^4 > 0$ a partir de un momento
o pueden ver que $\lim_{u\to+\infty} \frac{e^u}{u^4} = +\infty$.)

Concluimos que $e^{-\sqrt{n+1}} < \frac{1}{(n+1)^2}$ a partir de un N $\Rightarrow \sum e^{-\sqrt{n+1}} < \infty$.

3) Crit. de equivalencia: $\sum a_n$ y $\sum b_n$ tienen mismo comportamiento si $\frac{a_n}{b_n} \to L > 0$.

a) $\sum_{n=0}^{\infty} \frac{1}{n^2+1} \sim \sum_{1}^{\infty} \frac{1}{n^2}$ $\left(\text{Formalmente } \frac{1}{n^2+1} \Big/ \frac{1}{n^2} = \frac{n^2}{n^2+1} \to 1 \right)$

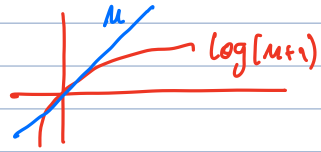
$\Rightarrow \sum \frac{1}{n^2+1} < \infty$ $\left(\text{pues } \sum_{0}^{\infty} \frac{1}{n^2} < \infty \right)$.

otro ejemplo: $\sum \frac{n^8+3n^7+2n}{2n^{10}+3n^5} \sim \sum_{1}^{\infty} \frac{1}{n^2}$ $\left(\text{Formalmente } a_n/b_n \text{ es } \left(\frac{n^8+3n^7+2n}{2n^{10}+3n^5}\right) n^2 \to \frac{1}{2} \right)$

¿Corolarios?
$\left(\begin{array}{l} \frac{a_n}{b_n} \to \infty \text{ y } \sum b_n = \infty \Rightarrow \sum a_n = \infty. \\ \frac{a_n}{b_n} \to 0 \text{ y } \sum b_n < \infty \Rightarrow \sum a_n < \infty. \end{array} \right)$

c) $\displaystyle\sum_{1}^{\infty} \frac{\log(n+1)-\log(n)}{10n+1} = \sum_{1}^{\infty} \frac{\log\left(\frac{n+1}{n}\right)}{10n+1} = \underbrace{\sum_{1}^{\infty} \frac{\log(1+1/n)}{10n+1}}_{a_n}$

$\left(\log(1+u) \underset{u\to 0}{\sim} u\right)$



* casi fácil.

En este caso, la serie es equiv. a $\displaystyle\underbrace{\sum_{1}^{\infty} \frac{1/n}{10n+1}}_{b_n}$ fácil.

que a su vez es equiv. a $\displaystyle\sum \frac{1}{n^2} < \infty$.

* $\displaystyle\frac{\log(1+1/n)}{1/n} \longrightarrow 1.$

Formalmente para probar equivalencia

$\displaystyle\frac{a_n}{b_n} = \left(\frac{\log(1+1/n)}{\cancel{10n+1}}\right)\cdot\frac{\cancel{10n+1}}{1/n} = \frac{\log(1+1/n)}{1/n}.$

Consecuencia de que

$\displaystyle\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$ (L'Hôpital).