• $D = \{(x,y) : y \in [c,d],\ x \in [a(y), b(y)]\}$

Práctico 10 :

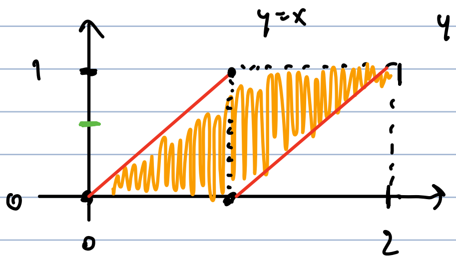
• $D = \{(x,y) : x \in [a,b],\ y \in [c(x), d(x)]\}$

3) c) $\int_D f$ donde $f(x,y) = xy^2$, $D = \{(x,y) : y \in [0,1],\ x \in [y, y+1]\}$

$$\int_0^1 \left[\int_y^{y+1} xy^2\,dx\right] dy = \int_0^1 \frac{x^2y^2}{2}\Big|_{x=y}^{x=y+1} dy = \int_0^1 \frac{(y+1)^2 \cdot y^2 - y^2y^2}{2}\,dy$$

$$= \frac{1}{2}\int_0^1 (y^2+2y+1)y^2 - y^4 = \frac{1}{2}\int_0^1 2y^3+y^2\,dy$$

$$= \frac{1}{2}\left(\frac{2y^4}{4} + \frac{y^3}{3}\right)\Big|_0^1$$



$D = \{(x,y) : y \in [0,1],\ x \in [y, y+1]\}$ ←

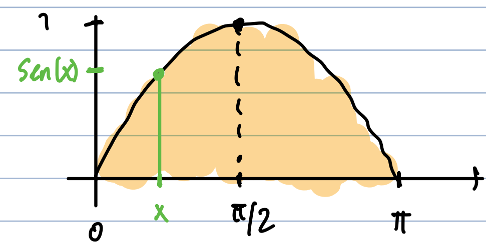
$D_1 = \{(x,y) : x \in [0,1],\ y \in [0,x]\}$

$D_2 = \{(x,y) : x \in [1,2],\ y \in [x-1, 1]\}$

$D = D_1 \cup D_2$ ←

$$\int_D f = \int_{D_1} f + \int_{D_2} f = \int_0^1\int_0^x xy^2\,dy\,dx + \int_1^2\int_{x-1}^1 xy^2\,dy\,dx.$$

3) d) $\int_0^\pi \int_0^{sen(x)} x^2 - y^2\,dy\,dx$ $\qquad D = \{(x,y) : x \in [0,\pi],\ y \in [0, sen(x)]\}$



$$\int_0^\pi \int_0^{sen(x)} x^2 - y^2\,dy\,dx = \int_0^\pi \left(x^2y - \frac{y^3}{3}\right)\Big|_{y=0}^{y=sen(x)} dx$$

$$= \int_0^\pi x^2 sen(x)\,dx - \int_0^\pi \frac{sen^3(x)}{3}\,dx$$

• $\int \underset{F}{\underset{\downarrow}{x^2}}\,\underset{g}{\underset{\downarrow}{sen(x)}}\,dx = -x^2\cos(x) + \int 2x\cos(x)\,dx$

$= -x^2\cos(x) + 2x\,sen(x) - \int 2\,sen(x)$

$= -x^2\cos(x) + 2x\,sen(x) + 2\cos(x)$

• $\int sen(x)\left(sen^2(x)\right)dx = \int sen(x)\left(1 - \cos^2(x)\right)dx$

$$\left(\begin{array}{l} u = \cos(x) \\ u' = -\operatorname{sen}(x) \end{array}\right) = -\int 1-u^2\,du = -\left(u - \frac{u^3}{3}\right) = -\left(\cos(x) - \frac{\cos(x)^3}{3}\right)$$

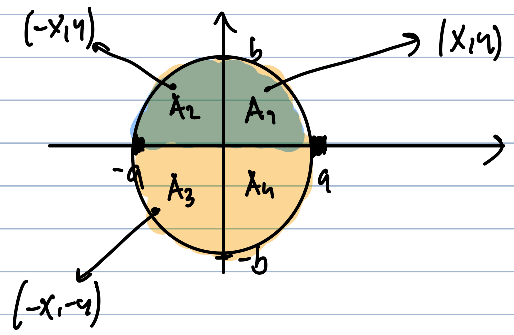
$$= \left(-x^2\cos(x) + 2x\operatorname{sen}(x) + 2\cos(x)\right) + \frac{1}{3}\left(\cos(x) - \frac{\cos(x)^3}{3}\right)\Bigg|_0^{\pi}$$

$$= \left[\pi^2 - 2 + \frac{1}{3}\left(-1 + \frac{1}{3}\right)\right] - \left[2 + \frac{1}{3}\left(1 - \frac{1}{3}\right)\right]$$

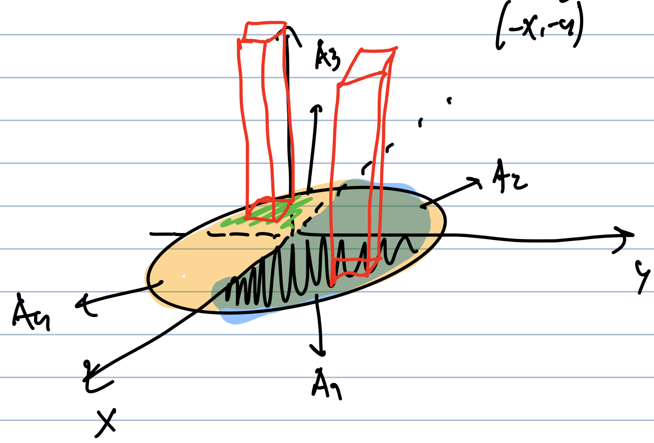
$$= \pi^2 - 4 + \frac{1}{3}\left(-2 + \frac{2}{3}\right) = \pi^2 - 4 + \frac{1}{3}\left(\frac{-6}{3} + \frac{2}{3}\right) = \pi^2 - 4 - \frac{4}{9}$$

$$= \boxed{\pi^2 - \frac{40}{9}}$$

3) e) $f(x,y) = xy$, $D_1 = \{(x,y) : \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \leq 1\}$, $D_2 = D_1 \cap \{(x,y) : y \geq 0\}$.



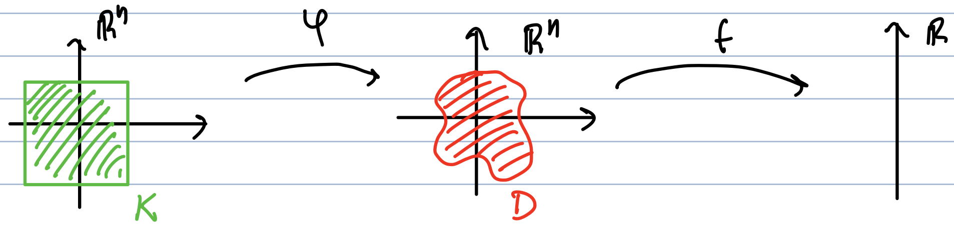
• Afirmación: $\displaystyle\int_{A_1} f = \int_{A_3} f = -\int_{A_2} f = -\int_{A_4} f$



Luego, por la afirmación • $\displaystyle\int_{D_1} f = \cancel{\int_{A_1} f} + \cancel{\int_{A_2} f} + \cancel{\int_{A_3} f} + \cancel{\int_{A_4} f} = 0$

• $\displaystyle\int_{D_2} f = \cancel{\int_{A_1} f} + \cancel{\int_{A_2} f} = 0$

Teorema cambio variable



$f: D \subset \mathbb{R}^n \to \mathbb{R}$ integrable, $\varphi: K \to D$ difeomorfismo (biyección diferenciable con inversa diferenciable)

Entonces
$$\boxed{\int_D f = \int_{K=\varphi^{-1}(D)} f\circ\varphi \cdot |\det J_\varphi|}$$

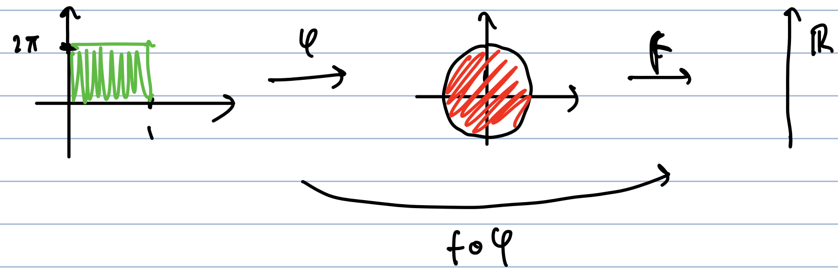
Por ejemplo:

$f: B(0,1) \to \mathbb{R}$

$\varphi: [0,1]\times[0,2\pi) \to B(0,1)$ polares

(Se tiene $|\det J_\varphi| = \rho$)

$$\int_{B(0,1)} f(x,y)\,dx\,dy = \int_{[0,1]\times[0,2\pi)} f(\rho\cos\theta, \rho\sin\theta)\,\rho\, d\rho\, d\theta$$



4) $T: \mathbb{R}^2 \to \mathbb{R}^2$ lineal,

· Comentarios: · T es continua. Más aún es diferenciable, y su diferencial coincide con ella $d_p T = T$

· En particular, si T isomorfismo lineal $\Rightarrow$ es difeomorfismo

Tenemos $D \subset \mathbb{R}^2$ medible Jordan (pensar en compacto)

Queremos calcular área de $T(D)$. para ciertos D.

· Si $D \subset \mathbb{R}^2$, $A(D) = \int_D 1$

Antes: ¿Qué pasa si T no es biyectiva? Si no es biyectiva $A(T(D)) = 0$

Entonces ahora suponemos T biyectiva.

Luego, por T.C.V $\quad A(T(D)) = \int_{T(D)} 1 = \int_D (1\circ T)\cdot|\det J_T|$

$$D \xrightarrow{T} T(D) \xrightarrow{1} \cdot$$

$$= \int_D 1\cdot|\det T| = |\det T|\int_D 1$$

Entonces $\boxed{A'(T(D)) = |\det T|\, A'(D).}$

a) $D = \{ (x,y) ;\ x \in [0,1],\ y \in [0,1] \} \rightarrow A'(D) = 1 \Rightarrow A'(T(D)) = |\det T|.$

b) $D = \{ (x,y) ;\ x^2+y^2 \leq 1 \}$, $A'(D) = \pi \Rightarrow A'(T(D)) = \pi \cdot |\det T|.$