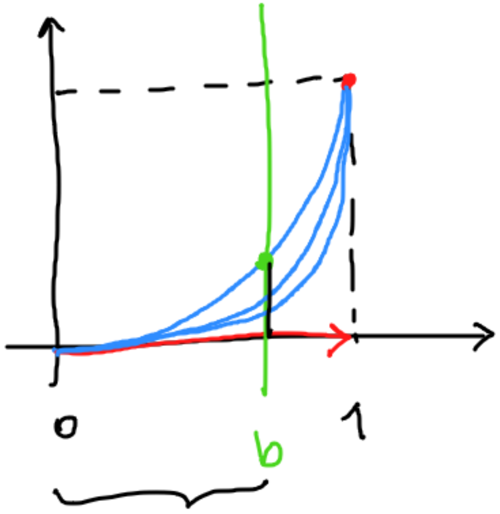
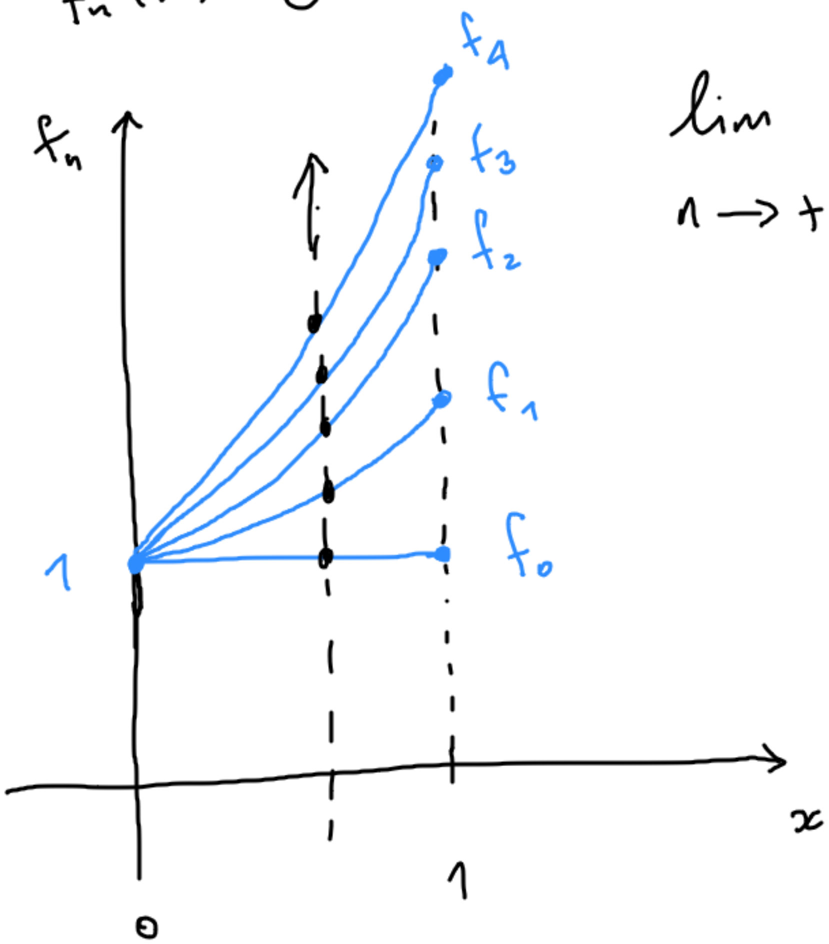
Observación:



$$\sup_{x\in[0,b]} |f_n(x) - f(x)| = b^n \longrightarrow 0$$

La convergencia depende del dominio de las funciones

(ii) $f_n : [0,1] \to \mathbb{R}$ $\quad f_n(x) = e^{nx}$ $\quad$ x fijo



$$\lim_{n \to +\infty} e^{nx} = \begin{cases} 1 & \text{si } x = 0 \\ +\infty & \text{si } x > 0 \end{cases}$$

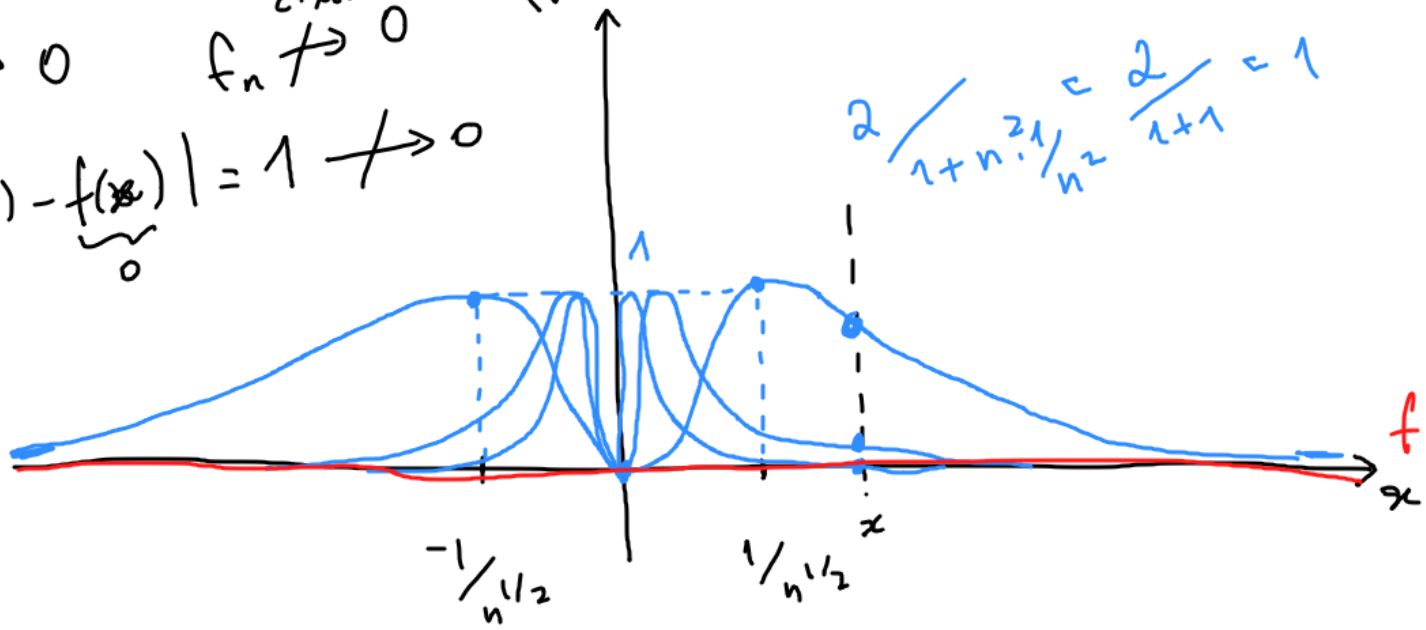
f_n no converge puntualmente a ninguna función.

(iii) $f_n : \mathbb{R} \to \mathbb{R}$ $\quad f_n(x) = \dfrac{2nx^2}{1+n^2x^4}$

$f_n'(x) = \dfrac{4nx(1+n^2x^4) - 4n^2x^3\,2nx^2}{(\quad)^2} = \dfrac{4nx + 4n^3x^5 - 8n^3x^5}{(\quad)^2} = \dfrac{4nx(1-n^2x^4)}{(\quad)^2}$

$f_n \xrightarrow{c.p.} 0 \qquad f_n \overset{c.u.}{\not\to} 0$

$\sup_{x\in\mathbb{R}} |f_n(x) - \underbrace{f(x)}_{0}| = 1 \not\to 0$



$\dfrac{2}{1+n^2 \cdot 1/n^2} = \dfrac{2}{1+1} = 1$

Fijamos x : $(x \neq 0)$ $\quad \lim_{n\to+\infty} \dfrac{2nx^2}{1+n^2x^4} = \lim_{n\to\infty} \dfrac{2nx^2}{n^2x^4} = \lim_{n\to\infty} \dfrac{2}{nx^2} = 0$

$x = 0$

$x = \pm 1/n^{1/2}$

(iv) $f_n : [0,1] \to \mathbb{R}$ $\qquad f_n(x) = n^{1/2}\, x\,(1-x^2)^n$ $\qquad \left(1+\frac{1}{n}\right)^n \to e$

$e^{\ln(\ )}$

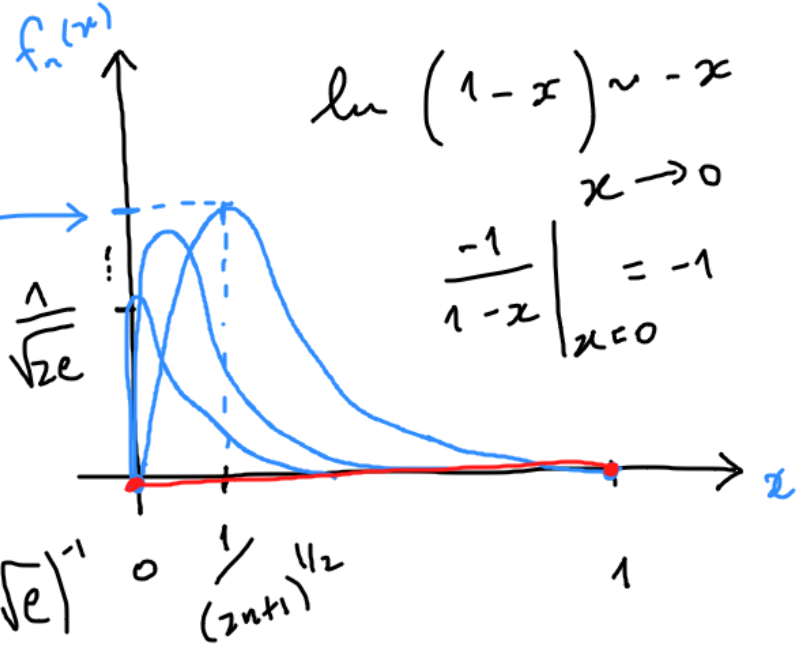
$\ln(1-x) \sim -x$

$x \to 0$

$\left.\frac{-1}{1-x}\right|_{x=0} = -1$

$$\underbrace{\left(\frac{n}{2n+1}\right)^{1/2}}_{\to \frac{1}{\sqrt{2}}} \overbrace{\left(\frac{2n}{2n+1}\right)^n}^{\to 1/\sqrt{e}} \to \frac{1}{\sqrt{2e}}$$

$$e^{n \ln\left(1-\frac{1}{2n+1}\right)} \sim e^{-n/(2n+1)} \to (\sqrt{e})^{-1}$$



$$f_n\left(\frac{1}{(2n+1)^{1/2}}\right) = n^{1/2}\frac{1}{(2n+1)^{1/2}} \cdot \left(1-\frac{1}{2n+1}\right)^n = \left(\frac{n}{2n+1}\right)^{1/2}\left(\frac{2n}{2n+1}\right)^n$$

$$f_n'(x) = n^{1/2}\left[(1-x^2)^n - x\,n\,(1-x^2)^{n-1}\,2x\right] = n^{1/2}(1-x^2)^{n-1}\left(1-x^2-2nx^2\right)$$

$$= n^{1/2}(1-x^2)(1-(2n+1)x^2)$$

$\to x = 1$

$\to x = \frac{1}{(2n+1)^{1/2}}$

$$f_n(x) = n^{1/2}\, x\,(1-x^2)^n \qquad f_n(x) \xrightarrow{c.p.} 0 = f(x)$$

$$\sup_{x\in[0,1]} |f_n(x) - \underbrace{f(x)}_{0}| = \left(\frac{n}{2n+1}\right)^{1/2} \cdot \left(\frac{2n}{2n+1}\right)^n \to \frac{1}{\sqrt{2e}} \neq 0$$

$$\|f_n - f\|_\infty \not\to 0 \qquad f_n \overset{c.u.}{\not\to} f$$

$$f_n(x) = x\underbrace{\Big[\underset{\downarrow\,\infty}{n^{1/2}}\ \underset{\downarrow\,0}{(1-x^2)^n}\Big]}_{a_n}$$

$$e^{n\left[\left(\frac{1}{2}\frac{\ln n}{n}\right) + \ln(1-x^2)\right]} \to 0$$

($\frac{1}{2}\frac{\ln n}{n} \to 0$; $\ln(1-x^2) < 0$)

$$a_n = e^{\ln a_n} = e^{\ln n^{1/2}(1-x^2)^n} = e^{\frac{1}{2}\ln n + \overbrace{n\ln(1-x^2)}^{<0}} \to 0$$

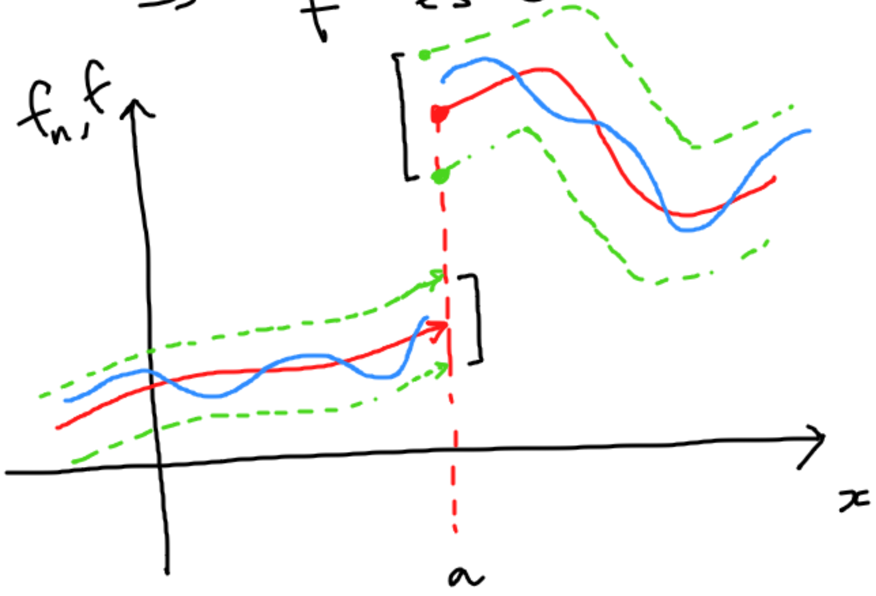
$\frac{1}{2}\ln n \to +\infty$ logarítmicamente; $n\ln(1-x^2) \to -\infty$ linealmente

Teorema 0.3: Sea $f_n : M \to \mathbb{R}$ continuas tales que

$$f_n \xrightarrow{c.u.} f \quad \text{con} \quad f : M \to \mathbb{R}.$$

Si las f_n son continuas en un punto $a \in M$

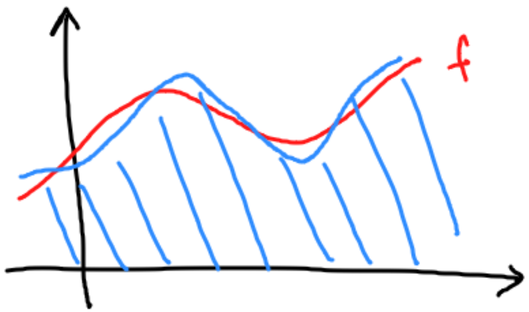
$\Rightarrow$ f es continua en a.



Teorema 0.4:

$f_n : [a,b] \longrightarrow \mathbb{R}$ continuas, $f_n \xrightarrow{c.u.} f$, $f : [a,b] \to \mathbb{R}$

$$F_n(x) = \int_a^x f_n(s)\,ds \qquad F(x) = \int_a^x f(s)\,ds$$



Entonces:

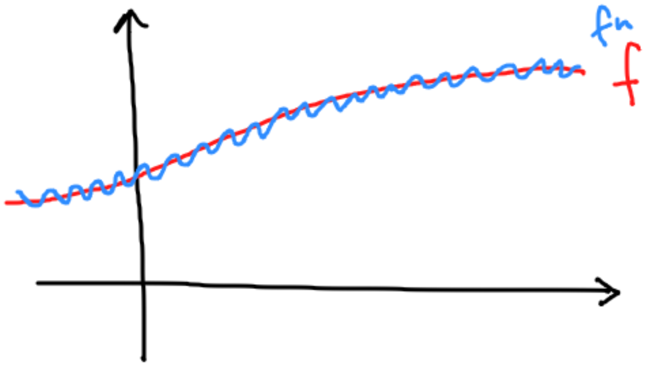
1) $F_n \xrightarrow{c.u.} F$

2) $$\underbrace{\int_a^b f_n(x)\,dx}_{F_n(b)} \longrightarrow \underbrace{\int_a^b f(x)\,dx}_{F(b)}$$

Dem:

$$\sup_{x\in[a,b]} |F_n(x) - F(x)| = \sup_{x\in[a,b]} \left| \int_a^x f_n(s) - f(s)\,ds \right| \leq \underbrace{|x-a|}_{\leq\, |b-a|} \cdot \overbrace{\sup_{x\in[a,b]} |f_n(x) - f(x)|}^{\to\, 0}$$

Observación: No es cierto que si $f_n \xrightarrow{c.u.} f$ entonces $f_n' \xrightarrow{c.u.} f'$.



$$f_n(x) = \underbrace{f_n(x_0)}_{\text{conv.}} + \underbrace{\int_{x_0}^{x} f_n'(s)\,ds}_{\text{Teo. 0.4}}$$

$$f(x) = f(x_0) + \int_{x_0}^{x} g(s)\,ds$$

Teorema 0.5: $f_n : (a,b) \to \mathbb{R}$ de clase C^1, $g : (a,b) \to \mathbb{R}$ tales que:

1) $f_n' \xrightarrow{c.u.} g$

2) $\exists\, x_0 \in (a,b) \,/\, \{f_n(x_0)\}$ converge.

$\Rightarrow \exists\, f : (a,b) \to \mathbb{R}$ clase C^1 / $f_n \xrightarrow{c.u.} f$ y $f' = g$.