CURSO DE COMPLEJIDAD COMPUTACIONAL 2021

Práctico 3

Temas : Clase PSPACE. Completitud en la clase PSPACE. Teorema de Savitch. Clases L y NL. Completitud en la clase NL.

3.1. Sipser

The Japanese game *go-moku* is played by two players, "X" and "O," on a 19×19 grid. Players take turns placing markers, and the first player to achieve 5 of his markers consecutively in a row, column, or diagonal, is the winner. Consider this game generalized to an $n \times n$ board. Let

$$GM = \{\langle B \rangle \mid B \text{ is a position in generalized go-moku, where player "X" has a winning strategy}\}.$$

By a *position* we mean a board with markers placed on it, such as may occur in the middle of a play of the game, together with an indication of which player moves next. Show that $GM \in$ PSPACE.

3.2. Ejercicio 4.12 Arora –Barak

Define **polyL** to be $\cup_{c>0}\mathbf{SPACE}(\log^c n)$. Steve's Class SC (named in honor of Steve Cook) is defined to be the set of languages that can be decided by deterministic machines that run in polynomial time and $\log^c n$ space for some $c > 0$.

It is an open problem whether PATH $\in$ SC. Why does Savitch's Theorem not resolve this question?

Is SC the same as **polyL** $\cap$ **P**?

3.3. Sipser

Show that, if every NP-hard language is also PSPACE-hard, then PSPACE = NP.

3.4. Ejercicio 4.3 Arora-Barak

Probar que todo lenguaje L que no sea el lenguaje vacío o $\{0,1\}^*$ es completo para **NL** si se usan reducciones de Karp polinomiales en el tiempo en la definición de completitud.

3.5. Sipser

a. Let $ADD = \{\langle x, y, z \rangle \mid x, y, z > 0$ are binary integers and $x + y = z\}$. Show that $ADD \in$ L.

b. Let $PAL\text{-}ADD = \{\langle x, y \rangle \mid x, y > 0$ are binary integers where $x + y$ is an integer whose binary representation is a palindrome$\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse). Show that $PAL\text{-}ADD \in$ L.

3.6. Ejercicio 4.5 Arora –Barak

Show that 2SAT is in **NL**.

Sugerencia: Mostrar que la satisfactibilidad de una fórmula $\phi \in 2CNF$ puede determinarse a partir de las respuestas a una cantidad polinomial de instancias del problema $PATH$ sobre el grafo dirigido G_ϕ, donde G_ϕ tiene una arista $(\bar{y}, x)$ y una arista $(\bar{x}, y)$ por cada cláusula $(x \vee y)$ de la fórmula ϕ.

3.7. Sipser

An undirected graph is ***bipartite*** if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it doesn't contain a cycle that has an odd number of nodes. Let $BIPARTITE = \{\langle G \rangle \mid G \text{ is a bipartite graph}\}$. Show that $BIPARTITE \in \text{NL}$.

3.8. Ejercicio 4.8 Arora –Barak

Define a function $f : \{0, 1\}^* \to \{0, 1\}^*$ to be write-once logspace computable if it can be computed by an $O(\log n)$-space TM M whose output tape is "write-once" in the sense that, in each step, M can either keep its head in the same position on that tape or write to it a symbol and move one location to the right. The used cells of the output tape are not counted against M's space bound.

Prove that f is write-once logspace computable if and only if it is implicitly logspace computable in the sense of Definition 4.16.

3.9. Ejercicio 4.7 Arora –Barak

Prove that in the certificate definition of **NL** (Section 4.3.1) if we allow the verifier machine to move its head back and forth on the certificate, then the class being defined changes to **NP**.

3.10. Ejercicio 4.4 Arora –Barak

Show that the following language is **NL**-complete:

$$\{ \llcorner G \lrcorner : G \text{ is a strongly connected digraph} \}$$