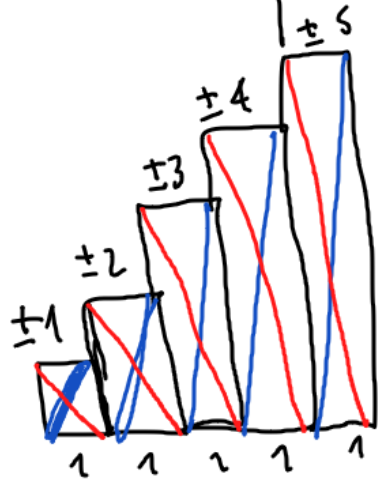
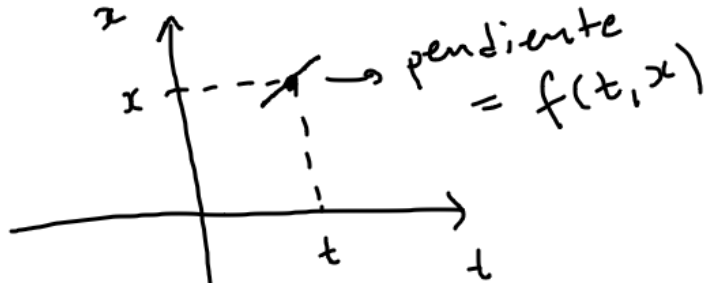


Campo de pendientes

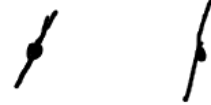
$$\dot{x} = f(t, x) \quad x: \mathbb{I} \rightarrow \mathbb{R}$$



Ejercicio 3 de la ficha 1

$$\dot{x} = 2t + x$$

$$f(t, x) = 2t + x$$



$\dot{x} = \sqrt{2t+x}$ usar 5 isoclinas (curvas de pendiente constante)

$\dot{x} = f(t, x) = 2t+x$
 $\uparrow f(t, x) = \text{const}$

Isoclinas:

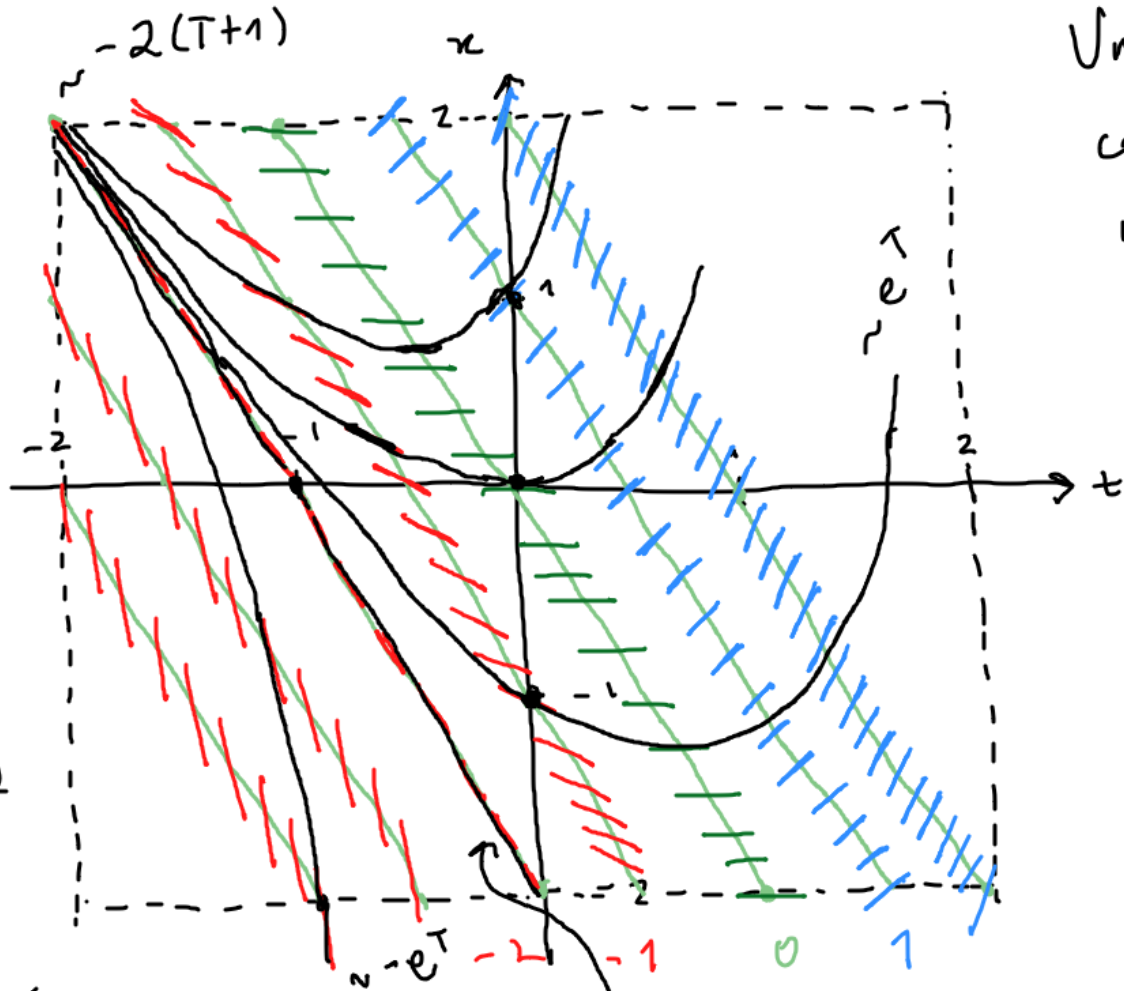
$2t+x=0, x=-2t$

$2t+x=1, x=-2t+1$

$2t+x=-1, x=-2t-1$

$2t+x=2, x=-2t+2$

$2t+x=-2, x=-2t-2$



Una solución que coincide con una isoclina:

$\dot{x} = \sqrt{2t+x}$

$2t+x=c$

$(x=c-2t)$

$\dot{x} = -2$

$-2 = 2t + (c-2t)$

$c=c$

$c=c-2$

$x = -2(t+1)$

una integral = gráfico de una solución

$$\dot{x} = 2t + x \quad (\text{linear 1}^{\text{er}} \text{ order})$$

$$\dot{x} + a(t)x = b(t)$$

$$\dot{x} - x = 2t$$

$$b(t) = 2t$$

$$a(t) = -1$$

$$(uv)' = \dot{u}v + u\dot{v}$$

$$(xv)' = \dot{x}v + x\dot{v}$$

$$\underbrace{\dot{x}e^{-t}}_v + x \underbrace{(-e^{-t})}_{\dot{v}} = 2te^{-t}$$

$$\boxed{e^{A(t)}} e^{-t}$$

$$\underbrace{(-te^{-t} - e^{-t})}' = \cancel{-e^{-t}} + te^{-t} + \cancel{e^{-t}} = te^{-t}$$

$$(xe^{-t})' = 2te^{-t}$$

partes

$$\int_{T_0}^T (xe^{-t})' dt = \int_{T_0}^T 2te^{-t} dt$$

$$\boxed{x(T)}e^{-T} - x(T_0)e^{-T_0} = -2 \left[te^{-t} + e^{-t} \right]_{T_0}^T = -2 \left(Te^{-T} + e^{-T} \right) + 2 \left(T_0 e^{-T_0} + e^{-T_0} \right)$$

$$x(T) = e^T \left[-2(Te^{-T} + e^{-T}) + 2(T_0 e^{-T_0} + e^{-T_0}) + x(T_0) e^{-T_0} \right]$$

$$x(T) = -2(T+1) + \left[2(T_0 e^{-T_0} + e^{-T_0}) + x(T_0) e^{-T_0} \right] e^T \quad \leftarrow \text{solución general}$$

$$x(T) = -2(T+1) + C e^T$$

La solución que coincide con la isoclina:

$$x(0) = -2 \quad (T_0 = 0)$$

$$x(T) = -2(T+1) + \left[2 - 2 \right] e^T = -2(T+1)$$

La solución que en $T_0 = 0$ vale -1 : $x(0) = -1$

$$x(T) = -2(T+1) + \left[2 + (-1) \right] e^T \Rightarrow x(T) = \overbrace{-2(T+1) + e^T}$$

$$\begin{cases} x(0) = -3 \\ x(T) = -2(T+1) - e^T \end{cases}$$

Ecuación autónoma: $\dot{x} = f(x)$ (no hay t en la ec.)

Isoclinas

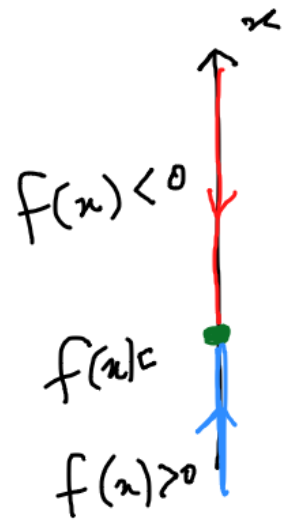
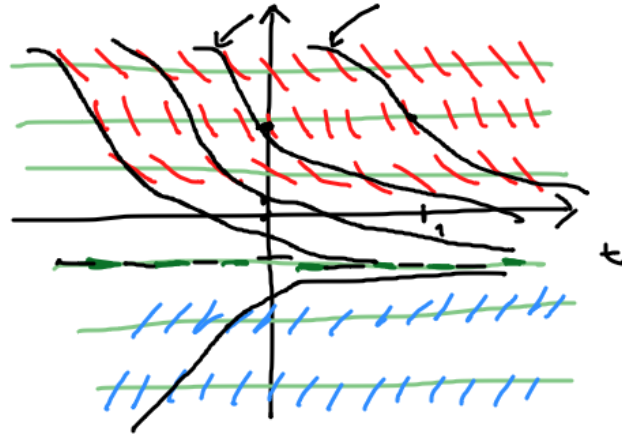
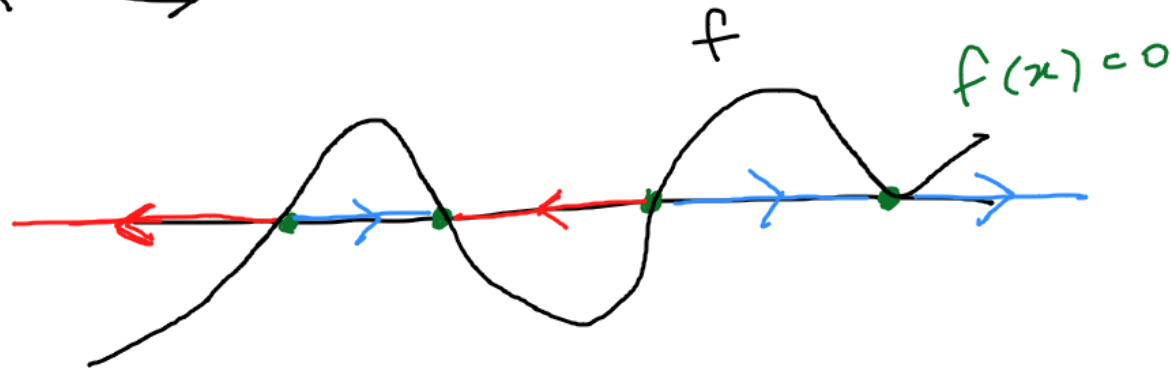
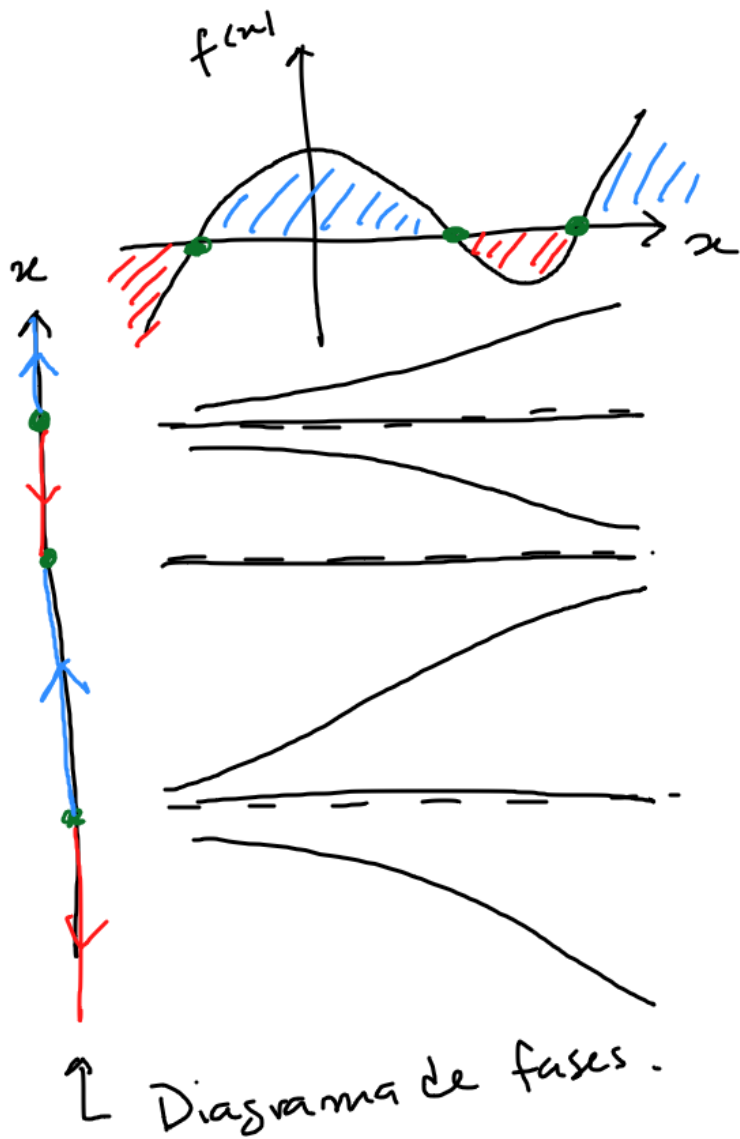
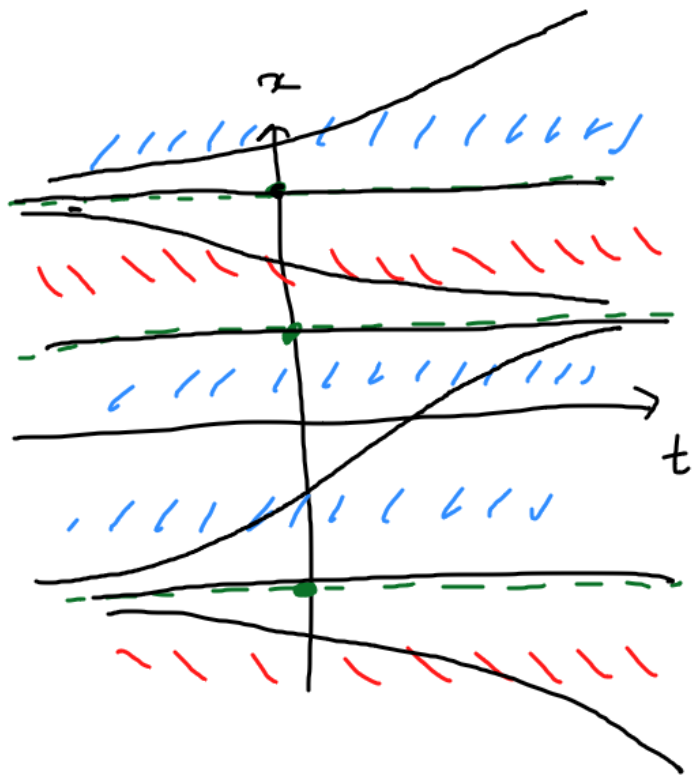


Diagrama de fase.

← punto de equilibrio
o sol. estacionaria



$$\begin{cases} \dot{x} = f(x) & \text{ec. autónoma} \\ x(0) = x_0 & (\text{c. i.}) \end{cases}$$



Ejercicio 4 de la ficha:

$$\dot{x} = k \left(1 - \frac{x}{M}\right) x$$

k, M Constantes positivas

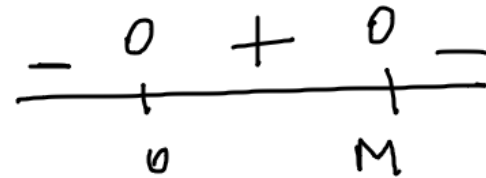


Diagrama de fases

