Ecuaciones en el plano

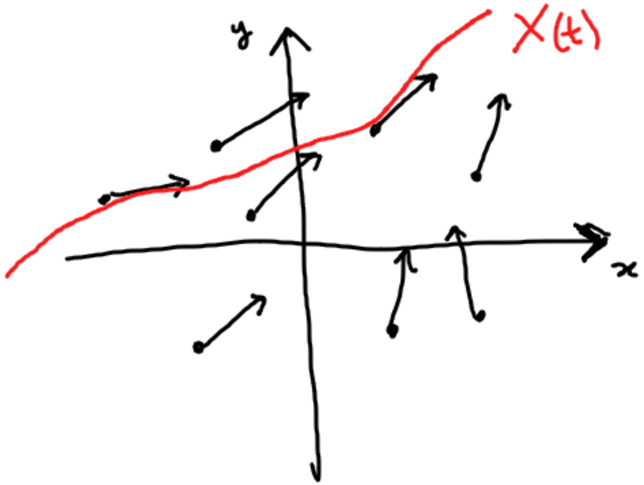
(x, y) coordenadas en el plano

$$\begin{cases} \dot{x} = f_1(t, x, y) \\ \dot{y} = f_2(\underset{t}{t}, \underbrace{x, y}_{X}) \end{cases}$$

Forma vectorial $\quad X = \begin{pmatrix} x \\ y \end{pmatrix} \quad F = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$$\dot{X} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

$$\dot{X} = F(t, X)$$



Autónoma $\dot{X} = F(X)$

Modelo de Lotka-Volterra (predador-presa)

Describe la evolución temporal de un ecosistema cos 2 especies → predador, → presa

predador = zorros (F) F = cantidad de zorros

presa = conejos (R) R = cantidad de conejos $X = \begin{pmatrix} R \\ F \end{pmatrix}$

$$\begin{cases} R' = aR - \overbrace{cRF} = \overbrace{(a-cF)R} \\ F' = -bF + \underbrace{dRF} = (-b+dR)F \end{cases}$$

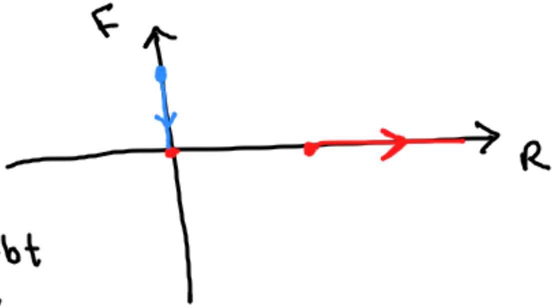
a, b, c, d son constantes
parámetros del modelos.

$F_0 = 0$

Observar: Si $F \equiv 0$ siempre

$$\begin{cases} R' = aR \\ F' = 0 \end{cases} \leftarrow R(t) = R_0 e^{at}$$

crecimiento exponencial de conejos tasa = a.



$R_0 = 0$

$$\begin{cases} R' = 0 \\ F' = -bF \end{cases} \quad F(t) = F_0 e^{-bt}$$

Ecuaciones de variables separables

$$\begin{cases} \dot{x} = f(t)g(x) \\ x(t_0) = x_0 \end{cases}$$

$$\frac{dx}{dt} = f(t)g(x)$$

Fórmula en x = Fórmula en t

$$\frac{1}{g(u)}\,du = f(s)\,ds$$

$$G(x) = \int_{x_0}^{x} \frac{1}{g(u)}\,du = \int_{t_0}^{t} f(s)\,ds$$

despejar la x.

$$\boxed{x = G^{-1}\left(\int_{t_0}^{t} f(s)\,ds\right)}$$

$$\frac{dx}{dt} = f(t)\, g(x(t))$$

$$\frac{1}{g(x(t))} \frac{dx}{dt} = f(t)$$

$$G(x(t)) = \int f(t)\,dt$$

$$\int \frac{1}{g(\underbrace{x(t)}_{u})} \overbrace{\frac{dx}{dt}\,dt}^{du} = \int f(t)\,dt$$

$$\boxed{x(t) = G^{-1}\left(\int f(t)\,dt\right)}$$

$$u = x(t)$$

$$du = \frac{dx}{dt}\,dt$$

$$\int \frac{1}{g(u)}\,du = G(u)$$

$$\underset{\overset{\vee}{0}}{(t^2+9)}\,\dot{x} = -tx \iff \dot{x} = \underbrace{\frac{-t}{t^2+9}}_{f(t)}\,\underbrace{x}_{g(x)}$$

$$\frac{dx}{dt} = \frac{-t}{t^2+9}\,x$$

$$\int_{x_0}^{x} \frac{du}{u} = \int_{t_0}^{t} \frac{-s}{s^2+9}\,ds = \begin{bmatrix} z = s^2+9 \\ dz = 2s\,ds \end{bmatrix} = -\frac{1}{2}\int_{t_0^2+9}^{t^2+9} \frac{1}{z}\,dz$$

$$\ln|u|\,\Big|_{x_0}^{x} = -\frac{1}{2}\ln|z|\,\Big|_{t_0^2+9}^{t^2+9} \iff \ln\left|\frac{x}{x_0}\right| = -\frac{1}{2}\ln\frac{t^2+9}{t_0^2+9}$$

$$\underbrace{\ln\left|\frac{x}{x_0}\right|}_{G(x)} = -\frac{1}{2}\ln\frac{t^2+9}{t_0^2+9} = \ln\left(\frac{t^2+9}{t_0^2+9}\right)^{-1/2}$$

$$\cancel{\left|\frac{x}{x_0}\right|} = \left(\frac{t^2+9}{t_0^2+9}\right)^{-1/2}$$

$$\boxed{x(t) = x_0\left(\frac{t^2+9}{t_0^2+9}\right)^{-1/2}}$$

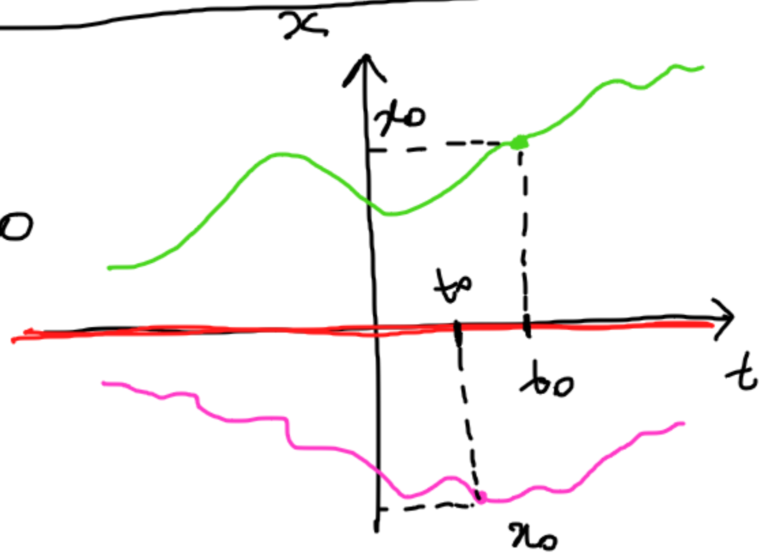
$$\boxed{\frac{x}{x_0}} > 0$$

$$x(t) \equiv 0$$

$$\dot{x} = \frac{-t}{t^2+9}\,x$$

$$|x| = \cdots \quad \underbrace{\quad}_{0} \quad \underbrace{\qquad\qquad}_{0}$$

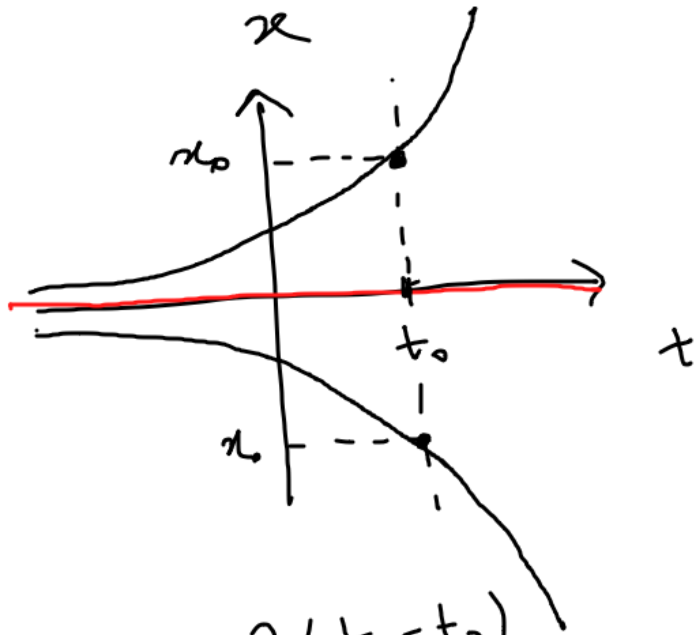
$$x = \pm \cdots$$



Ejemplo: $\dot{x} = 2x$

$$\frac{dx}{dt} = 2x$$

$$\frac{dx}{x} = 2\,dt$$



$$\int_{x_0}^{x} \frac{du}{u} = 2\int_{t_0}^{t} ds = 2s\Big|_{t_0}^{t}$$

$$\ln\left|\frac{x}{x_0}\right| = 2(t-t_0)$$

$$\left|\frac{x}{x_0}\right| = e^{2(t-t_0)}$$

$$x = x_0 e^{2(t-t_0)}$$

<u>Ecuaciones lineales de 1^{er} orden</u>: $\dot{x} + p(t)x = q(t)$

Método del factor integrante de Euler

$$\dot{x} + px = q$$

Consideremos una función μ que vamos a elegir después

$$\underbrace{\mu\dot{x} + \mu p x} = \mu q$$

$$(\mu x)^{\cdot} = \mu q$$

$$\int (\mu x)^{\cdot}\, dt = \int \mu q\, dt + c$$

$$\mu x = \int \mu q\, dt + c$$

Derivada del producto:

$$(\mu x)^{\cdot} = \dot{\mu} x + \cancel{\mu \dot{x}}$$

$$= \cancel{\mu \dot{x}} + \mu x p$$

$$\dot{\mu} \cancel{x} = \mu \cancel{x} p$$

$$\boxed{\dot{\mu} = \mu p}$$

$$\boxed{x = \frac{1}{\mu}\left(\int \mu q\, dt + c\right)}$$

Si elijo el factor integrante $\mu(t)$ de forma que $\dot{\mu} = p\mu$

entonces $$x = \frac{1}{\mu}\left(\int \mu q \, dt + C\right)$$ $\left[\dot{\mu} = p(t)\mu\right]$

Para el factor integrante: $$\boxed{\frac{\dot{\mu}}{\mu} = p(t)}$$ $\underline{\mu(t) = e^{\int p(t)dt}}$

$$\boxed{x(t) = \frac{1}{\mu}\left(\int \mu q \, dt + c\right) \text{ con } \mu = e^{\int p(t)dt}}$$

Principio de superposición:

$$\dot{x} + px = q_1 \quad (x_1)$$

$$\dot{x} + px = q_2 \quad (x_2)$$

$\underline{c_1 x_1 + c_2 x_2}$ es solución de $\underbrace{\dot{x} + px = c_1 q_1 + c_2 q_2}$

$$q = c_1 q_1 + c_2 q_2$$

$$\dot{x} + px = 0$$

$$\dot{x} = -p(t)\, x$$

$$x = x_p + x_h$$

$$\underbrace{\dot{x} + px = q}$$