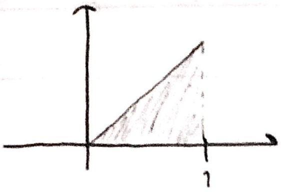
PRÁCTICO 10.

→ Ejercicio 1:

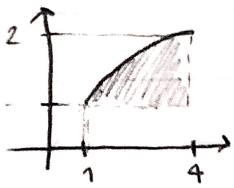
a. $\int_0^1 \int_0^y f(x,y)\,dx\,dy$ — $D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le y \,,\, 0 \le y \le 1\}$



Reescribimos $D = \{(x,y) \in \mathbb{R}^2 ,\ 0 \le x \le 1 ,\ x \le y \le 1\}$

$$\Rightarrow \int_0^1 \int_0^y f(x,y)\,dx\,dy = \int_0^1 \int_x^1 f(x,y)\,dy\,dx$$

b. $D = \{(x,y) : x \in [1,4] ,\ y \in [\sqrt{x},2]\} = \{(x,y) : y \in [1,2] ,\ x \in [y^2,4]\}$

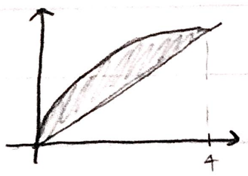


$$\int_1^4 \int_{\sqrt{x}}^2 f(x,y)\,dy\,dx = \int_1^2 \int_{y^2}^4 f(x,y)\,dx\,dy$$

c. $D = \{(x,y) : 0 \le y \le 2 ,\ y^2 \le x \le 2y\} = \{(x,y) : 0 \le x \le 4 ,\ x/2 \le y \le \sqrt{x}\}$

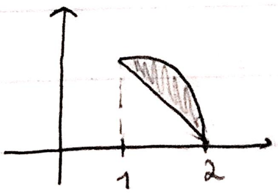
$y \ge x/2$

$y \le \sqrt{x}$



$$\Rightarrow \int_0^2 \int_{y^2}^{2y} f(x,y)\,dx\,dy$$

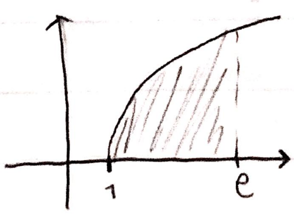
d. $D = \{(x,y) : 1 \le x \le 2 ,\ 2-x \le y \le \sqrt{2x-x^2}\} = \{(x,y) : 0 \le y \le 1 ,\ 2-y \le x \le 1+\sqrt{1-y^2}\}$



$$2x - x^2 \ge y^2 \Leftrightarrow x^2 - 2x + y^2 \le 0 \Leftrightarrow x = \frac{2 \pm \sqrt{4-4y^2}}{2}$$

$$\Rightarrow \int_1^2 \int_{2-x}^{\sqrt{2x-x^2}} f(x,y)\,dy\,dx = \int_0^1 \int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

e. $D = \{(x,y) : 1 \le x \le e ,\ 0 \le y \le \log(x)\} = \{(x,y) : 0 \le y \le 1 ,\ e^y \le x \le e\}$



$$\int_1^e \int_0^{\log(x)} f(x,y)\,dy\,dx = \int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy$$

→ Ejercicio 2.

a. $\int_0^1 \left(\int_0^{\sqrt{1-y^2}} y x^2 \, dx \right) dy \to D = \{(x,y): 0 \leq y \leq 1, 0 \leq x \leq \sqrt{1-y^2}\}$
$= \{(x,y): 0 \leq y \leq \sqrt{1-x^2}, 0 \leq x \leq 1\}$

$\to \int_0^1 \left(\int_0^{\sqrt{1-x^2}} y x^2 \, dy \right) dx =$

$= \int_0^1 x^2 \left(\int_0^{\sqrt{1-x^2}} y \, dy \right) dx = \int_0^1 x^2 \cdot \left(\frac{y^2}{2} \Big|_0^{\sqrt{1-x^2}} \right) dx = \int_0^1 x^2 \cdot \frac{(\sqrt{1-x^2})^2}{2} \, dx$

$= \frac{1}{2} \int_0^1 x^2 - x^4 \, dx = \frac{1}{2} \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1 = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2}{2 \cdot 15} = \frac{1}{15}$

b. $D = \{(x,y): 1 \leq y \leq 2, 1/2 \leq x \leq 1/y\} = \{(x,y): 1/2 \leq x \leq 1, 1 \leq y \leq 1/x\}$

$\int_1^2 \left(\int_{1/2}^{1/y} xy \, dx \right) dy = \int_{1/2}^1 \left(\int_1^{1/x} xy \, dy \right) dx = \int_{1/2}^1 x \left(\frac{y^2}{2} \Big|_1^{1/x} \right) dx =$

$= \int_{1/2}^1 \frac{x}{2} \cdot \left(\frac{1}{x^2} - 1 \right) dx = \int_{1/2}^1 \frac{1}{2x} - \frac{x}{2} \, dx = \frac{1}{2} \left(\log(x) - \frac{x^2}{2} \right) \Big|_{1/2}^1 =$

$= \frac{1}{2} \left(\log(1) - \log(1/2) - \frac{1}{2} + \frac{1}{16} \right) = \frac{1}{2} \left(\log(2) - 7/16 \right)$

→ Ejercicio 3.

a. $f(x,y) = 2x - y, \; D = \{(x,y): 1 \leq x \leq 4, 0 \leq y \leq 3\}$

$\int_1^4 \left(\int_0^3 (2x - y) \, dy \right) dx = \int_1^4 \left(2xy - y^2/2 \Big|_0^3 \right) dx = \int_1^4 (6x - 9/2) \, dx =$

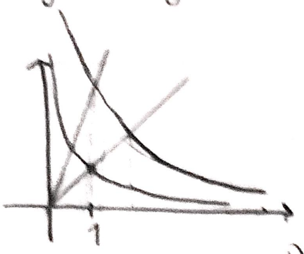
$= \left(3x^2 - \frac{9}{2} x \right) \Big|_1^4 = 48 - 3 - 18 + \frac{9}{2} = \frac{83}{2}$

d. $f(x,y) = x^2 - y^2, \; D = \{(x,y): 0 \leq x \leq \pi, 0 \leq y \leq \operatorname{sen} x\}$

$\int_0^\pi \left(\int_0^{\operatorname{sen} x} (x^2 - y^2) \, dy \right) dx = \int_0^\pi \left(x^2 y - y^3/3 \Big|_0^{\operatorname{sen} x} \right) dx = \int_0^\pi (\underbrace{x^2 \operatorname{sen} x}_{\text{partes}} - \underbrace{\operatorname{sen}^3(x)}_{(1-\cos^2(x))\operatorname{sen}(x)}/3) \, dx$

$= \left(-x^2 \cos(x) + 2(x \operatorname{sen}(x) + \cos x) - \left(-\cos(x) + \frac{\cos^3(x)}{3} \right) \right) \Big|_0^\pi = \pi^2 - 4 - 2 + \frac{2}{3} = \pi^2$

F. $f(x,y) = x^2y^2$. Divididmos el dominio en distintas zonas hallando los puntos de intersección entre las curvas



$x = 1/x \Leftrightarrow x = 1$, $4x = 1/x \Leftrightarrow x = 1/2$

$x = 2/x \Leftrightarrow x = \sqrt{2}$ $\quad 4x = 2/x \Leftrightarrow x = 1/\sqrt{2}$

$\Rightarrow D = \{ 1/2 \le x \le 1/\sqrt{2},\ 1/x \le y \le 4x \} \cup \{ 1/\sqrt{2} \le x \le 1,\ 1/x \le y \le 2/x \}$
$\cup \{ 1 \le x \le \sqrt{2},\ x \le y \le 2/x \}$

$$\Rightarrow \int_D f(x,y) = \int_{1/2}^{1/\sqrt{2}} \left(\int_{1/x}^{4x} x^2y^2\, dy \right) dx + \int_{1/\sqrt{2}}^{1} \left(\int_{1/x}^{2/x} x^2y^2\, dy \right) dx + \int_{1}^{\sqrt{2}} \left(\int_{x}^{2/x} x^2y^2\, dy \right) dx$$

$$= \int_{1/2}^{1/\sqrt{2}} \frac{x^2}{3}\left((4x)^3 - \frac{1}{x^3} \right) dx + \int_{1/\sqrt{2}}^{1} \frac{x^2}{3} \cdot \left(\frac{8}{x^3} - \frac{1}{x^3} \right) dx + \int_{1}^{\sqrt{2}} \frac{x^2}{3} \left(\frac{8}{x^3} - x^3 \right) dx$$

$$= \int_{1/2}^{1/\sqrt{2}} \frac{4^3}{3} \cdot x^5 - \frac{1}{3x}\, dx + \int_{1/\sqrt{2}}^{1} \frac{7}{3} \frac{1}{x}\, dx + \int_{1}^{\sqrt{2}} \frac{8}{3}\frac{1}{x} - \frac{x^5}{3}\, dx$$

$$= \left(\frac{4^3}{3} \cdot \frac{x^6}{6} - \frac{1}{3}\log(x) \right) \Bigg|_{1/2}^{1/\sqrt{2}} + \frac{7}{3}\log(x) \Bigg|_{1/\sqrt{2}}^{1} + \left(\frac{8}{3}\log(x) - \frac{x^6}{18} \right) \Bigg|_{1}^{\sqrt{2}}$$

$$= \frac{4^3}{3.6} \cdot \left(\frac{1}{2^3} - \frac{1}{4^3} \right) + \frac{1}{3}\log(1/2) - \frac{1}{3}\log(1/\sqrt{2}) - \frac{7}{3}\log(1/\sqrt{2}) + \frac{8}{3}\log(\sqrt{2}) - \frac{\sqrt{2}^6}{18} + \frac{1}{18}$$

$$= \cancel{\frac{2^3}{3.6}} - \cancel{\frac{1}{18}} + \frac{1}{3}\log(2^{-1}) - \frac{8}{3}\log(2^{-1/2}) + \frac{8}{3}\log(2^{1/2}) - \cancel{\frac{2^3}{18}} + \cancel{\frac{1}{18}}$$

$$= -\frac{1}{3}\log(2) + \frac{4}{3}\log(2) + \frac{4}{3}\log(2) = \frac{7}{3}\log(2)$$