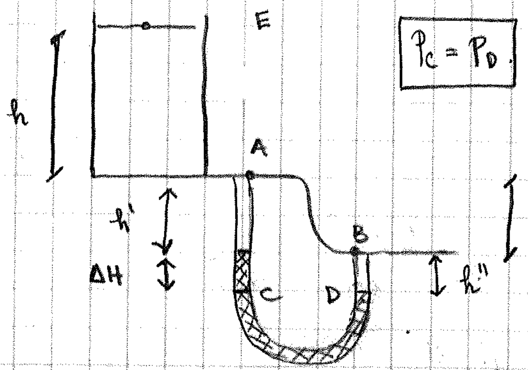
PROB 1 PARCIAL 1/10/2014



$\boxed{P_C = P_D}$

PARTE A

Cuando está en reposo...

$$\left.\begin{array}{l} P_A = P_0 + \rho g h \\ P_B = P_0 + \rho g (h+L) \end{array}\right\rangle \quad P_B - P_A = \rho g L$$

$$\left.\begin{array}{l} P_C = P_A + \rho g h' + \rho' g \Delta H \\ P_D = P_B + \rho g h'' \end{array}\right\rangle \quad P_D - P_C = P_B - P_A + \rho g (h'' - h') - \rho' g \Delta H = 0$$

$$P_D - P_C = \rho g (L + h'' - h') - \rho' g \Delta H = (\rho - \rho') g \Delta H = 0 \rightarrow \boxed{\Delta H = 0}$$

$(\rho \neq \rho')$

Por geometría: $h' + \Delta H = L + h'' \longrightarrow \Delta H = L + h'' - h'$

PARTE B

i) Aplico Bernoulli entre el punto E (de entrada) y el punto B (de salida)

$$P_0 + \rho g (h+L) + \rho \frac{v_E^2}{2} = P_0 + \rho \frac{v_B^2}{2} \qquad v_E A_E = v_B A_B \longrightarrow (A_E >> A_B)$$

$v_E << v_B$.

$$v_B = (2g(h+L))^{1/2} = 7{,}67 \text{ m/s}$$

ii) Aplico Bernoulli entre el punto A y el punto B.

$$P_A + \rho g L + \rho \frac{v_A^2}{2} = P_B + \rho \frac{v_B^2}{2} \qquad v_A A_A = v_B A_B \longrightarrow v_A = \frac{A_B}{A_A} v_B$$

$$v_A = \frac{25}{16} v_B = 12 \text{ m/s}$$

$$P_A = P_B + \frac{\rho}{2}(v_B^2 - v_A^2) - \rho g L \quad \longrightarrow \quad P_B - P_A = \rho g L + \frac{\rho}{2}(v_A^2 - v_B^2) = 52{,}4 \text{ kPa}$$

$$P_D - P_C = P_B - P_A + \rho g (h'' - h') - \rho' g \Delta H = 0 \qquad h'' - h' = \Delta H - L$$

$$\cancel{\rho g L} + \frac{\rho}{2}(v_A^2 - v_B^2) - \cancel{\rho g L} + (\rho - \rho') g \Delta H = 0$$

$$(\rho' - \rho)\Delta H = \frac{\rho}{2g}(v_A^2 - v_B^2) \rightarrow \boxed{\Delta H = 34{,}5 \text{ cm}}$$

$D >> d_2 \rightarrow v_E << v_B$.

PARTE C Bernoulli entre E y B $\longrightarrow$ $v_B^2 = 2g(h+L) = \left(\frac{A_E}{A_B}\right)^2 v_E^2 = \alpha^2 \dot{h}^2$

$$\alpha^2 = \left(\frac{A_E}{A_B}\right)^2$$

$$\frac{dh}{dt} = \frac{\sqrt{2g(h+L)}}{\alpha} \longrightarrow \frac{dh}{(h+L)^{1/2}} = \frac{(2g)^{1/2}}{\alpha} dt$$

$$2(h+L)^{1/2} - 2L^{1/2} = (2g)^{1/2} \frac{t}{\alpha}$$

$$t = (2/g)^{1/2} \alpha \left[3^{1/2} - 1^{1/2}\right] = 33 \text{ s.} \longrightarrow \boxed{t = 33 s}$$

$$\alpha = \frac{d_E^2}{d_2^2}$$

PROB. 2

Parte A $\quad v_s = 343$ m/s

$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

i) $\Delta P_1(x,t) = \Delta P_0 \sin(kx - \omega t)$

$\Delta P_2(x,t) = \Delta P_0 \sin(kx + \omega t)$

$\Delta P_T(x,t) = 2\Delta P_0 \sin(kx)\cos(\omega t)$

$\Delta P(x=0,t) = 0 \quad \forall t$

$\Delta P(x=L,t) = 0 \quad \forall t$

$kL = n\pi$

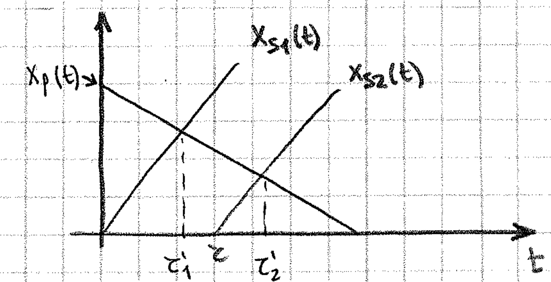
$$k_n = \frac{n\pi}{L} = \frac{\omega_n}{v_s} = \frac{2\pi f_n}{v_s} \rightarrow \boxed{f_n = \frac{n v_s}{2L}}$$

$$v_s = \frac{\omega_n}{k_n}$$

ii) $n = 1 \rightarrow f = 440$ Hz $\rightarrow \boxed{L = 39\text{ cm}}$

iii) $f_n \leq 20$ kHz $\rightarrow n \leq 45 \rightarrow$ 44 nodos en el interior

Parte B



$X_t = 0 \quad \forall t$

$X_p = X_{p0} - v_p t$

$X_{S1}(t) = v_s t$

$X_{S2}(t) = v_s(t - \tau)$

$\tau'_1: \quad X_p(\tau'_1) = X_{p0} - v_p \tau'_1$

$X_{S1}(\tau'_1) = v_s \tau'_1$

$X_p(\tau'_1) = X_{S1}(\tau'_1)$

$$\tau'_1 = \frac{X_{p0}}{v_s + v_p}$$

$\tau'_2: \quad X_p(\tau'_2) = X_{p0} - v_p \tau'_2$

$X_{S2}(\tau'_2) = v_s(\tau'_2 - \tau)$

$$\tau'_2 = \frac{X_{p0} + v_s \tau}{v_s + v_p}$$

$$\tau' = \tau'_2 - \tau'_1 = \frac{v_s \tau}{v_s + v_p}$$

$$\frac{1}{f'} = \frac{1}{f}\frac{v_s}{(v_s + v_p)} \rightarrow f' = (1 + v_p/v_s) f \quad (i)$$

$f' = 452$ Hz, $f = 440$ Hz $\rightarrow \boxed{v_p = 9{,}35\text{ m/s}}$ (ii)

Parte C

TUBO ABIERTO - CERRADO $\quad v'_s = 972$ m/s

$$f_n = \frac{n v'_s}{4L'} \rightarrow L'_1 = 0{,}538\text{ m}, \quad L'_2 = 0{,}568\text{ m}$$

$f_{B_1} = f_2 - f_1 = 12$ Hz $\rightarrow f_2 = 452$ Hz

$f_{B_2} = f_1 - f_2 = 12$ Hz $\rightarrow f_2 = 428$ Hz