PRÁCTICO 2: ECUACIONES DIFERENCIALES.

→ Ejercicio 1.

pues y depende de x

c. $xe^{2y}y' - (1+e^{2y}) = 0 \Rightarrow xe^{2y}y' = (1+e^{2y}) \Rightarrow \frac{e^{2y}y'}{(1+e^{2y})} = \frac{1}{x} \Rightarrow \frac{e^{2y(x)}}{1+e^{2y(x)}}y'(x) = \frac{1}{x}$

$\Rightarrow \int \frac{e^{2y(x)}y'(x)\,dx}{1+e^{2y(x)}} = \int \frac{1}{x}dx \Rightarrow \int \frac{e^{2y}}{1+e^{2y}}dy = \int \frac{1}{x}dx$

haciendo el cdv

$y = y(x) \Rightarrow dy = y'(x)dx$

$\Rightarrow \frac{1}{2}\int \frac{e^{u}}{1+e^{u}}du = \int \frac{1}{x}dx \Rightarrow \frac{1}{2}Ln(1+e^{u}) = Ln(x) + k$, llamando $k = Ln(c)$

cdv:

$u = 2y$

$du = 2dy$

$\Rightarrow Ln(1+e^{2y(x)}) = Ln(cx^2) \Rightarrow 1+e^{2y(x)} = cx^2$

$\Rightarrow e^{2y(x)} = cx^2 - 1 \Rightarrow 2y(x) = Ln(cx^2-1) \Rightarrow y(x) = \frac{1}{2}Ln(cx^2-1)$

→ Ejercicio

a. $x^2y' + y(y-x) = 0 \Rightarrow x^2y'(x) + y(x)(y(x)-x) = 0$, haciendo el cdv: $y(x) = xu(x)$

$y'(x) = u'(x)x + u(x)$

$\Rightarrow x^2(u'(x)x + u(x)) + u(x)x(u(x)x - x) = 0$

$\Rightarrow u'(x)x^3 + \cancel{u(x)x^2} + u(x)^2x^2 - \cancel{u(x)x^2} = 0$ → si $x=0$, se cumple trivialmente Sino

$u'(x)x + u(x)^2 = 0 \Rightarrow \frac{u'(x)}{u(x)^2} = -\frac{1}{x} \Rightarrow \int \frac{u'(x)}{u(x)^2}dx = -\int \frac{1}{x}dx$

$\Rightarrow \int \frac{du}{u^2} = -\int \frac{1}{x}dx \Rightarrow -\frac{1}{u} = -Ln(x) + k \Rightarrow u = \frac{1}{Ln(x)-k} \Rightarrow y(x) = \frac{x}{Ln(x)-k}$

$u = u(x)$

$du = u'(x)dx$

deshaciendo el cdv

→ Ejercicio 3.

b. 2) $x(x-1)y' + (1-2x)y + x^2 = 0 \rightarrow (x^2-x)y' + (1-2x)y = -x^2$

HOMOGÉNEA: $(x^2-x)y_H' + (1-2x)y_H = 0 \rightarrow \frac{y_H'}{y_H} = \frac{2x-1}{x^2-x} \Rightarrow \int \frac{y_H'(x)}{y_H(x)}dx = \int \frac{2x-1}{x^2-x}dx$

$\rightarrow \int \frac{dy_H}{y_H} = \int \frac{2x-1}{x^2-x}dx \Rightarrow Ln(y_H) = Ln(x^2-x) + k \Rightarrow y_H(x) = (x^2-x)C$

$C = e^k$

PARTICULAR: tomando $y_P(x) = C(x)(x^2-x) \rightarrow y_P'(x) = C'(x)(x^2-x) + C(x)(2x-1)$

$\Rightarrow x(x-1)\left(C'(x)(x^2-x) + C(x)(2x-1)\right) + (1-2x)C(x)(x^2-x) + x^2 = 0$

$\rightarrow C'(x)(x^2-x)^2 + \cancel{C(x)(2x-1)(x^2-x)} + \cancel{C(x)(x^2-x)(1-2x)} + x^2 = 0$

$\Rightarrow C'(x)(x^2-x)^2 = -x^2 \Rightarrow C'(x) = \frac{-x^2}{(x^2-x)^2} = \frac{-x^2}{x^2(x-1)^2} = -\frac{1}{(x-1)^2}$

$\Rightarrow \int C'(x)dx = -\int \frac{1}{(x-1)^2}dx \Rightarrow \int C'(x)dx = -\int \frac{du}{u^2} \Rightarrow C(x) = \frac{1}{u} + k = \frac{1}{x-1} + k$

$u = x-1$

$du = dx$

$\Rightarrow y(x) = y_H(x) + y_P(x) = C(x^2-x) + \left(\frac{1}{x-1} + k\right)(x^2-x) = C(x^2-x) + x + k(x^2-x)$

$= A(x^2-x) + x$