2 f) Asumiendo $\frac{d}{ds}\int_0^{+\infty} f(t)e^{-st}\,dt \overset{(*)}{=} \int_0^{+\infty} \frac{\partial}{\partial s}\left(f(t)e^{-st}\right)dt$

(*) Regla de Leibniz

Demostrar $F'(s) = \mathcal{L}\left(\underbrace{-t}\, f(t)\right)(s)$

$$F'(s) = \frac{d}{ds}\int_0^{+\infty} f(t)e^{-st}\,dt = \int_0^{+\infty} \frac{\partial}{\partial s}\left(f(t)\,\underbrace{e^{-st}}\right)dt = \int_0^{+\infty} f(t)\cdot[-t\cdot e^{-st}]\,dt$$

$(s\in\mathbb{C})$
$$= \int_0^{+\infty} \underbrace{-t\cdot f(t)}\, e^{-st}\,dt = \mathcal{L}\left(-t f(t)\right)(s)$$

g) $\mathcal{L}\left(H(t)\cdot e^{2t}\underbrace{t^2}_{(-t)\cdot(-t)}\right)(s) = \mathcal{L}\left(-t\, H(t)e^{2t}\cdot(-t)\right)(s) = \frac{d}{ds}\left(\mathcal{L}\left(H(t)e^{2t}\cdot(-t)\right)(s)\right) = \frac{d^2}{ds^2}\left(\mathcal{L}\left(H(t)e^{2t}\right)\right)$

$$= \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right) = \frac{d}{ds}\left(\frac{-1}{(s-2)^2}\right) = \frac{2}{(s-2)^3} \quad //$$

3) (v) $\mathcal{L}\left(\underbrace{\cos(\alpha t)}_{\frac{e^{i\alpha t}+e^{-i\alpha t}}{2}}\right)(s) = \frac{1}{2}\mathcal{L}\left(e^{i\alpha t}\right) + \frac{1}{2}\mathcal{L}\left(e^{-i\alpha t}\right)$

$$= \boxed{\frac{1}{2}\frac{1}{s-i\alpha} + \frac{1}{2}\frac{1}{s+i\alpha}} = \frac{1}{2}\,\frac{s+i\alpha+s-i\alpha}{(s+i\alpha)(s-i\alpha)}$$

$$= \frac{1}{\cancel{2}}\,\frac{\cancel{2}s}{s^2+\alpha^2} = \frac{s}{s^2+\alpha^2}$$

(x) $\mathcal{L}\left(t^n \sin(\alpha t)\right)(s) = (-1)^n \frac{d^n}{ds^n}\left(\mathcal{L}(\sin\alpha t)(s)\right) = (-1)^n\frac{d^n}{ds^n}\left(\frac{\alpha}{s^2+\alpha^2}\right) \quad //$

$\dot{x} + x = e^{+2t}$, x_0 cond inicial

$\Downarrow$

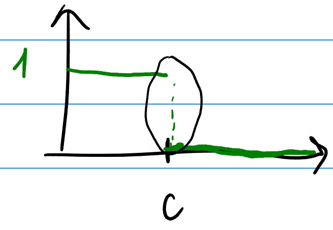
$$sX(s) - x_0 + X(s) = \frac{1}{s-2} \Rightarrow X(s)(s+1) = \frac{1}{s-2} + x_0$$

$$\Rightarrow X(s) = \frac{x_0}{s+1} + \frac{1}{(s-2)(s+1)} = \frac{x_0}{s+1} + \frac{1/3}{s-2} - \frac{1/3}{s+1}$$

$$\Rightarrow x(t) = x_0 e^{-t} + \frac{1}{3}e^{2t} - \frac{1}{3}e^{-t}, \text{ sol general}$$

¿$f(t)$?

7 e) $\begin{cases} f'' + 2f' - 3f = g, \\ f(0) = f'(0) = 0 \end{cases}$ $\quad g(t) = \begin{cases} 1, & \text{si } 0 \leq t < c \\ 0, & \text{si } t \geq c. \end{cases}$



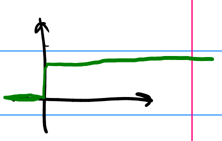
$F(s) = \mathcal{L}(f)$

$\Rightarrow \quad s^2 F(s) + 2s F(s) - 3F(s) = G(s)$

$\Rightarrow F(s) = \dfrac{G(s)}{s^2+2s-3}, \quad 1 \text{ y } -3. \quad \Big| \quad e^t, \, e^{-3t}$

- $g(t) = H(t) - H(t-c)$

H(t)

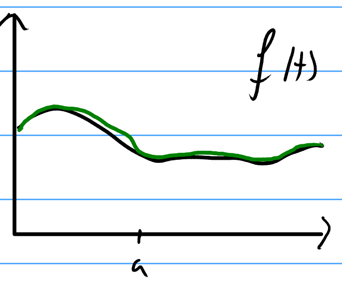


$\Rightarrow \quad G(s) = \dfrac{1}{s} - \mathcal{L}(H(t-c))(s) = \dfrac{1}{s} - e^{-cs} \cdot \dfrac{1}{s} = \dfrac{1-e^{-cs}}{s}$

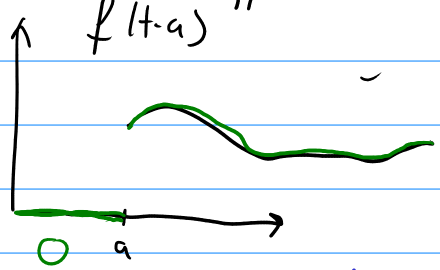
$F(s) = \dfrac{1-e^{-cs}}{s(s^2+2s-3)}.$

ACLARACIÓN: $\quad \mathcal{L}(f(t-a))(s) = e^{-as}\mathcal{L}(f(t))(s)$

$a > 0$



$\Rightarrow$

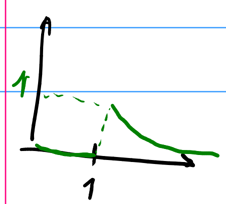


NO ES TRASLACIÓN EN EL TIEMPO

× $\mathcal{L}(e^{(t-1)})(s) \neq e^{-s} \cdot \mathcal{L}(e^t)(s) = \dfrac{e^{-s}}{s-1}$ $\quad \Big| \quad \mathcal{L}(e^{(t-1)})(s) = \mathcal{L}(e^t \cdot e^{-1})(s)$

$= e^{-1} \cdot \mathcal{L}(e^t)(s) = \dfrac{e^{-1}}{s-1}$

✓ $\mathcal{L}(H(t-1)e^{(t-1)})(s) = e^{-s} \cdot \mathcal{L}(\underbrace{H(t)}_{1 \, \forall t} e^t)(s) = \dfrac{e^{-s}}{s}$



① $\times\ \mathcal{L}(e^t)(s) = \frac{1}{s}$ $\qquad f:[0,+\infty)\to\mathbb{R},\ f(t)=e^t$

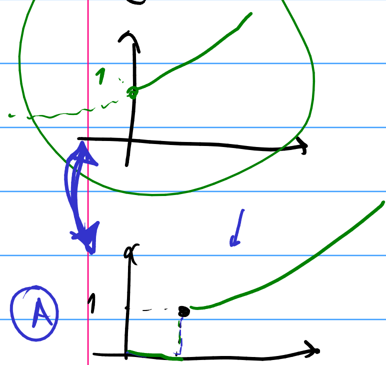
"$f(t-1)$"

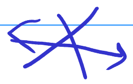
$$\mathcal{L}(g(t))(s) = e^{-s}\mathcal{L}(f)(s) \qquad g:[0,+\infty)\to\mathbb{R},\quad g(t)=\begin{cases} f(t-1), & \text{si } t\geq 1\\ 0, & \text{si } t\leq 1\end{cases}$$

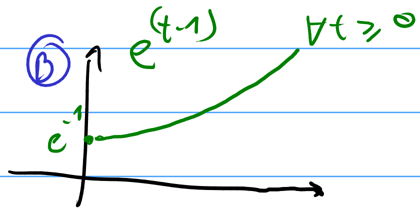
EN EL 1ER PO

$$\mathcal{L}(e^{(t-1)})(s) \cancel{=} e^{-s}\cdot\mathcal{L}(e^t)(s) = \frac{e^{-s}}{s-1}$$

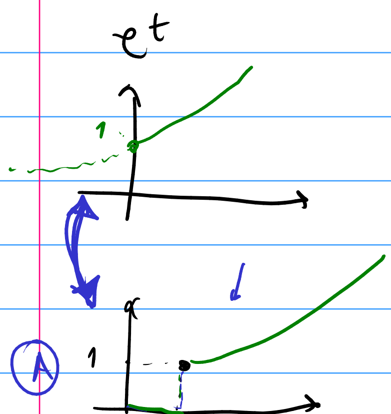
$e^t \qquad e^{(t-1)}\ \forall t\geq 0$







$$f:[0,+\infty)\to\mathbb{R},\qquad \mathcal{L}(\underbrace{H(t-1)f(t-1)}_{\begin{cases}0 & \text{si } t\leq 1\\ f(t-1) & \text{si } t\geq 1\end{cases}})(s) = e^{-s}\mathcal{L}(f(t))(s).$$



La propiedad habla de esta "traslación"

$$F(s) = \frac{1-e^{-cs}}{s(s^2+2s-3)} = \frac{1}{s(s^2+2s-3)} - e^{-cs} \cdot \frac{1}{s(s^2+2s-3)}$$

$$\frac{1}{s(s-1)(s+3)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+3} = \frac{-1/3}{s} + \frac{1/4}{s-1} + \frac{1/12}{s+3}$$

1 y -3 $\mathcal{L}^{-1}\left(\frac{1}{s(s-1)(s+3)}\right)(t) = \underbrace{\frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3}}_{h(t)} \quad \forall t \geq 0$

$$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s(s^2+2s-3)}\right) - \mathcal{L}^{-1}\left(e^{-cs} \cdot \frac{1}{s(s^2+2s+3)}\right)$$

$$= h(t) - h_{\text{corrido}}(t) \qquad \underline{\forall t \geq 0}$$

$$= \left(\frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3}\right) - \left(H(t-c) \cdot \left[\frac{1}{4}e^{t-c} + \frac{1}{12}e^{-3(t-c)} - \frac{1}{3}\right]\right) \quad \forall t \geq 0$$

$$= \begin{cases} \frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3} - 0 & \text{si } t \leq c \\ \frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3} - \left(\frac{1}{4}e^{t-c} + \frac{1}{12}e^{-3(t-c)} - \frac{1}{3}\right) & \text{si no} \end{cases}$$

MAL $f(t) = \frac{1}{4}e^{t} + \frac{1}{12}e^{-3t} - \frac{1}{3} - \left(\frac{1}{4}e^{(t-c)} + \frac{1}{12}e^{-3(t-c)} - \frac{1}{3}\right) \quad \forall t \geq 0$

$$\dot{x} + 3x - y = u(t) \qquad sX(s) + 3X(s) - Y(s) = U(s)$$

$$\dot{y} - 2y = 2u(t) \qquad sY(s) - 2Y(s) = 2U(s).$$

$$\frac{X(s)}{U(s)}$$