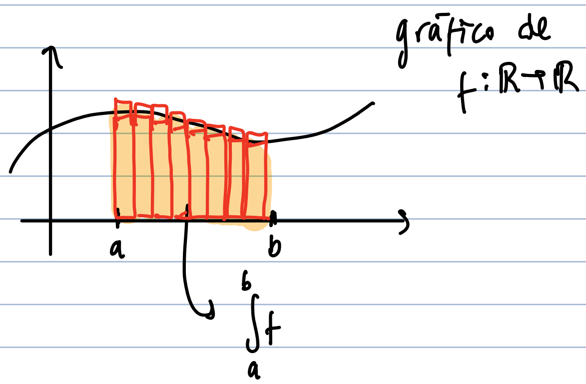
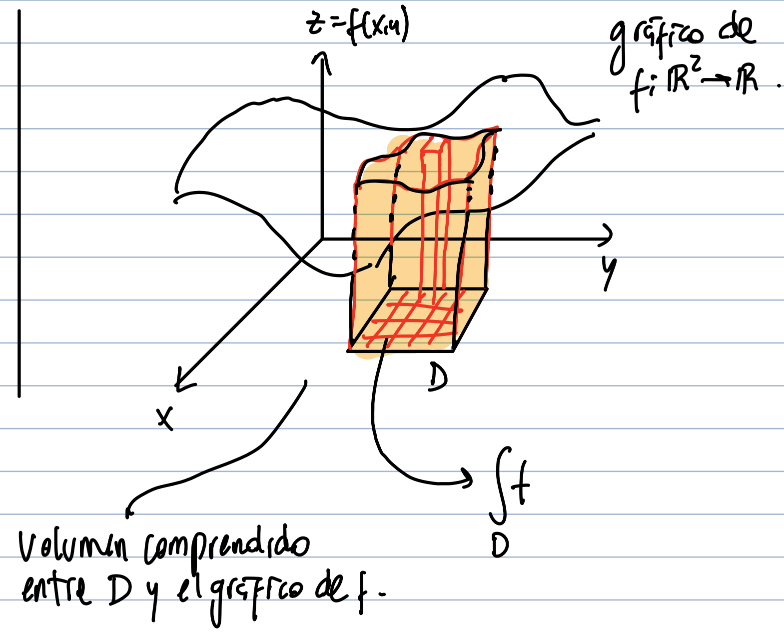
Práctico 10 : Integrales.

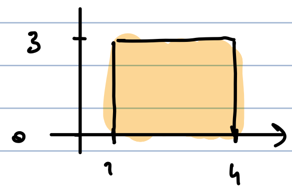




Si escribimos $D\subset\mathbb{R}^2$ así $D=\{(x,y) : x\in[a,b],\ y\in[c,d]\}$
y $f:D\to\mathbb{R}$ integrable, entonces

$$\int_D f = \int_a^b \Big(\underline{\int_c^d f(x,y)\,dy} \Big)\,dx = \int_c^d \Big(\int_a^b f(x,y)\,dx \Big)\,dy$$

3) a) $f(x,y)=2x-y$, $D=\{(x,y) : x\in[1,4],\ y\in[0,3]\}$



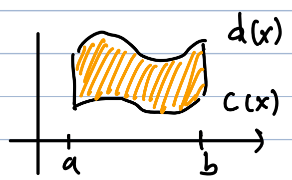
$$\int_0^3\int_1^4 f(x,y)\,dx\,dy = \int_0^3 \Big(\int_1^4 2x-y\,dx \Big)\,dy$$

$$= \int_0^3 \left(x^2-yx\right)\Big|_{x=1}^{x=4} dy = \int_0^3 (16-4y-1+y)\,dy$$

$$= \int_0^3 -3y+15\,dy = \left(-\frac{3y^2}{2}+15y\right)\Big|_0^3$$

¿Si los dominios D no son rectángulos?

Si podemos escribir D así : $D=\{(x,y) : x\in[a,b],\ y\in[c(x),d(x)]\}$



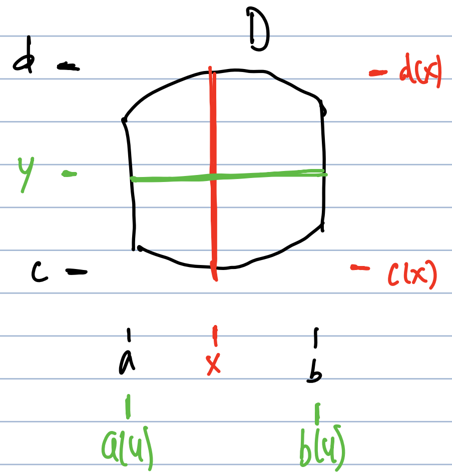
Entonces calculo f así $\displaystyle\int_D f = \int_a^b \Big(\int_{c(x)}^{d(x)} f(x,y)\,dy \Big)\,dx$

Aquí no se pueden dar vuelta los lím. de integración (no tiene sentido)

Pero sí podemos reescribir a D (a veces) para invertir el orden de integración.

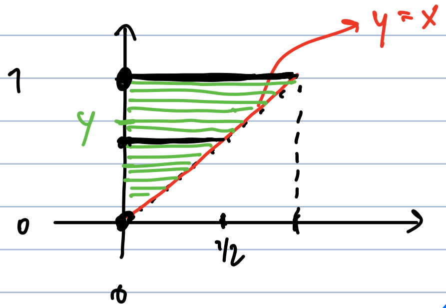
Si podemos reescribir D así $D = \{(x,y) : y \in [c,d],\ x \in [a(y), b(y)]\}$

entonces
$$\boxed{\int_D f = \int_a^b \int_{c(x)}^{d(x)} f(x,y)\,dy\,dx = \int_c^d \int_{a(y)}^{b(y)} f(x,y)\,dx\,dy.}$$



- $\underbrace{x \in [a,b]}_{\text{fijo}}$, $y \in [c(x), d(x)]$ pintar con líneas verticales
- $y \in [c,d]$, $x \in [a(y), b(y)]$ pintar con líneas horizontales

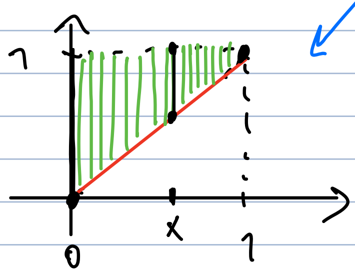
1) a) $\displaystyle\int_0^1 \left[\int_0^y f(x,y)\,dx\right] dy \rightarrow D = \{(x,y) : y \in [0,1],\ x \in [\overset{a(y)}{0}, \overset{b(y)}{y}]\} \rightarrow 0 \leq x \leq y.$



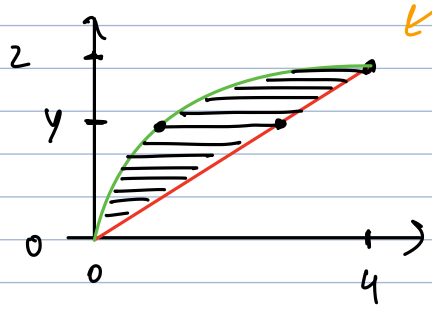
$\rightarrow D = \{(x,y) : x \in [0,1],\ y \in [x, 1]\}.$

Por lo tanto puedo expresar la integral así

$$\int_0^1 \left[\int_x^1 f(x,y)\,dy\right] dx$$



1) c) $\displaystyle\int_0^2 \left[\int_{y^2}^{2y} f(x,y)\,dx\right] dy$ $D=\{(x,y) : y\in[0,2],\ x\in[y^2,2y]\}.$

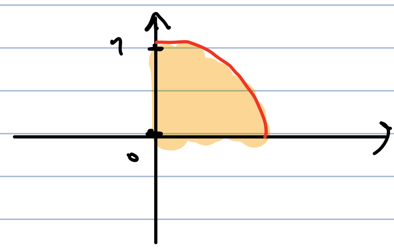


$x=2y \rightsquigarrow y=\frac{x}{2}$, $x=y^2 \rightarrow y=\sqrt{x}$

$D=\{(x,y) : x\in[0,4],\ y\in[\frac{x}{2},\sqrt{x}]\}.$

Luego la integral se expresa $\displaystyle\int_0^4\int_{x/2}^{\sqrt{x}} f(x,y)\,dy\,dx$

2) a) $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}} yx^2\,dx\,dy$



D: $\boxed{\begin{array}{l} y\in[0,1] \\ x\in[0,\sqrt{1-y^2}] \end{array}}$

$\displaystyle\int_0^1 \left(\frac{yx^3}{3}\right)\Big|_{x=0}^{x=\sqrt{1-y^2}} dy = \frac{1}{3}\int_0^1 \frac{-2y}{-2}\cdot(1-y^2)^{3/2}\,dy.$

$u=1-y^2$
$u'=-2y$

$x=\sqrt{1-y^2}$
$x^2=1-y^2$
$x^2+y^2=1.$

$\displaystyle=\frac{1}{3}\frac{1}{-2}\int_1^0 (u)^{3/2}\,du$

$\displaystyle=\frac{1}{3}\frac{1}{-2}\left(u^{5/2}\cdot\frac{2}{5}\right)\Big|_1^0 = \frac{1}{3}\cdot\frac{1}{5} = \frac{1}{15}$