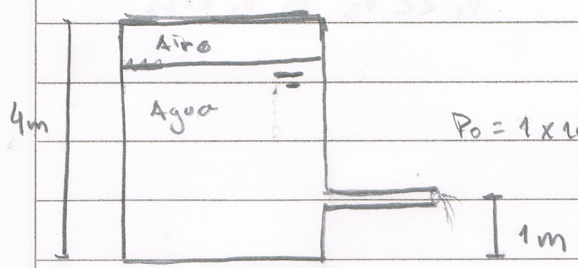
Problema 1

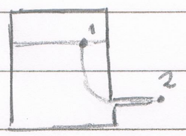


$P_0 = 1 \times 10^5\, Pa$

$\Rightarrow$ Si $h = 3{,}5\ m \Rightarrow P = 4{,}20 \times 10^5\, Pa$

Aire $\Rightarrow T = cte$

a)



$$P_1 + \rho g h_1 + \rho \frac{v_1^2}{2} = P_2 + \rho g h_2 + \rho \frac{v_2^2}{2}$$

$h_1 = 3{,}5\ m \qquad h_2 = 1m \qquad P_1 = 4{,}20 \times 10^5\, Pa$

$P_2 = 1{,}0 \times 10^5\, Pa$

como $A_1 >> A_2 \Rightarrow v_1 << v_2$

$\Rightarrow v_1^2 << v_2^2$

$$\Rightarrow (P_1 - P_2) + \rho g (h_1 - h_2) = \rho \frac{v_2^2}{2} \qquad \Rightarrow \qquad v_2 = \sqrt{2\left[\frac{(P_1 - P_2)}{\rho} + g(h_1 - h_2)\right]}$$

$\rho = 1000 kg/m^2$

$$v_2 = \sqrt{2\left[(420 - 100) + 9{,}8(3{,}5 - 1)\right]}$$

$$\Rightarrow \boxed{v_2 = 26{,}25\ m/s}$$

b) v_2 si $h = 3m$ o si $h = 2m$

$\Rightarrow$ Aire $T = cte \Rightarrow PV = cte \qquad V = A_1 \times (4m - h)$

$\Rightarrow P . A_1 (4 - h) = cte \qquad \Rightarrow P(4-h) = C$

si $h = 3{,}5 \Rightarrow P = 4{,}20 \times 10^5\, Pa \qquad \Rightarrow 4{,}20 \times 10^5\, Pa\,(4 - 3{,}5)m = C$

$\Rightarrow C = 2{,}10 \times 10^5\, Pa.m$

$\Rightarrow$ Si $h = 3m \Rightarrow P(4-3)m = 2{,}10 \times 10^5\, Pa.m$

$\Rightarrow P = 2{,}10 \times 10^5\, Pa$

Si $h = 2m \Rightarrow P(4-2)m = 2{,}10 \times 10^5\, Pa.m \Rightarrow \boxed{P = 1{,}05 \times 10^5\, Pa}$

Si $h = 3$ m $\quad P = 2{,}10 \times 10^5$ Pa

$\Rightarrow \quad P_1 + \rho g h_1 + \rho \frac{v_1^2}{2} = P_2 + \rho g h_2 + \rho \frac{v_2^2}{2} \qquad v_1 << v_2 \Rightarrow v_1^2 << v_2^2$

$\Rightarrow \quad v_2 = \sqrt{2\frac{(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)}$

$\boxed{h=3} \quad \Rightarrow \quad v_2 = \sqrt{2\left[110 + 9{,}8 \times 2\right]} = \boxed{16{,}1 \text{ m/s}}$

$\boxed{h=2} \quad \Rightarrow \quad v_2 = \sqrt{2\left[5 + 9{,}8 \times 1\right]} = \boxed{5{,}44 \text{ m/s}}$

c) h para que se detenga.

$\Rightarrow$ si $v = 0 \Rightarrow$ hidrostática.

$\Rightarrow \quad P_1 + \rho g h_1 = P_2 + \rho g h_2$

$P_1(4 - h_1) = C = 2{,}10 \times 10^5$ Pa

$\frac{2{,}10 \times 10^5}{(4-h)} + 9800\, h = 1{,}0 \times 10^5 + 9800$

$\Rightarrow \frac{210}{(4-h)} + 9{,}8h = 109{,}8 \quad \Rightarrow \quad 210 + 9{,}8h(4-h) = 109{,}8(4-h)$

$\Rightarrow \frac{210}{9{,}8} + 4h - h^2 = -\frac{109{,}8}{9{,}8}h + \frac{4 \times 109{,}8}{9{,}8}$

$\Rightarrow \quad h^2 - 4h - \frac{109{,}8}{9{,}8}h + \frac{4 \times 109{,}8 - 210}{9{,}8} = 0$

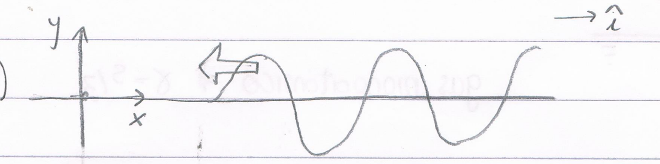
$\Rightarrow h^2 - 15{,}2h + 23{,}4 = 0 \qquad h = \frac{15{,}2 \pm \sqrt{(15{,}2)^2 - 4 \times 23{,}4}}{2}$

$h = \frac{15{,}2 \pm 11{,}73}{2} \quad \Rightarrow \quad h = \begin{cases} \rightarrow 1{,}74 \text{ m} \\ \rightarrow 13{,}5 \text{ m} \quad \times \end{cases} \quad \Rightarrow \boxed{h = 1{,}74 \text{ m}}$

$\hookrightarrow > 4$!

Problema 2)

a)



$A = 7{,}5 \times 10^{-4}\, m$

$f = 440\ Hz$

$v = 143{,}0\ m/s$

$y_1(x,t) = A\,sen(Kx + \omega t + \Phi_1)$ onda original

$y_2(x,t) = -A\,sen(Kx - \omega t + \Phi_2)$ onda reflejada.

Como en $x=0$ hay un extremo fijo $\Rightarrow y_1(x=0,t) + y_2(x=0,t) = 0\ \forall t$

$$y_T(x,t) = A\,sen(Kx + \omega t + \Phi_1) - A\,sen(Kx - \omega t + \Phi_2)$$
$$= A\left[sen(Kx + \omega t + \Phi_1) + sen(-Kx + \omega t - \Phi_2)\right]$$
$$= 2A\,sen\left(\omega t + \frac{\Phi_1 - \Phi_2}{2}\right)\cdot \cos\left(Kx + \frac{\Phi_1 + \Phi_2}{2}\right)$$

$$y_T(x=0,t) = 0 \Rightarrow \cos\left(\frac{\Phi_1 + \Phi_2}{2}\right) = 0 \Rightarrow \Phi_1 + \Phi_2 = (2n-1)\pi$$

$$\Rightarrow y_T(x,t) = 2A\,sen\left(\omega t + \frac{\Phi_1 - \Phi_2}{2}\right)\cos\left(Kx + \frac{(2n-1)\pi}{2}\right) \quad n \in \mathbb{N}^*$$

con $\omega = 2\pi f = 2764{,}6\ rad/s$

$K = \frac{\omega}{v} = 19{,}33\ rad/m$

b) NODOS $\Rightarrow y_T(x,t) = 0\ \forall t \Rightarrow Kx + \frac{(2n-1)\pi}{2} = \frac{(2m-1)\pi}{2}$ $\quad m \in \mathbb{N}^*$

$$\Rightarrow Kx = (m-n)\pi \Rightarrow x = \frac{(m-n)\pi}{K} \geq 0$$

Si $m-n = 0 \Rightarrow x_0 = 0\,m$

$m-n = 1 \Rightarrow x_1 = 0{,}163\,m$

$m-n = 2 \Rightarrow x_2 = 0{,}325\,m$

$m-n = 3 \Rightarrow x_3 = 0{,}488\,m$

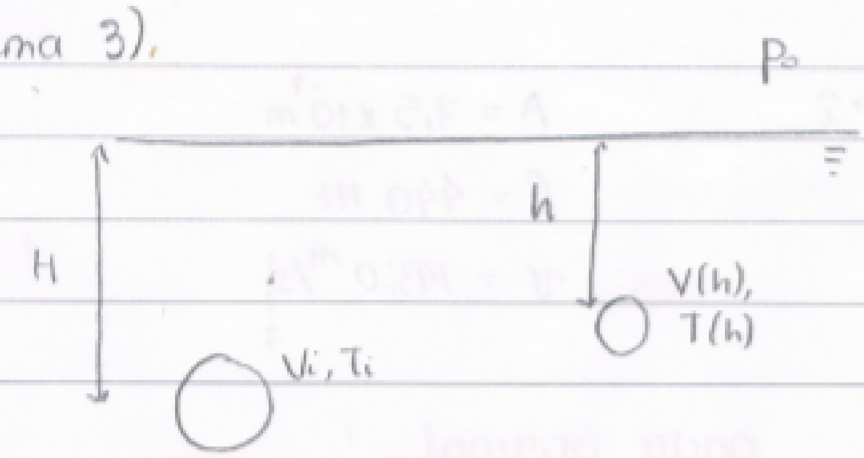
c) $y_{T\,máx} = 2A = 1{,}5 \times 10^{-3}\,m$

$$u_T(x,t) = \frac{\partial y_T(x,t)}{\partial t} = 2A\omega\cos\left(\omega t + \frac{\Phi_1 - \Phi_2}{2}\right)\cos\left(Kx + \frac{(2n-1)\pi}{2}\right)$$

$$\Rightarrow u_{T\,máx} = 2A\omega = 4{,}15\ m/s$$

$$a_T(x,t) = \frac{\partial^2 y_T}{\partial t^2} \Rightarrow a_{T\,máx} = 2A\omega^2 = 1{,}15 \times 10^4\ m^2/s$$

Problema 3).



gas monoatómico $\Rightarrow \gamma = 5/3$

a)

$Q = 0 \Rightarrow$ proceso adiabático $\Rightarrow pV^{\gamma}$ cte

por hidrostática: $p(h) = p_0 + \rho g h$

$\Rightarrow (p_0 + \rho g h) \cdot V(h)^{\gamma} = (p_0 + \rho g H) V_i^{\gamma} \Rightarrow V(h) = V_i \left(\dfrac{p_0 + \rho g H}{p_0 + \rho g h} \right)^{3/5}$

b) $\left. \begin{array}{l} pV^{\gamma} \text{ cte.} \\ p = \dfrac{nRT}{V} \end{array} \right\}$ $T \cdot V^{\gamma - 1}$ cte $\Rightarrow T_i V_i^{2/3} = T(h) V(h)^{2/3}$

$\Rightarrow T(h) = T_i \left(\dfrac{p_0 + \rho g h}{p_0 + \rho g H} \right)^{2/5}$

c) $\Delta U = n c_v \Delta T = \dfrac{3}{2} nR (T - T_i) = \dfrac{3}{2} \dfrac{p_i V_i}{T_i} (T - T_i)$

$\Rightarrow \Delta U = \dfrac{3}{2} (p_0 + \rho g H) V_i \left[\left(\dfrac{p_0 + \rho g h}{p_0 + \rho g H} \right)^{2/5} - 1 \right]$