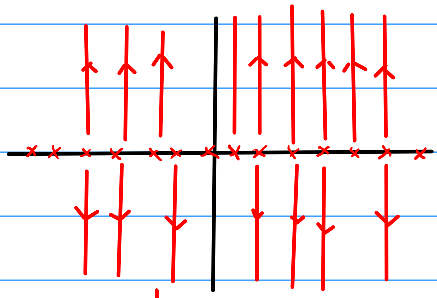
- $\dot{x} = Ax$

Estudio 2x2:

<u>Caso $A = D$ (diagonal)</u> , $A = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$

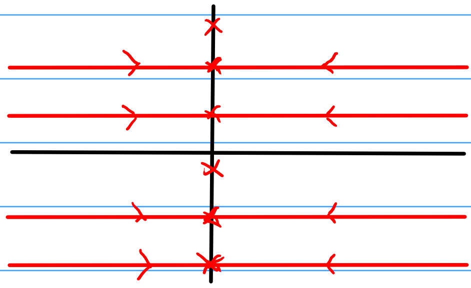
<u>$a = 0, d \neq 0$</u> :



$d > 0$

Si $d < 0$, es igual pero con las flechas invertidas

<u>$a \neq 0, d = 0$</u>



$a < 0$

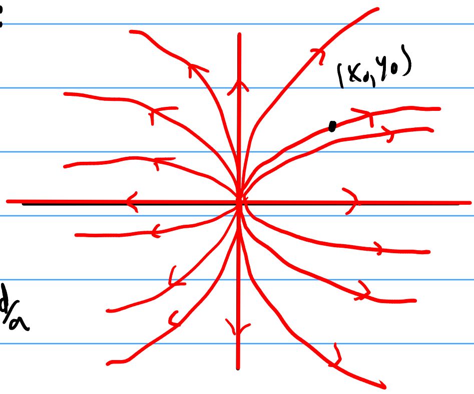
- Si $a > 0$, es igual pero con las flechas invertidas

<u>$a \neq 0, d \neq 0$</u> :

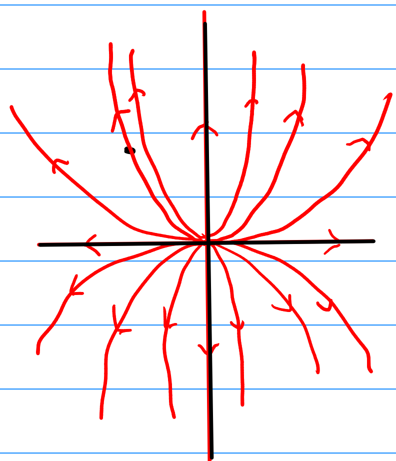
<u>$a, d > 0$</u> :

$$y = y_0 \left(\frac{x}{x_0}\right)^{d/a}$$

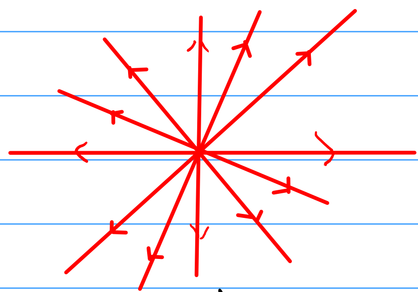
$0 < \frac{d}{a} < 1$



$a > d$



$d > a$

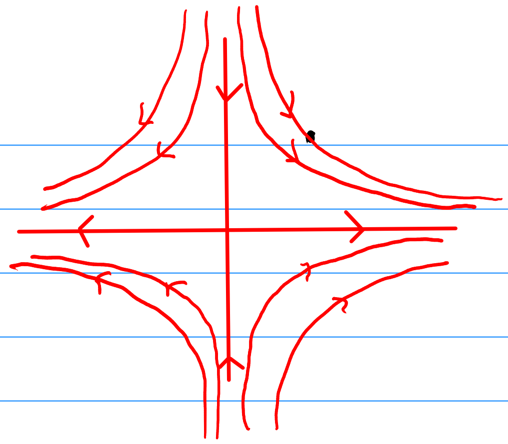


$a = d$

<u>$a, d < 0$</u> : Análogo pero con las flechas hacia adentro

$a>0, d<0$:

$$y = y_0 \left(\frac{x}{x_0}\right)^{\overset{<0}{d/a}}$$



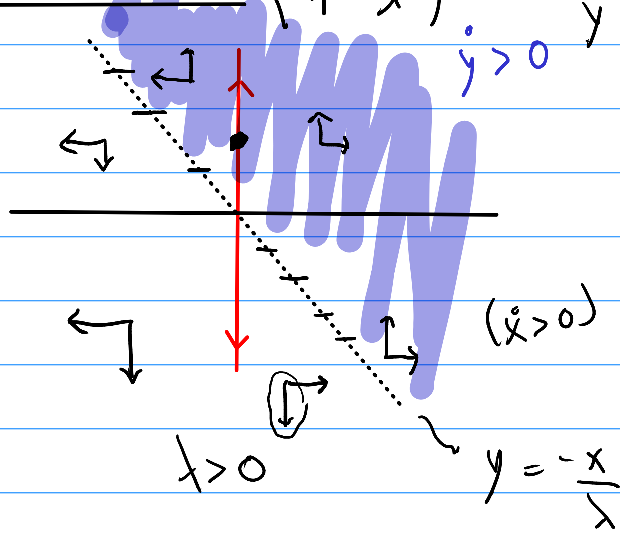
$a<0, d>0$: Análogo pero con las flechas invertidas.

Caso $A = J$, $J = \begin{pmatrix} \lambda & 0 \\ 1 & \lambda \end{pmatrix}$

$\dot{x} = \lambda x \longrightarrow x(t) = x_0 e^{\lambda t}$

$\dot{y} = x + \underbrace{\lambda y} \longrightarrow y(t) = e^{\lambda t}(y_0 + x_0 t)$

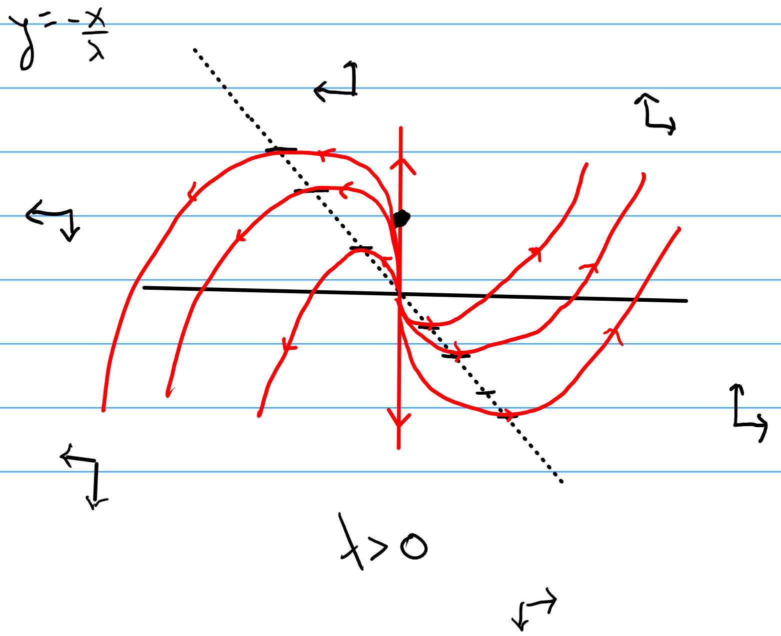
$\lambda \neq 0$



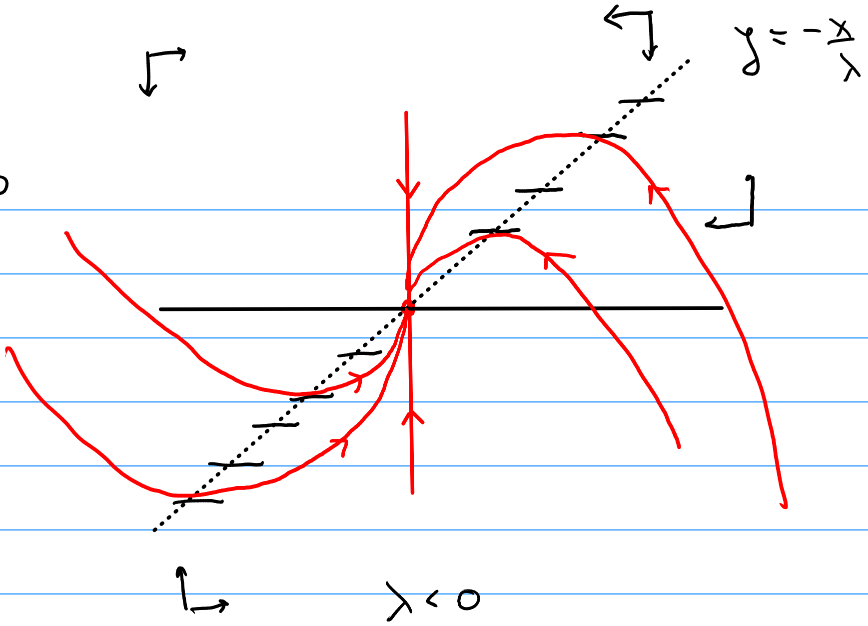
$\mathrm{sign}(\dot{y})$: $\dot{y} > 0 \iff x + \lambda y > 0 \iff y > -\frac{x}{\lambda}$

¿Cómo se relaciona $x(t)$ con $y(t)$? $\frac{1}{\lambda}\ln\frac{x(t)}{x_0} = t$

$$\Rightarrow y(t) = \frac{x(t)}{x_0}\left(y_0 + \frac{x_0}{\lambda}\ln\left(\frac{x(t)}{x_0}\right)\right)$$



Caso $\lambda < 0$



$\dot{x} = \lambda x$

$\dot{y} = x + \lambda y$

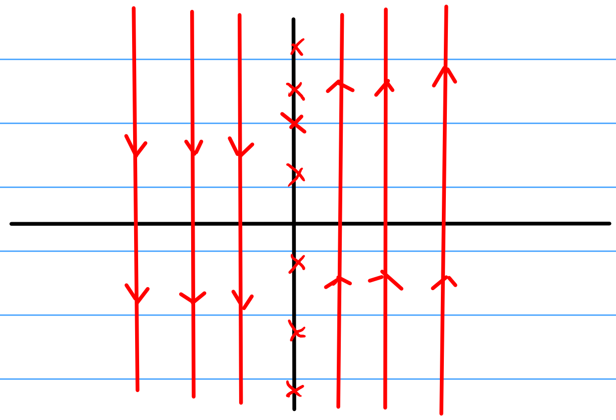
$\dot{y} > 0$

$x + \lambda y > 0$

$\lambda y > -x$

$y < -\frac{x}{\lambda}$

Caso $\lambda = 0$

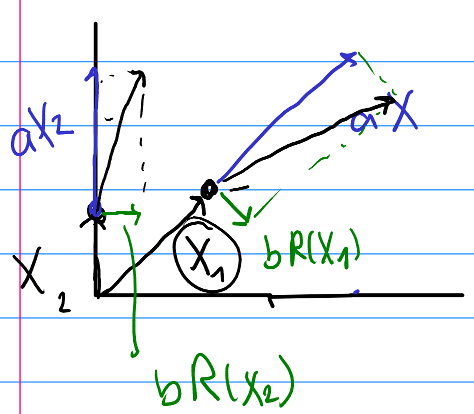


$\dot{x} = 0$

$\dot{y} = x$

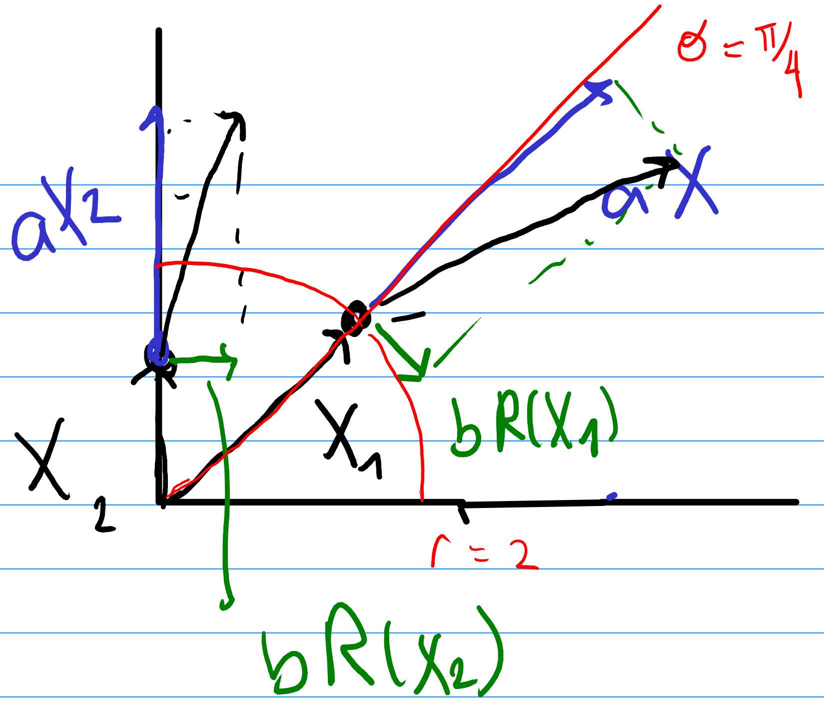
Caso $\lambda = a \pm ib$. Es un resultado que $A \sim \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

Entonces estudio el caso $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} = aI + b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$



$\dot{x} = AX = aIX + b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}X$

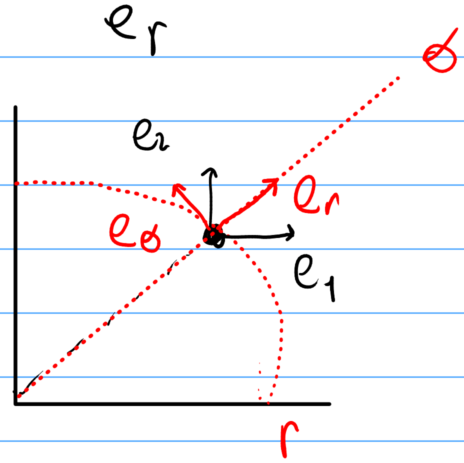
AX_2



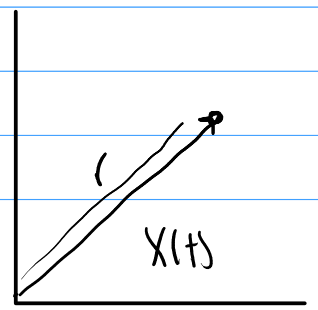
CV: polares $\begin{cases} x = r\cos\theta \\ y = r \operatorname{sen}\theta \end{cases}$ $\qquad X(t) = x(t)e_1 + y(t)e_2 \quad , \quad e_1 = (1,0) \quad e_2 = (0,1)$

$$\Rightarrow \dot{X}(t) = \dot{x}(t)e_1 + \dot{y}(t)e_2 = (\dot{r}\cos\theta - r\dot{\theta}\operatorname{sen}\theta)\, e_1 + (\dot{r}\operatorname{sen}\theta + r\dot{\theta}\cos\theta)\, e_2$$

$$= \dot{r}\underbrace{(\cos\theta\, e_1 + \operatorname{sen}\theta\, e_2)}_{e_r} + r\dot{\theta}\underbrace{(-\operatorname{sen}\theta\, e_1 + \cos\theta\, e_2)}_{e_\theta} = \dot{r}e_r + r\dot{\theta}e_\theta$$



$$\underbrace{\dot{X}(t)}_{=\ \dot{r}e_r + r\dot{\theta}e_\theta} = T_A(X(t)) \quad , \quad (T_A)_{\mathcal{e}\,\mathcal{e}} = A$$



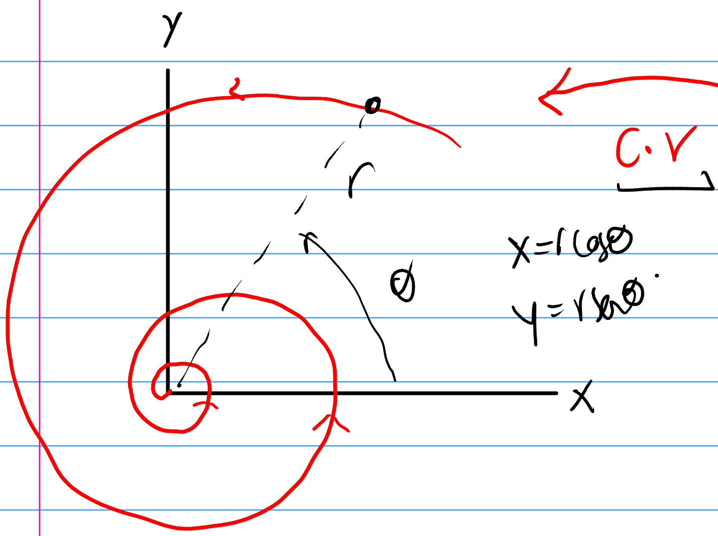
$$T_A(X(t)) = T_A(r\underbrace{e_r}_{r}) = a r e_r - b r e_\theta$$

$\dot{x} = T_A(x)$, $\dot{r} e_r + r\dot{\theta} e_\theta = a r e_r - b r e_\theta$

$$\Rightarrow \begin{cases} ar = \dot{r} \\ r\dot{\theta} = -br \end{cases} \Rightarrow \boxed{\begin{cases} \dot{r} = ar \\ \dot{\theta} = -b \end{cases}}$$

$r(t) = r_0 e^{at}$

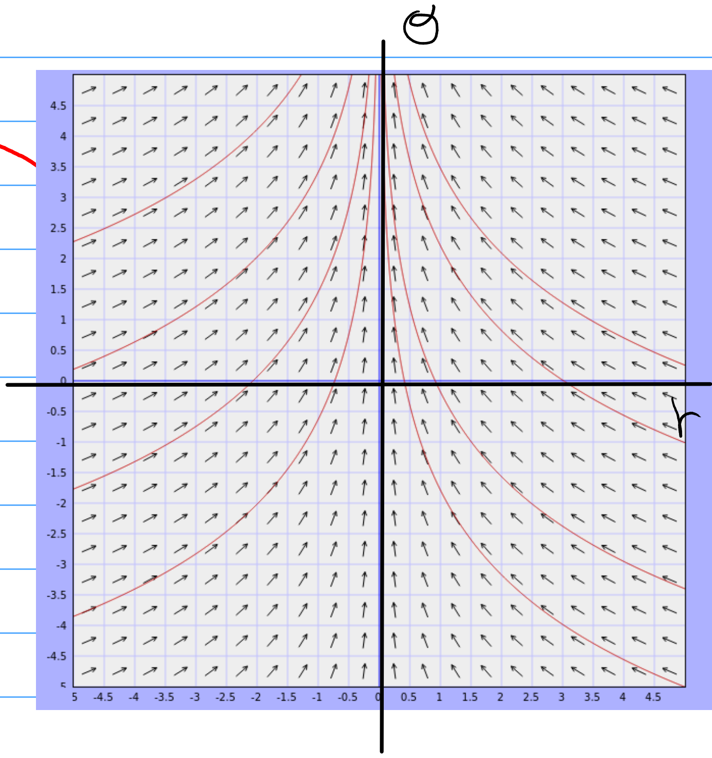
$\theta(t) = -bt + \theta_0$



$x = r\cos\theta$

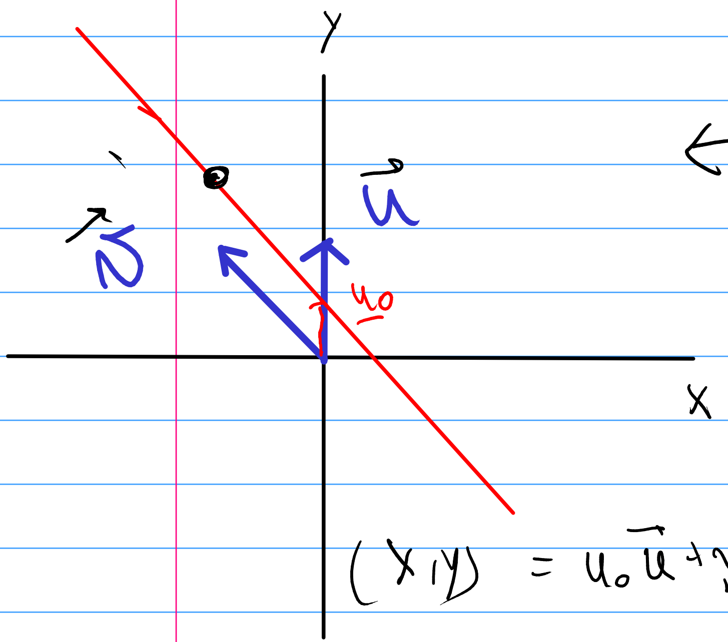
$y = r\sin\theta$

C.V

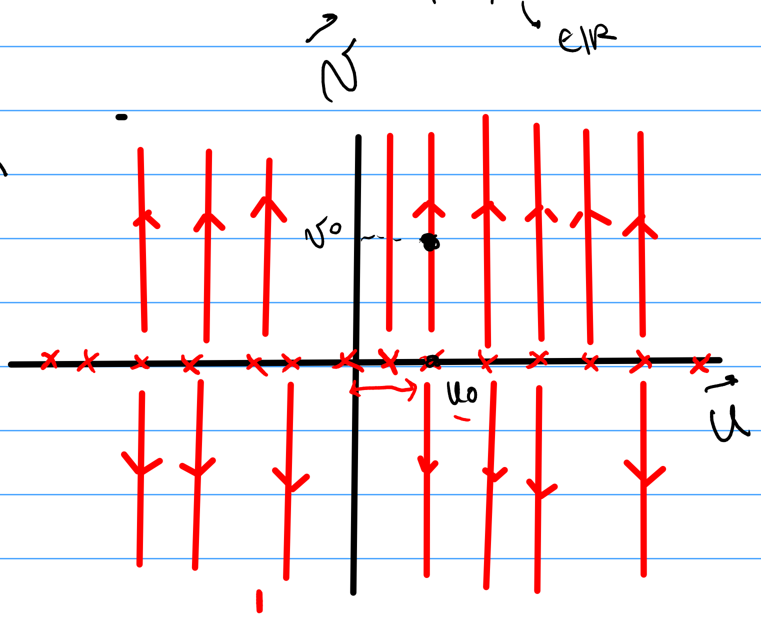


$b<0,\ a<0$

$\vec{u} = (0,1)$, $\vec{v} = (-1,1)$



$(u_0, v) \in \mathbb{R}$



$(x,y) = u_0 \vec{u} + v \vec{v}$