→ Ejercicio 1. g. $\sum_{n=1}^{\infty} \frac{n \operatorname{arctg}(n+1) - (n+1)\operatorname{arctg}(n)}{n(n+1)}$

tenemos que $S_N = \sum_{n=1}^{\infty} \frac{n \operatorname{arctg}(n+1) - (n+1)\operatorname{arctg}(n)}{n(n+1)}$

Miremos sus términos: $n=1 \rightarrow \frac{\operatorname{arctg}(2) - 2\operatorname{arctg}(1)}{2} = \frac{\operatorname{arctg}(2)}{2} - \operatorname{arctg}(1)$

" $n=2 \rightarrow \frac{2\operatorname{arctg}(3) - 3\operatorname{arctg}(2)}{6} = \frac{\operatorname{arctg}(3)}{3} - \frac{\operatorname{arctg}(2)}{2}$

$n=3 \Rightarrow \frac{3\operatorname{arctg}(4) - 4\operatorname{arctg}(3)}{12} = \frac{\operatorname{arctg}(4)}{4} - \frac{\operatorname{arctg}(3)}{3}$

Si repetimos hasta N, observando que es telescópica, resulta que

$S_N = \frac{\operatorname{Arctg}(N+1)}{N+1} - \operatorname{arctg}(1)$

$\Rightarrow \sum_{n=1}^{\infty} \frac{n \operatorname{arctg}(n+1) - (n+1)\operatorname{arctg}(n)}{n(n+1)} = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{\overbrace{\operatorname{arctg}(N+1)}^{\text{acotado}}}{N+1} - \operatorname{arctg}(1)$

$= -\operatorname{arctg}(1) = -\frac{\pi}{4}.$