[Solo la derivada primera]

Contexto: Vamos a estudiar una ecuación de la forma $\dot{X} = f(X,t)$ donde $X:(a,b)\to\mathbb{R}^n$ es mi incógnita y $f:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$.

Sistema de ecuaciones diferenciales

$X(t) = (\underbrace{X_1(t)},\underbrace{X_2(t)},\dots,\underbrace{X_n(t)})$ n funciones incógnitas

Ejemplo: $x:(a,b)\to\mathbb{R}$, $\begin{cases}\ddot{x}+x=0\\ x(0)=x_0\\ \dot{x}(0)=v_0\end{cases}$

Si defino $X(t)=(x(t),\dot{x}(t)) \Rightarrow$

$$\dot{X}(t) = (\dot{x}(t),\ddot{x}(t)) = (\dot{x}(t),-x(t)) = \begin{pmatrix}0&1\\-1&0\end{pmatrix}\cdot\overbrace{\begin{pmatrix}x(t)\\ \dot{x}(t)\end{pmatrix}}^{X(t)}$$

$$\Rightarrow \dot{X}(t) = f(X(t)),\quad f(X) = \begin{pmatrix}0&1\\-1&0\end{pmatrix}X$$

Conclusión: Toda ecuación lineal de orden n, con incógnita $x:(a,b)\to\mathbb{R}^1$ se puede escribir como un sistema de ecuaciones diferenciales de orden 1 y lineal: $\dot{X} = AX$ con $X:(a,b)\to\mathbb{R}^n$

Sistema de ecuaciones lineales $\dot{X} = AX$, con $A\in M_{n\times n}$.

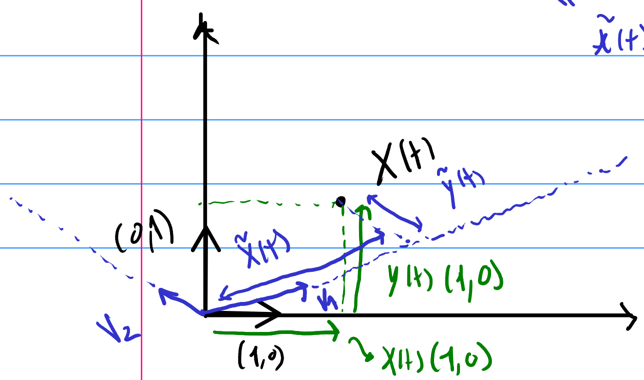
1 a) $\begin{cases}\dot{x} = 2x-y\\ \dot{y} = -2x+3y\end{cases}$ $(x_0,y_0)=(1,1)$

$X(t) = (x(t),y(t))$: $\dot{X}(t) = \overbrace{\begin{pmatrix}2&-1\\-2&3\end{pmatrix}}^{A}X(t)$

$= \underline{x(t)(1,0)} + y(t)(0,1)$, $\mathcal{C}=\{(1,0),(0,1)\}$: $(x(t),y(t)) = \mathrm{Coord}_{\mathcal{C}}(X(t))$

$\tilde{x}(t)v_1 + \tilde{y}(t)v_2 \to (\tilde{x}(t),\tilde{y}(t)) = \mathrm{Coord}_{\beta}(X(t))$

$\beta=\{v_1,v_2\}$



Idea: En lugar de usar las coordenadas x,y usar las coordenadas $\tilde{x},\tilde{y}$.

¿Como se relaciona $(\tilde{x}, \tilde{y})$ con (x, y)?

$$Y(t) = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = (I)_{\mathcal{B}\,\mathcal{C}} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = P^{-1} X(t)$$

Voy a buscar una base de vectores propios, $\mathcal{B}$.

$$\Rightarrow \quad \dot{Y}(t) = (\dot{P^{-1} X(t)}) = P^{-1} \cdot \dot{X}(t) = P^{-1} A X(t) = P^{-1} A P Y(t) = D\, Y(t)$$

$$\begin{cases} \dot{\tilde{x}}(t) = \lambda_1 \tilde{x}(t) \\ \dot{\tilde{y}}(t) = \lambda_2 \tilde{y}(t) \end{cases}$$

$$A = \begin{pmatrix} 2 & -1 \\ -2 & 3 \end{pmatrix}, \qquad \det(A - \lambda I) = 0 \Leftrightarrow \lambda \text{ es vap}$$

$$\begin{vmatrix} 2-\lambda & -1 \\ -2 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2 = \lambda^2 - 5\lambda + 4 \quad \begin{matrix} \lambda = 4 \\ \lambda = 1 \end{matrix}$$

$$S_1 : \operatorname{Ker}(A - I) \quad \left(\begin{array}{cc|c} 1 & -1 & 0 \\ \cancel{-2} & \cancel{2} & \cancel{0} \end{array}\right) \Rightarrow \underline{x = y}$$

$$v = (x, y) : (A - I)v = 0$$

$$\Rightarrow S_1 = [(1,1)].$$

$$S_4 : \operatorname{Ker}(A - 4I) \quad \left(\begin{array}{cc|c} -2 & -1 & 0 \\ \cancel{-2} & \cancel{-1} & \cancel{0} \end{array}\right) \Rightarrow -2x = y$$

$$S_4 = [(-1, 2)]$$

$$\mathcal{B} = \{(1,1), (-1,2)\}$$

$$(A)_{\mathcal{B}\,\mathcal{B}} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \Rightarrow \begin{cases} \dot{\tilde{x}} = \tilde{x} \\ \dot{\tilde{y}} = 4\tilde{y} \end{cases} \Rightarrow \begin{matrix} \tilde{x}(t) = \tilde{x}_0 e^{t} \\ \tilde{y}(t) = \tilde{y}_0 e^{4t} \end{matrix}$$

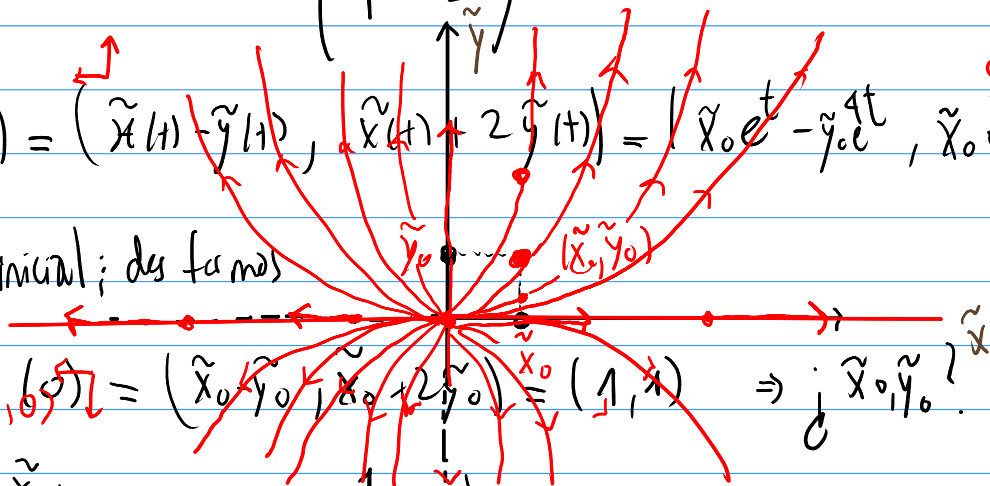
$Y = P^{-1}X$

Deshacemos el C.V.: $\begin{pmatrix}x\\ y\end{pmatrix} = P\underbrace{\begin{pmatrix}\tilde{x}\\ \tilde{y}\end{pmatrix}}$

con $P = (I)_{\mathcal{C}\mathcal{B}} = \begin{pmatrix}1 & -1\\ 1 & 2\end{pmatrix}$

$$\Rightarrow (\overset{X(t)}{x(t), y(t)}) = (\tilde{x}(t) - \tilde{y}(t), \tilde{x}(t) + 2\tilde{y}(t)) = (\tilde{x}_0 e^{t} - \tilde{y}_0 e^{4t}, \tilde{x}_0 e^{t} + 2\tilde{y}_0 e^{4t})$$

Sig($\dot{\tilde{y}}$)



Condición inicial; dos formas

1) $X(0) = (\tilde{x}_0 - \tilde{y}_0, \tilde{x}_0 + 2\tilde{y}_0) = (1,1) \Rightarrow ¿\tilde{x}_0, \tilde{y}_0?$ Despejo

$(e^t x_0, 0)$

$$\begin{pmatrix}\tilde{x}_0\\ \tilde{y}_0\end{pmatrix} = \underbrace{\begin{pmatrix}1 & -1\\ 1 & 2\end{pmatrix}^{-1}}_{P^{-1}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

2) $(\tilde{x}_0, \tilde{y}_0) = Y(0) = P^{-1}X(0) = P^{-1}\cdot\begin{pmatrix}1\\ 1\end{pmatrix}$

$$P^{-1} = \begin{pmatrix}1 & -1\\ 1 & 2\end{pmatrix}^{-1} = \frac{1}{3}\begin{pmatrix}2 & 1\\ -1 & 1\end{pmatrix}$$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

Diagrama de fase de $\dot{Y} = DY$ (OJO, NO ES EL QUE QUIERO)

$D = \begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}$

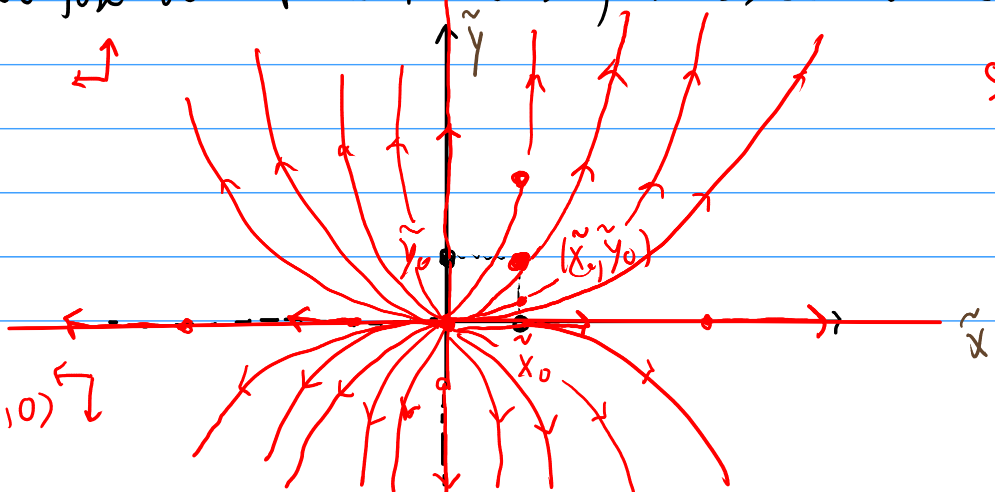
Sig($\dot{\tilde{y}}$) = sg(4y) > 0

Si y > 0

$\dot{Y} = DY$

$Y(0) = (x_0, 0)$

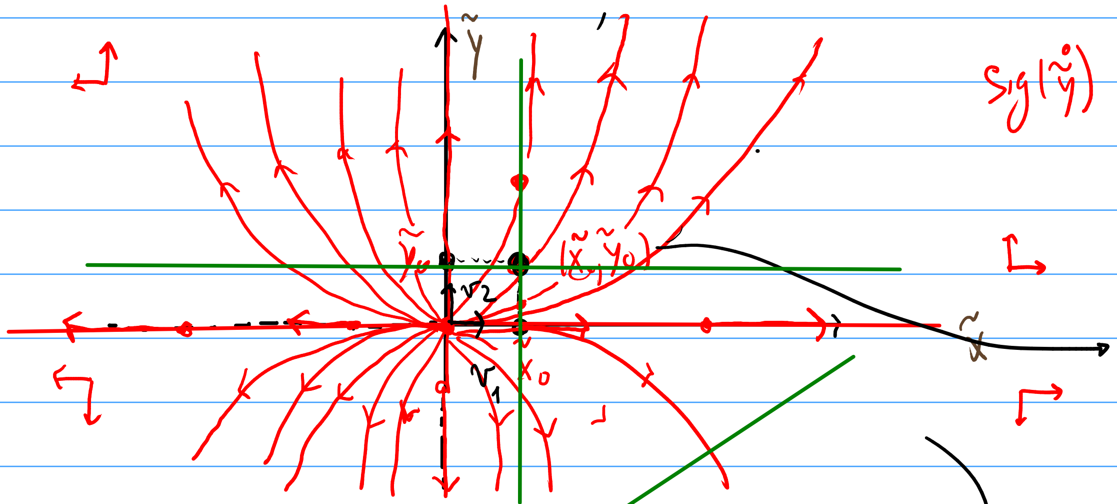
$\Rightarrow Y(t) = (e^t x_0, 0)$



$$\Rightarrow Y(t) = (\underbrace{\tilde{x}_0 e^{t}}_{\tilde{x}(t)}, \underbrace{\tilde{y}_0 e^{4t}}_{\tilde{y}(t)})$$

$$\tilde{y}(t) = \tilde{y}_0 \cdot \left(\frac{\tilde{x}(t)}{\tilde{x}_0}\right)^4 = \frac{\tilde{y}_0}{\tilde{x}_0^4} \cdot \tilde{x}(t)^4$$

$$\Rightarrow \tilde{y} = C \cdot \tilde{x}^4$$



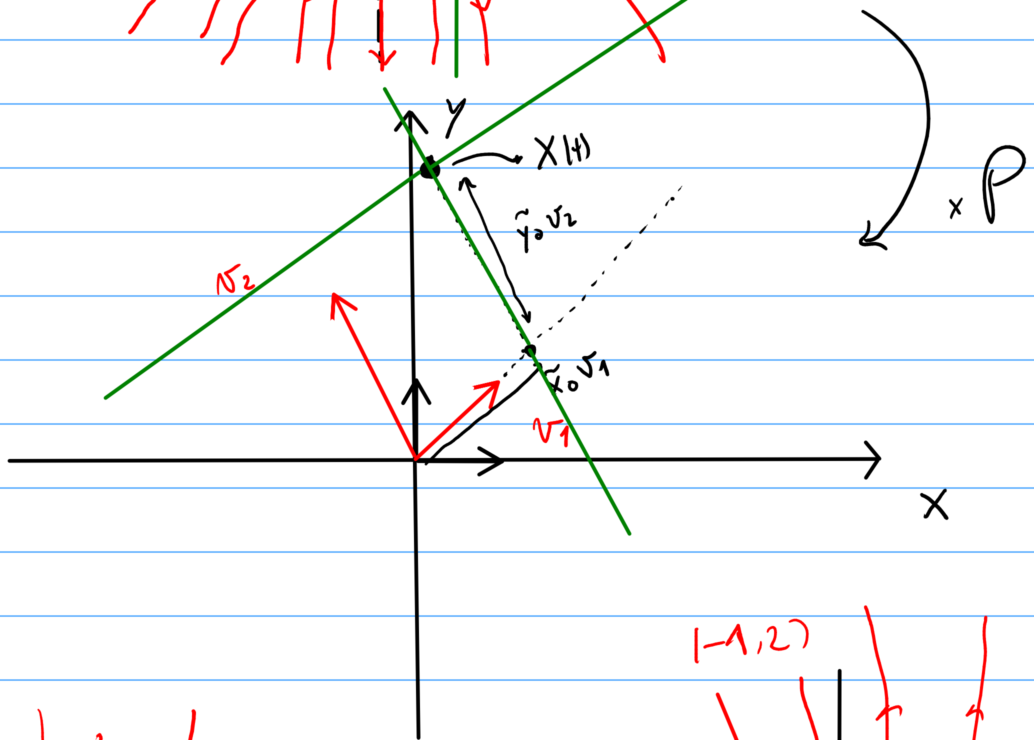
$\mathrm{sig}(\dot{\tilde{y}}) = \mathrm{sg}(4y) > 0$ si $y > 0$

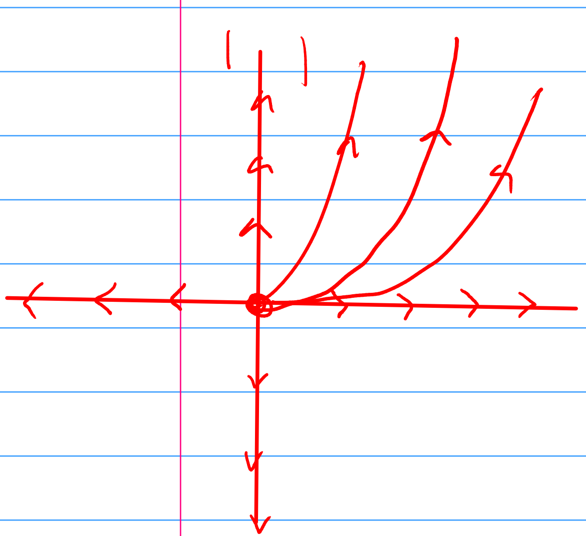
$X(0) = \tilde{x}_0 v_1 + \tilde{y}_0 v_2$

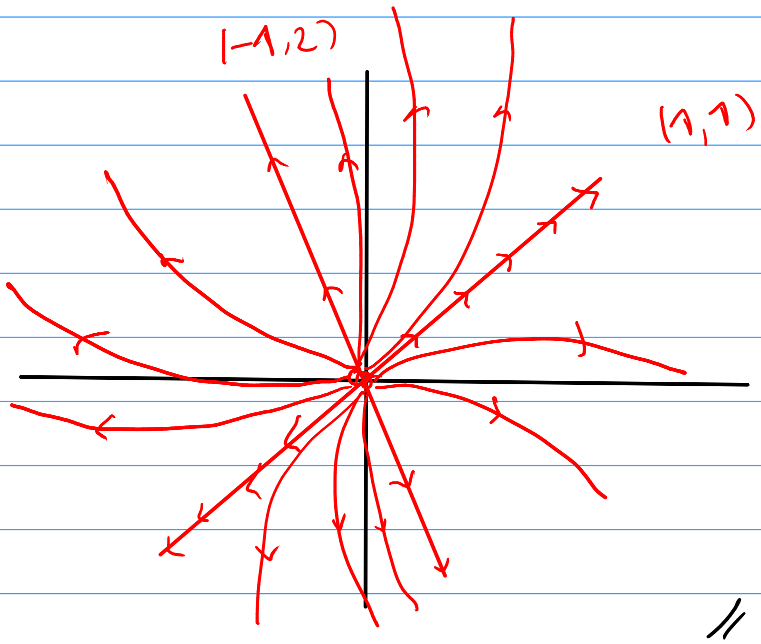
$X = PY$

$\dot{X} = AX$

$B = \{v_1, v_2\}$
$(1,1)$ $(-1,2)$







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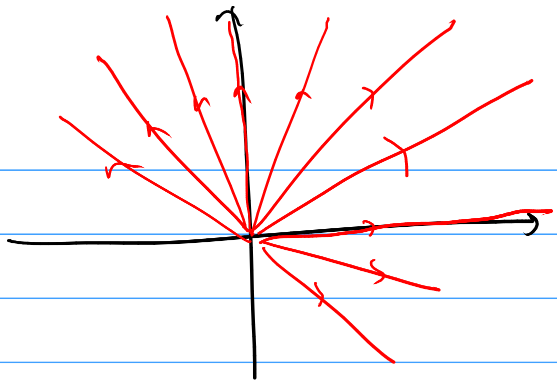

$$(\tilde{x}_0 e^{\lambda t}, \underbrace{\tilde{y}_0 e^{\lambda t}}\,)$$

$$\tilde{y} = \frac{\tilde{y}_0}{\tilde{x}_0}\tilde{x}$$