Práctica 7:

Sea $f: \mathbb{R}^2 \to \mathbb{R}$, definimos el conjunto de nivel $a \in \mathbb{R}$ para f como

$$C_a = \{ (x,y) \in \mathbb{R}^2 : f(x,y) = a \} = f^{-1}(\{a\})$$

1) a) $f(x,y) = x^2+y^2$

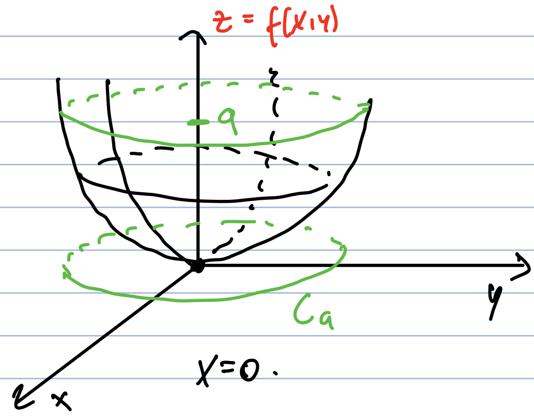
- Obs: Si $a<0$, $C_a(f) = \{(x,y) : f(x,y) = a\} = \phi$.
- Si $a=0$, $f(x,y)=0$ sii $x^2+y^2=0$ sii $(x,y)=(0,0)$

  Entonces $C_0(f) = \{(0,0)\}$.

- Si $a>0$, $f(x,y) = a$ sii $x^2+y^2 = a$.

  $C_a = \{(x,y) : x^2+y^2 = a\}$.

  ↳ circunferencia de radio $\sqrt{a}$

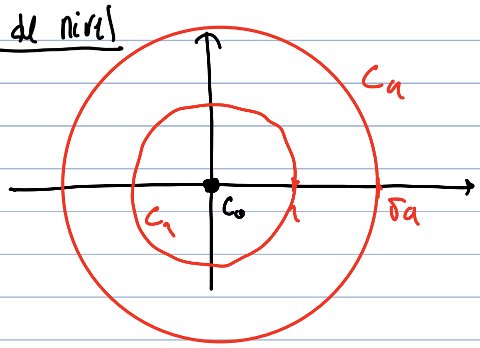


$X=0$.

$f(0,y) = y^2$.

$f(x,0) = x^2$.

Dibujo de conj de nivel



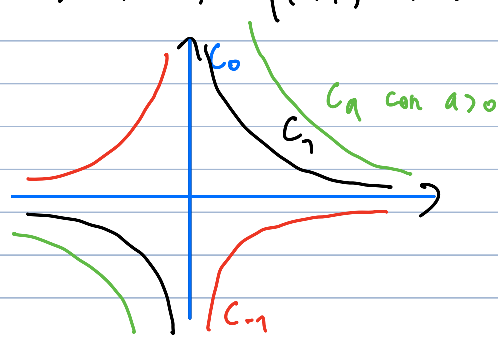
e) $f(x,y) = xy$.

Fijamos $a \in \mathbb{R}$.

- Si $a=0$, $f(x,y)=0$ sii $xy=0$ sii $x=0$ ó $y=0$.

  $C_a = \{(0,y) : y \in \mathbb{R}\} \cup \{(x,0) : x \in \mathbb{R}\}$.

- Si $a \neq 0$, $f(x,y)=a$ sii $xy=a$ si $y = \frac{a}{x}$. $C_a = \{(x,y) : y = \frac{a}{x}\}$.



Dibujo de (algunos) conj de nivel.

3) c) $f(x,y) = \cosh(x^2-y^2)$

$$\boxed{\cosh(r) = \frac{e^r + e^{-r}}{2}}$$

$$\boxed{\cosh^{-1}(a) = \pm\log\left(a+\sqrt{a^2-1}\right).}$$

Fijamos a.

(Observar que $\cosh(r) \geq 1 \quad \forall r$.

$\hookrightarrow C_a = \emptyset$ si $a < 1$)

- $\underline{\text{Si } a=1}$: $f(x,y) = 1$ sii $\cosh(x^2-y^2) = 1$ sii $x^2-y^2=0$ sii $x^2=y^2$
  sii $x=\pm y$.

- $\underline{\text{Si } a>1}$: $f(x,y) = a$ sii $\cosh(\underbrace{x^2-y^2}_{r}) = a$.

  sii $x^2-y^2 = \cosh^{-1}(a) = \pm\log\left(a+\sqrt{a^2-1}\right) = \pm b$, $b>0$.

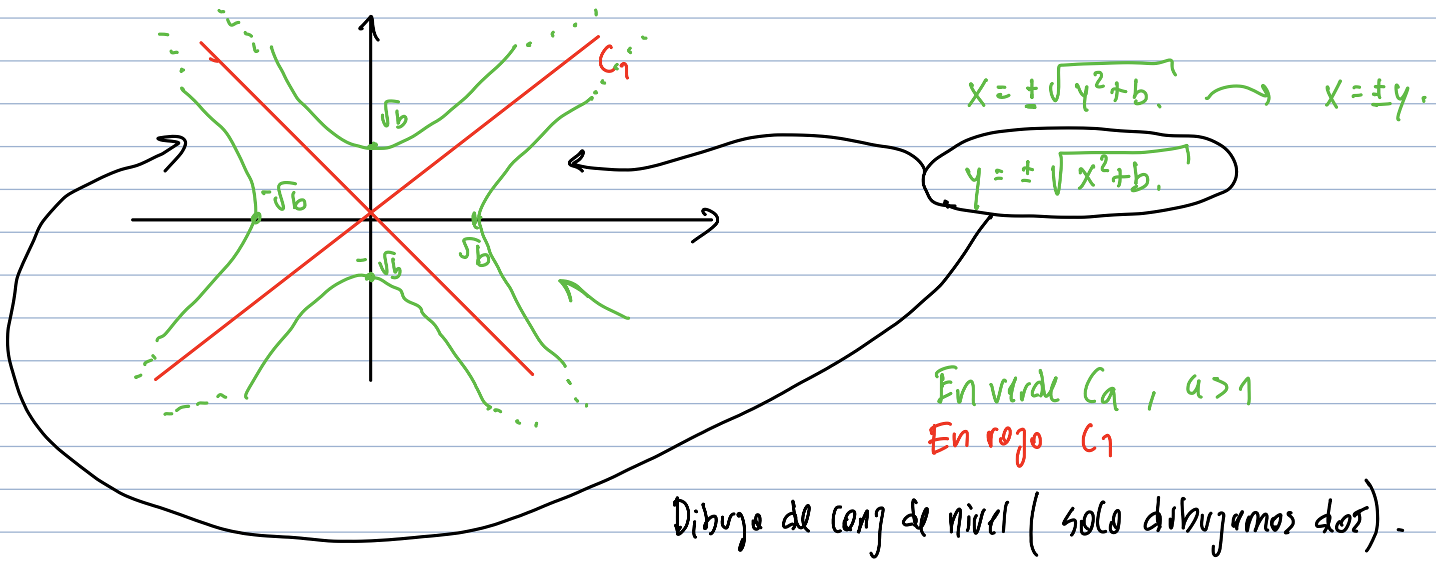
  sii $x^2-y^2 = \pm b$. $\to$ $x^2-y^2 = b$. / $x^2-y^2=-b$.

$f(x,y) = a$ sii $x^2 = y^2+b$ ó $y^2 = x^2+b$.

sii $x = \pm\sqrt{y^2+b}$ ó $y = \pm\sqrt{x^2+b}$.

$$C_a = \left\{(x,y)\in\mathbb{R}^2 : x = \pm\sqrt{y^2+b}\right\} \cup \left\{(x,y)\in\mathbb{R}^2 : y = \pm\sqrt{x^2+b}\right\}$$

donde $b = \log\left(a+\sqrt{a^2-1}\right)$



En verde C_a, $a>1$

En rojo C_1

Dibujo de conj de nivel (solo dibujamos dos).