Pr 1

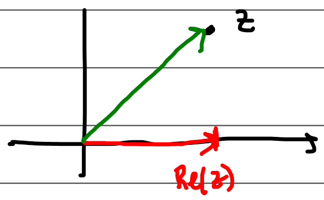
$(|z_1+z_2| \leq |z_1|+|z_2|)$

4) c) $|z_1+z_2|^2 \leq (|z_1|+|z_2|)^2$

$\left(\begin{array}{l}\overline{ab} = \bar{a}.\bar{b} \\ \bar{\bar{b}} = b.\end{array}\right)$

· Recordar $z.\bar{z} = |z|^2$ $\quad ((a+bi)(a-bi) = a^2+b^2)$

$$|z_1+z_2|^2 = (z_1+z_2)(\overline{z_1+z_2}) = (z_1+z_2)(\bar{z_1}+\bar{z_2})$$
$$= z_1.\bar{z_1} + z_2.\bar{z_2} + z_1.\bar{z_2} + \bar{z_1}z_2$$
$$= |z_1|^2+|z_2|^2 + \underline{z_1.\bar{z_2} + \overline{(z_1\bar{z_2})}}$$
$$= |z_1|^2+|z_2|^2 + 2Re(z_1\bar{z_2}) \quad *$$
$$\leq |z_1|^2+|z_2|^2 + 2|z_1.\bar{z_2}|$$



En gral, $Re(z) \leq 2|z|$

(¿Por qué? Si $z = a+bi$, $Re(z) = a$
$|z| = \sqrt{a^2+b^2} \geq a$)

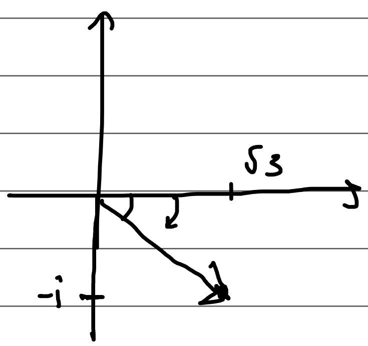
$* \quad z+\bar{z} = 2Re(z).$

$$= |z_1|^2+|z_2|^2 + 2|z_1|.|z_2|$$
$$= (|z_1|+|z_2|)^2$$

✓

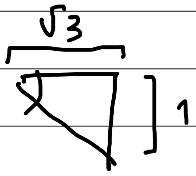
5) d) $z^6 = 8(\sqrt{3}-i)$

Traducimos a polares: $\underbrace{z = re^{i\theta}}$, $\underbrace{8(\sqrt{3}-i) = \rho e^{i\alpha}}$



· $\rho = |8(\sqrt{3}-i)| = 8.|(\sqrt{3}-i)| = 8.\sqrt{3+1}$
$= 16.$

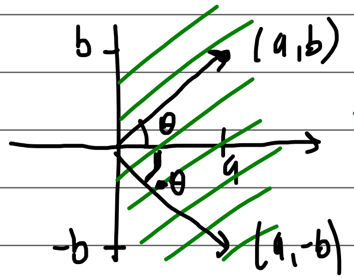
· α?



$tg(\alpha) = 1/\sqrt{3}$

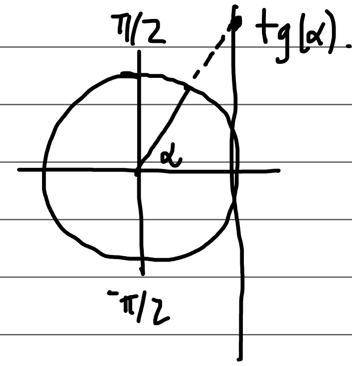
$\alpha = arctg(1/\sqrt{3})$
$= \pi/6.$

Por el sentido en el que cuento ángulos, escribo $\boxed{\alpha = -\pi/6}$ → negativo!



$\theta = \operatorname{arctg}(b/a)$.

$-\theta = \operatorname{arctg}(-b/a)$



- $z^6 = 8(\sqrt{3}-i)$ $\qquad \left[z^6 - 8(\sqrt{3}-i) = P(z) \right]$

$$\boxed{(re^{i\theta})^6 = 16\, e^{-i\pi/6}} \rightsquigarrow r^6 e^{i6\theta} = 16\, e^{i(-\pi/6 + 2\pi n)} \; *$$

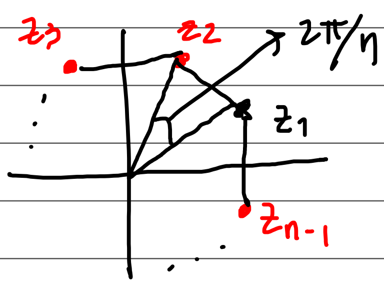
$$\begin{cases} r^6 = 16 \\ 6\theta = 2\pi n - \pi/6. \end{cases} \rightarrow \begin{matrix} r = 16^{1/6} \\ \theta = \frac{2\pi}{6} n - \pi/36 \end{matrix}$$

Obtengo seis posibles ángulos $\theta_n = \frac{2\pi}{6} n - \pi/36$ con $n = 0,1,2,3,4,5$.

y las soluciones son $\boxed{16^{1/6} \cdot e^{i\theta_n}}$

Si dibujo las raíces obtengo un hexágono.

- En gral : z^n = num. complejo tiene n soluciones que forman un n-ágono, donde tengo que "ir sumando $\frac{2\pi}{n}$ ángulos"

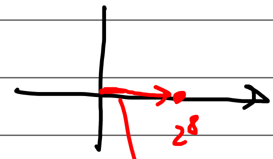


* Cuando igualo ángulos pensando en reales, la igualdad siempre es a menos de $2\pi n$.

"$\theta_1 = \theta_2$" $\rightsquigarrow$ $\theta_1 + 2\pi m = \theta_1 + 2\pi n$

(para n, m enteros)

8) $z^8 = 2^8.$



ángulo 0

= ángulo $2\pi n$

Las soluciones de esta ecuación son:

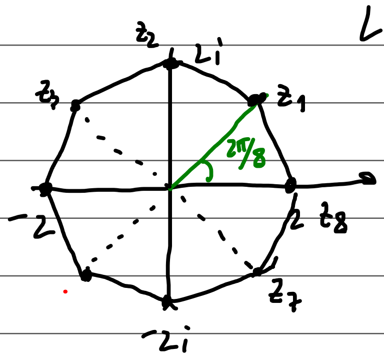
- traducimos ecuación: $(re^{i\theta})^8 = 2^8.e^{i(0+2\pi n)}$

$$r^8 e^{i8\theta} = 2^8 e^{i(2\pi n)}$$

$\rightarrow r = 2$

$\rightarrow 8\theta = 2\pi n \Rightarrow \theta_n = \frac{2\pi}{8}.n \ , \ n = 1...8.$

Las soluciones son $\left[z_n = 2.e^{i\frac{2\pi n}{8}} \text{ con } n = 1...8 \right]$



1) Falso) $\quad 9\pi = \pi$

2) Verdad $\quad 9\pi = 2\pi.4 + \pi$

3) Verdad.

4) $z_1.z_2 ... z_8 = 2^8.e^{i\left(\frac{2\pi}{8}1 + \frac{2\pi}{8}.2 + \cdots \frac{2\pi}{8}8\right)}$

$1+2+3+4+5+6+7+8$

$$= 2^8 e^{i\frac{2\pi}{8}(1+2+3+\cdots 8)}$$
$$= 2^8 e^{i\frac{2\pi}{8}(36)}$$
$$= 2^8 e^{i\frac{2\pi}{2}(9)}$$
$$= 2^8.e^{i9\pi} = 2^8 e^{i\pi} = -2^8$$

$\rightarrow$ 4 es Falsa

10) · P polinomio con coef. reales. $P(z) = a_d z^d + a_{d-1} z^{d-1} + \cdots + a_0$

con a_i reales

a) Probar $P(\bar{z}) = \overline{P(z)}$

Prueba: $\overline{P(z)} = \overline{a_d z^d + \cdots + a_1 z + a_0} = \overline{a_d z^d} + \overline{a_{d-1} z^{d-1}} + \cdots + \overline{a_0}$

$$= \overline{a_d}.\overline{z^d} + \overline{a_{d-1}}.\overline{z^{d-1}} + \cdots + \overline{a_0}$$
$$= a_d.\bar{z}^d + a_{d-1}.\bar{z}^{d-1} + \cdots + a_0$$
$$= P(\bar{z}).$$

b) Si z_0 es raíz de $P \Rightarrow \bar{z_0}$ también.

Por parte a) si z_0 raíz, $\overline{0} = \overline{P(z_0)} = P(\bar{z_0})$

$\overline{0} = 0$

✓