2f) Admitiendo que $F(s) = \mathcal{L}(f(t))(s)$ es derivable y que

$$\frac{d}{ds}\left(\int_0^{+\infty} f(t) e^{-st} dt\right) = \int_0^{+\infty} \frac{\partial}{\partial s}\left(f(t) e^{-st}\right) dt$$

$$\Rightarrow F'(s) = \mathcal{L}(-t f(t))(s).$$

$$F'(s) = \frac{d}{ds} F(s) = \frac{d}{ds}\left(\int_0^{+\infty} f(t) e^{-st} dt\right) = \int_0^{+\infty} \frac{\partial}{\partial s}\left(f(t) e^{-st}\right) dt$$

$$= \int_0^{+\infty} f(t) \cdot (-t)\, e^{-st} dt = \int_0^{+\infty} (-t f(t)) \cdot e^{-st} dt = \mathcal{L}(-t f(t))(s)$$

$-t \cdot -t$

g) $$\mathcal{L}\left(e^{2t} \cdot t^2\right)(s) \underset{\uparrow}{=} \mathcal{L}(t^2)(s-2) = \frac{d}{ds}\left(\mathcal{L}(-t)(s-2)\right) = \frac{d^2}{ds^2}\left(\mathcal{L}(1)(s-2)\right)$$

Traslación en frecuencia

$$= \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right) = \frac{d}{ds}\left(\frac{-1}{(s-2)^2}\right) = \frac{2}{(s-2)^3}$$

$$\mathcal{L}(e^{2t} \cdot t^2)(s) = \mathcal{L}(-t \cdot (e^{2t} (-t)))(s) = \frac{d}{ds}\mathcal{L}(e^{2t} \cdot (-t))(s)$$

$$= \frac{d^2}{ds^2}\mathcal{L}(e^{2t})(s) = \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right) = \dots$$

3) (v) $$\mathcal{L}(\underbrace{\cos(\alpha t)}_{\shortparallel}) = \frac{1}{2}\mathcal{L}(e^{i\alpha t} \cdot 1) + \frac{1}{2}\mathcal{L}(e^{-i\alpha t})$$

$$\frac{e^{i\alpha t} + e^{-i\alpha t}}{2}$$

$$\underset{\uparrow}{=} \frac{1}{2}\frac{1}{s-i\alpha} + \frac{1}{2}\frac{1}{s+i\alpha} = \frac{1}{2}\frac{2s}{(s-i\alpha)(s+i\alpha)} = \frac{s}{s^2+\alpha^2}$$

Traslación

binomio conjugado

$s^2 - (i\alpha)^2 = s^2 + \alpha^2$

CC

$\mathcal{L}(e^{at} \cdot 1)(s)$

$= \mathcal{L}(1)(s-a) = \dfrac{1}{s-a}$

(x) $$\mathcal{L}(t^n \sin(t))(s) = (-1)^n \frac{d^n}{ds^n}\mathcal{L}(\sin t)(s) = (-1)^n \cdot \frac{d^n}{ds^n}\left(\frac{1}{s^2+1}\right)$$

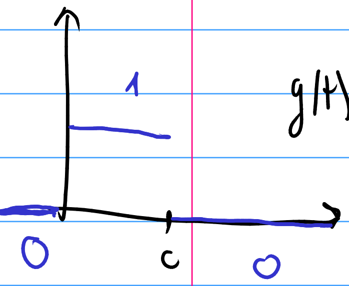
[Solo piqué].

MULTIPLICANDO un t

$\cdot\ \gamma f$

7e) $f''+2f'-3f=\begin{cases}1, & 0\le t<c\\ 0, & t\ge c\end{cases}$ $g(t)$, $f(0)=f'(0)=0$

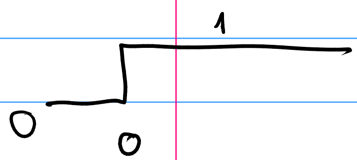
Primer problema: Calcular la transformada de $g(t)=\begin{cases}1 & \text{si } 0\le t<c\\ 0 & \text{si } t\ge c\end{cases}$



$$g(t)=H(t)-H(t-c)$$

$$\Rightarrow \mathcal{L}(g(t))(s)=\mathcal{L}(H(t))(s)-\mathcal{L}(H(t-c))(s)$$

$$=\frac{1}{s}-e^{-cs}\cdot\mathcal{L}(H(t))(s)=\frac{1-e^{-cs}}{s}$$



2) Despejar $F(s)$: $\quad F(s)\cdot(s^2+2s-3)=\dfrac{1-e^{-cs}}{s}$

$\mathcal{L}(f''(t))(s)$

$$F(s)=\frac{1-e^{-cs}}{s(s^2+2s-3)}=\boxed{\frac{1}{s(s^2+2s-3)}}-e^{-cs}\cdot\boxed{\frac{1}{s(s^2+2s-3)}}$$

$$\frac{1}{s(s^2+2s-3)}=\frac{1}{s(s-1)(s+3)}=\frac{A}{s}+\frac{B}{s-1}+\frac{C}{s+3}\qquad \forall s\neq 0,1,-3$$

Raíz 1 y -3.

Tapadita: - Multiplico x s y tomo $s\to 0$ $\quad A=\dfrac{-1}{3}$

- Multiplico por x $(s-1)$ y $s\to 1$: $\dfrac{1}{4}=B$

- Mult por $s+3$ y tomo límite $s\to -3$: $C=\dfrac{1}{12}$

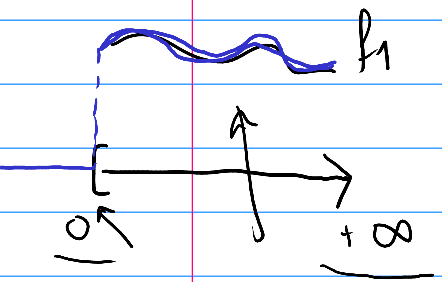
$$F(s)=\frac{1}{4(s-1)}+\frac{1}{12(s+3)}-\frac{1}{3s}-e^{-cs}\cdot\left(\frac{1}{4(s-1)}+\frac{1}{12(s+3)}-\frac{1}{3s}\right)$$

$$\Rightarrow f(t)=\frac{1}{4}e^{t}+\frac{1}{12}e^{-3t}-\frac{1}{3}-\mathcal{L}^{-1}\left(\underbrace{e^{-cs}\left(\frac{1}{4(s-1)}+\frac{1}{12(s+3)}-\frac{1}{3s}\right)}\right)(t)$$

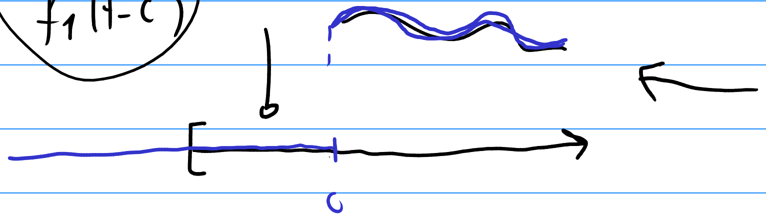
Le llamo $f_1(t) = \frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3}$

$$\mathcal{L}(f_1(t-c))(s) = e^{-cs}\mathcal{L}(f_1(t))(s) = e^{-cs}\left(\frac{1}{4(s-1)} + \frac{1}{12(s+3)} - \frac{1}{3s}\right)$$

$$\| \quad f_1(t-c) \quad \|$$



$\Rightarrow$

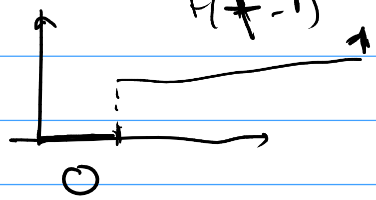


VOLVIENDO

$$F(s) = \frac{1}{4(s-1)} + \frac{1}{12(s+3)} - \frac{1}{3s} - e^{-cs}\cdot\left(\frac{1}{4(s-1)} + \frac{1}{12(s+3)} - \frac{1}{3s}\right)$$

$$\Rightarrow f(t) = \underbrace{\left(\frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3}\right)}_{H(t)} - H(t-c)\left(\frac{1}{4}e^{(t-c)} + \frac{1}{12}e^{-3(t-c)} - \frac{1}{3}\right) \quad //$$

$f' = H(t-1)$, $f(0) = 0$ ... Es razonable que la solución valga 0 antes de $t=1$.



$$sF(s) = \frac{e^{-s}}{s} \Rightarrow$$

$$F(s) = \frac{e^{-s}}{s^2}$$

$$\rightarrow f(t) = (t-1)H(t-1)$$