stresses and other mechanical treatment; such cores require careful heat treatment, which can be more easily applied to strip laminations and to strip-wound cores in their finished condition than to other forms of core, see p. 110. The winding of ring cores with uniformly distributed toroidal coils presents difficulty since it must be done by hand; consequently it is usually a troublesome, slow, and rather expensive process, though machinery has been invented to enable the winding to be done more quickly and cheaply.

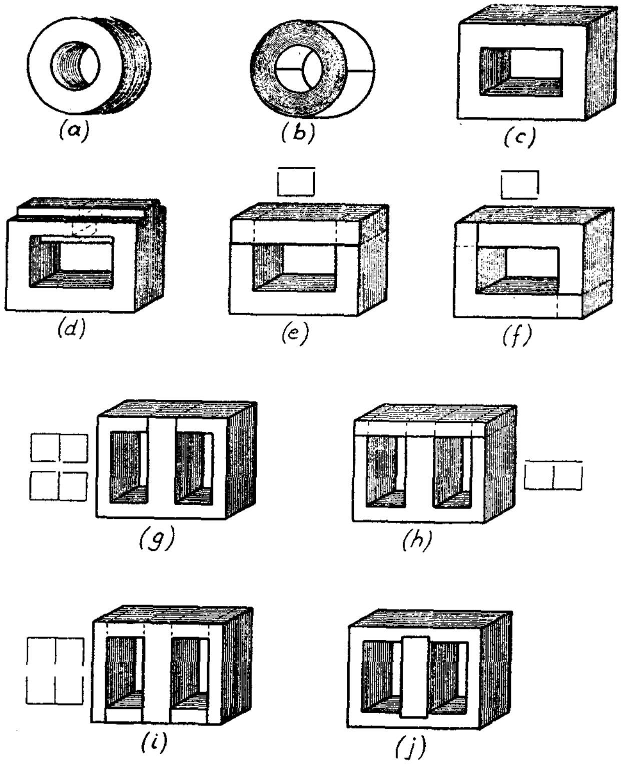


FIG. 39. TYPES OF CURRENT TRANSFORMER CORES

Rectangular cores are used in practically all cases where the primary winding consists of more than a single turn; they may be of the core-type or the shell-type, as in power transformer construction. The jointless core-type, Fig. 39 (*c*), is preferred in the best quality transformers; the limb upon which

the coils are accommodated is sometimes made of cruciform section by the use of two sizes of plates, Fig. 39 (*d*). As in all jointless cores, winding must be done by hand. Winding is made easier and the use of former-wound coils is made possible by providing a removable yoke, Fig. 39 (*e*), or by using L-shaped stampings, Fig. 39 (*f*). Sometimes the core is built up of simple limb and yoke strips, but though this method is very simple from the punching point of view it has the disadvantage of introducing four joints instead of two into the magnetic circuit. Shell-type cores are made up in a variety of ways, the simplest, and magnetically the best, being the jointless core with or without a cruciform central limb; this again necessitates hand winding. Winding is facilitated, especially in h.v. transformers with the windings on porcelain or other spools, by several devices. In Fig. 39 (*g*) the central limb is a tongue, enabling the stamping to be threaded through the coils; in Fig. 39 (*h*) a single E-shaped stamping with a removable yoke, while in Fig. 39 (*i*) two sets of similar E-shaped stampings are used. (The relative positions of successive plates are shown by the small line sketches). The cores are firmly bolted together after assembly. Fig. 39 (*j*) shows a type of construction used in Germany; the outer jointless yokes and the inner limb are independently assembled, the latter complete with its windings being pressed into the slots provided.

Although all due care may have been exercised in the actual construction of the core it is possible to reduce I_m still further by using one or both of the following artifices: (i) Operation of the core at a flux-density corresponding with maximum permeability. (ii) Using a core material in which the permeability at low densities is higher than in silicon-iron alloy plates. These subjects will be discussed on pp. 95 and 103.

Wirz[*] has given an interesting mathematical treatment of the effects of I_m and I_w, reaching conclusions similar to those obtained in Chapter II. Writing $E_p = E_s/K_T$ and putting

$$I_m = bE_p,\ I_w = gE_p \text{ and } b/g = I_m/I_w = \cot \xi,$$

where g is the excitation conductance and b the excitation susceptance, we find after some reductions

$$K_c \doteqdot K_T[1 + R'_s g(1 + (b/g) \tan \phi_s)],$$
$$\tan \beta \doteqdot R'_s g[(b/g) - \tan \phi_s],$$

---

[*] E. Wirz, "Untersuchungen über die möglichen Fehlerquellen bei Stromwandlern," *Arch. f. Elekt.*, vol. 6, pp. 23–72 (1918).

where $R'_s = R_s/K_T^2$ is the total secondary resistance referred to the primary side. Let at_m be the magnetizing ampere-turns per cm. (r.m.s.) required for the value of B_{max} used in the core; then if the core is without air-gaps and of constant cross-section,

$$I_m = l_i at_m/T_p \text{ and } I_w = A_i l_i w_i 10^8/4{\cdot}44 T_p f B_{max} A_i$$

using the notation of p. 71. Hence

$$b = l_i at_m 10^8/4{\cdot}44 T_p^2 f B_{max} A_i \text{ and } g = l_i w_i 10^{16}/A_i(4{\cdot}44 T_p f B_{max})^2,$$

which enables these quantities to be calculated from the $at_m - B_{max}$ and $w_i - B_{max}$ characteristic curves for the iron. For given transformers Wirz plots b, g and b/g as functions of B_{max}. The b-curve has a minimum value near the knee of the saturation curve, i.e. in the region of maximum permeability, showing the desirability of working the iron near that point if possible. The g-curve is of hyperbolic shape, remaining substantially constant from $B_{max} = 3\,000$ and upwards. The b/g-curve is similar to a saturation curve, b/g decreasing very rapidly at low inductions. Hence, if the core is worked at too low a flux density K_c will tend to vary widely with I_s; Wirz recommends for this reason that B_{max} should exceed 1 000. The curves are plotted for various secondary burdens, and calculations of K_c and β are made for transformers working under different conditions; in addition the effect of butt joints in increasing b is strikingly shown.

The question of the proportions of the core is one of some interest, since the weights of iron and copper, the cost of active material, the accuracy of the transformer and its output are all dependent thereon. The appropriate proportions to be used in any given case are somewhat difficult to state in general terms, since so many factors not capable of simple expression have to be taken into account, such as accommodating the transformer in a "line" for economic production, meeting guarantees for mechanical strength, and so on. A sketch of the simple process of design is given on p. 111, and a preliminary design of core would be modified until all the necessary conditions for accuracy, cost of materials, economic manufacture, and the like were satisfactorily fulfilled; a graphical method such as that of Fleischhauer (see p. 58) is of very great assistance in enabling the influence of individual factors to be easily determined. Certain writers have attacked some of the simpler aspects of the problem from a theoretical standpoint. Krutsch* has shown that the ratio error and phase-angle are least when a quantity defined as the "shape factor" is a minimum, this being $(ll_i/AA_i)v^{2/3}$; where l_i and A_i are the mean path length and cross-sectional area of the core, A is the total section of primary and secondary copper with

---

* J. Krutsch, "Der Einfluss der Gestalt der Messwandler auf ihre Fehler," *Arch. f. Elekt.*, vol. 24, pp. 593–611 (1930).

insulation and clearance spaces, l is the mean length of turn of this total section, and v is the volume of a rectangular parallelepiped enclosing the transformer. He applies this to several types of core; for example, in the plain core of Fig. 39 (c) with concentric cylindrical coils wound on one limb, if a be the width of the square section of the limbs, the breadth of the window should be $4a$ and its depth $0{\cdot}9a$. If the windings occupy both of the long limbs, then the proportions of the window are $4{\cdot}1a$ wide by $1{\cdot}5a$ deep. Billig* has worked out a similar problem for a ring core with toroidal secondary and a bar primary. He proves that the transformer errors are proportional to $P^{\alpha} l_i^{1+\alpha}$, where α is about 2/3 for normal transformers and 1 for a transformer with additional magnetization (see Section 8), and P is the sum of the output in volt-amperes plus the secondary copper loss. By finding the conditions to make this quantity a minimum Billig proves that for a fixed core weight W_i kilogrammes, the radial breadth of the ring is given by

$$b_1 = \sqrt{[AT\,.\,W_i/300\sqrt{(VA\,.\,W_c)}]}\text{ cm.},$$

where AT = full rated ampere-turns, VA = burden in volt-amperes, and the copper weight in kilogrammes is

$$W_c = (0{\cdot}23\,AT/\delta)\sqrt{(W_i/D)},$$

where δ is the current density in the secondary winding (150 to 200 amperes per sq. cm.) and D is the inner diameter of ring in cm.

The following numerical example illustrates Billig's theory. A transformer with additional magnetization has $AT = 1\,200$, $VA = 40$, $D = 25$ cm., $W_i = 30$ kg., $\delta = 200$ A/cm.², errors proportional to Pl^2_i. Then $W_c = 1{\cdot}51$ kg. and $b_1 = 4$ cm., the axial depth of the ring being 11 cm. Fig. 40 shows the way l_i, P, Pl_i^2 and VA vary with the ratio of depth to radial breadth of the ring in this example.

8. **The use of additional magnetization.** The normal value of the maximum flux-density in a silicon-iron core lies between 500 and 1 500 lines per sq. cm., in order that the specific loss shall be minimized and I_w kept small. The maximum permeability of the material occurs, however, at a density of 4 000 to 6 000 lines per sq. cm. A mere increase of the working density to this figure, however, will only result in an increase both in I_m and in I_w; consequently, an improvement of the working

---

* E. Billig, "Die günstigste Form von Stromwandlerkernen," *Bull. Schw. Elekt. Verein*, vol. 24, pp. 70–72 (1933).

permeability can only be secured by some artifice designed to put the iron into a better magnetic condition.

The voltage E_s required to circulate a given rated current in the secondary circuit of a current transformer necessitates a core flux-density which varies harmonically between the limits $\pm B_{max}$, where $B_{max}A_i$ is the flux inducing E_s. This flux-density is set up by the magnetizing resultant of the primary and secondary ampere-turns, namely, I_mT_p, which varies between the limits $\pm (\sqrt{2})I_mT_p$. Neglecting hysteresis, the magnetization and permeability curves are drawn in Fig. 41 (*a*); the range of variation *ab* of the magnetizing ampere-turns for a symmetrical variation of $\pm B_{max}$ about the origin *O* is shown in the diagram.

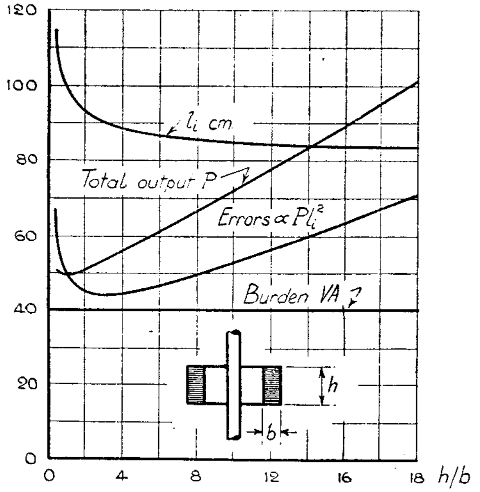


FIG. 40. THE PROPORTIONS OF RING CORES FOR MINIMUM ERRORS

Now assume that by means of an additional tertiary winding an auxiliary flux is superposed upon the core and, as a preliminary, suppose this auxiliary field to be steady. Then the working variation $\pm B_{max}$ takes place about some point, such as *A* or *B*, higher up the magnetization curve. As the auxiliary flux is increased from zero the range of excitation demanded from the transformer magnetizing ampere-turns gradually decreases to a minimum, as at *cd* when the working cycle is performed in the neighbourhood of maximum permeability, and thereafter increases, at first slowly and then rapidly as at

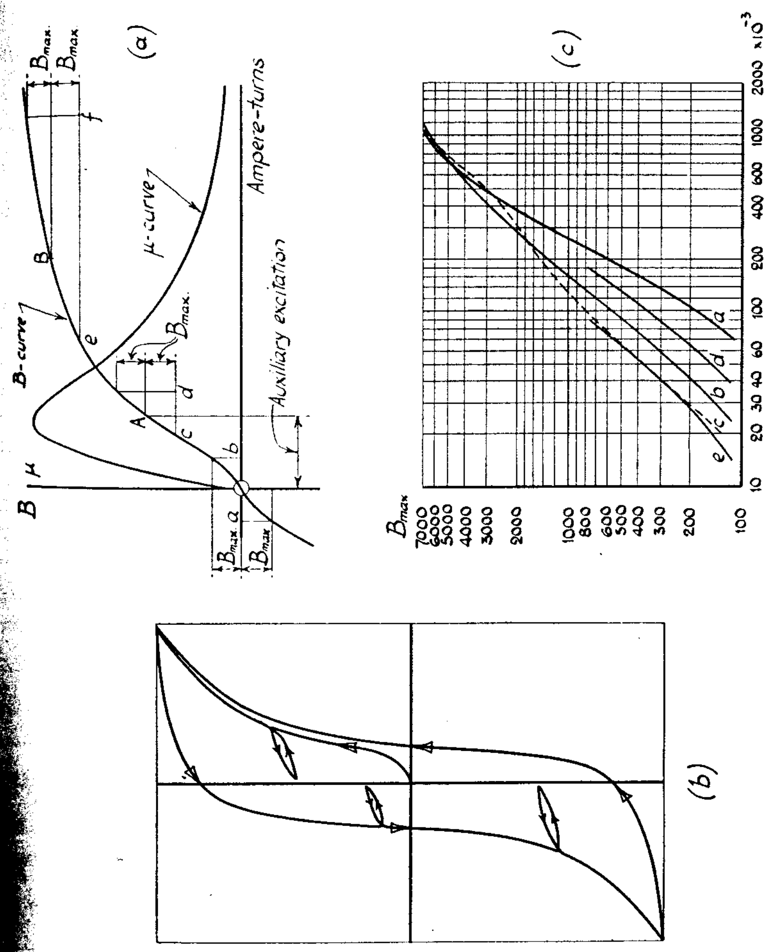


FIG. 41. ADDITIONAL MAGNETIZATION

ef. Thus, assuming no hysteresis, moving the working flux variation up to the region of maximum μ by the addition of a steady flux would reduce the value of I_m for a given range of variation $\pm B_{max}$.

Unfortunately, this desirable effect of a steady additional magnetization is entirely vitiated by the influence of hysteresis.* Referring to Fig. 41 (*b*), the hysteresis loop is shown for an additional d.c. magnetization applied in the order indicated by the open arrow-heads. Stopping the d.c. magnetization at any point on the loop, the working alternating flux follows the small loops in the way originally described by Ewing, who says, "Every loop in these diagrams shows that whenever the process of altering the magnetic force is reversed from a process of increment to a process of decrement, or vice versa, the magnetism begins to change very slowly relatively to the change of H, no matter how fast it may have been changing (in the opposite direction) immediately before." Again even at the steepest part of the main loop if we "begin to remove the magnetic force, the gradient of the new curve may be some 70 or 80 times less steep than that of the curve from which it springs." In other words, the effective permeability of the a.c. magnetization, represented by the average slope of the small loop, is always less than the permeability at the point on the d.c. loop giving the auxiliary magnetization. Moreover, it is well-known (see Gans, loc. cit.) that the a.c. permeability with such a d.c. auxiliary field is smaller than the initial permeability for the material and decreases as the auxiliary field is increased.

If, however, the additional magnetization is produced by an alternating flux of the same frequency as the working flux of the transformer, the phenomenon is quite different.† Fig. 41 (*c*) shows in logarithmic co-ordinates a series of a.c. magnetization curves, taken from Vahl's paper, for 4 per cent silicon-iron

---

* See J. A. Ewing, *Magnetic Induction in Iron and Other Metals*, Ch. V (1892). Also R. Gans, "Magnetisch Korrespondierende Zustände," *Phys. Zeits.*, vol. 11, pp. 988–991 (1910); "Die Gleichung der Kurve der reversiblen Suszeptibilität," *idem.*, vol. 12, pp. 1053–1054 (1911).

† For a full discussion of the problem, see G. Stein, *Zeits. f. tech. Phys.*, vol. 14, pp. 495–499 (1933); "Iron biased by a.c.—Magnetic properties and their importance in the development of current transformer technique," *Elecn.*, vol. 112, pp. 391–393 (1934). J. Goldstein, "Die Wechselstromvormagnetisierung im Stromwandlerbau und ihre auschauliche Wirkungsweise," *Bull. Schw. Elekt. Verein*, vol. 25, pp. 229–232 (1934). H. Vahl, "Vor- und Gegenmagnetisierung bei den Stromwandlern," *V.D.E. Fachberichte*, pp. 38–41 (1934).

plates, giving B_{max} as a function of the r.m.s. ampere-turns per cm. with various conditions of additional a.c. magnetization. Curve *a* is the simple a.c. magnetization curve for the material. The effect of additional a.c. magnetization is to shift the curve to the left, i.e. for a given density to reduce the working ampere-turns per cm. or, in other words, to increase the working permeability. The degree of displacement depends upon the amount of the additional superposed ampere-turns and their phase relationship, ϕ_a, to the secondary voltage E_s. The shift is greatest for lower densities and falls off gradually until at a density of 5 000 any amount of premagnetization produces no improvement in the working permeability, and above this density may even reduce it. For the steel to which Fig. 41 (*c*) refers, an auxiliary magnetization of about 0·32 ampere-turns per cm. produces maximum working permeability. With this premagnetization and $\phi_a = 0°$ curve *b* is obtained; curve *c* relates to $\phi_a = 45°$, showing the importance of the phase adjustment. Curve *d* is for an auxiliary magnetization of 0·16 ampere-turns per cm. with $\phi_a = 0$ and a similar curve is obtained for a corresponding increase over 0·32 ampere-turns per cm. Finally, curve *e* is obtained with the most favourable adjustment of the premagnetization both in magnitude and phase. The auxiliary source must be sufficiently constant and the correct adjustment of its phase has a by no means negligible effect on the working permeability, as both Vahl and Stein have shown. It is interesting to observe that with B_{max} about 500 the working ampere-turns per cm. for silicon-iron are reduced from 180 without to 63 with premagnetization, resulting in a corresponding reduction in the errors for the same density or in weight for the same errors in the two cases.

Premagnetization can be obtained in two ways, (i) by separate-excitation, as in the transformers of Iliovici and of Wellings and Mayo; and (ii) by self-excitation, as introduced by Goldstein and Vahl. These two types will now be considered in detail.

The principle of separate-excitation was introduced by Iliovici* as early as 1914 in a series of transformers made by the Compagnie pour la Fabrication des Compteurs, but without any provision being made for the phase adjustment of the

---

* A. Iliovici, "Transformateurs d'intensité compoundés," *Bull. Soc. Franç. des Elecns.*, vol. 3, pp. 55–70 (1923); "Tendences actuelles dans la construction et l'utilisation des transformateurs de mesure," *idem.*, vol. 10, pp. 1191–1215 (1930); "Transformateurs de mesure. Progrés constructifs récents. Discussion des Régles de Normalisation," *idem.*, vol. 2, pp. 1117–1131 (1932).

superposed flux. The necessary compounding is supplied by an auxiliary winding excited at a constant voltage and frequency from the supply network. Since the operation of the transformer must depend only upon the interaction of the primary upon the secondary winding via the working flux, the additional flux must induce no e.m.f. in either of these windings; in other words, the auxiliary winding must have no mutual inductance with respect to the primary and secondary windings. Fig. 42 (*a*) shows an arrangement, introduced about 1914, for core- or shell-type cores[*] in which the limb carrying the primary and secondary coils is divided into two equal branches traversed in opposite directions by the compounding flux, as shown by the arrows. This design lends itself readily to the ring-type construction for bar-type transformers, as shown in Fig. 42 (*b*) where only the auxiliary windings are indicated; the holes ensure the appropriate division of the compounding flux. The primary is a single conductor passed through the central opening and the secondary is uniformly wound over the whole ring. A further design is shown in Fig. 42 (*c*) and is now usually preferred to Fig. 42 (*a*). In all cases the auxiliary winding is supplied with current at 100 volts from the secondary of a small voltage transformer and consumes only about 4 watts. Excellent results are obtained.

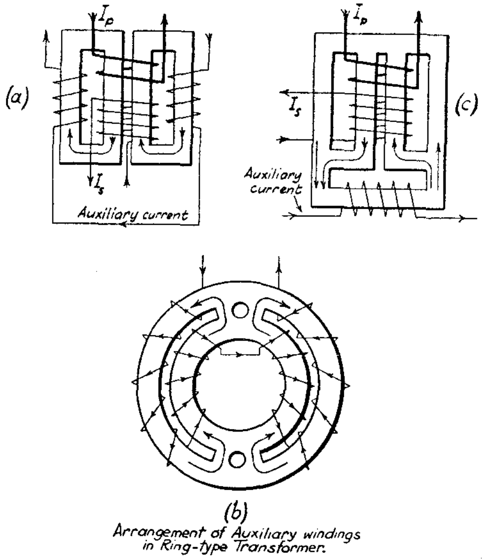


FIG. 42. ILIOVICI'S COMPOUNDED CURRENT TRANSFORMERS

---

[*] Fig. 42 (*a*) shows the actual arrangement of a shell-type transformer. It gives also a diagrammatic representation of a core-type transformer, the two equal packets of plates being supposed laid flat in the plane of the paper, one to the left and one to the right, instead of standing side by side.

Wellings and Mayo (loc. cit. on p. 76) obtain the auxiliary current from an additional small current transformer in the way shown in Fig. 43 (*a*) applied to a bar-type transformer. The main core is divided into two equal parts which are subjected to additional magnetization in opposite directions by the crossed winding energized by the auxiliary core, the latter and the main cores being linked in common upon the primary conductor. By this simple device the compounding flux induces no e.m.f. in the secondary winding and is of a sufficient magnitude to bring the iron of the main cores into the region of maximum working permeability. Bar-type transformers for primary currents as low as 100 amperes have been constructed on this principle with silicon-iron cores, in which additional magnetization and the compensating devices described on p. 85 give surprisingly high accuracy; with nickel-iron cores adequate accuracy can be secured with still lower currents. The following tables give typical figures.

Bar-type transformer with Stalloy core. Rated primary current 200 amperes. Burden 50 volt-amperes.

Primary current, per cent .	100	20	10
ε_c per cent	+ 0·7	+ 0·4	+ 0·7
β minutes	45	36	30

Bar-type transformer with Mumetal core. Rated primary current 50 amperes. Burden 10 volt-amperes.

Primary current, per cent	125	100	20	10	5
ε_c per cent . . .	+ 0·7	+ 0·6	+ 0·9	+ 0·9	+ 0·9
β minutes . . .	37	40	47	38	35

In the transformers manufactured by the A.E.G.[*] the additional magnetization is ingeniously derived from the secondary

---

[*] J. Goldstein, "Die neuste Entwicklung im Stromwandlerbau," *Elekt. Zeits.*, vol. 53, pp. 377–380, 428–431, 503–505 (1932); *Bull. Schw. Elekt. Verein*, vol. 24, pp. 100–104 (1933); "Neue Wege im Stromwandlerbau," *E.u.M.*, vol. 51, pp. 489–492 (1933).

current itself by an unbalancing of magneto-motive forces, resulting in unequal magnetization of two cores arranged as in Fig. 43 (*b*). The bulk of the secondary turns T_1 is put on the middle limb, while unequal portions T_2, T_3 are wound upon the yokes. The values of T_1, T_2 and T_3 are chosen so as to

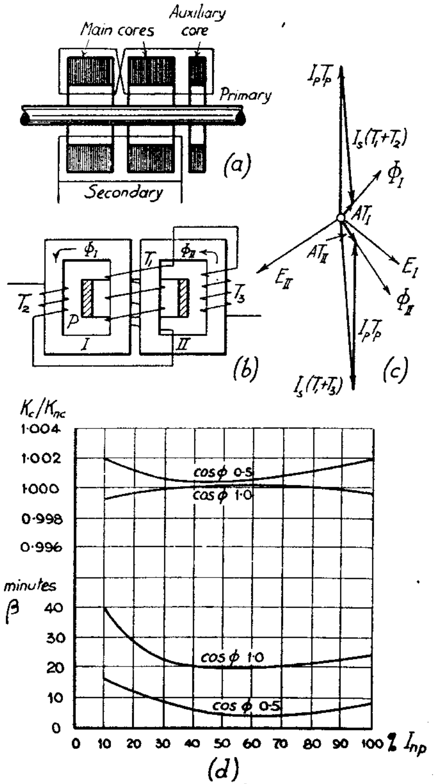


FIG. 43. WELLINGS AND MAYO'S, AND GOLDSTEIN'S COMPOUNDED CURRENT TRANSFORMERS

produce the most favourable effect. On Core I the primar ampere-turns preponderate over the secondary ampere-turns while the secondary is preponderant on Core II; the tw fluxes in the middle limbs are, therefore, unequal and approximately in opposition. In vector notation,

$$\begin{aligned} AT_{\text{I}} &= \boldsymbol{i}_pT_p + \boldsymbol{i}_s(T_1 + T_2), \\ AT_{\text{II}} &= \boldsymbol{i}_s(T_1 + T_3) + \boldsymbol{i}_pT_p, \end{aligned}$$

for the two cores, these equations being illustrated by the vector diagram in Fig. 43 (*c*). The fluxes Φ_{I} and Φ_{II} induce voltages E_{I} and E_{II}, their sum being the total secondary voltage E_s circulating I_s. Regarding Cores I and II as separate transformers, I_s leads on E_{II} and lags on E_{I}; hence, the ratio for Core I is too low (ε_c positive) and for Core II is too high (ε_c negative). The combination can be adjusted so that the overall ratio at a given load is correct. Moreover, Core I is worked, at rated current, a little above the point of maximum permeability and Core II at a density about 20 per cent below it; the value of I_m will be in the neighbourhood of its minimum and, as the resultant permeability curve for the two cores is in these circumstances nearly flat, will change linearly with I_s; hence ε_c and β will be both small and constant. The curves in Fig. 43 (*d*) show that the accuracy attainable in a large transformer for 125 kV lies within the limits for class 0·5, the burden being 100 VA.

9. **The application of nickel-iron alloys.** In Section 7 it has been pointed out that a possible way to improve the characteristics of a current transformer would be to substitute some better core material for the silicon-iron plates that are usually employed.* Though the ordinary 4 per cent alloy has high permeability and low losses, the values of I_m and I_w are still sufficient to cause quite appreciable ratio and phase errors. If, however, the desired flux could be set up in the core with a lower number of ampere-turns and with the production of smaller iron losses, then either the accuracy of a current transformer for a given output could be materially improved or, retaining the same limits of accuracy as before, the output could be increased. The discovery that the desired properties are possessed by alloys of iron and nickel,† with or without additional constituents, and the application of these alloys to current transformer core construction, is the most important recent development in the practice of electrical measurement;

---

* For an improved silicon-iron alloy produced by the American Rolling Mill Co., see "Tran-Cor. Hochsiliziertes Spezialblech für Transformatoren und Messwandler," *Arch. f. tech. Mess.*, Z911–3, Apr. (1933). This material has a μ about double that of ordinary silicon steel up to $B_{max} = 100$ and has very low total losses. An improved German material of the same type is made by the Eisen und Hüttenwerke A.G. of Bochum, see "EHW Stromwandlerblech," *Arch. f. Tech. Mess.*, Z911–4, Apr. (1934).

† For a complete summary of the physical, metallurgical and magnetic properties of modern ferrous alloys used for electrotechnical purposes, the reader should consult A. Kussmann, "Stand der Forschung und Entwicklung auf dem Gebiet der ferromagnetischen Werkstoffe," *Arch. f. Elekt.*, vol. 297–332 (1935).

results have been obtained that can rightly be described as revolutionary.

Alloys of nickel and iron have long been known in nature, since they are found in certain meteorites; their first preparation by synthesis appears to be due to Faraday and Stodart[*] in the year 1820, in the course of a now classic research upon the improvement of steel. Most of the earlier work on the alloys had reference to their mechanical properties, but in 1889 J. Hopkinson[†] tested the magnetic properties of ferrous alloys containing various proportions of nickel. He found, in particular, that an alloy containing about 25 per cent of nickel was non-magnetic at air temperatures. Systematic studies of the magnetic properties of the whole series of alloys were undertaken by Burgess and Aston[‡] in 1912 and by R. A. Hadfield and B. Hopkinson[§] in 1911, the latter workers paying particular attention to the effects of strong fields and observing many interesting features. Materials of commercial quality were used. Unfortunately, the most remarkable properties of nickel-iron alloys are not obtained unless the constituents are very pure, and remain uncontaminated during the process of fusion, and the resulting alloy must be subsequently subjected to special annealing and heat-treatment. In 1913 Yensen prepared pure iron by melting electrolytic iron *in vacuo* in the high frequency electric induction furnace and found that this material had very high magnetic qualities. Applying the same metallurgical technique, Yensen investigated a large number of alloys of iron with other metals in an attempt to find a material with a higher saturation intensity than pure iron. In 1920 he[||] published an exhaustive account of the magnetic properties of the nickel-iron series and drew particular attention to the 50 per cent alloy, to which the name of "Hipernik" has been given. About the same period research was in progress in the laboratories of the American Telegraph and Telephone Co. upon the increase in the speed of working on submarine telegraph cables by the application of continuous magnetic loading. The required magnetic material must have a permeability in very small fields far exceeding that of any material hitherto known, even pure iron, and it was found that a ferro-nickel with about 78 per cent of nickel had the required properties. This alloy was described by Arnold and Elmen[¶] in 1923 and given the name "Permalloy."

Although the nickel-iron alloys were originally developed for other purposes, their higher permeability in low fields and smaller iron losses specially recommend them as improvements upon silicon-iron for current transformer cores, as pointed out by Drysdale.[*] Since the permeability is at least ten times greater than that of good silicon core steel and the specific loss about one-sixth, it follows that the exciting current will be reduced to about a tenth of its value with silicon steel, with a corresponding reduction in ratio and phase-angle error and greater perfection in the transformer.

Before discussing the individual alloys and their applications it is advantageous to examine their general properties to see how these depend upon the nickel content. Confining attention for the present to the binary alloy Ni-Fe, Yensen[†] has made tests on a series of alloys varying from 100 per cent Fe to 100 per cent Ni. Electrolytic iron and electrolytic nickel were melted in magnesia crucibles in a high-frequency furnace under a pressure of 1 to 2 mm. of mercury; specimens were prepared from the cast ingots and annealed from 900° C. in a high vacuum or in a hydrogen atmosphere. The entire elimination of impurities, particularly carbon and oxygen, is essential to the development of high magnetic qualities. The results of subsequent tests on the specimens are summarized in Fig. 44. The density of the alloy varies uniformly from 7·9 for pure iron to 8·9 for pure nickel. The resistivity attains a maximum of about 82 microhms per cm. cube with a nickel content of 30 per cent, about double that of manganin; over the range of alloys which have magnetic characteristics useful for the present purpose, i.e. those with 40 to 80 per cent of nickel, the resistivity lies between 72 and 20 microhms per cm. cube, thus ensuring low specific eddy current loss. The hysteresis loss in pure iron is very small, being about 600 ergs per c.c. per cycle at $B_{max} = 10\,000$; small amounts of nickel increase the loss very rapidly until a maximum of about 50 000 ergs per c.c. per cycle is attained in an alloy containing 25 per cent of

---

[*] M. Faraday and J. Stodart, *Phil. Mag.*, vol. 56, pp. 26–35 (1820); *Phil. Trans. Roy. Soc.*, p. 253 (1822).

[†] J. Hopkinson, "Magnetic properties of alloys of nickel and iron," *Proc. Roy. Soc. A.*, vol. 47, pp. 23–24 (1889); vol. 48, pp. 1–13 (1890).

[‡] *Metallurgical and Chemical Engineering*, vol. 8, p. 23 (1910).

[§] R. A. Hadfield and B. Hopkinson, "The magnetic properties of iron and its alloys in intense fields," *Journal I.E.E.*, vol. 46, pp. 235–284 (1911).

[||] J. D. Yensen, "Magnetic and electrical properties of iron-nickel alloys," *Trans. Amer. I.E.E.*, vol. 39, pp. 791–815 (1920).

[¶] H. D. Arnold and G. W. Elmen, "Permalloy, an alloy of remarkable magnetic properties," *Journal Frank. Inst.*, vol. 195, pp. 621–631 (1923); *Bell Syst. Tech. J.*, vol. 2, No. 3, pp. 101–111 (1923).

[*] C. V. Drysdale, "The application of high permeability alloys to current transformers," *Journal Sci. Insts.*, vol. 3, p. 58 (1925). "Progress in the design and construction of electrical instruments," *Journal Sci. Insts.*, vol. 4, pp. 177–184, 209–217, 241–251, 288–299 (1927). B. Hague, "Über die Anwendung von Nickeleisenlegierungen bei neuzeitlichen Stromwandlern," *E.u.M.*, vol. 51, pp. 208–211 (1933).

[†] J. D. Yensen, "Magnetic properties of the fifty per cent iron-nickel alloy," *Journal Frank. Inst.*, vol. 199, pp. 333–342 (1925). "Magnetism and magnetic materials," *Elect. J.*, vol. 27, pp. 214–218 (1930); see also Report 14*a* before Section 1 of the International Electrical Congress in Paris (1932), and *Arch. f. tech. Mess.*, Z911–2, Nov. (1932). G. Keinath, "Hochmagnetische Legierungen aus Nickeleisen," *Arch. f. tech. Mess.*, Z913–5, Nov. (1932).

Ni. Further addition of nickel causes the hysteresis loss to fall rapidly; between 30 and 70 per cent of Ni the loss is as low or lower than that of pure iron or silicon steel, and increases thereafter to 20 000 ergs per c.c. per cycle for pure nickel.

Although the addition of nickel has a remarkable influence upon the losses, the effect upon the magnetic properties is even more striking. Varying proportions of nickel affect (i the saturation flux density, (ii) the initial permeability, and (iii) the maximum permeability in a marked degree. Referring again to Fig. 44, a series of curves has been plotted showing the variation of B with nickel content for various constant values of H. It will first be noticed that the induction obtained for a given H is always less in the alloys than in pure iron, and

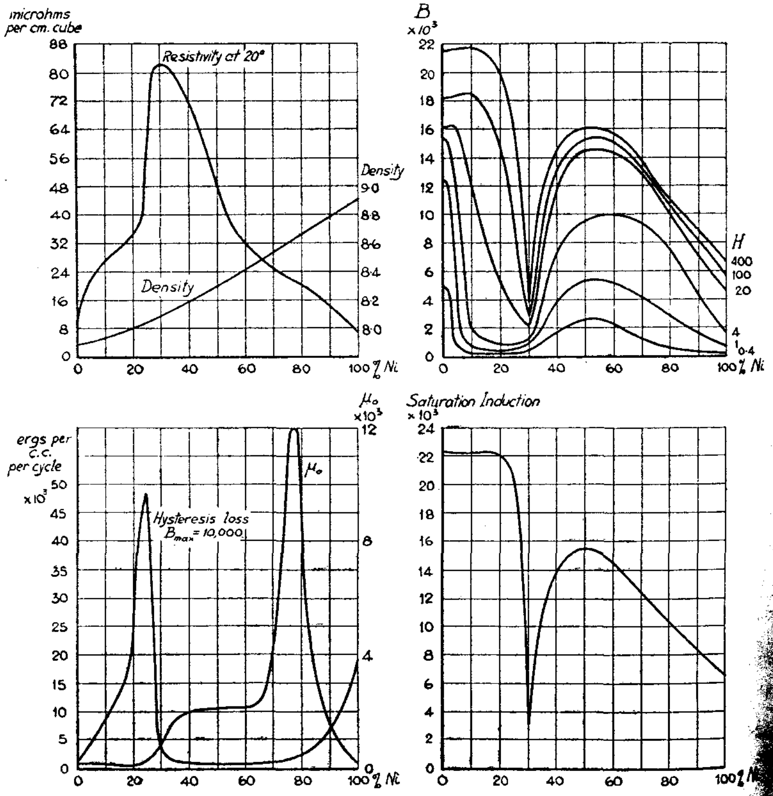


FIG. 44. PHYSICAL PROPERTIES OF THE BINARY NICKEL-IRON ALLOYS

this is also true of the saturation density, i.e. the induction toward which the B–H curve becomes asymptotic for high magnetizing forces. Hence nickel has, in general, a depressant effect upon the saturation value. As the nickel content increases from zero the value of B for a constant H falls, at first slowly and then rapidly, until with about 30 per cent of nickel a marked minimum is reached. The relatively non-magnetic quality of these alloys was observed by J. Hopkinson as long ago as 1899 (loc. cit. *ante*) and is nowadays exploited commercially in the production of non-magnetic cast irons. Further addition of nickel causes the induction and the saturation value to rise to a maximum in the 50 per cent alloy, and thereafter to fall steadily toward the characteristic for pure nickel.

Fig. 45 gives a series of magnetization curves showing B as a function of H for a number of materials, the standard of reference being vacuum-fused, electrolytic iron, which is probably the purest iron obtainable. The region of greatest practical interest for our present purpose is the lower portion of the curves comprised within the range of 0 to 5000 for B and of H from 0 to 1 c.g.s. unit. The curve for "Armco" iron represents the properties of commercially obtainable pure iron containing 99·9 per cent of Fe, prepared by the American Rolling Mill Corporation; the influence of the slight impurities on the magnetizability is very striking, as also is the improvement effected at the lower flux-densities by the addition of silicon. At higher densities silicon is disadvantageous, depressing the saturation value of commercial pure iron; the main advantage of silicon is, as is well-known, its great effect upon the resistivity and consequent reduction of the eddy current losses. The remaining curves refer to nickel-iron alloys and all show a notable improvement over silicon steel, especially at low densities. The curve marked "Hipernik" relates to the 50 per cent Ni–Fe alloys; those marked "Mumetal" and "Permalloy C" relate to alloys containing 76–79 per cent of nickel, but with additional constituents, these materials not being simple binary alloys.

It is well known* that at very low values of the magnetizing force H, the induction B may be approximately expressed by a relation of the form

$$B = \mu_0 H + \nu H^2,$$

or

$$\mu = \mu_0 + \nu H.$$

---

* See J. A. Ewing, *Magnetic Induction in Iron and Other Metals*, Chapter VI, (1892).

In the neighbourhood of $H = 0$ the magnetization curve is a straight line (the term in H^2 then being negligible) of slope μ_0; μ_0 is called the "initial permeability" of the material. As H increases from zero the curve assumes the form of a parabola having an initial inclination of μ_0 to the axis of H. Since in current transformers we are interested particularly in the region of low magnetizing forces, the importance of high initial permeability will be appreciated. Referring again to Fig. 44, the initial permeability is plotted from results given by Arnold and Elmen* (loc. cit.) as a function of nickel content. The value of μ_0 for pure iron† is about 200, addition of nickel causing a gradual fall to a lowest value in an alloy with 25 per cent of Ni; i.e. the least μ_0 corresponds approximately with rapidly increasing resistivity, maximum hysteresis loss and minimum saturation density. Higher proportions of nickel increase the initial permeability rapidly to about 2 000, about 10 times that of pure iron, at which value it remains roughly constant for alloys containing 40 to 65 per cent of nickel. Further addition of nickel causes a surprisingly sudden increase in μ_0 to the enormous value of about 12 000 in an alloy with 78 per cent Ni, after which there is a steep decline to the figure of about 200 in pure nickel. This extraordinarily high value of initial permeability, far greater than that found in any pure magnetic metal or in any other alloy, can only be secured by special heat treatment. The alloy is annealed for one hour at 900° C. and allowed to cool slowly to the magnetic transformation point 625° C., at which temperature it is retained for 15 minutes in a nitrogen atmosphere. It is then removed from the furnace and cooled rapidly by motion in the air or by contact with a copper plate at air temperature.

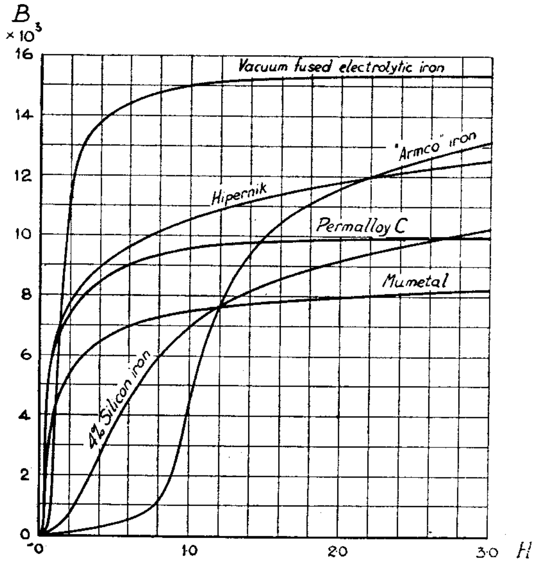


FIG. 45. *B–H* CURVES FOR VARIOUS MAGNETIC MATERIALS

---

* See further, G. W. Elmen, "Magnetic alloys of iron, nickel and cobalt," *Journal Frank. Inst.*, vol. 207, pp. 583–617 (1929); *Bell Syst. Tech. J.*, vol. 8, pp. 435–465 (1929).

† For commercial soft iron, Ewing gives a value of $\mu_0 = 183$; its value for 4 per cent silicon steel is about 500.

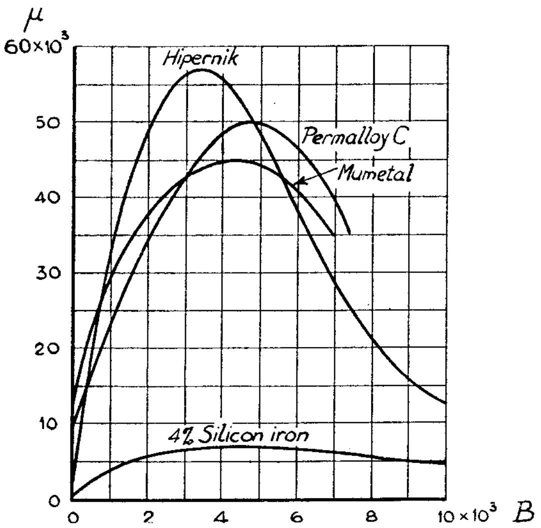


FIG. 46. PERMEABILITY CURVES FOR NICKEL- AND SILICON-IRON ALLOYS

The nickel content affects not only the initial permeability but also the maximum permeability, as Fig. 46 shows for three materials in common use. The enormous advantage of nickel-iron over silicon-iron alloys is clearly shown by these curves and is further emphasized in Fig. 47, where the initial part of the *B–H* curve has been plotted in semi-logarithmic coordinates.

These general remarks will make it clear that the low losses and high permeability of certain nickel-iron alloys make them valuable materials for core construction; two types of material have received practical application, namely, the 50 per cent or "Hipernik" group and the 78 per cent or "Permalloy" group. To obtain full advantage from them the material should be used in jointless rings, preferably made up of strip of the desired width wound up spirally like a clockspring into the form of a ring, thus avoiding waste. After all mechanical working is completed the core should be subjected to the appropriate heat-treatment, which depends not only on the alloy and the properties that are to be imparted to it, but on the size and form of the completed core. The one disadvantage of nickel-iron alloys that at present debars them from general use is their relatively high cost. They have been applied, therefore, only in two important practical cases: (i) in the production of standard transformers having the highest possible degree of precision, where the perfection of the transformer must be secured regardless of cost; (ii) in the design of transformers with a bar-type or single-turn primary and a toroidal secondary wound on a ring-type core, especially for currents under 150 amperes. It is practically impossible to make such bar-type transformers with silicon-iron cores with an accuracy of ratio and phase-angle adequate for metering purposes, unless more or less elaborate compensating devices are employed. Such transformers are most desirable in large high-voltage installations on account of their cheapness, simplicity, ease of insulation and mechanical strength against short-circuit forces; yet it is often necessary to abandon these advantages and to

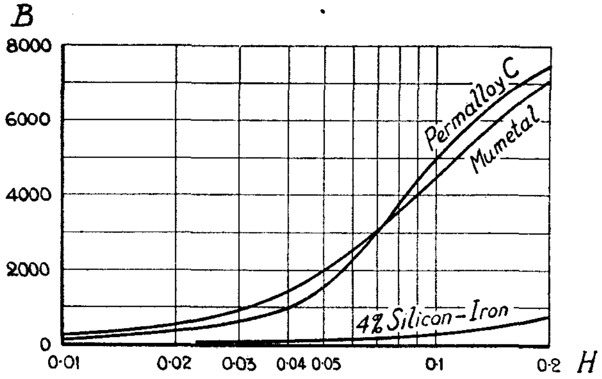


FIG. 47. *B–H* CURVES AT LOW DENSITIES FOR NICKEL- AND SILICON-IRON ALLOYS

install a more expensive transformer with a wound-type primary in order that the required precision for accurate metering may be secured. The use of nickel-iron cores in simple bar-type transformers, however, enables the advantages of this type to be realized and adequate accuracy to be obtained at the same time. The extra cost of a nickel-iron core is much less than the difference in price between a wound-type transformer and a bar-type transformer with a silicon-iron core and much inferior characteristics.

The fundamental expressions for the design of a current transformer are those given on pp. 41 and 42, namely,

$$K_c \doteqdot K_T\left[1 + \frac{I_m \sin \phi_s + I_w \cos \phi_s}{K_T I_s}\right],$$

and

$$\tan \beta \doteqdot \frac{I_m \cos \phi_s - I_w \sin \phi_s}{K_T I_s}$$

If $Z_s = (R_s^2 + X_s^2)^{\frac{1}{2}}$ is the total impedance of the secondary circuit inclusive of the winding and the external burden, the e.m.f. to be induced in the secondary winding to circulate a given current I_s is $E_s = Z_s I_s = (\sqrt{2})\pi\, T_s f\, B_{max} A_i\, 10^{-8}$ volts, assuming a sinusoidal flux variation. If the core dimensions have been tentatively settled in such a way that B_{max} is retained in the region of low total loss and high permeability, B_{max} is determined and the values of K_c and β can be calculated provided that I_m and I_w are known as functions of B_{max}. It is usual to express the necessary characteristics of the core material in the form of curves, derived from measurements made upon specimens of the laminated iron, giving the magnetizing volt-amperes w_m and the total loss (hysteresis and eddy currents together) w_w in watts, both per kilogramme of the material at the given frequency, plotted to a base of B_{max}. These quantities may be measured either by a suitable low-reading wattmeter, or preferably by a bridge method or by the use of an a.c. potentiometer;* appropriate methods are described on pp. 411–415. Let $W_i = \gamma_i A_i l_i 10^{-3}$ be the weight of the core in kg., γ_i being its specific gravity, A_i its cross-section in sq. cm., and l_i its mean path length in cm.; then remembering that I_m and I_w are associated with the primary side,

$$I_m = W_i w_m K_T / E_s \text{ and } I_w = W_i w_w K_T / E_s.$$

---

* See F. E. J. Ockenden, "Nickel iron alloys and their application to instrument construction," *Journal Sci. Insts.*, vol. 8, pp. 113–117 (1931); D. C. Gall, "Testing nickel iron alloy by means of the a.c. potentiometer," *idem.*, vol. 9, pp. 219–222 (1932).

Since $E_s I_s = VA$, the total secondary volt-amperes, the ratio and phase-angle are easily seen to be

$$K_c \doteq K_T[1 + (W_i/VA)\,(w_m \sin\phi_s + w_w \cos\phi_s)],$$

$$\tan\beta \doteq (W_i/VA)\,(w_m \cos\phi_s - w_w \sin\phi_s),$$

where w_m and w_w are read off from the curves for the value of B_{max} calculated from E_s.

Before reviewing the specific properties and applications of the various alloys it will be of interest first to refer to a new method recently introduced by the I.G. Farbenindustrie A.G. of Oppau[*] for the preparation of pure iron and nickel. Some metals form compounds with carbon monoxide, known as *carbonyls*, which have the property when heated of decomposing into the pure metal and CO; there is usually at least one volatile carbonyl of each metal that can be purified by distillation. Iron pentacarbonyl $Fe(CO)_5$ is formed when finely-divided iron is in contact with CO at ordinary temperatures; it is a pale yellow liquid which distils at about 103° C. When the vapour is passed through a hot tube at 180° C., iron of very great purity is deposited in the form of a powder consisting of minute spheres about 10^{-5} cm. in diameter. The powder is sintered into a solid mass by heating in a reducing atmosphere to 1 200 °C., forged into a billet and rolled into sheet. Its magnetic properties are those of pure iron, namely $\mu_0 \doteq 3\,000$, $\mu_{max} \doteq 20\,000$ at $B = 5\,500$ and $H = 0{\cdot}9$, the saturation density being 22 000 and the resistivity 10 microhms per cm. cube. Nickel tetracarbonyl $Ni(CO)_4$ is a somewhat similar nickel compound, a clear liquid volatilizing at 43° C. Its vapour decomposes at 180°–200° into pure nickel and CO, the metal being put into practical form by sintering and rolling.

Alloys have been prepared in Germany on a commercial scale from these constituents, and show all the behaviour just described; their saturation densities are somewhat higher and the resistivities rather lower than for the alloys prepared in the usual way from electrolytic constituents.[†] An alloy of iron with 4 per cent Si, known as Hyperm 4, has $\mu_0 \doteq 1\,000$, $\mu_{max} \doteq 20\,000$ and a total loss per kg. of about 0·7 watt for $B_{max} = 10\,000$; it is a useful intermediate step between ordinary silicon-iron plates and the nickel-iron alloys.

9*a*. HIPERNIK. This material has been developed specially for current transformer cores by Yensen in the research laboratories of the Westinghouse Electric and Manufacturing Co., Pittsburg, U.S.A. Similar materials are made on the Continent under various names, such as Invariant, Hyperm 50, etc. As mentioned earlier in this section, it is a simple binary alloy containing 50 per cent each of nickel and iron, and is prepared by the vacuum fusion of electrolytic nickel and iron in the induction furnace, all traces of carbon and oxygen being removed in the process; the cast ingots may be rolled into the form of ribbon or sheet from which the core is assembled. In Germany the carbonyl process is used; since no fusion is required contamination is avoided and alloys of great purity are produced. Coiled ribbon[*] is preferable to punchings as there is much less mechanical handling of the material, absence of punching stresses, and entire elimination of waste.

As initially prepared, Hipernik has a maximum permeability μ_{max} below 10 000 and a hysteresis loss for $B_{max} = 10\,000$ of 1 000 ergs per c.c. per cycle. By annealing in a hydrogen or nitrogen atmosphere, to avoid deleterious oxidation, μ_{max} is raised to about 80 000 and the hysteresis loss reduced to about 220 ergs per c.c. per cycle; the value of μ_0 is about 3 000. By a special annealing from 1 000–1 300° C. it is possible to raise μ_{max} to the extraordinary value[†] of 167 000 and to diminish the hysteresis loss by one-third.

Hipernik has many advantages over alloys of the simple Permalloy or 78 per cent Ni group. It has more than double their electrical resistivity with consequently lower eddy-current loss; its hysteresis loss is the lowest and the saturation value the highest of the whole range of alloys containing 30–100 per cent of Ni. The Permalloy group, however, possesses the highest initial permeability. These properties have been clearly shown in the preceding diagrams and are further emphasized by the figures in the following table; this has been compiled in part from data published by Yensen and in part from figures kindly furnished by the Heraeus Vacuumschmelze A.G. and the Telegraph Construction and Maintenance Co.

The superiority of Hipernik over the ordinary 4 per cent silicon-iron is clearly shown by the curves in Fig. 48, which exhibit w_w and w_m as functions of B_{max} for ring stampings 0·35 mm. thick tested at 60 cycles per sec.; though the range of induction is limited, the reduction of total loss and magnetizing volt-amperes is sufficiently indicated to show the improvement

---

[*] L. Schlecht, W. Schubardt and F. Duftschmid, "Ueber die Verfestigung von Pulverförmigem Carbonyleisen durch Wärme und Druckbehandlung," *Zeits. f. Elektrochemie und ang. phys. Chem.*, vol. 37, pp. 485–491 (1931). "Die technische Verarbeitung von pulverförmigem Carbonyleisen nach dem Sinterverfahren," *Stahlu. Eisen*, vol. 52, pp. 845–849 (1932). F. Stäblein, "Technische Werkstoffe grosser magnetische Weichheit," *Zeits. f. tech. Phys.*, vol. 13, pp. 532–534 (1932).

[†] See also A. Kussmann, loc. cit., *ante*.

[*] Too tight coiling of a strip core can greatly influence the hysteresis loss, see Kussmann, loc. cit., *ante*.

[†] T. D. Yensen, "Permeability of Hipernik reaches 167 000," *Elect. J.*, vol. 28, pp. 386–388 (1931). For the properties of alloys of the Hipernik type made from carbonyl iron and nickel, see the papers cited on p. 112.

COMPARATIVE TABLE OF MAGNETIC ALLOYS

Description	Pure Iron	4% Silicon Steel	Hipernik	Permalloy	Permalloy C	Mumetal
Composition		4% Si 96% Fe	50% Ni 50% Fe	78·5% Ni 21·5% Fe	78·5% Ni 18·0% Fe 3·0% Mo 0·5% Mn	76% Ni 17% Fe 5% Cu 2% Cr
μ_0	700	440–500	3 000	5 850	6 000– 10 000	12 000– 30 000
μ_{max}	26 000	8 000–9 000	70 000	74 000	50 000– 100 000	45 000– 100 000
Saturation density	22 600	16 000– 20 000	15 500	10 500	9 000	8 500
Hysteresis loss in ergs per c.c. per cycle $B_{max} = 10\,000$	600	500	220	200		
Coercive force from $B_{max} = 10\,000$ . .	0·20	0·5–0·6	0·05	0·05	0·035	0·03
Remanence from $B_{max} = 10\,000$ . .	8 600	5 200–9 000	7 300	5 500	4 500	
Resistivity in microhms per cm. cube at 20° C.	10	55	46	21	55	42–45
Specific gravity	7·9	7·6	8·3	8·6	8·4	8·5

From tests on punchings 0·35 mm. thick.

to be expected by substitution of the nickel-iron for the silicon-iron in a current transformer. In further illustration* Fig. 49 gives the ratio and phase-angle curves for two identical ring-type transformers for a nominal current-ratio of 1 500/5, except that one has a core of silicon steel weighing 4·4 kg. and the other an equal weight of Hipernik. Although the fluxes in the two cores are the same B_{max} is slightly different, since the specific gravities of the two materials are 7·6 and 8·3 respectively. The secondary burden is 3 volt-amperes at full rated current with a power-factor of 0·8 at 60 cycles per sec.; the secondary has 300 turns. It will be seen that the ratio error and phase-angle of the Hipernik transformer are about one-third of those with the silicon-iron core and that the curves are much flatter; hence turns-compensation will be more effective in bringing the whole ratio curve nearer to unity when nickel-iron is used.

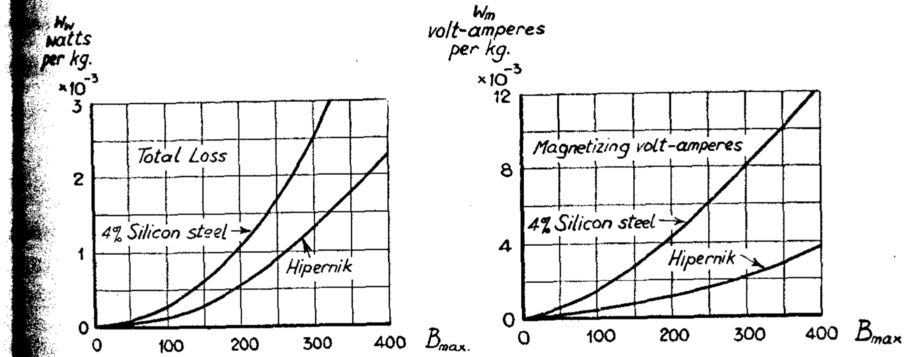


FIG. 48. TOTAL LOSS PER KG. AND MAGNETIZING VOLT-AMPERES FOR HIPERNIK AND 4 PER CENT SILICON STEEL

9*b*. PERMALLOY. An important group of alloys containing about 78 per cent of nickel has been developed in the laboratories of the International Standard Electric Corporation, based upon the work of Arnold and Elmen† (loc. cit. *ante*). These

---

[*] T. Spooner, "Current transformers with nickel-iron cores," *Journal Amer. I.E.E.*, vol. 45, pp. 540–545 (1926). A. M. Wiggins, "Ring-type current transformers," *Elect. J.*, vol. 24, pp. 621–625 (1927); "Improved current transformers with nickel-iron cores," *idem.*, vol. 26, pp. 152–154 (1929). E. C. Wentz, "Current transformers for low voltage networks," *idem.*, vol. 27, pp. 509–510 (1930); "Hipernik—its uses and limitations," *idem.*, vol. 29, pp. 227–229 (1932).

[†] Further treatment of Permalloy will be found in L. W. McKeehan and G. G. Cioffi, "Magnetic hysteresis loops in Permalloy," *Phys. Rev.*, vol. 23, p. 305 (1924); "Magnetostriction in Permalloy," *idem.*, vol. 28, pp. 146–157 (1926).

investigators found that a binary alloy containing 78·5 per cent Ni and 21·5 per cent Fe, named Permalloy, possessed astonishingly high initial permeability; its use as continuous magnetic loading for telegraph and telephone cables has been one of the most important developments in modern communication engineering. To secure the best results the material must be given a special heat treatment; it is, however, peculiarly sensitive to thermal and mechanical influences. In order to render the material more suitable for purposes other than cable loading, e.g. for transformer cores, an alloy containing rather less iron with addition of small quantities of molybdenum and manganese has been prepared under the name of Permalloy C.* This material is far less sensitive than the binary alloy and is much more easily worked into strip or sheet form; it has received some application to current transformers in the United States and is made in Europe by the Heraeus Vacuumschmelze. Its general features are given in the table on p. 114.

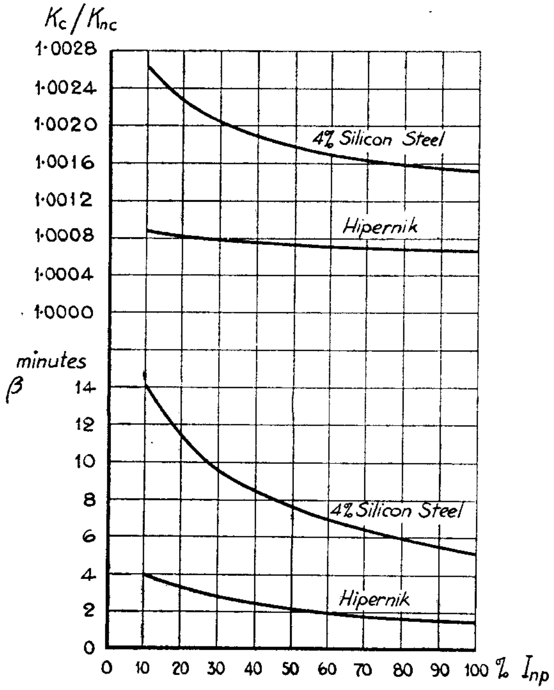


FIG. 49. CHARACTERISTICS OF CURRENT TRANSFORMERS WITH EQUAL-WEIGHT CORES OF HIPERNIK AND 4 PER CENT SILICON STEEL

---

* "Das Permalloy C," *E.N.T.*, vol. 7, pp. 251–254 (1930).

In Germany* a number of alloys for use in submarine telegraphy and telephony were investigated by the Deutsch-Atlantische Telegraphengesellschaft and the Physikalische Technische Reichsanstalt; some of these alloys have also been used for our present purpose.

9*c*. MUMETAL. This material, which is a modified Permalloy, is almost exclusively used in Great Britain and very considerably on the European continent; it was introduced by Smith and Garnett and is made under the patents held by the Telegraph Construction and Maintenance Co. It is a quaternary alloy containing 76 per cent Ni and 17 per cent Fe with the addition of 5 per cent Cu and 2 per cent Cr or Mn to improve its mechanical properties and reduce thermal sensitiveness; its general features are tabulated on p. 114, from which it will be seen generally to resemble Permalloy but with a resistivity as high as that of Hipernik. It can be supplied in sheet or other form, and may be worked to shape hot or cold; a simple heat treatment after manufacture puts the material into the best magnetic condition.

The magnetizing volt-amperes and total loss as a function of B_{max} for 0·35 mm. plates at 50 cycles per sec. are plotted in the upper diagram of Fig. 50, again showing a great improvement over silicon steel. For design purposes the curves for w_m and w_w have been replotted in the lower diagram of Fig. 50 using double-logarithmic co-ordinates, whereby a much wider range of values can be exhibited with an open scale and without confusion; interpolation is also rendered easy by the closely linear shape taken by the curves when plotted in this way.

The application of Mumetal in the construction of normal types of current transformer has been made by many manufacturers, notably by Elliott Brothers† and by Everett, Edgcumbe and Co.‡ Results have been obtained which show that

---

* E. Gumlich, W. Steinhaus, A. Kussmann and B. Scharnow, "Ueber Materialen mit hoher Anfangspermeabilität," *E.N.T.*, vol. 5, pp. 83–100 (1928), vol. 7, pp. 231–235 (1930); G. Keinath, "Hochmagnetische Legierungen aus Nickel-Eisen," *Arch. f. tech. Mess.*, Z913–1, T63 (1931); Heraeus-Vacuumschmelze A.G., Hanau a.M., "Hochmagnetische Legierung aus Nickel-Eisen für Messgeräte und Messwandler," *Arch. f. tech. Mess.*, Z913–2, F7 (1931).

† W. Phillips, "High permeability and low loss alloys for use in current transformers and electrical instruments," *Meter Eng. Tech. Assn.*, 9th meeting, Nov. 25th (1927); *El. Rev.*, vol. 101, p. 956 (1927).

‡ K. Edgcumbe and F. E. J. Ockenden, "Some recent advances in alternating current measuring instruments," *Journal I.E.E.*, vol. 65, pp. 553–586 (1927).

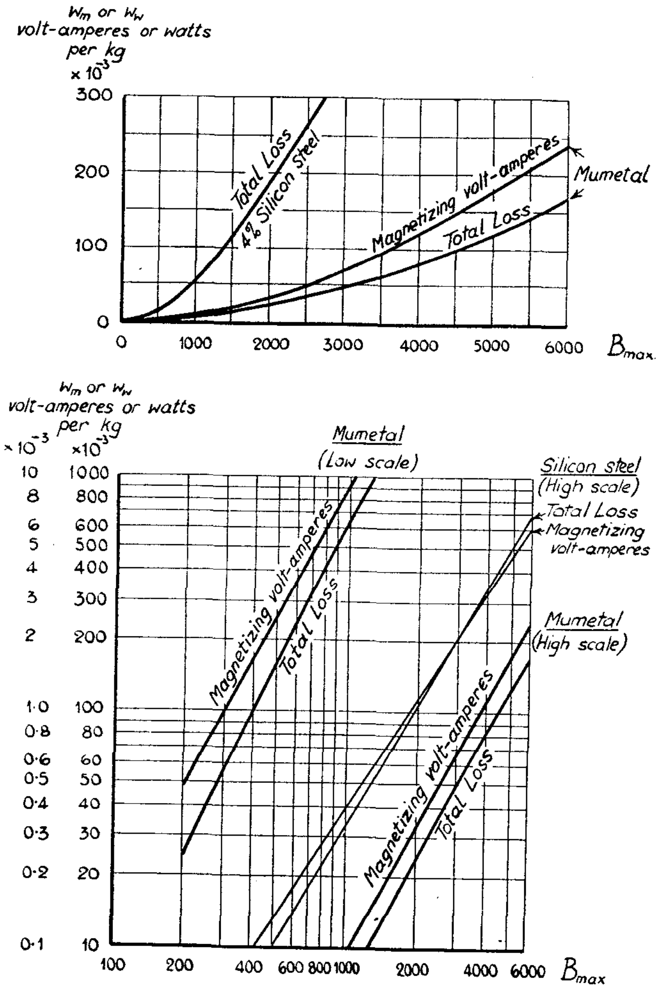


FIG. 50. MAGNETIZING VOLT-AMPERES AND TOTAL LOSS PER KG. FOR MUMETAL AND 4 PER CENT SILICON STEEL

for primary currents as low as 100 amperes, bar-type transformers can be made to conform to the B.S.I. specification for accuracy; with currents from 200–500 amperes the use of Mumetal enables these limits of accuracy to be attained with from 6 to 2½ times the usual secondary output.*

---

* "Breaker," "Developments in current-transformer design," *El. Rev.* vol. 102, pp. 816–818 (1928).

An interesting and important application of Mumetal is that made at the National Physical Laboratory in the design of transformers having the greatest possible precision for laboratory purposes. In the N.P.L., currents have hitherto been measured by passing them through four-terminal resistors, which are air-cooled up to 200 amperes and water-cooled for larger currents; descriptions of these will be found on p. 340. The resistors for the large currents are bulky, difficult to construct with a sufficiently low residual inductance, and dissipate a considerable amount of power; thus, a 5 000 ampere resistor to give a 2 volt drop wastes 10 kilowatts. If, however, a resistor of 0·4 ohm is connected to the secondary of a 5 000/5 precision type current transformer, the dissipation is only 10 watts for the same volt-drop; the combination will behave like a pure resistor, provided the ratio of the transformer remains constant and its phase-angle is negligibly small. These advantages have been realized by Spilsbury and Arnold* in a transformer consisting of Mumetal rings wound with a uniformly-distributed toroidal coil of 999 turns. The whole is enclosed in a teak case 11 in. × 11 in. × 6 in. with a 3½ in. hole through which one or more turns of the primary cable may be passed; the ratio with one turn is almost exactly the nominal rated

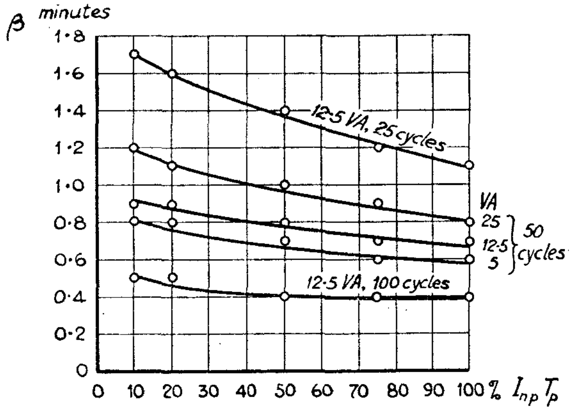


FIG. 51. PHASE-ANGLE CURVES FOR NATIONAL PHYSICAL LABORATORY STANDARD TRANSFORMER WITH MUMETAL CORE

---

* R. S. J. Spilsbury and A. H. M. Arnold, "Some accessory apparatus for precise measurements of alternating current," *Journal I.E.E.*, vol. 68, pp. 889–897 (1930).

value, 5 000/5. It is found that the ratio does not change more than 0·01 per cent with 1, 2, 5, 10 or 25 turns; with 1 or 2 turns the phase-angle does not change more than 0·2 minute, and for 5, 10 and 25 turns by more than 0·1 minute. The ratio is independent of the burden, which at full secondary current is 12·5 volt-amperes, consisting of the 0·4 ohm resistor and 0·1 ohm leads; it is also independent of frequency. The phase-angle β is less than 1 minute over the whole range of primary current at rated burden; its change with frequency and burden is shown in Fig. 51. From this it will be seen that the angle is usually negligible and can be made zero for any desired primary current by shunting the secondary terminals with a suitable condenser; one of 1·4 μF capacitance makes β zero at half the rated ampere-turns and reduces it to − 0·1 minute for greater values and + 0·1 minute for smaller values. The transformer is quite stable, showing no signs of magnetic ageing when tested over a considerable period, and it is not affected by mechanical shock. Arnold has recently described some multi-range transformers with even better characteristics; some details are given on p. 136.

10. **Influence of law of iron-loss upon shape of characteristics.** The reader will have observed that the normal ratio and phase-angle curves for a current transformer with a silicon-iron core slope downward, with an upward concavity, as the current increases; with a nickel-iron core the curves have usually the same general character, but the slope and concavity are much less and in some cases the characteristics are practically horizontal. With both materials certain abnormal results are sometimes encountered and were first noticed as far back as 1910 by Edgcumbe,* the explanation being given by Agnew† in the following year.

Fig. 52 is taken from Agnew's paper and illustrates very clearly the particular feature to be discussed. The phase-angle curve follows the normal course, decreasing with increased current. The ratio curve, on the other hand, at first increases to a maximum and then decreases as full load is approached. Such a curve is abnormal for a silicon-iron core, and Agnew has shown that its shape depends entirely upon the law of variation of the total iron losses with the maximum flux density

---

* K. Edgcumbe, "Some notes on the use of instrument transformers," *Elec. Rev.*, vol. 67, pp. 163–165 (1910).

† P. G. Agnew, "A study of the current transformer with particular reference to iron loss," *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911).

B_{max} in the core. These losses consist of the hysteresis loss due to cyclic magnetization and the Jouleian heat produced by eddy currents, the former being preponderant and usually 75 to 80 per cent of the whole. With a constant frequency the eddy current loss depends upon B^2_{max}, while the hysteresis loss can be represented by the so-called "Steinmetz law," which, for the range of density used in power transformers, expresses the loss as proportional to the 1·6th power of the maximum density. The approximate character of this "law" is well recognized and the law is known to fail at low flux densities.

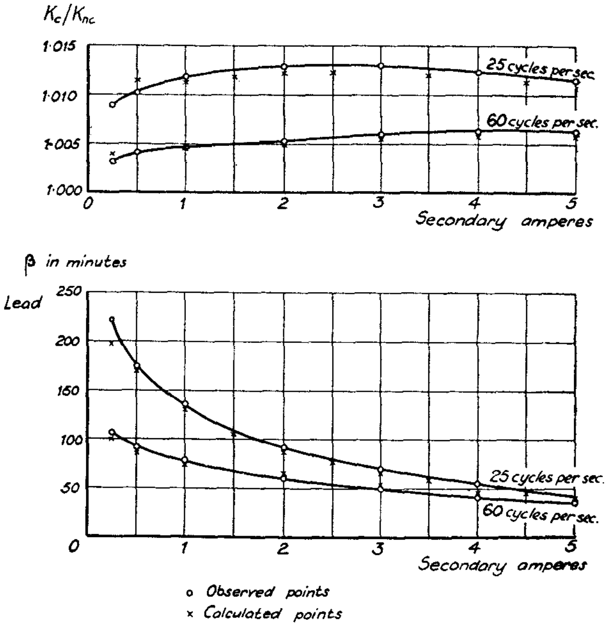


FIG. 52. ABNORMAL CHARACTERISTIC CURVES FOR A CURRENT TRANSFORMER

If a graph of the total iron loss per kg, w_w, as a function of B_{max} is made in double-logarithmic co-ordinates, the resulting curve is nearly linear (see Fig. 50), so that as a first approximation we can write $w_w = C_w B^{\sigma}_{max}$, where C_w and σ are constants for plates of given thickness, provided the range of density is not large. In the region where the Steinmetz hysteresis expression is roughly true, σ is somewhat less than 2; it has long been known* that at the low densities used in current transformers σ may be more nearly 3, and it is this fact that has an important bearing upon the shape of the ratio curve.

From page 112 the current ratio is

$$K_c \fallingdotseq K_T[1 + (W_i/VA)\,(w_m \sin\phi_s + w_w \cos\phi_s)];$$

w_w has its greatest effect upon the ratio when the secondary circuit has unity power-factor, the ratio then being

$$K_c \fallingdotseq K_T[1 + (W_i/VA)\,w_w].$$

Since the secondary circuit is non-reactive,

$$E_s = R_s I_s = C_s B_{max},$$

and $\quad VA = E_s I_s = I^2{}_s R_s,$

where C_s is a constant. Substitution of VA and w_w in K_c gives

$$K_c \fallingdotseq K_T\,[1 + K\,I_s{}^{\sigma-2}],$$

where K is a constant. The slope of the ratio curve is

$$dK_c/dI_s = K_T\,K(\sigma-2)I_s{}^{\sigma-3},$$

and its curvature is proportional to

$$d^2K_c/dI^2{}_s = K_T\,K(\sigma-2)\,(\sigma-3)\,I_s{}^{\sigma-4}.$$

The ratio curve takes various forms, according to the values of σ, which are summarized in the table.

Exponent	Slope	Curvature	Shape	Remarks
$\sigma < 2$	$-$	$+$	Slope down, concave up	Normal type
$\sigma = 2$	Zero	Zero	Horizontal straight line	Found in high grade transformers
$2 < \sigma < 3$	$+$	$-$	Slope up, concave down	Abnormal type
$\sigma = 3$	$+$	Zero	Straight line, slope up	Have never been observed
$\sigma > 3$	$+$	$+$	Slope up, concave up	

While these general conclusions are substantially correct they do not provide a complete explanation of the phenomena, e.g. the maximum in the abnormal curve is unaccounted for, since they are based upon the tacit assumption that σ is independent of B_{max}, which is not true. Agnew has shown that the

---

* L. W. Wild, "Series transformers for wattmeters," *Elecn.*, vol. 56, pp. 705–706 (1906).

necessary amendment to the theory can be made in the following manner: Beginning with the equation

$$K_c \fallingdotseq K_T\left[1 + \frac{W_i}{R_s}\cdot\frac{w_w}{I^2{}_s}\right]$$

with w_w a function of B_{max}, i.e. of I_s, which can be graphically expressed, we have

$$\frac{dK_c}{dI_s} = \frac{K_T W_i}{R_s}\cdot\frac{I^2{}_s\,(dw_w/dI_s) - 2I_s\,w_w}{I^4{}_s}$$

$$= \frac{K_T W_i}{R_s}\cdot\frac{w_w}{I^3{}_s}\left[\frac{(dw_w/w_w)}{(dI_s/I_s)} - 2\right].$$

Hence, no matter what the law of iron loss may be, so long as the first term in the bracket is greater than 2 the curve will rise; when it is equal to 2 the curve will be horizontal; and if it is less than 2 the curve will fall. Agnew names this term the "ratio of variation" of the iron-loss, and he has shown by experiment that it exceeds 2 at low densities and diminishes as the density increases, even becoming less than 2 at the higher values. Thus, according to the density at which the core is worked for the lower values of I_s, the ratio curve may start with falling slope—the normal case; or it may rise initially and ultimately fall—the abnormal case. A horizontal curve is not uncommon in transformers with nickel-iron cores, showing that the ratio of variation for this material is very nearly 2 and accounting for the very flat characteristic usually found in practice; in other words, the volt-amperes expended in the secondary circuit and the loss in the core both vary in proportion to the square of the secondary current, with resulting constancy of ratio.

A simple interpretation can be given to the ratio of variation by putting the total loss per k.g. in the form

$$w_w = [C_w B^{\varrho}{}_{max} \equiv C'_w I^{\sigma}{}_s],$$

where C'_w is a new coefficient and σ an exponent, which may be a function of I_s. Plot w_w as a function of I_s on double-logarithmic paper, i.e.

$$\log w_w = \log C'_w + \sigma \log I_s$$

or

$$Y = A + \sigma X$$

which, in general, is not a straight line. The slope of this curve on the log paper will be

$$\frac{dY}{dX}=\frac{dY/dI_s}{dX/dI_s}=\frac{dY}{dw_w}\cdot\frac{dw_w/dI_s}{1/I_s}=\frac{1}{w_w}\cdot\frac{dw_w/dI_s}{1/I_s}$$

or

$$\frac{dY}{dX}=\frac{dw_w/w_w}{dI_s/I_s}=\text{ratio of variation};$$

so that the ratio of variation is the logarithmic derivative of the iron loss curve and can be easily found by simple graphical methods since the curve is closely linear on double logarithmic paper.

11. **The effects of remanence on ratio and phase-angle.** It is well-known that the secondary circuit of a current transformer should not be opened while current is flowing in the primary winding.[*] The ampere-turns I_0T_p required to excite the core is only a small proportion of the total primary ampere-turns I_pT_p, usually less than 1 per cent. The vector difference of I_pT_p and I_0T_p is balanced by the secondary ampere-turns I_sT_s; should the secondary circuit be opened, however, the whole of the primary ampere-turns I_pT_p becomes available to magnetize the core and may set up a high flux density in it. In an average transformer with about 1 000 primary ampere-turns at rated current and a mean iron path of about 35 cm., the r.m.s. ampere-turns per cm. will be about 30, which produces in a jointless silicon-iron core a peak density of about 15 000 lines per sq. cm. in place of the normal value of about 500. This abnormally high density has three main effects. First, it results in much increased iron losses, but as these are only then of the order of 4–5 watts per kg. at $B_{max} \doteqdot 15\,000$, they are much too small to cause overheating; moreover, as a density of 15 000 is not far from the saturation value for silicon steel the density and the accompanying iron losses when the primary is carrying quite considerable overload current will not be much further increased. Hence, the danger of overheating the core is not serious. Second, the high density results in an abnormal secondary voltage with a high peak value; this is a much more serious matter and will be dealt with in Section 26. Third, the abnormally high flux density may leave the core permanently magnetized to a considerable

extent, greatly reducing its permeability and increasing the ratio and phase-angle of the transformer; the magnitude of the effect clearly depends upon the point in the cycle at which the primary current is interrupted. A similar effect is obtained if the core has been left with initial remanent magnetism by the passage of a direct current in either of its windings, e.g. from the use of a Wheatstone bridge to measure the winding resistances or by applying certain polarity tests which employ direct currents (see p. 586, for example).

Fig. 53 shows the effect of such magnetization upon the ratio and phase-angle of a 200/5, 40 volt-ampere transformer tested by Agnew and Fitch.[*] The increase in ratio at full load is about 0·1 per cent, but at $\frac{1}{10}$ of full load is nearly 3 per cent and in some instances may be much greater. The increase in phase-angle is, as would be expected, even more marked than that in ratio. The effects are greater as the secondary impedance becomes larger, whether as the result of added inductance in the burden or of a rise in frequency, as the curves clearly indicate.

Fortunately the effects of magnetization can be removed by demagnetizing the core, which should always be done before the transformer is further used. This can be achieved in several ways, the most obvious of which is to pass an alternating current slightly in excess of full-load current through the primary, the secondary being open, and then gradually to reduce the current to zero; this can be most easily done by allowing the alternator to slow down gradually to rest. Alternatively, it is often more convenient, on account of the smaller currents required for the purpose, to open the primary and pass the demagnetizing current through the secondary, decreasing it to zero as before. A current of about 0·2–0·5 ampere has been shown by Engelhardt[†] to be sufficient for satisfactory demagnetization. A further method is to pass full rated current through the primary while a high resistance connected across the secondary terminals is reduced to zero.

Another case in which magnetization may occur arises when

---

[*] The I.E.C. and the Swiss Rules recommend that a transformer shall withstand this condition for one minute without damage, but nevertheless the characteristics of such a transformer should be held suspect until demagnetization and retesting have been effected.

[*] P. G. Agnew and T. T. Fitch, "The determination of the constants of instrument transformers," *Bull. Bur. Stds.*, vol. 6, pp. 281–299 (1910). The American Standard Rules contain a specification for the permissible change in ratio and phase-angle when a transformer has been magnetized by a d.c. of full rated value.

[†] V. Engelhardt, "Ueber den Einfluss der remanenten Magnetisierung auf die Angaben von Stromwandlern und über deren Beseitigung," *Elekt. Zeits.*, vol. 41, pp. 647–650 (1920).

a current transformer is subjected to a transient short-circuit current, especially one displaced from the normal zero line. In such a current the unidirectional, exponentially-decaying component acts in the same ways as a d.c. magnetization of the core and has a corresponding effect in increasing the ratio and phase-angle of the transformer. These asymmetrical transient currents occur in two practical conditions, which can be distinguished as accidental or intentional in their origin.

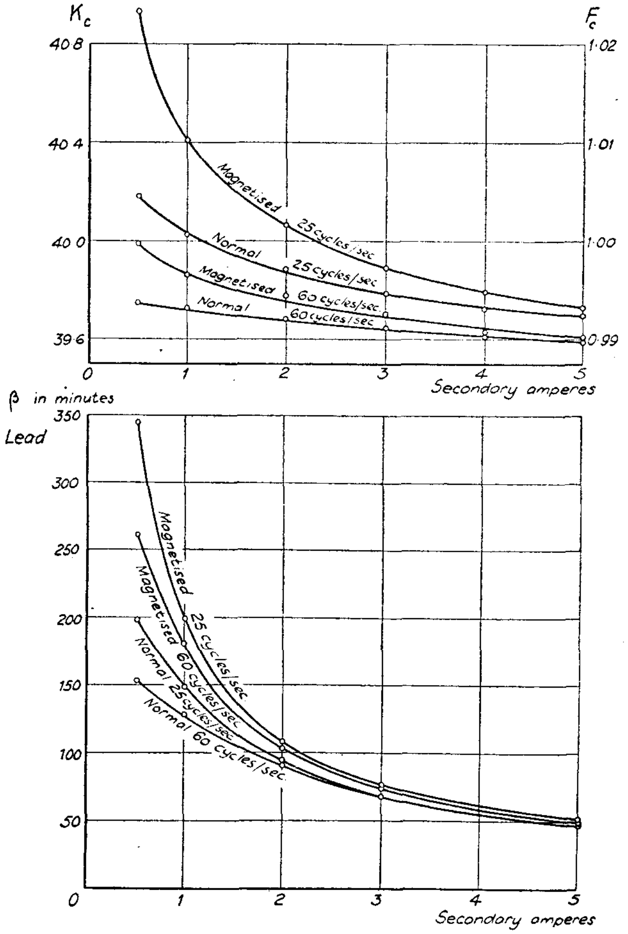


FIG. 53 EFFECT OF REMANENT MAGNETISM UPON CHARACTERISTICS OF A CURRENT TRANSFORMER

The accidental transients arise when a short-circuit occurs on the primary network of a power system, influencing all the measuring or protective current transformers through which the short-circuit current flows.* The intentional transients are encountered in the testing of circuit breakers or other apparatus under short-circuit conditions. Oscillograms of the testing current are often taken by interposing a current transformer between the test circuit and the oscillograph, and the remanence caused by the unidirectional component may make considerable errors in the wave-forms recorded in successive "shots."† After a severe short-circuit on a power system it is very desirable to demagnetize the measuring transformers before reliance can be placed on their subsequent performance. Similarly, in oscillography of transient currents it is necessary first to select a transformer in which an excessive degree of saturation does not occur and then to eliminate the effect of unidirectional magnetization by demagnetizing the core after the recording of each oscillogram; the conditions to be satisfied are fully discussed by Marshall and Langguth in the paper cited.‡

12. **Magnetic leakage.** The magnetic flux in a transformer is usually analysed into three components; the main or mutual flux in the iron core, linked in common by the entire primary and secondary windings and inducing voltages in them proportional to their numbers of turns; the primary leakage flux passing through the primary turns only; and the secondary leakage flux linked only with the secondary winding. The leakage fluxes give rise in the respective windings to reactance voltages. As pointed out on p. 39 the secondary leakage reactance forms a part of the total impedance of the secondary circuit and necessarily has an influence upon the magnitude of the voltage that must be induced in the secondary winding by the main flux in order to drive a given current round the circuit; hence it influences the amount of the main flux, the exciting ampere-turns and both the ratio and phase-angle

---

* "Accuracy test for transformers," *Elec. World*, vol. 82, p. 284 (1923).

† D. E. Marshall and P. O. Langguth, "Current transformers excitation under transient conditions," *Journal Amer. I.E.E.*, vol. 48, pp. 884–888 (1929).

‡ The unsuitability of an iron-cored transformer for the measurement of asymmetric transients was pointed out originally by A. R. Anderson and H. R. Woodrow, "Characteristics and limitations of the series transformer," *Univ. of Illinois, Bull.*, vol. 8, No. 61, pp. 1–45 (1912). They suggest the use of an air-cored transformer as originally described by Campbell in 1896, see p. 3. Current transformers are now replaced by four-terminal resistors in modern switchgear testing stations.

of the transformer, as has been fully discussed on p. 42. The primary leakage reactance has no direct effect on the ratio, but as the primary and secondary leakage fluxes are more or less interdependent, changes in the one necessarily influence the other; this fact is important in the case of transformers with unsymmetrically distributed primaries and will be more fully examined below. Since the leakage fluxes may, and usually do, take a portion of their paths through some part of the iron core, it follows that the total core flux will be different from point to point round the core.

The importance of magnetic leakage has long been known,* but it is surprising how little precise information as to its properties has been available until quite recently. The first thorough investigation of the subject is due to Price and Duff;† as their memoir is published in a journal that is not readily accessible to the general reader, it will be useful to summarize their conclusions. The distribution and amount of the leakage fluxes in a number of different types of current transformer was investigated by winding search coils upon various parts of their cores. The voltages induced in the search coils were measured with a separately-excited, reflecting-dynamometer voltmeter so that their magnitude and phase were determined. The reactances were measured by the following artifice. Considering first the secondary winding, a desired value of current was passed through it; a current in phase-opposition was applied to the primary, of such a magnitude that the secondary and primary ampere-turns were equal, thus reducing the main flux to zero. This could be tested by a search coil wound round the core. Since in the actual use of a transformer the primary and secondary ampere-turns are nearly equal and opposite, it follows that the actual leakage fluxes and those under the test conditions will be practically identical and the measured reactance of the secondary, computed from ammeter, voltmeter and resistance readings, will be that due to its leakage field. Observations were made at the same time upon the primary winding in the same way. The following table gives a summary of the results at 60 cycles per sec. for the types of transformer diagrammatically represented in Fig. 54.

---

* See J. Görner, "Stromwandler," *Schw. Elekt. Zeits.*, vol. 3, pp. 434–435, 444–445, 455–457, 474–475, 489–490, 504–506 (1906); *Elekt. Zeits.*, vol. 27, pp. 208–209 (1906), for an investigation of the leakage field by filings diagrams.

† H. W. Price and C. K. Duff, "Effects of magnetic leakage in current transformers," *Univ. of Toronto Eng. Res. Bull.*, No. 2, pp. 167–190 (1921).

Type of Transf. in Fig. 54	K_{nc}	Type of Core	Type of Windings	Secondary Leakage Reactance in Ohms	Primary Leakage Reactance referred to Secondary
a	5/5	Shell	Superposed link-shaped coils on central limb	0·54	0·75
b	5/5	,,	Link-shaped secondary. Long rectangular primary	1·14	5·2
b	75/5	,,	,, ,,	2·0	9·0
c	5/5	Rectangular	Superposed link-shaped coils on limb	0·87	1·1
d	1 600/5	,,	One secondary coil on one side of core. Bar primary	3 to 7	—
e	1 600/5	,,	Two secondary coils, on opposite sides of core. Bar primary	0·86 to 1·07	—
—	1 200/5	Circular ring	Uniformily distributed secondary. Bar primary	Negligible	—

A short-circuit test on a transformer, as is well-known, measures the sum of the reactance of the winding connected to the supply together with that of the short-circuited winding reduced to the turns of the former; no information is obtainable by this means as to the proportion of the measured reactance to be attributed to each winding. The above table shows that when the two windings are superposed on a common limb and are not very dissimilar in shape, the secondary and reduced primary reactances are roughly of the same order (see *a* and *c*), a fact which is referred to further on p. 420. When the windings are very dissimilar, as in *b*, as also are the two reactances, and the common assumption that the total short-circuit reactance can be divided into two equal parts is considerably in error (see p. 53).

Price and Duff found that both the primary and secondary leakage fluxes use parts of the core in their path and are nearly in phase with the currents in the respective windings and proportional to these currents; the fluxes lag slightly on the currents on account of the iron-loss produced by the passage of the leakage fields through the core, though the bulk of their path is in air. There is more leakage flux in the outer laminations of the core than in the inner, and the fluxes may pass not only along the length of the core but also transversely and even at right-angles to the plane of lamination.

The total flux in the core is, because of the leakage fluxes, different in different parts of the core, and this fact makes the prediction of ratio and phase-angle curves a matter of no little difficulty, especially in those cases where the leakages are considerable, as in transformer (*d*). The exciting current estimated from the characteristic curves of the iron or measured in an open-circuit test on the transformer itself is dependent on a uniform flux in the core, and will necessarily be different from that required by the transformer under operating conditions when different sections of the core carry widely different fluxes. Consequently, observed and calculated ratio and phase-angle curves cannot, in general, be expected to agree when the leakages are large. Price and Duff illustrate this point by numerous tests and curves of considerable practical interest; the calculated ratio is usually too high and the phase-angle β too great. In an extreme case the calculated ε_c was 6·5 per cent, neglecting the effect of leakage, the corresponding value of β being 80 min.; the actual values were 3 per cent and 40 min. respectively.

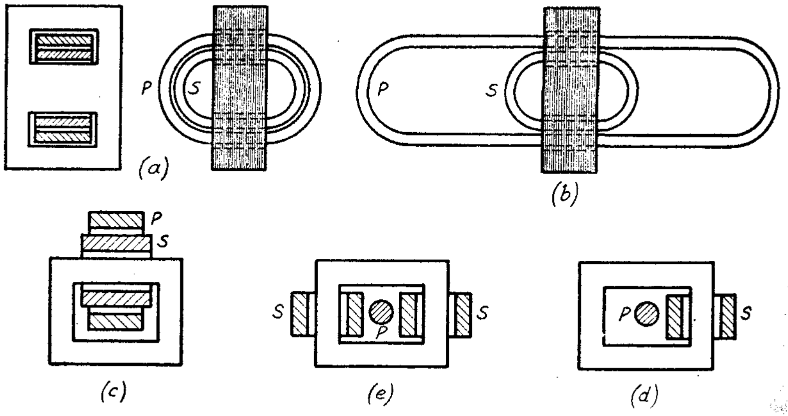


FIG. 54. TYPES OF CURRENT TRANSFORMERS TESTED FOR MAGNETIC LEAKAGE BY PRICE AND DUFF

In the case of a transformer with a single-bar primary or with a multi-turn primary of cable passed through the core opening, the secondary leakage, and with it the values of K_c and β, depends very considerably upon the type of secondary winding and the situation of the primary winding. The variations with the position of the primary are most marked when the secondary is concentrated in a single coil (example (*d*)) and are much reduced by dividing the secondary into two symmetrically disposed coils (example (*e*)). When the transformer has a ring core provided with a uniformly-distributed toroidal secondary winding the characteristics are practically independent of the position of the primary conductor,[*] and the secondary leakage is negligibly small. It follows, therefore, that the calculated and measured characteristics of a ring-type transformer will be practically coincident, as has been shown to be the case on p. 49. On p. 77 it has been shown that an increase in the total secondary reactance usually reduces β but increases K_c and the slope of the ratio curve. Consequently, placing the primary conductor through the hole in such a way as to make the greatest secondary leakage gives a minimum value for β; the position for least leakage reduces K_c but increases β. With a ring core provided with a uniformly distributed secondary the differences are usually extremely small.

The tests made by Price and Duff extended only to currents of 1 600 amperes. Park[†] has recently repeated their experiments on transformers of various designs for primary currents from 7 500 to 12 000 amperes and has also investigated the effect of the return conductors upon the magnetic leakage. Transformers for such large currents usually have a single-bar primary passing through the central opening or window of the core; the return lead may consist of one or more conductors at different distances from and in various situations about the transformer. The cores may be rectangular or circular. In the case of rectangular cores the secondary winding may be accommodated (i) on all four limbs, (ii) on two opposite limbs as in Fig. 54 (*e*), or (iii) on one limb only as in Fig. 54 (*d*). In the case of circular ring cores the secondary is a uniformly distributed toroid.

[*] For a series of curves taken with different arrangements of primary, see also G. F. Shotter, "A new null method of testing instrument transformers and its application," *Journal I.E.E.*, vol. 68, pp. 873–888 (1930).

[†] J. H. Park, "Accuracy of high range current transformers," *Bur. Stds., Journal of Res.*, vol. 14, pp. 367–392 (1935).

According to the situation of the primary bar in the window, the arrangement of the return leads, the shape of the core and the distribution of the secondary winding upon it the ratio and phase-angle characteristics will be considerably influenced.

It has been pointed out above that the total flux at any section of the core consists of the working or main flux, confined within the core, constant at all sections and linking mutually the entire primary and secondary windings, together with the primary and secondary leakage fluxes. The latter vary at different sections of the core and their magnitudes and distributions depend upon the relative arrangements of the primary and secondary circuits with respect to the core and to one another. If there were no leakage flux, the flux density in the core would be uniform and due only to the main flux; the presence of leakage makes the flux density greater than when there is no leakage and different at various sections. Hence the effect of leakage is like that of superposed magnetization (see p. 99), changing the ampere-turns required to set up the main flux and with this alteration in I_m to alter K_c and β. If the addition of the leakage to the main flux does not bring the total density at any section above the value for maximum permeability the ampere-turns will be decreased; if, on the other hand, the total density is above the point for maximum permeability the ampere-turns will be increased. Thus leakage may increase or decrease K_c and β relative to the values without leakage. Moreover, as the leakage varies with the value of I_p its effect will be different at different loads and the shape of the ratio and phase-angle curves may be considerably changed.

It is usual to assume in transformer theory that the two leakage fluxes are of a strictly local kind, the primary leakage flux being proportional to and linked only with the primary current and the secondary leakage being similarly associated with the secondary current alone. While this is substantially true, it by no means represents a complete view of the problem. The primary and secondary ampere-turns act not only upon the core, in which their resultant sets up the main flux, but also upon the leakage space in which they set up a third leakage flux linked mutually with both windings or portions of them. This is the well-known "doubly-linked" leakage (the *doppelverkettete Streuung* of German technical literature). It passes through both windings in much the same way as the main flux, but while the latter is confined to the core the leakage does not enter the core except for a small part of its path, the rest of which is in the space between the windings. It is, naturally, most in evidence when there is considerable dissymmetry in the arrangement of the windings, e.g. when the primary is on one limb and the secondary on the opposite limb of a rectangular core. It is not difficult to see that such doubly-linked leakage flux passes through the secondary in such a way as to oppose the secondary self-leakage flux. We may then define the total secondary leakage inductance as the integrated value per unit current taken over the whole winding of the secondary leakage flux linkages with the secondary turns minus the linkages of the doubly-linked leakage with the secondary turns. The effect of the secondary flux is usually the larger, giving the winding a positive or inductive reactance which has the same effect on K_c and β as adding inductance to the burden. With certain extreme arrangements of the primary circuit the contribution of the doubly-linked leakage may become the greater, giving the secondary winding an apparently negative reactance, which has the same effect as reducing the inductance of the burden.

Park's general conclusions are in agreement with those of Price and Duff, but the magnitude of the observed changes is necessarily larger. With rectangular cores it is shown that with a central primary bar and a return consisting of four conductors parallel to it and symmetrically arranged outside the transformer, one opposite each limb, the curves of K_c and β are normal, i.e. falling with increasing primary current. So long as the return is arranged in this symmetrical way the curves are very little altered either in magnitude or shape by the distance of the return conductors from the transformer; the like is also true when the primary consists of uniformly-distributed loops. If, with a symmetrical distribution of the return conductor and a secondary coil on each limb, the primary bar is displaced from the centre of the window the modification in the leakage flux is very slight, unless the bar is quite close to one limb. If, however, the return is not arranged symmetrically, displacing the primary bar may considerably affect the leakage and the values of K_c and β; in the extreme case the ratio curve may actually rise. With a single loop primary round one limb the effect of the leakage flux on the core-magnetization and the sign of the leakage reactance may alter the shape of the K_c and β curves; these fall in the normal way for the lower currents, reach a minimum and then rapidly turn upward for larger currents; an example of such curves will be found in Fig. 77. All these phenomena are observed in greater degree with two secondary coils, and still more markedly with only one coil. Hence, the more uniformly the secondary is distributed the less will the configuration of the primary circuit affect the transformer's performance. In practice a bar-type transformer and the return conductors are usually fairly symmetrically arranged. In such a case the characteristics depend but little upon the actual arrangement of the primary circuit, and it follows that in testing these transformers it is by no means necessary to duplicate exactly the arrangement of the return conductors. An arrangement is usually chosen in testing work which will make the stray magnetic field as small as possible, see p. 524. Park has also tested two ring-type cores with uniformly-distributed secondary windings; one core was of Hipernik, for a primary current of 3 000 amperes and the other of silicon-iron for 5 000 amperes. He shows their almost complete freedom from changes in K_c and β with variations in the arrangement of the primary circuit.

While these conclusions regarding the performance of ring-type transformers are true for currents under 5 000 amperes, especially with silicon-iron cores, it is not safe to assume them true for larger currents, particularly with nickel-iron cores since the density for maximum permeability and the saturation density of this material are both much lower than for silicon-iron. The use of nickel-iron cores in current transformers with a high standard of performance has led Arnold* to make a careful investigation of the effects of leakage due to changes in the position of the primary conductors in ring-type transformers. Tests on Mumetal-cored transformers for ratios of 5 000/5 and 2 000/5 showed that the ratio was altered by less than 1 part in 10 000 and the phase-angle by less than 0·0001 radian (0·34 min.) with any change in the position of the primary conductor likely to occur in practice. In the case of transformers for 10 000/5 and 20 000/5 it was found that with the primary winding concentrated upon a small part of the ring the ratio error increased enormously when the primary ampere-turns exceeded 4 000 in the former and 2 000 in the latter. At full-load in the larger transformer the ratio error reached the astonishing value of 67 per cent, the secondary current being too low. These large errors were due to the considerable leakage fluxes causing saturation in the nickel-iron core and enormously increasing the magnetizing current required to produce the main flux, as well as to the production of negative secondary leakage reactance.

To investigate the problem more fully, Arnold made a series of tests at 50 cycles per sec. with a non-reactive burden of 0·5 ohm upon six transformers with ratios from 2 000/5 to 20 000/5; five had ring cores of Mumetal and one of Stalloy. With the primary uniformly-distributed round the ring it was found that the measured and calculated ratio and phase-angle agreed in all cases fairly closely, confirming the observations of Price and Duff. With the primary winding wound closely over a small fraction of the core it was noticed that the secondary current wave-form was much distorted when the value of ε_c exceeded 0·2 per cent, and that the performance was changed from the normal in the following ways: (i) With small currents the ratio of primary to secondary ampere-turns is less than the normal, indicating negative leakage reactance, while β is equal to or slightly less than normal. (ii) As the current increases β decreases more rapidly than normally and may become negative; at high currents it tends to become positive again. The ratio increases faster than the normal rate and above a certain current is greater than the normal value. For the 2 000/5 and 5 000/5 transformers, although the actual values of ε_c and β are quite different with the two arrangements of the primary, they are in both cases negligibly small over the working range. This is by no means the case in the larger transformers, in which the leakage flux with the concentrated primary is very considerable; the consequent early saturation of parts of the Mumetal core is the cause of the large ratio errors, since there is a large increase in the magnetizing current required to produce the flux. Arnold has measured the distribution of the fluxes round the core and has shown that if the distribution of the leakage flux is known it is possible to calculate ε_c and β quite accurately.

In laboratory transformers for large currents it is often necessary to wind various numbers of primary turns through the ring; it is convenient, therefore, to have a large central aperture to enable the cables to be inserted easily and quickly. It is difficult in such cases to avoid some concentration of primary ampere-turns, with the possibility of attendant leakage fluxes and large errors. Arnold has shown that the leakage can be appreciably reduced by enclosing the core and secondary

---

* A. H. M. Arnold, "Leakage phenomena in ring-type current transformers," *Journal I.E.E.*, vol. 74, pp. 413–423 (1934).

within a massive copper shield, the eddy currents set up in which will act in such a way as to annul the leakage fluxes. The drawing in the lower part of Fig. 55 shows the type of shield used; it is composed of two copper castings fitted together to surround the core and having a small circumferential slit which prevents the shield acting as a short-circuited turn. By providing a sufficient section of copper, so that the resistance of the eddy paths is as low as possible in comparison with their reactance, the shield is exceedingly effective, as the following figures will show. They relate to the 20 000/5 Mumetal-cored transformer; the thickness of metal in the shield is 0·625 inch and the diameter of the aperture through which the primary winding is inserted is 7·5 inches.

	Primary Ampere-turns	Primary A.T. / Secondary A.T.		Phase Angle in Minutes	
		Primary Uniformly Distributed	Primary Concentrated	Primary Uniformly Distributed	Primary Concentrated
Without Shield	20 000	1·0000	1·67	+ 0·3	—
	10 000	1·0000	1·47	+ 0·3	—
	4 000	1·0000	1·06	+ 0·4	—
	1 000	1·0000	1·0002	+ 0·6	+ 0·6
With Shield	20 000	1·0000	1·0000	+ 0·4	+ 0·4
	10 000	1·0000	1·0000	+ 0·5	+ 0·3
	4 000	1·0000	1·0000	+ 0·6	+ 0·4
	1 000	1·0000	1·0000	+ 0·7	+ 0·5

It will be seen from this table that the presence of the shield has no effect on the performance of the transformer when the primary is uniformly-distributed. With the concentrated winding the shielded transformer has the same ratio as with uniform distribution and there is only very slight change in the phase-angle; in other words, the shielded transformer behaves independently of the arrangement of its primary winding.

A further interesting influence of leakage is encountered in shell-type transformers in which the primary leads come on opposite sides of the core; in this case one window contains one primary conductor more than the other, as shown in the upper-part of Fig. 55, giving the effect upon the central limb of a number of primary turns equal to an integer + $\frac{1}{2}$.* Since the primary and secondary ampere-turns on the limb balance *on the average*, it follows that the local resultant ampere-conductors in the left-hand window exceed those through the right-hand window. Consequently, more flux will pass into the left yoke than into the right, and the flux unbalance increases the leakage flux. The final result is to increase the exciting current of the transformer and to increase both ε_c and β. The effect can be allowed for by the addition of a short-circuited tertiary circuit consisting of two equal coils in series-opposition

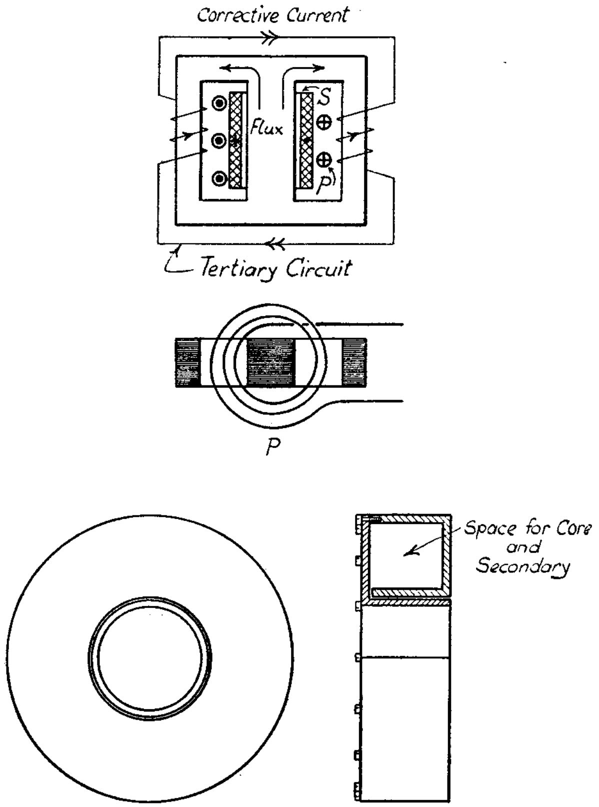


FIG. 55. AUXILIARY WINDING FOR CANCELLING EFFECT OF LEAKAGE IN SHELL-TYPE CURRENT TRANSFORMERS—ARNOLD'S LEAKAGE SHIELD FOR RING-TYPE TRANSFORMERS

---

* R. H. Chadwick, "Transformer windings with fractional windings," *Gen. Elec. Rev.*, vol. 30, pp. 342–345 (1927).

upon the yokes, as shown. Assuming the core flux to be increasing, the instantaneous directions of the e.m.f. induced in the two coils will be as indicated by the single-barbed arrows marked upon them; in consequence of the greater flux the e.m.f. in the left-hand coil exceeds that in the right-hand coil, the resultant driving a corrective current in the direction of the double-barbed arrows in such a way as to demagnetize the left-hand and magnetize the right-hand yoke. Since this current is produced by the originally assumed inequality of division of the flux into the yokes, it follows that the effect of the corrective current is to equalize that division and hence to improve the values of K_c and β by the beneficial effect upon the leakage and the exciting current. By using unequal yoke coils and a series resistance it is possible to use the method to improve the characteristics by a kind of auxiliary magnetization resembling that discussed on p. 102.

13. **The effects of frequency.** The subjects discussed in the preceding sections are concerned with the influence of internal features of design and construction upon the performance of a current transformer; we shall now consider the effects of various external factors, the first being the supply frequency.

With a given current I_s in the secondary circuit of resistance R_s and inductance L_s the secondary induced voltage is

$$E_s = Z_s I_s = I_s\sqrt{[R^2_s + (2\pi f L_s)^2]} \propto f\,B_{max}$$

or

$$\sqrt{[(R_s/f)^2 + (2\pi L_s)^2]} \propto B_{max}.$$

Hence if the frequency is lowered, B_{max}, and with it the exciting current, will be increased. The general effect of this in a normal transformer is to increase K_c and β, and also to increase the rate at which these quantities vary with I_s.

These effects are shown in Fig. 56, taken from Agnew and Fitch's paper previously cited. The curves relate to a 200/5 transformer for a 15 000 volt circuit and show the effect of frequency with various burdens, the values of R_B and L_B being marked on the curves.

As has been shown on p. 121 the iron loss is mostly due to hysteresis, eddy currents being relatively unimportant; hence the iron loss is nearly proportional to the frequency over a considerable range. But the induced voltage in the primary is also proportional to the frequency; hence I_w is practically independent of moderate changes in frequency. If the total secondary impedance Z_s is changed in the same proportion as the frequency, B_{max} remains constant and with it I_w and I_m also; thus the vector diagram will remain as before and the values of K_s and β will be unaltered. To secure this result it will be noted that it is the total secondary burden that is to be reduced to compensate for a reduction in frequency; on account of the internal burden of the secondary winding it follows, therefore, that the external burden must be reduced to a somewhat greater extent than the frequency if the same values of K_s and β are to be preserved at the lower as at the higher frequency. Hence the available external burden will be less. For this reason the B.S. Specification 81 prescribes that a transformer satisfying the requirements for accuracy at 50 cycles per second shall also do so at 25 cycles per second

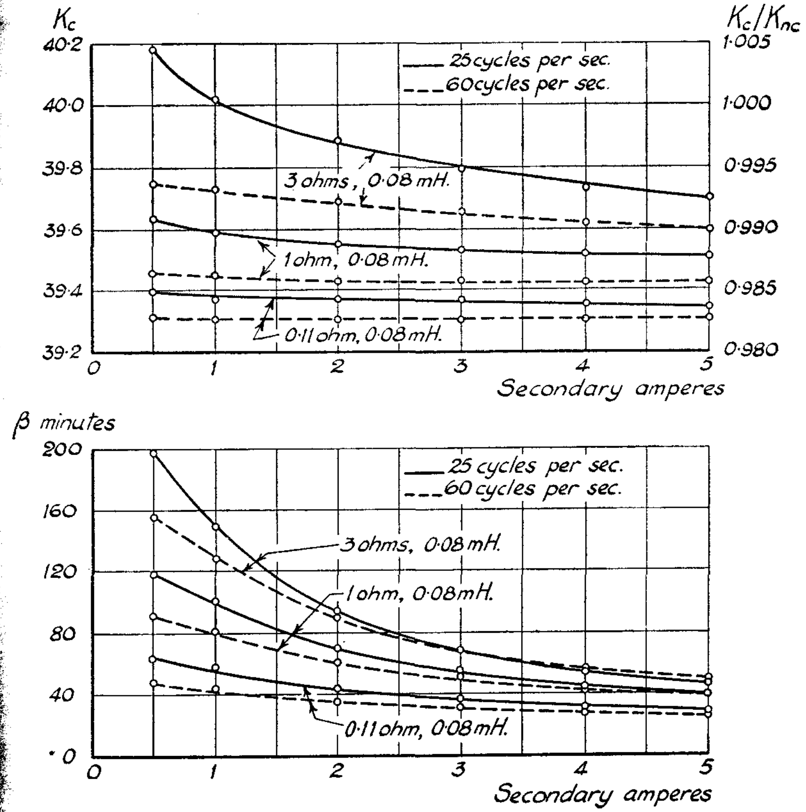


FIG. 56. THE EFFECT OF FREQUENCY AND BURDEN UPON THE CHARACTERISTICS OF A CURRENT TRANSFORMER

if the external burden is reduced to about 40 per cent of the former value.

14. **The effects of wave-form.** Closely related to the influence of frequency is that of wave-form. This problem was investigated by Roessler* in 1896 for power transformers and in 1908 by Lloyd† for the special case of instrument transformers. He showed that the precise effect of the primary current wave-shape upon the ratio and phase-angle depends largely upon whether the primary resistance drop preponderates over the primary leakage reactance drop or the reverse, despite the fact that a flat wave gives higher iron losses than a peaked one. With predominant ohmic drop a peaked primary current wave reduces the ratio and a dimpled wave increases it; this is the usual case, particularly in bar-type ring-wound transformers. The effect is usually negligible, unless the harmonic becomes abnormally great, i.e. at least half the fundamental. The magnitude of the change in ratio is illustrated by the following figures and is of the same order as that due to small changes in frequency.

Shape of Primary Wave	Third Harmonic in per Cent	Per Cent Increase in Ratio Relative to Sine Wave
Peak	11	0
Flat	11	0
Peak	30	+ 0·06
Dimple	30	− 0·06
Peak	68	+ 0·16
Dimple	68	− 0·16

These results were obtained from tests on a 5/5 transformer at 60 cycles per sec. with a secondary current of 4 amperes; the primary current was supplied by two independent sine-wave generators one of which had three times the frequency of the other.

A further interesting question is to what extent the secondary current wave may be regarded as a reduced copy of the primary current wave. By taking oscillograms of the primary and

---

* G. Roessler, "The behaviour of transformers under the influence of alternating currents of different wave-forms," *Elecn.*, vol. 36, pp. 124–126, 150–153, 184–185, 219–222 (1896).

† M. G. Lloyd, "The effect of wave-form upon the voltage ratio of transformers," *Elec. World*, vol. 52, pp. 845–846 (1908); "Effect of wave-form upon iron losses in transformers," *Bull. Bur. Stds.*, vol. 4, pp. 477–510 (1908). See also P. G. Agnew, loc. cit. *ante* (1911).

secondary currents Robinson* was able to show that any difference of shape between the two waves must be very small, even with quite distorted waves. Agnew (loc. cit.) using refined methods of wave analysis has shown the differences to be negligible in all practical cases, since a 20 per cent harmonic in the primary current is reproduced with a distortion of only 1 part in 2 500 in the secondary wave. Hence we can assume the primary and secondary wave-shapes to be identical.

This is true when the primary current wave is determined by the nature and magnitude of the load impedance in the primary circuit, i.e. when the volt-drop across the primary winding is only a small proportion of the total voltage of the primary network, as is invariably the case for a transformer under normal operating conditions. It is not always true, however, when the transformer forms the sole load on the primary network as may occur, for example, when a transformer is under test and supplied from an alternator and step-down transformer at a voltage much smaller than that of the network in which it will normally work; for in this case the primary current wave is determined by the transformer itself and the secondary wave may be of an entirely different form, depending on the degree of saturation in the core. It is necessary in testing current transformers, therefore, to use a sufficiently high voltage and to swamp the effect of the transformer itself by the inclusion in series with its primary of sufficient external resistance.†

The identity of primary and secondary wave-forms is a direct consequence of the low magnetic induction in the core under normal operating conditions. It will not be so when the core becomes saturated, e.g. under overload conditions or with unsymmetrical transients on the primary side, or with excessive leakage fluxes.

15. **The effect of secondary burden.** An increase in the impedance of the secondary burden requires an increase in the induced secondary voltage if a given secondary current is to be maintained at a given frequency, and this increased voltage necessitates a proportionate increase in the main flux and in the exciting current. Hence, in general, an increase of secondary burden has the same influence as a decrease of frequency, increasing both K_c and β, as is shown by the curves in Fig. 56. Not only so, but the change of burden modifies the slope of the characteristics by an amount depending on the

---

* See L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–1039 (1910), and C. H. Sharp and W. W. Crawford, *ibid.*, vol. 29, pp. 1517–1541 (1911).

† W. B. Buchanan, "Overload limitations of high-voltage single-turn primary current transformers," *Univ. of Toronto Eng. Res. Bull.*, No. 1, pp. 191–239 (1919).

changes in resistance *and* reactance that have been made, burdens with the higher power-factor giving the greater slope. For this reason it is essential to state in addition to the volt-amperes to be delivered by the transformer to its external burden at the rated current, the power-factor of that burden; this fact is recognized in the standardizing rules issued by the various national authorities (see p. 10).

16. **The effect of secondary current rating.** The normal rated secondary current for current transformers is 5 amperes, a figure which has been internationally adopted for general use; there are, however, circumstances in which a much smaller current would be advantageous.* In large modern power stations the distance from the control room to the switch cubicles is often considerable, runs of 100 ft. or more being not uncommon. Since 200 ft. of 7/0·028 cable has a resistance of nearly 0·4 ohm, the connecting leads from the switchboard to the current transformer represent a burden of about 10 volt-amperes on the secondary of the latter at 5 amperes rated current. There would be a distinct advantage in such cases in reducing the rating to 1 ampere, or even to 0·5 ampere, thereby greatly diminishing the power wasted in the leads—in the case cited to 0·4 volt-ampere—and enabling transformers of lower rated output to be used.† With an output as low as 5 volt-amperes it is possible to design bar-type transformers for a 1 ampere secondary rating having an accuracy adequate for metering purposes and possessing the advantages of higher mechanical and thermal strength under primary overload than is obtainable from the wound-primary transformers that would be required to obtain the same accuracy with a rating of 5 amperes. The use of 1 ampere increases, for the same number of ampere-turns, the total number of secondary turns, and thus enables more exact adjustment of the turns-ratio to be made than formerly. The reduced secondary current has the disadvantage that the voltage induced in the secondary if it is accidentally open-circuited may be very much higher than with the 5 amperes rating and become dangerous to life (see p. 205). To avoid this Pfiffner‡ has suggested that a portion of the iron circuit should be reduced in cross-section so that it will rapidly saturate when the exciting ampere-turns exceed the normal rated value. In transformers with nickel-iron cores the excessive rise of flux is automatically checked by the fact that the saturation density is lower than with silicon-iron.

17. **Multi-range transformers.** The normal type of current transformer used for switchboard or similar purposes is designed to work with a single value of nominal ratio and a fixed secondary burden. Measurements in the laboratory and test-room must be made over a wide range of currents with a high degree of accuracy and preferably with the use of a minimum amount of expensive apparatus. It is modern practice to use for this purpose a precision ammeter, rated for full-scale deflection with 5 amperes, operated from the secondary of a multi-range current transformer. By the choice of a suitable ratio it is possible to arrange for the readings to be taken at the upper part of the scale, where the precision is highest, for all values of the primary current.

Such multi-range transformers must operate on all ranges within limits of error laid down by the national rules as permissible for precision-type transformers, and maintain their high accuracy over a wide range of burden and frequency. They invariably take the form of a ring core either of silicon-iron plates or, in the most modern types, of nickel-iron strip, carefully assembled, annealed, and insulated; the secondary winding is uniformly distributed round the ring in toroidal form. The primary winding may either be toroidally wound over the secondary, or it may consist of the primary conductor looped once or more through the central hole; alternatively, a combination of both kinds of primary may be used. Since the secondary leakage flux of a ring-type transformer is very small, as shown on p. 131, the ratio and phase-angle characteristics are, for the same primary ampere-turns, independent of the number and position of the primary turns, except in high-ratio transformers with nickel-iron cores, see p. 134. The core with its windings is assembled in a wooden or moulded-insulation case provided with the necessary terminals; the whole arrangement is of a sufficiently small weight and bulk to be readily portable.

A variable ratio can be obtained either by (*a*) various groupings of primary and secondary windings upon the ring, or by (*b*) a cable primary looped through the ring; or by combinations of these.

---

* Edgcumbe and Ockenden, *loc. cit.* (1927).

† G. Keinath, "Zur Frage der Mindestleistung der Messwandler," *Elekt. Zeits.*, vol. 51, pp. 1738–1739 (1930).

‡ E. Pfiffner, "Stromwandler mit kleiner induzierter Spannung bei offenem Sekundärstromkreis," *E.u.M.*, vol. 33, pp. 289–291 (1915).

Five different methods may be distinguished—

(i) Series-parallel connection of the primary winding;
(ii) Tapping of the primary winding;
(iii) Series-parallel connection of the secondary winding;
(iv) Tapping of the secondary winding;
(v) Use of cable primary.

In (i), (ii) and (v) the ratio can only be changed when the primary circuit is open. In (iii) and (iv) it is not necessary to open the primary circuit when altering the ratio, though care should be taken to make the change when the current has a small value or alternatively to short-circuit the secondary circuit if the change is made with full primary current flowing; to this end many transformers are provided with a secondary short-circuiting device, or are designed in such a way as to suffer no ill effects from a temporary opening of the secondary circuit.

Primary series-parallel connection has the advantage that with different rated currents the full ampere-turns are always employed and the copper space is fully utilized; the accuracy of the transformer is, therefore, the same on all ranges. This method is usually limited to changes in the ratio 1 : 2, using a two-section primary, or 1 : 2 : 4, using a four-section primary. The change of connection is made by movable links, by contact plugs or by knife switches. A wider range of ratios is not easily arranged since the links, etc., become numerous, unwieldy, and inconvenient. It is obvious that all the sections of the primary must have exactly equal resistances and reactances if they are to share the current in accurate proportion when they are joined in parallel grouping.

The use of simple links is preferred by many makers, e.g. le Compagnie des Compteurs, while others, such as Messrs. Everett Edgcumbe, use plugs, which are lighter and more convenient. The Siemens & Halske A.G.* employ an ingenious system of contact blades, composed partly of copper and partly of insulation, inserted between the primary section terminals, as indicated for a three-range transformer in Fig. 57.

By tapping the primary winding any desired ratio can be obtained, with full ampere-turns and identical accuracy on each range. The disadvantage over the first method is that the copper space is not fully utilized on all ranges, so that such transformers are somewhat more bulky.

A typical example of the connections for a Siemens & Halske transformer tapped for four ratios is given in Fig. 58, the rating being 1 500 ampere-turns.

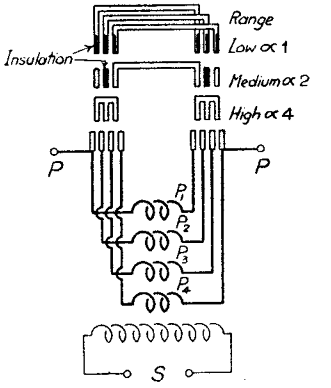


FIG. 57. MULTI-RANGE CURRENT TRANSFORMER WITH SERIES-PARALLEL PRIMARY

Secondary series-parallel connection is not often employed on account of its inflexibility; tapping of the secondary is much to be preferred where a large number of ratios is desired. Since

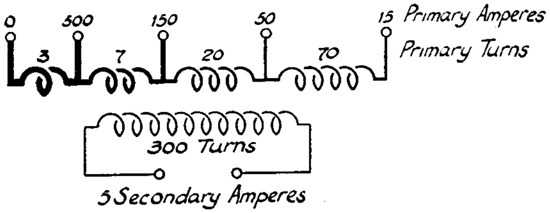


FIG. 58. MULTI-RANGE CURRENT TRANSFORMER WITH TAPPED PRIMARY

the rated secondary current is fixed, changing the ratio by alteration of the secondary turns will vary the rated ampere-turns. As the errors in a transformer vary approximately in proportion to the inverse square of the ampere-turns, it follows that the accuracy changes as the secondary turns are altered; consequently, if the lower ranges are to be of adequate accuracy the higher ranges must operate with a large value of rated ampere-turns.

---

* G. Keinath, "Umschaltbare Stromwandler," *Elekt. Wirts.*, vol. 25, pp. 331–335 (1926); "Vielfach umschaltbare Stromwandler," *Elekt. Zeits.*, vol. 48, pp. 693–694 (1927). For some interesting early examples of multi-range transformers, consult "Präzisions-Stromwandler mit mehreren Messbereichen," *Siemens Nach.*, vol. 8, p. 178 (1906); and K. A. Sterzel, "Stromwandler für Wechselstrom-Leistungsmessungen," *Elekt. Zeits.*, vol. 30, pp. 489–491 (1909).

Most modern multi-range transformers combine the use of methods (ii) and (iv) with (v), obtaining thereby a large variety of ranges; the lower ratios are secured by changing the primary and secondary tappings, while the higher ones are provided by the use of a cable primary looped once or more through the central hole in the core. A typical example of such a transformer* (Siemens & Halske A.G.) is shown in Fig. 59. Two sizes are made, the first giving eight ranges from 15 to 600 amperes, the second eleven ranges from 15 to 1 500 amperes; in both cases the ranges 15, 50, 150 are given by primary tappings. The higher ratios are given with cable primary. Similar transformers are manufactured by other makers, e.g. Messrs. Everett, Edgcumbe in their "Omni-range" series and Messrs. Elliott in their "Multiversal" and "Nikron" types, all of which give a great variety of ranges by similar means.

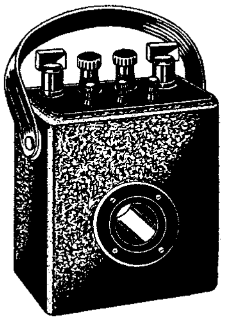


Fig. 59. Portable Multi-range Current Transformer (S. & H.)

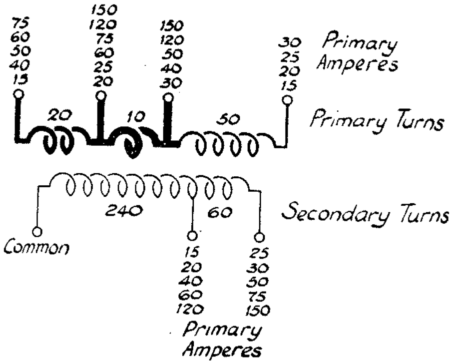


Fig. 60. Multi-range Current Transformer with Tapped Primary and Secondary

As an example of the extreme flexibility of the multi-range principle the transformer manufactured by Messrs. Hartmann and Braun,† giving no less than 41 ranges between 15 and 1 900 amperes, may be

mentioned. Ten ranges are given by selection of appropriate tappings of the primary and secondary windings provided on the core, as Fig. 60 indicates. The remaining ranges from 200 to 1 900 amperes by steps of 50 amperes are given with from 6 to 1 turns of cable usng as a secondary winding either sections of the secondary alone or portions thereof joined in series with parts of the wound primary in order to secure the required number of ampere-turns. The output depends on the primary range and varies from 15 to 60 volt-amperes for an accuracy of the 0·5 class. The weight is 11 kg. (24 lb.), and the transformer may be used in circuits up to 750 volts. By the provision of an insulating tube through the central hole the safe voltage may be raised to 6 000.

When measuring apparatus is required for tests on site its easy portability becomes an important factor; this has been realized by several manufacturers in the design of miniature portable instruments with their accompanying transformers. Messrs. Ferranti Ltd. have developed a series of 2½ in. scale instruments in conjunction with which they supply a multi-range current transformer for 0·5 ampere secondary current with primary windings for 2·5, 10 and 25 amperes, higher values being dealt with by the use of a cable primary, one turn sufficing for 200 amperes. The transformer is contained in a moulded case measuring 4¾ in. × 3¾ in. × 1⅜ in. (12 × 9·5 × 3·5 cm.). Messrs. Everett, Edgcumbe manufacture the "Cadet" transformer, measuring 4¾ in. × 4⅛ in. × 2⅛ in., giving ranges of 5, 25, 50, 125 and 250 amperes with the primary winding. In both these examples the dimensions are about half those of transformers designed for laboratory use, and the weight is considerably reduced. One of the most interesting examples of this modern tendency to produce miniature portable precision instruments is provided by the 11·5 × 10·8 × 4·6 cm. (4·5 in. × 4·25 in. × 1·8 in.) series of the A.E.G., accompanied by a current transformer of the same size weighing barely one kilogramme.* The secondary current is 5 amperes, two built-in primary ranges, 15 and 50 amperes, being provided; ranges up to 600 amperes are obtained by the use of cable loops. The output is 5 volt-amperes and the accuracy is within the 0·5 class. Since the instruments are identical in size and shape and are so small, any six of them with suitable leads can easily be accommodated in a fitted carrying case no larger than an ordinary attaché case.

18. **Current transformers in parallel. Totalizing transformers.** The summation of the current or power delivered by a number of parallel feeders from a generating station is a measurement of great practical importance; a discussion of the various methods, both mechanical and electrical, used for such multiple metering is, however, outside the scope of the present volume. Certain of the methods involve the use of current transformers in special ways, which may have a considerable effect upon their accuracy; it is within our purpose to discuss only this aspect of the problem, leaving the reader

---

* G. Keinath, loc. cit. (1927); also see "Stromwandler-Tragbare Wandler," *Arch. f. tech. Mess.*, Z285–1 (1931); "Umschaltbare Stromwandler," *idem.*, Z288–1 (1932); "Tragbare Stromwandler," *idem.*, Z285–2 (1932).

† A. Keller, "Neuer vielfach umschaltbarer tragbarer Präzisionstromwandler," *Elekt. Zeits.*, vol. 48, pp. 1795–1797 (1927); "Universal-Stromwandler," *E.u.M.*, vol. 46, pp. 1072–1074 (1928); "Präzisionsstromwandler," *Elekt. Zeits.*, vol. 54, pp. 1258–1259 (1933); also see "H und B Präzisions-Stromwandler," *Arch. f. tech. Mess.*, Z285–3, Apr. (1934), for a description of the newest transformers with nickel-iron cores.

* H. Vahl, "Tragbarer Präzisions-Stromwandler mit mehreren Messbereichen," *A.E.G. Mitt.*, part 7, pp. 497–499 (1930).

to consult the technical literature for practical details of the various methods.

The idea of using current transformers to sum the currents in a number of circuits appears to be due originally to Ayrton and Mather[*] who in 1893 accomplished the summation[†] by leading the various feeders through the central hole of a single ring-type transformer provided with an instrument in its secondary circuit scaled to read the desired sum. Their invention was before its time and it is only recently, with the tremendous growth in power distribution by alternating currents,

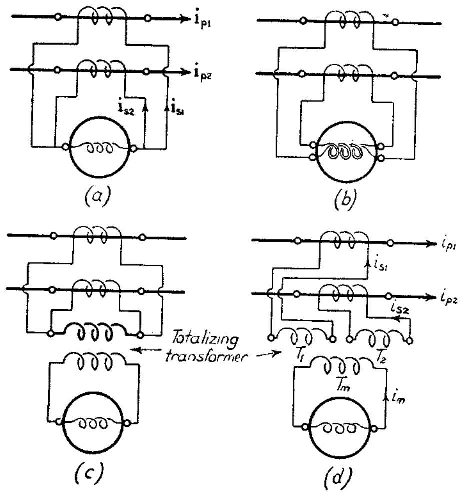


Fig. 61. Current Summation with Current Transformers

that the question has become one of practical urgency. The simplest method now used is shown for the case of two feeders in Fig. 61 (*a*). Each feeder[‡] is provided with its own transformer, thereby securing adequate dielectric and mechanical strength together with high accuracy for each individual transformer, the secondary windings being connected in parallel to the current element of a suitable meter; the transformers

---

[*] W. E. Ayrton and T. Mather, *British Patent*, No. 24 217 (1893).

[†] The word "summation" is derived from the verb "to sum," to denote either the act of summing or the resulting aggregate. The author wishes strongly to protest against the growing use, by a false etymology, of the pseudo-verb "to summate."

[‡] It will be understood that the discussion in the text refers to the feeders of one phase in a polyphase system.

must be of "precision" class and should be liberally rated, as will be explained below. Successful parallel operation demands the satisfaction of certain conditions.[*] First, the feeders must all belong to the same phase and terminate at the same bus-bars; for convenience sake the feeders should not be too numerous. Second, since the transformer primaries are in parallel at the bus-bars and the secondaries are in parallel at the meter, it follows, as in the parallel-working of power transformers, that the transformers should have the same nominal ratio; they should, moreover, have similar ratio errors and phase-angles. If all the secondaries are designed for 5 amperes, equality of ratio implies that the full rated currents of all the feeders must be equal; alternatively, if the feeders have different ratings the rated secondary currents must be different if equality of ratio is to be maintained. It is further necessary that the secondaries should be paralleled *at the meter terminals* and that their connecting leads thereto should be of equal resistance. The importance of these conditions will be apparent in the extreme case when one primary circuit carries no current, so that the secondary circuit is in parallel with the meter coil as the burden on the active transformer; consequently the errors of this transformer will be quite different when one feeder is inactive from their values when both transformers are sharing the load. It is not difficult to show that the "paralleling effect" will be least when the secondaries are joined at the meter and greatest when paralleled at the transformers.[†] With non-inductive burdens the ratio is not much affected but the phase-angle may be considerably changed; the converse is true with inductive burdens. Since the effect depends upon the magnitudes of the exciting currents of the transformers, it is smaller with transformers of higher volt-ampere rating since the exciting currents in them are of less relative importance than is the case in transformers of lower rating. With proper care in the choice and installation of the transformers the total error in totalizing two or more circuits should not exceed the error of an individual

---

[*] "Totalizing the output of two or more a.c. circuits on one watt-hour meter by paralleling the secondaries of current transformers," *Proc. N.E.L.A.*, vol. 85, pp. 1001–1014 (1928). An excellent discussion of the whole subject of summation is given by H. Vahl, "Summierung mit Stromwandlern," *Arch. f. tech. Mess.*, V3224–1 (May, 1933).

[†] O. Howarth, "The metering of three-phase supplies," *Journal I.E.E.*, vol. 69, pp. 381–393 (1931). E. W. Hill and G. F. Shotter, "Current transformer summations," *Journal I.E.E.*, vol. 69, pp. 1251–1264 (1931).

transformer,* except in the more extreme cases of unequal load division.

The errors in ratio and phase-angle are easily worked out; as an example consider the two transformers shown in Fig. 61 (*a*). Let I_{p1}, I_{p2} be the r.m.s. primary currents, $\boldsymbol{i}_{p1}$ and $\boldsymbol{i}_{p2}$ the corresponding harmonic vectors; then if the second current leads on the first by ψ,

$$\boldsymbol{i}_{p2} = (I_{p2}/I_{p1})\,(\cos\psi + j\sin\psi)\boldsymbol{i}_{p1} = K\varepsilon^{j\psi}\boldsymbol{i}_{p1}$$

expresses the relation between them. Further let K_{c1}, K_{c2} be the current ratios, ε_{c1}, ε_{c2} the fractional ratio errors, β_1, β_2 the phase-angles of the transformers; then if the secondary harmonic vectors are $\boldsymbol{i}_{s1}$ and $\boldsymbol{i}_{s2}$ and K_{nc} is the nominal ratio of either transformer, from p. xxiv

$$\begin{aligned}\boldsymbol{i}_{s1} = -(1/K_c)\varepsilon_1^{\,j\beta_1}\boldsymbol{i}_{p1} &\fallingdotseq -(1/K_{nc})\,(1 + \varepsilon_{c1})\,(1 + j\beta_1)\boldsymbol{i}_{p1} \\ &\fallingdotseq -(1/K_{nc})\,(1 + \varepsilon_{c1} + j\beta_1)\boldsymbol{i}_{p1}\end{aligned}$$

since ε_{c1} and β_1 are small quantities such that their product is negligible in comparison with unity; likewise,

$$\boldsymbol{i}_{s2} \fallingdotseq -(1/K_{nc})\,(1 + \varepsilon_{c2} + j\beta_2)\boldsymbol{i}_{p2} \fallingdotseq -(1/K_{nc})\,(1 + \varepsilon_{c2} + j\beta_2)K\varepsilon^{j\psi}\boldsymbol{i}_{p1}.$$

The ratio of the total secondary vector to the total primary vector is

$$\begin{aligned}\frac{\boldsymbol{i}_{s1} + \boldsymbol{i}_{s2}}{\boldsymbol{i}_{p1} + \boldsymbol{i}_{p2}} = -\frac{1}{K_c}\varepsilon^{j\beta} &\fallingdotseq -\frac{1}{K_{nc}}(1 + \varepsilon_c)\,(1 + j\beta) \fallingdotseq -\frac{1}{K_{nc}}\,(1 + \varepsilon_c + j\beta)\ \text{say}, \\ &\fallingdotseq -\frac{(1 + \varepsilon_{c1} + j\beta_1) + K(1 + \varepsilon_{c2} + j\beta_2)\,(\cos\psi + j\sin\psi)}{(1 + K\cos\psi) + jK\sin\psi}\cdot\frac{1}{K_{nc}}\end{aligned}$$

Hence,

$$1 + \varepsilon_c + j\beta \fallingdotseq \frac{\left\{\begin{matrix}[(1 + K\cos\psi) + \varepsilon_{c1} \\ + K(\varepsilon_{c2}\cos\psi - \beta_2\sin\psi)] + j[\beta_1 \\ + K\sin\psi + K(\varepsilon_{c2}\sin\psi + \beta_2\cos\psi)]\end{matrix}\right\}[(1 + K\cos\psi) - jK\sin\psi]}{(1 + K^2 + 2K\cos\psi)}$$

Separating the components of these operators gives

$$\begin{aligned}(1 + K^2 + 2K\cos\psi)\,(1 + \varepsilon_c) &= (1 + K^2 + 2K\cos\psi) + (1 + K\cos\psi)\varepsilon_{c1} \\ &\quad + K(\varepsilon_{c2}\cos\psi - \beta_2\sin\psi) + K\beta_1\sin\psi + K^2\varepsilon_{c2}; \\ (1 + K^2 + 2K\cos\psi)\beta &= (1 + K\cos\psi)\beta_1 + K(\varepsilon_{c2}\sin\psi + \beta_2\cos\psi) \\ &\quad - K\varepsilon_{c1}\sin\psi + K^2\beta_2,\end{aligned}$$

expressing the overall fractional ratio error ε_c and phase-angle β of the combination in terms of those of the individual transformers. When the loads on the feeders have approximately the same power-factor, as is commonly the case in practice, put $\psi = 0$, $\cos\psi = 1$ and $\sin\psi = 0$; then by simple reductions,

$$\varepsilon_c = \frac{I_{p1}}{I_{p1} + I_{p2}}\,\varepsilon_{c1} + \frac{I_{p2}}{I_{p1} + I_{p2}}\,\varepsilon_{c2},$$

$$\beta = \frac{I_{p1}}{I_{p1} + I_{p2}}\,\beta_1 + \frac{I_{p2}}{I_{p1} + I_{p2}}\,\beta_2.$$

By an exactly similar process for n transformers it may be shown that

$$\varepsilon_c = \sum_1^n \frac{I_{px}}{\Sigma I_{px}}\,\varepsilon_{cx} \text{ and } \beta = \sum_1^n \frac{I_{px}}{\Sigma I_{px}}\,\beta_x.$$

As a numerical example of the properties of these expressions take the case of two transformers with $\varepsilon_{c1}/\varepsilon_{c2} = \frac{1}{2}$, 1 and 2; then

I_{p1}/I_{p2}	0	0·5	1·0	1·5	2·0
Value of $\dfrac{\varepsilon_c}{\varepsilon_{c2}}$ for $\varepsilon_{c1}/\varepsilon_{c2} = \frac{1}{2}$.	1·000	0·833	0·750	0·700	0·667
$\varepsilon_{c1}/\varepsilon_{c2} = 1$.	1·000	1·000	1·000	1·000	1·000
$\varepsilon_{c1}/\varepsilon_{c2} = 2$.	1·000	1·333	1·500	1·600	1·667

This table shows that the total error in this case at no time exceeds the greater value of the individual errors of the transformers.

Various methods have been devised to minimize the "paralleling effect" by removing the direct coupling of the secondary windings. Hill and Shotter (loc. cit.) have used the arrangement of separate interlaced meter coils, as shown in Fig. 61 (*b*), with very marked success. It is clear, however, that this method becomes impracticable when the feeders are numerous; this difficulty may be overcome by the use of an intermediate summing or totalizing transformer, the introduction of which appears to be due to M. B. Field.* In Field's arrangement the secondaries are paralleled at the primary of the totalizing transformer, as in Fig. 61 (*c*), the secondary of which is designed for a normal current of 5 amperes and supplies this rated current to a standard type of meter; the method is analogous to Fig. 61 (*a*) and is subject to similar sources of error. Hill and Shotter, and independently Vahl,† have described the arrangement of Fig. 61 (*d*), which is the analogue of Fig. 61 (*b*), in which the paralleling effect is eliminated. The primary of the totalizing transformer is divided up into sections, one for each of the feeders to be totalized. The main current

---

* A. M. Wiggins, "Parallel operation of current transformers for totalizing two or more circuits," *Elect. J.*, vol. 26, pp. 379–381 (1929); K. Gocht, "Messfehler bei Summenschaltung von Stromwandlern," *Elekt. Wirts.*, vol. 31, pp. 485–487 (1932), *E.u.M.*, vol. 51, p. 149 (1931).

* *British Patent*, No. 143 160, 31st Dec. (1919).

† H. Vahl, "Summation durch Summenstromwandler," *Elekt. Wirts*, vol. 30, pp. 256–258 (1931).

transformers need not have the same nominal ratio; all that is necessary is that the overall ratio of a given transformer and its section of the totalizing transformer shall be the same for all feeders. Moreover, no part of the secondary or tertiary circuit need carry a greater rated current than 5 amperes. Since the ampere-turns on the intermediate transformer can be made large its contributory errors are very small, and can be still further reduced by the use of a nickel-iron alloy core. A single totalizing transformer can deal with a large number of feeders, since its primary can be subdivided into any desired number of sections. If the number of feeders is very great it may be more convenient to divide them into groups, each group having its own totalizing transformer; the secondary currents of these can then be summed in turn by another totalizing transformer which supplies the meter.

By analogy with Fig. 61 (*d*), let the instantaneous values of the primary currents in n feeders be $i_{p1}, i_{p2}, i_{p3}, \ldots i_{pn}$, the corresponding secondary currents in the main current transformers being $i_{s1}, i_{s2}, i_{s3}, \ldots i_{sn}$. Let $T_1, T_2, T_3 \ldots T_n$ be the turns in the n sections of the totalizing transformer's primary and T_m its secondary turns, i_m being the instantaneous current in the meter. Equating ampere-turns on the primary and secondary sides of the totalizing transformer,

$$\begin{aligned} i_m T_m &= i_{s1}T_1 + i_{s2}T_2 + i_{s3}T_3 + \ldots i_{sn}T_n \\ &= (i_{p1}/K_{c1})T_1 + (i_{p2}/K_{c2})T_2 + (i_{p3}/K_{c3})T_3 + \ldots (i_{pn}/K_{cn})T_n \end{aligned}$$

where K_{c1}, K_{c2}, etc. are the ratios of the transformers. If the values of T_1, T_2, etc. are adjusted to make

$$T_1/K_{c1} = T_2/K_{c2} = T_3/K_{c3}, \text{ etc.,}$$

and we use the well-known theorem

$$a/b = c/d = e/f = \ldots \equiv (a + c + e \ldots)/(b + d + f \ldots),$$

then

$$i_m = \frac{T_1 + T_2 + T_3 + \ldots T_n}{T_m} \cdot \frac{1}{K_{c1} + K_{c2} + K_{c3} + \ldots K_{cn}} \cdot (i_{p1} + i_{p2} + i_{p3} + \ldots i_{pn});$$

i.e. the meter current is a copy to scale of the sum of the currents the feeders. If the main transformer and totalizing transformer se ondaries are rated at 5 amperes it follows that for this common ca

$$T_m = T_1 + T_2 + T_3 + \ldots T_n.$$

As a numerical example consider a two-feeder system with tran formers rated at 150/5 and 50/5, and the meter at the overal rated ratio of 200/5. Taking the rated values of the totalizin transformer ampere-turns at 1000, then $T_m = 200$ turns and $T_1 +$

= 200. It will suffice to take the actual ratios equal to the nominal ratios; then

$$\begin{aligned} 200 i_m &= \frac{5}{150}T_1 i_{p1} + \frac{5}{50}T_2 i_{p2}, \text{ from the first equation,} \\ &= 200 . \frac{5}{200}(i_{p1} + i_{p2}), \text{ from the last equation,} \end{aligned}$$

from which, $T_1 = 150$ turns, $T_2 = 50$ turns. The contributions of each section to the meter current are 3·75 and 1·25 amperes respectively.

If ε_m is the fractional ratio error of the totalizing transformer itself, it is not difficult to show in the way adopted for Fig. 61 (*a*) that the overall ratio error in metering the currents in n feeders is

$$\varepsilon_c = \frac{I_{p1}}{I_{p1} + I_{p2} + \ldots I_{pn}}\varepsilon_1 + \frac{I_{p2}}{I_{p1} + I_{p2} + \ldots I_{pn}}\varepsilon_2 + \ldots \frac{I_{pn}}{I_{p1} + I_{p2} + \ldots I_{pn}}\varepsilon_n + \varepsilon_m,$$

with an exactly similar expression for the overall phase-angle.

The totalizing transformer lends itself readily to the metering of currents in interconnected systems at different voltages. Fig. 62 shows the arrangement for a single phase only, feeders 1 and 2 coming from generators at a rated voltage of V_{I} and feeders 3 and 4 at a voltage V_{II}; the system can be extended to any number of feeders.

If the metering is to be done at the voltage V_{I}, the current at the V_{I} bus-bars is $i_1 + i_2 + (V_{\mathrm{II}}/V_{\mathrm{I}})(i_3 + i_4)$. Equating ampere-turns in the totalizing transformer,

$$\begin{aligned} i_m &= \frac{1}{T_m}(i_{s1}T_1 + i_{s2}T_2 + i_{s3}T_3 + i_{s4}T_4) \\ &\equiv \frac{1}{T_m}\left(\frac{i_1}{K_{c1}}T_1 + \frac{i_2}{K_{c2}}T_2 + \frac{i_3}{K_{c3}}T_3 + \frac{i_4}{K_{c4}}T_4\right) \end{aligned}$$

If we now put $T_1/K_{c1} = T_2/K_{c2}$ and $T_3/K_{c3} = T_4/K_{c4} = T_{\mathrm{I}}V_{\mathrm{II}}/K_{c1}V_{\mathrm{I}}$, then

$$i_m = \frac{1}{T_m} \cdot \frac{T_1}{K_{c1}}\left[i_1 + i_2 + \frac{V_{\mathrm{II}}}{V_1}(i_3 + i_4)\right] = \frac{T_1 + T_2 + T_3 + T_4}{T_m} \cdot \frac{1}{(K_{c1} + K_{c2}) + (V_{\mathrm{II}}/V_{\mathrm{I}})(K_{c3} + K_{c4})}\left[i_1 + i_2 + \frac{V_{\mathrm{II}}}{V_{\mathrm{I}}}(i_3 + i_4)\right],$$

as required. These expressions are easily generalized for any number of feeders on each bus-bar.

A further interesting case of parallel operation has recently been discussed by Krüzner.* It may sometimes happen that a

---

* H. Krüzner, "Die Parallelschaltmöglichkeit von Stromwandlern," *E.u.M.*, vol. 53, pp. 133–136 (1935).

large current is to be measured and that a suitable current transformer is not available. In such a case two smaller transformers may be used as a substitute, their primary windings being connected in parallel and their secondaries in series. If the transformers are similar in construction and rating, calculation and experiment show that the overall ratio-error and phase-angle of the group are no greater than would be obtained with a single large transformer of the same total rating. The errors are calculable as special cases of the expressions on p. 150 with $I_{p1} = I_{p2}$. Krüzner further shows that quite considerable inequality of the two transformers does not make the accuracy of measurement appreciably worse.

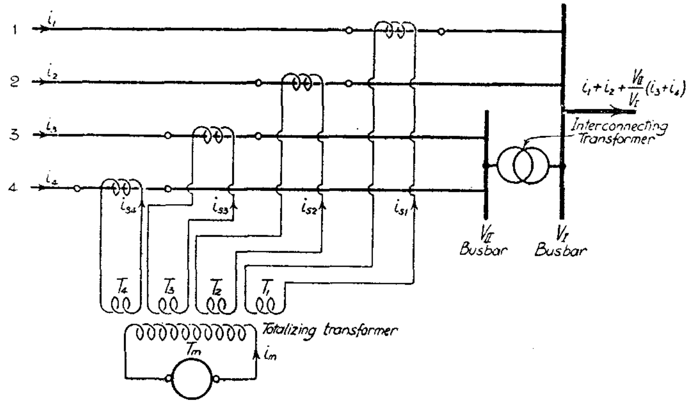
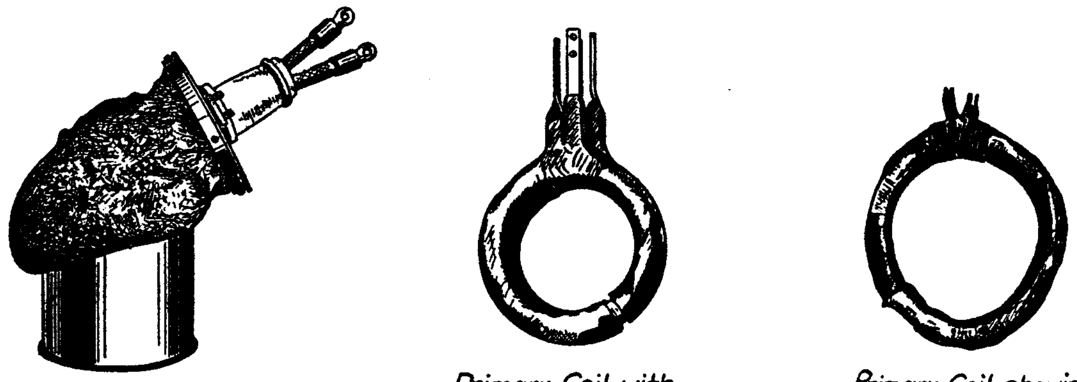


FIG. 62. TOTALIZING CURRENTS IN FEEDERS AT DIFFERENT VOLTAGES

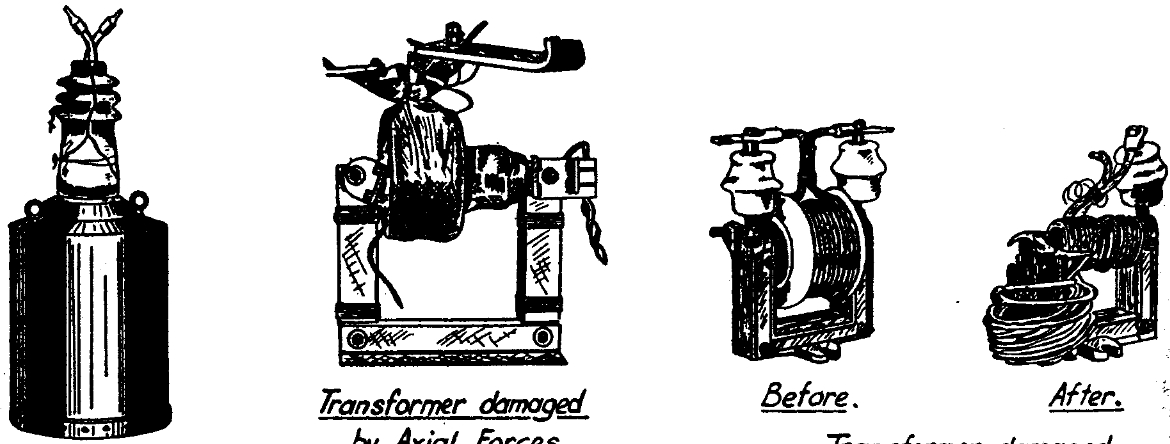
19. **The effects of short-circuit currents (general).** Current transformers are designed to operate continuously at full rated current and to withstand a reasonable overload for a definite interval, in both cases without undue temperature rise; appropriate thermal limits are prescribed in the various national standards and will be referred to on p. 591. When short-circuit faults occur on the primary network, currents of many times the normal magnitude may flow, not only in large power systems but also in small systems and in branch circuits. Although the duration of the short-circuit currents is very limited, since the fault is quickly cleared by the opening of the main oil-switch in the affected section, their effects are so severe that the current transformers may be seriously damaged or even completely destroyed by the abnormal mechanical and thermal conditions imposed upon them; some typical



Exploded Compound-filled Transformer.

Primary Coil with Insulation ruptured by Hoop Tension.

Primary Coil showing deformation of parts near Iron Core.



Insulator fractured by force of repulsion between lead-in conductors.

Transformer damaged by Axial Forces.

Before. After.

Transformer damaged by Axial Forces.

FIG. 63. CURRENT TRANSFORMERS DAMAGED BY EXCESSIVE OVERLOAD CURRENTS

instances are shown in Fig. 63.
considerable practical interest in
transformers under short-circuit
the advent of super-power statio
where the energy that may be mon
may reach an enormous value.

Short-circuit currents have four i
transformers: (*a*) The production o
angle. (*b*) Excessive heating. (*c*) T
ical forces sufficiently great to defo
(*d*) The generation of transient volt
consider these problems here; t
examined on p. 61; the detailed t
will form the topics of Sections 20,

It has been shown in Section 11
of a current transformer is appro
increased from zero up to several t
being due to the fact that the flux d
very low. In other words, so long
the secondary current will grow ne
the primary current. When the
large multiple of its rated value,
short-circuit conditions, the straigh
characteristic is passed, saturation s
away" from approximate consta
i.e. the secondary current is small
had saturation not occurred and t
These facts are well illustrated in
striking way by Fig. 64; these cur
published by Edgcumbe and Ocke
transformer with a silicon-iron rin
primary and secondary windings ha
tests being made at 50 cycles per se
burden of 30 volt-amperes. It wil
away" point should occur well outs
apparatus operated by the transfor
controlled by suitable choice of the

---

* L. Dorfman, "The choice of instrumen
pp. 341–342 (1920). C. O. Werres, "Curren
tion from the overcurrent standpoint," *Ge*
(1932). J. Grillet, "La déstruction des tr
Schw. Elekt. Verein, vol. 24, pp. 97–100 (1
circuit rating of current transformers," *B.T*
(1933); *Elec. Times*, vol. 84, pp. 507–508 (1

instances are shown in Fig. 63. There is, in consequence, considerable practical interest in the behaviour of current transformers under short-circuit conditions, especially since the advent of super-power stations and interlinked systems where the energy that may be momentarily passed into a fault may reach an enormous value.

Short-circuit currents have four important effects on current transformers: (*a*) The production of large ratio error and phase-angle. (*b*) Excessive heating. (*c*) The development of mechanical forces sufficiently great to deform the windings and leads. (*d*) The generation of transient voltage rises.* We shall briefly consider these problems here; the theory of (*a*) has been examined on p. 61; the detailed treatment of (*b*), (*c*) and (*d*) will form the topics of Sections 20, 21 and 26 respectively.

It has been shown in Section 11 of Chapter II that the ratio of a current transformer is approximately constant as I_p is increased from zero up to several times rated current I_{np}, this being due to the fact that the flux density in the core is normally very low. In other words, so long as the core is unsaturated the secondary current will grow nearly in linear proportion to the primary current. When the primary current becomes a large multiple of its rated value, such as would occur under short-circuit conditions, the straight part of the magnetization characteristic is passed, saturation sets in and the ratio "breaks away" from approximate constancy and rapidly increases, i.e. the secondary current is smaller than it would have been had saturation not occurred and the ratio remained constant. These facts are well illustrated in Fig. 25, but in a still more striking way by Fig. 64; these curves are plotted from results published by Edgcumbe and Ockenden (loc. cit.) on a special transformer with a silicon-iron ring core provided with equal primary and secondary windings having 1 000 ampere-turns, the tests being made at 50 cycles per sec. with a non-reactive rated burden of 30 volt-amperes. It will be clear that the "break-away" point should occur well outside the working range of the apparatus operated by the transformer; its position is readily controlled by suitable choice of the primary ampere-turns and

---

* L. Dorfman, "The choice of instrument transformers," *Elect. J.*, vol. 17, pp. 341–342 (1920). C. O. Werres, "Current transformer design and application from the overcurrent standpoint," *Gen. Elec. Rev.*, vol. 35, pp. 544–549 (1932). J. Grillet, "La déstruction des transformateurs de courant," *Bull. Schw. Elekt. Verein*, vol. 24, pp. 97–100 (1933). J. G. Wellings, "The short-circuit rating of current transformers," *B.T.H. Activities*, vol. 9, pp. 151–154 (1933); *Elec. Times*, vol. 84, pp. 507–508 (1933).

the sectional area of the core, i.e. by regulating the onset of saturation. In transformers operating measuring instruments the falling-off of secondary current with very large primary current is a natural protection to the instruments; the "break-away" may conveniently occur not far above the rated primary ampere-turns. In transformers operating protective gear, especially those actuating some form of current-balance system, correctness of ratio must be maintained up to large overloads, even up to 100 or more times the rated primary current; this can be secured by the use of a core of sufficiently large section.

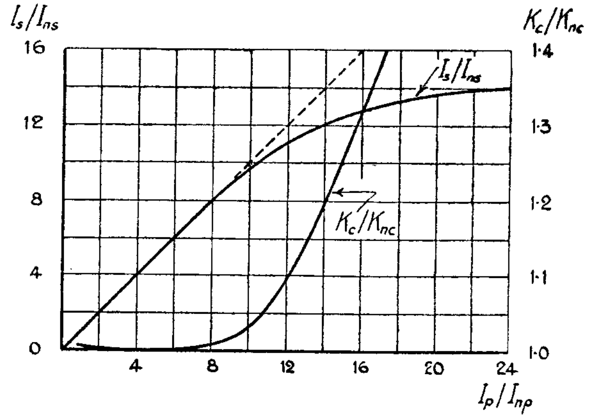


FIG. 64. OVERLOAD RATIO CURVES FOR CURRENT TRANSFORMER

The thermal effects of the short-circuit current depend upon the r.m.s. value of the current and upon the duration of the short-circuit. The mechanical effects depend upon the initial maximum peak-value attained by the current. Of the two effects the thermal effect is the more dangerous, on account of the deleterious action of heat upon the insulation of the windings; the mechanical effect may, nevertheless, be quite serious in certain cases. For this reason much attention has recently been given to the thermal rating of transformers as well as to their mechanical strength; the various problems involved will be considered in the following sections.

20. **The thermal effects of short-circuit currents.** There are three sources of heat in a current transformer, viz. the core, the secondary winding, and the primary winding; of these, the heat generated in the core is usually quite negligible, even with the abnormal flux set up during short-circuit conditions. The current density in the secondary copper is usually less than

that in the primary and, as we have seen, the ratio of the transformer increases rapidly with large values of primary current; hence, as a general rule, the heating of the primary copper is much more important than that of the secondary and will be exclusively considered here. With rapid-action protective devices a short-circuit is cleared by the opening of the main switchgear within about one-half second; with time-lag protection the fault may be maintained for a few seconds. In such short intervals the heat generated in the primary is unable to escape by conduction through the mass of the winding to its cooling surface, and is all utilized in raising the temperature of the winding. Since the amount of heat produced is large the temperature rise may be considerable. In an extreme case, where the cross-section of the primary copper has been unduly economized, the winding may even melt and the transformer act as an efficient fuse! In other cases the rise of temperature may reduce the mechanical strength of the copper to such an extent that the mechanical forces produced by the short-circuit current can easily deform or even rupture the windings.

Thermal effects may be a particular source of danger in transformers which are compound-filled. The heat may cause the compound to melt and expand, or to decompose with the production of inflammable gases; high internal pressures may be thus set up in the transformer case, with risk of explosion, ignition of the melted compound and considerable fire hazard. An example is shown in Fig. 63. These defects are to some extent shared by oil-immersed transformers;* for this reason both types are rated for much lower temperature rise than air-cooled transformers. There has been a recent tendency to remove these dangers by the use of non-inflammable insulating materials, such as porcelain, and incombustible fillings, such as sand; examples will be encountered later.

The earlier types of air-cooled current transformers were usually unsatisfactory from the point of view of thermal security. Numerous tests† showed that though the transformers

---

* An interesting instance of failure of oil-filled transformers occurred at the Taylor's Lane Power Station of the North Metropolitan Electric Power Supply Co. on 6th Dec., 1933. *See Elec. Times*, vol. 84, pp. 770–771, 816 (1933).

† P. Torchio, "High current tests on high tension switchgear," *Trans. Amer. I.E.E.*, vol. 40, pp. 61–86 (1921). J. B. Gibbs and L. Dorfman, "Temperature and mechanical stresses in current transformers," *Elec. World*, vol. 79, pp. 221–223 (1922). "Tests for rating current transformers," *Elec. World*, vol. 82, pp. 169–173 (1923).

withstood the mechanical forces on short-circuit, failure was frequently of a thermal nature. With the growth of large power schemes the problem of thermal design has recently received considerable attention, transformers often being designed in the same way as circuit-breakers* for definite short-circuit ratings. In air-cooled transformers this involves the provision of a primary conductor with a sufficiently large section, such that a specified short-circuit current may be carried for a given interval without the temperature rising to a value that will cause permanent damage to the organic insulating materials, such as cotton, paper, varnish or compound used in the construction of the windings.

Let τ_1 be the temperature of the winding initially, τ_2 its temperature at the end of the period of short-circuit. When the temperature has some intermediate value τ the resistance of the winding is $R = R_0(1 + \alpha\tau)$, where R_0 is the resistance at 0° C., and $\alpha = 1/234{\cdot}5 = 42{\cdot}8 \times 10^{-4}$ is the temperature coefficient of resistance for copper. If I_{sc} is the r.m.s. value of the current during the whole interval of short-circuit, t seconds, the power turned into heat is $I_{sc}{}^2R$ watts, I_{sc} being in amperes and R in ohms. Let M be the mass of the winding in grammes and σ its specific heat in calories per gramme per degree C. Then, since t is small, no heat is radiated and we may equate the rate of generation of heat to its rate of absorption; in calories,

$$M\sigma\int_{\tau_1}^{\tau_2}\frac{d\tau}{1+\alpha\,\tau} = 0{\cdot}24\; I_{sc}{}^2R_0\int_0^t dt$$

If we write,

$$\tau_1' = \tau_1 + (1/\alpha) \text{ and } \tau_2' = \tau_2 + (1/\alpha),$$

then,

$$(M\sigma/\alpha)\log_\varepsilon(\tau_2'/\tau_1') = 0{\cdot}24\; I_{sc}{}^2R_0t.$$

Now let l_c be the length of copper in cm., A_c its cross-sectional area in sq. cm., and γ_c its density, then

$$M = l_c\,A_c\,\gamma_c \text{ and } R_0 = \rho_0 l_c/A_c,$$

where ρ_0 is the resistivity of copper at 0° C. Substituting these expressions and writing $\delta = I_{sc}/A_c$ for the current-density in amperes per sq. cm.,

$$\log_\varepsilon\frac{\tau_2'}{\tau_1'} = \frac{0{\cdot}24\;\rho_0\alpha}{\sigma\gamma_c}\,\delta^2t.$$

---

* G. L. E. Metz, "Short-circuit effects upon current transformers," *Elec. Rev.*, vol. 101, pp. 892–894 (1927).

As average values we may take $\rho_0 = 1{\cdot}6 \times 10^{-6}$ ohm-cm., $\alpha = 42{\cdot}8 \times 10^{-4}$, $\sigma = 0{\cdot}095$, and $\gamma_c = 8{\cdot}93$; converting from Naperian to common logarithms,

$$\log_{10}(\tau_2'/\tau_1') = 8{\cdot}44\;\delta^2t\;10^{-10},$$

which is in agreement with a formula given by Grillet.* Let δ_n denote the current-density with normal rated primary current, so that $\delta = k_{sc}\delta_n$ is the current-density with the sustained short-circuit current I_{sc}; then

$$k_{sc} = \frac{3{\cdot}35 \times 10^4}{\delta_n\sqrt{t}}\sqrt{\log_{10}\frac{\tau_2'}{\tau_1'}}$$

It is provided in many standardizing rules that the ambient air temperature shall not exceed 40° C. The average temperature rise of an open-type transformer with normal rated load may be taken as about 50° C. After working some time under normal conditions the temperature of the windings will be not more than 90° C. If a short-circuit now occurs, the temperature of the copper may attain say 250° C. for a short period without the insulation being permanently damaged.† Taking $\tau_2 = 250$, $\tau_1 = 90$, i.e. $\tau_2' = 484{\cdot}5$ and $\tau_1' = 324{\cdot}5$, the above formula becomes

$$k_{sc} = 144(1/\delta_n'\sqrt{t})$$

where δ_n' is the normal rated current-density in amperes per sq. mm. The curves in Fig. 65 are plotted from this expression for $\delta_n' = 1{\cdot}5$, 2 and 2·5 amperes per sq. mm. (968, 1 290 and 1 610 amperes per sq. in.). It will be seen from the curves that with a normal current-density of 2 amperes per sq. mm. a current 100 times the rated value can be carried for ½ second, 71 times for 1 second, or 50 times for 2 seconds, assuming the temperature of the transformer to rise by 160° C. from a working copper

---

* J. Grillet, "La déstruction des transformateurs de courant en cas de court-circuit sur le réseau," *Rev. Gén. de l'Él.*, vol. 26, pp. 841–852 (1929). Grillet gives the numerical factor as 8, doubtless due to the choice of a higher specific heat to take account of the presence of the insulating materials. See also, J. Grillet, "Resistance of current transformers to short circuits," *Int. Conf. H.T.E.S.*, vol. 2, pp. 309–331 (1931). C. Bresson, "Théorie et construction des transformateurs de mesure de courant," *Écl. et Force Mot.*, Aug., Sept., Oct. (1931).

† The carbonizing temperature of cotton and paper is about 220° C.; a copper temperature of 250° is permissible, however, since there is a lag between the heating of the copper and its insulation. At the end of the short-circuit period the copper temperature begins to fall, but the stored heat continues to cause the insulation temperature to rise; the copper and insulation tend to equalize, therefore, at some temperature considerably lower than 220°.

temperature of 90° C. That such values may be safely attained in practice is confirmed by numerous tests, such as those cited on p. 157.

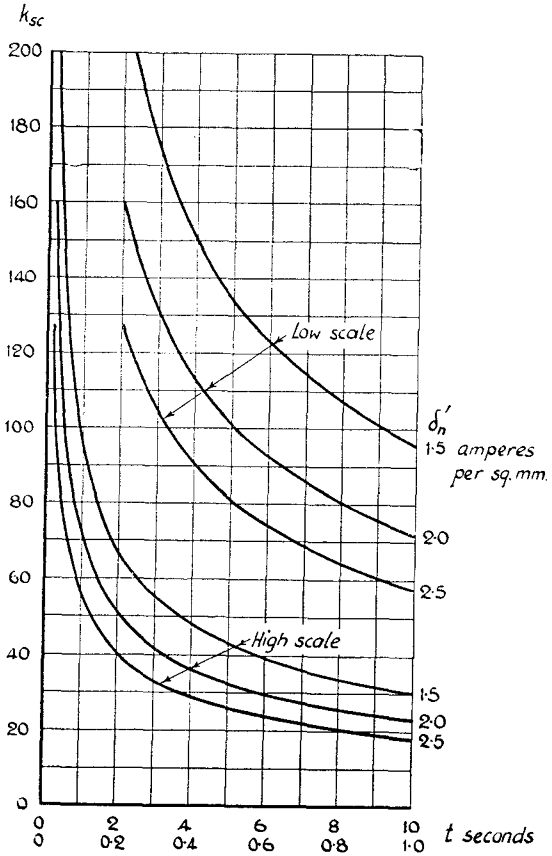


FIG. 65. THERMAL OVERLOAD FACTOR FOR CURRENT TRANSFORMER

The various standard specifications make some interesting recommendations for thermal security. The British rules state that 120 000 amperes per sq. in., i.e. 186 amperes per sq. mm., can be tolerated for ½ second; with the above assumptions this corresponds with k_{sc} = 93 times normal current when δ_n' has the very usual value of 2 amperes per sq. mm., or a 1 second rating of 66 times normal current. The French rules are more severe, giving a 1 second rating of k_{sc} = 80 for Class 2 transformers (used for switchboard watt-hour meters) and k_{sc} = 200 for Class 3 transformers (used for switchboard ammeters), in both cases without "undue heating," a rather vague term. Italy requires a 1 second rating of k_{sc} = 75, while the U.S.A. adopts the very conservative figure of k_{sc} = 40. In all cases the secondary is supposed to be closed on its rated burden. The German rules define the current that may be carried without damage for 1 second as the *thermal current limit* (*Thermischer Grenzstrom*) and base it upon a final temperature of 200° C. It is empirically defined that the thermal limit in kilo-amperes is given by the formula

$$Therm = 180\,A_c'/1\,000$$

where A_c' is the copper section in sq. mm. This corresponds with an expression for k_{sc} in the form

$$k_{sc} = 180(1/\delta_n'\sqrt{t})$$

which, for the same normal current-density and time of short-circuit, gives values of k_{sc} about 25 per cent higher than the formula deduced above. According to the German rule each 180 amperes of the 1 second rating requires a copper section of 1 sq. mm.; hence if *Therm* = 100 kilo-amperes, then—

For t =	1	2	4	9	16	sec.
A_c' =	60	85	120	180	240	sq. mm.

This table shows that the section of primary conductor should be determined by the time-limit of the tripping gear; the transformers with the greater section should be installed nearer to the power station, at which the time-limit of the protective devices is of longer duration than at the distant parts of the system.

21. **The mechanical effects of short-circuit currents.** The mechanical forces set up within a current transformer under primary short-circuit conditions may attain very large values, sufficient in many cases to deform or displace the windings* and not infrequently resulting in the complete destruction of the transformer. The forces have their greatest value when the current arrives at its first amplitude, which may be many times the amplitude of the normal rated primary current; this occurs

---

* R. Rüdenberg, "Kurzschlussströme beim Betrieb grosser Kraftwerke," *E.u.M.*, vol. 43, pp. 77–90, 98–106 (1925). Also see the papers by Grillet and by Bresson already cited.

at a very short interval after the incidence of the short-circuit, say about 1/200 second, and the forces thereafter decline rapidly as the current falls towards its steady short-circuit value, as shown in Fig. 66. This short duration of the period during which the maximum forces occur gives to the forces something of an impulsive or "hammer-blow" character; one may safely conclude that if the transformer is sufficiently strong to withstand the forces due to the first amplitude, the remainder of the short-circuit period is of no further dynamical interest, though it is of prime importance from the thermal point of view.

The mechanical effect depends on the square of the first current peak, the square of the number of turns, and upon the shape of the coils. It is well-known that the forces acting on a coil of any shape tend to deform it in such a way that its area becomes a maximum for a given perimeter, i.e. if the coil is not circular it tends to become so. Hence coils of circular shape with the fewest possible number of turns will be best from the mechanical point of view. The ideal transformer for mechanical security will have a single-bar primary surrounded by a ring core with a toroidal secondary, such an arrangement being practically immune from dangerous forces: see Section 22.

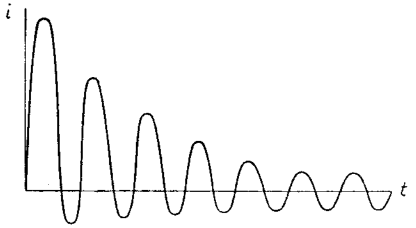


Fig. 66. Current Wave with Initial Transient

The forces fall into two groups, (i) those acting upon the windings, more particularly the primary; and (ii) those stressing the primary leads. Dealing first with the former group, these forces are most serious in the core- or shell-type transformer with coaxial primary and secondary windings, as in Fig. 67 (*a*). If the transformer is symmetrically constructed, the system of forces consists* of (*a*) a radial force acting outwards on the primary coil, tending to burst it and putting its conductors into a state of peripheral hoop tension; (*b*) a radial force acting inwards on the secondary coil, subjecting it to compression; and (*c*) forces compressing the windings in the axial direction. If the coils are not originally circular, the peripheral forces tend to change them to the circular shape; thus, windings with rectangular, oval or link-shaped coils are least satisfactory, the ideal shape of coil being circular, since this is least liable to deformation. Even if the coil is originally circular it may suffer deformation if the leakage field is not distributed symmetrically around it, as when one portion of the coil is nearer the iron core than the rest; an example is shown in Fig. 63. When there is a radial and axial dissymmetry, whether due to constructional imperfection or otherwise, as in Fig. 67 (*b*), there are added to the preceding system two additional forces (*d*) acting radially, tending to restore the coils to co-axial symmetry and (*e*) tending to displace the coils in

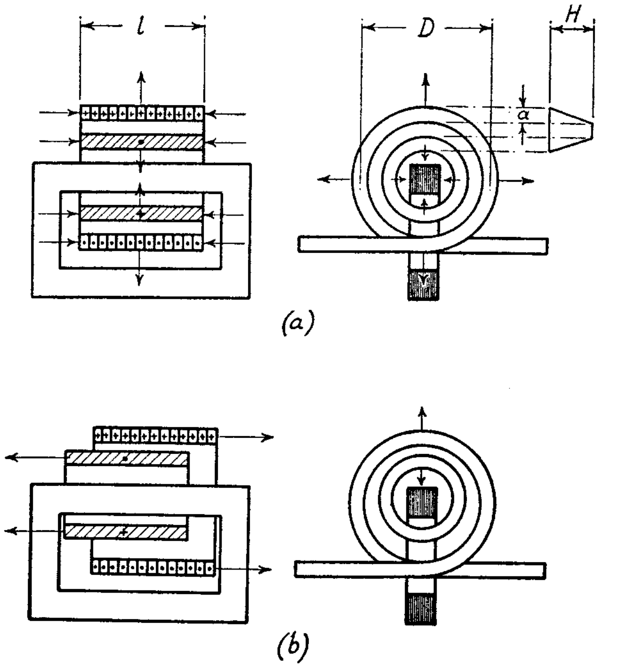


Fig. 67. Mechanical Forces in Current Transformer with Coaxial Coils

(*a*) when symmetrical; (*b*) when there is axial and radial dissymmetry

---

* *See* Grillet, loc. cit. *ante*; also W. Reiche, "Über die Kurzschlussfestigkeit von Stromwandlern," *Elekt. Zeits.*, vol. 49, pp. 1772–1776 (1928). The formulae given in Reiche's paper for the radial force are defective; their correct form is given later in this Section.

the axial direction, increasing thereby the longitudinal dissymmetry.

The accurate calculation of the forces is a matter of some difficulty and resembles in some respects the corresponding problem in the design of power transformers; this matter has been discussed in some detail by the writer* in an earlier publication, to which the reader is referred. We shall content ourselves here by noting certain methods of calculation specially adapted to current transformers.

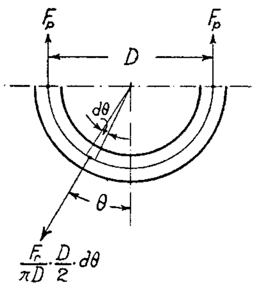


FIG. 68. CALCULATION OF HOOP TENSION IN TRANSFORMER COIL

The axial forces (*c*) and (*e*) tend to cause crushing of the insulation, slipping of the layers one over the other, and piling up of the winding against the ends of the core window; their calculation is not easy, though approximate formulae have been given by Grillet (loc. cit.) and Liebold,† and an exact solution by Clem.‡ For small axial displacements Grillet suggests that the magnitude of the total axial force can be estimated from the formula

$$F_a = \tfrac{1}{2}(i_{sc}{}^2\, T_p{}^2\, D\Delta/l^2)10^{-6} \text{ kilogramme,}$$

where i_{sc} is the first peak of the short-circuit current in amperes, D the mean diameter of the coil in cm., l its axial length in cm., and Δ is the amount of axial displacement in cm. between the primary and secondary coils, assuming these to have the same axial length. As Fig. 63 shows, the axial forces may be very destructive. The one example is a 30 VA, 200/5 transformer damaged by a current peak of 100 000 amperes i.e. 350 times the amplitude of the normal current. In the other example a small porcelain-insulated transformer has been completely wrecked by a peak only 75 times the normal value. In both

---

* B. Hague, *Electromagnetic Problems in Electrical Engineering*, pp. 298–309, 313–325; Oxford University Press (1929), contains a full treatment o the exact solutions of Rogowski and of Roth on this subject and a shor bibliography of related literature is given on p. 343.

† R. Liebold, "Kurzschlusskräfte an Wandlern und Transformatoren und der Verlauf der Feldstärke im Streuraum," *Elekt. Zeits.*, vol. 49, pp. 134–138 (1928).

‡ J. E. Clem, "Mechanical forces in transformers," *Journal Amer. I.E.E.* vol. 46, pp. 814–817 (1927). See also, I. Schigyo, *Journal I.E.E. Japan*, vol. 54, pp. 98–99, 819–828 (1934); Galmiche, *Bull. Soc. Franç. des Elecns.*, vol. 4 pp. 885–900 (1934).

cases the relative axial movement of the windings and the piling up of their turns is clearly shown. The only practical remedy is to have windings of equal lengths, to centre them accurately and clamp them securely in position.

Consider now the radial force on a coil such as the primary in Fig. 67 (*a*) tending to burst it from within, in much the same way as a cylinder subjected to internal fluid pressure. Imagine the coil cut in two by a diametral plane, as in Fig. 68, the equilibrium being maintained by applying forces F_p at the cut surfaces. If F_r is the total radial force, the force upon an element of periphery $D\,.\,d\theta/2$ will be $D\,.\,d\theta\,.\,F_r/2\pi D$ or $F_r d\theta/2\pi$, assuming F_r to be uniformly distributed. The horizontal components of these elementary forces balance on the right and left sides of the line $\theta = 0$; the sum of the vertical components is equilibrated by $2F_p$, i.e.

$$2\int_0^{\pi/2} (F_r/2\pi)\cos\theta\, d\theta = 2F_p$$

or

$$F_p = F_r/2\pi$$

In the actual coil, F_p is provided by a peripheral or hoop tension and results in the conductors being subjected to tensile stress.

Exact formulae have been given by Roth and others for the radial force but they are somewhat difficult to use; we can, however, easily obtain simple expressions that will provide upper and lower limits to the magnitude of the force. Assume first that the primary and secondary ampere-turns are numerically equal. Since they are in opposition with respect to the core flux, they act together in producing leakage flux in the interspace between the windings. We shall assume that the leakage flux in the space between the coils is of uniform density H from end to end of the coils and that the tubes of induction therein are uniform, straight and parallel to the length of the coils; within the coils the field is supposed to fall off linearly as shown in Fig. 67 (*a*). These assumptions are untrue in actual transformers and virtually postulate that the coils are of infinite axial length. If i_{sc} is the first peak of the short-circuit current in amperes, D the mean diameter and l the length of the coil in cm., the total radial force on the external primary coil is

$$F_r = \frac{H}{2}\cdot\frac{i_{sc}T_p}{10}\,.\,\pi D = \tfrac{1}{2}\left(\frac{4\pi}{10}\,\frac{i_{sc}T_p}{l}\right)\cdot\frac{i_{sc}T_p}{10}\,.\,\pi D \text{ dynes.}$$

Dividing by 981 000 gives

$$F_r = 202 \times 10^{-9} \, (i_{sc}{}^2 T_p{}^2 D/l) \text{ kilogramme.}$$

Now let δ_{sc} be the current-density in amperes per sq. cm. due to the current i_{sc} and δ_n the normal r.m.s. current density due to rated r.m.s. current I_{np}. If a is the winding depth and σ the space factor of the winding

$$i_{sc} T_p = a l \sigma \, \delta_{sc},$$

and putting $\delta_{sc} = k \delta_n$

makes $F_r = 202 \times 10^{-9} \, a^2 l \sigma^2 D k^2 \delta_n{}^2$ kilogramme,

and $F_p = F_r / 2\pi = 32 \times 10^{-9} \, a^2 l \sigma^2 D k^2 \delta_n{}^2$ kilogramme.

This peripheral force is carried by a copper area $\sigma a l$, so that the hoop tensile stress in the copper is

$$p_p = 32 \times 10^{-9} \, a \sigma D k^2 \delta_n{}^2 \text{ kg. per sq. cm.,}$$

whence

$$k = (5\,600/\delta_n) \sqrt{(p_p / a \sigma D)} = (56/\delta_n{}') \sqrt{(p_p / a \sigma D)}$$

where $\delta_n{}'$ is the normal current density in amperes per sq. mm. Similar expressions may be deduced for the internal secondary coil.

These equations certainly over-estimate the forces. We have seen on p. 155 that with short-circuit conditions the primary and secondary ampere-turns are by no means equal, since the secondary current falls away considerably from proportionality to the primary current as saturation occurs in the core. In the extreme limit, the secondary ampere-turns may be neglected in comparison with the primary ampere-turns and, at the resulting high saturation, we can regard the core permeability as being roughly unity. Treating the primary a an air-cored inductance, the force acting radially upon it will b

$$F_r = \tfrac{1}{2} i_{sc}{}^2 \frac{\partial L}{\partial r} 10^7 = i_{sc}{}^2 \frac{\partial L}{\partial D} 10^7 \text{ dynes,}$$

where L is the self inductance of the coil in henries comput from any suitable formula. Using Nagaoka's formula,* we ca write

$$L = N \pi^2 T_p{}^2 \, (D^2/l) \, 10^{-9} \text{ henries,}$$

where N is a function of D/l which has been computed and is tabulated in the publication cited and elsewhere. Differentiating

$$\frac{\partial L}{\partial D} = \pi^2 \frac{T_p{}^2}{l} D \left[2N + D \frac{\partial N}{\partial D} \right] 10^{-9};$$

which makes

$$F_r = 101 \times 10^{-9} [2N + D(\partial N / \partial D)] \, a^2 l \, \sigma^2 D k^2 \delta_n{}^2 \text{ kg.,}$$

$$F_p = 16 \times 10^{-9} [2N + D(\partial N / \partial D)] a^2 l \sigma^2 D k^2 \delta_n{}^2 \text{ kg.,}$$

and $p_p = 16 \times 10^{-9} [2N + D(\partial N / \partial D)] \, a \sigma D k^2 \delta_n{}^2$ kg. per sq. cm.,

with a corresponding expression for k, giving a lower limit for the force.

If the primary is not to be permanently damaged the peripheral stress set up by the first current peak must not exceed the elastic limit of copper;* this, however, falls considerably with a rise of temperature, as also does the breaking stress, in the way shown in Fig. 69. At 250° C., the value assumed in short-circuit temperature-rise calculations in Section 20, the elastic limit may be taken as 1 400 kg. per sq. cm. Even if the copper is not actually ruptured by the hoop tension it may stretch sufficiently to burst the insulating wrappings, as shown in Fig. 63 for the primary coil of a 200/5 transformer damaged by a current peak of 59 800 amperes.

To illustrate the properties of the formulae we shall calculate k for the primary of a 50 kV transformer having $D = 13{\cdot}3$ cm., $a = 1{\cdot}5$ cm., $l = 15$ cm., $\sigma = 0{\cdot}33$. Taking $p_p = 1\,400$ kg. per sq. cm. we obtain for the assumption $I_p T_p = I_s T_s$,

$$k = 814/\delta_n{}';$$

---

* E. B. Rosa and F. W. Grover, "Formulas and tables for the calculati of mutual and self inductance (revised)," *Bull. Bur. Stds.*, vol. 8, pp. 1–2 (1913).

* The yield point of copper, especially at high temperatures, is a somewhat indefinite matter since this metal exhibits in a marked degree the phenomenon of "creep," i.e. increasing strain with continued application of a given constant stress. Consequently, it is very difficult to decide at what stress Hooke's law ceases to hold; somewhat analogous difficulties occur in connection with the breaking stress. Moreover, any values that are obtained are much affected by the presence of small impurities and the mechanical handling of the material, such as drawing, etc. The curves given are from figures given by Reiche (breaking stress) and Grillet (elastic limit), though it should be noted that these differ in considerable degree from results for annealed electrolytic copper published by A. K. Huntington, "The effect of temperatures higher than atmospheric on tensile tests of copper and its alloys and a comparison with wrought iron and steel," *Journal Inst. of Metals*, vol. 8, pp. 126–144 1912). We shall use the term "elastic limit" in the sense of the stress which if exceeded will produce creep which tends to breakdown.

and for the assumption of an air-cored coil and $I_s = 0$

$$k = 1\,035/\delta_n',$$

since for this coil $N = 0{\cdot}714$ and $\partial N/\partial D = -\,0{\cdot}0157$. These expressions are plotted in Fig. 70, from which it will be seen that with a normal r.m.s. current density of 2 amperes per sq. mm. the initial peak may reach the mean value of 460 times the rated r.m.s. current without the hoop stress exceeding the elastic limit of copper at 250° C. If this peak value is maintained with a sine-shaped wave, then for this final temperature

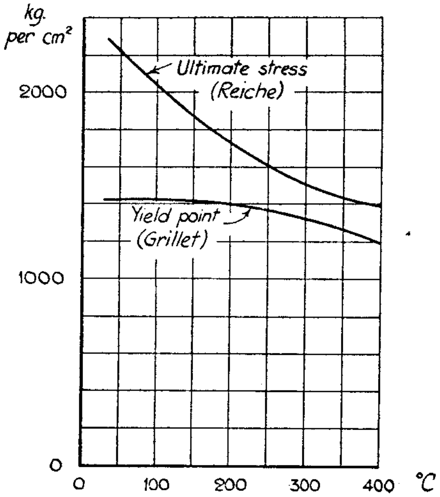


FIG. 69. ULTIMATE STRESS AND YIELD POINT OF COPPER AS A FUNCTION OF TEMPERATURE

the thermal multiplier is 326 times rated current, which can be carried for only about 0·05 second, i.e. for about 2·5 cycles of a 50 cycle supply. In practice, of course, the initial peak is not maintained but is rapidly damped out, so that the therma limit is not reached for a much longer time than this. Th British, Italian and American standards regard a transforme as mechanically satisfactory if it is not damaged by the curren corresponding with 1 second thermal rating given on p. 160. The German rules define the *dynamic current limit* (*Dynamische Grenzstrom*) as the first amplitude which can be carried withou damage when the secondary winding is short-circuited, bu gives no numerical values for the limit. The French rules state that for Class 2 transformers a current having a maximum

value equal to 240 times the r.m.s. rated value ($k = 240$) shall be carried for 0·1 second without mechanical deformation; for Class 3 transformers a corresponding figure of $k = 600$ is specified, the secondary in both cases being closed on its rated burden.

Turning now to the forces on the primary leads, the most important case occurs when both leads pass together as parallel conductors inside a common porcelain insulator, a construction

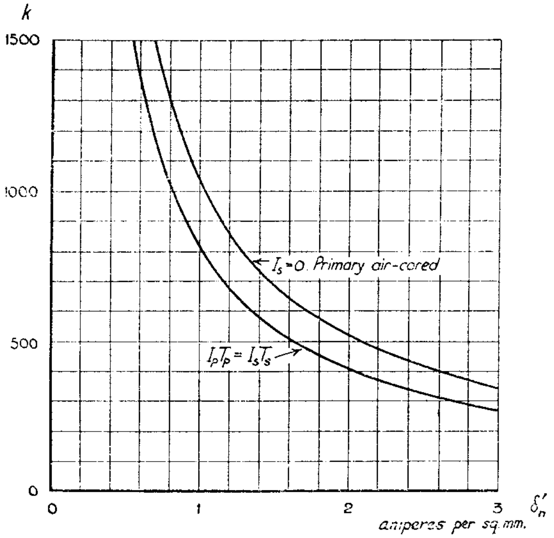


FIG. 70. MECHANICAL OVERLOAD FACTOR FOR RADIAL FORCE IN CURRENT TRANSFORMER

frequently adopted in large current transformers of the oil-immersed or compound-filled types for high voltages. Treating the leads as parallel wires of length l and distance apart d, the force of repulsion in kilogrammes is given by the formula (Hague, *Electromagnetic Problems*, p. 327)

$$F = 2{\cdot}04 \times 10^{-8} i_{sc}^2 \, [\sqrt{(1 + l^2/d^2)} - 1]$$

which is plotted in Fig. 71 for a current of 1 000 amperes; for any current all that is necessary is to multiply the ordinates by the square of the current in kiloamperes. For comparison the dotted line gives the value of the force on the assumption that the conductors form a section of length l taken from infinitely long wires, and demonstrates that this commonly-used formula may be seriously in error, especially for the smaller values of l/d. As an example of the force to be expected consider two leads 30 cm. long and 2 cm. apart carrying a peak current 400 times the rated r.m.s. current of 100 amperes, i.e. 40 kiloamperes. The force per kA is 0·287 kg., so that the total force of repulsion is $40^2 \times 0{\cdot}287 = 459{\cdot}2$ kg. or 0·41 ton. Such large forces necessitate the conductors being securely bound together with string or wire if this force is not to separate them and crack the porcelain insulator by internal pressure. An actual example of such a fracture is shown in Fig. 63 taken from Rüdenberg's paper cited on p. 161. Alternatively, the conductors may be confined within a steel tube which can safely withstand the stress, the tube acting as a lining to the insulator.

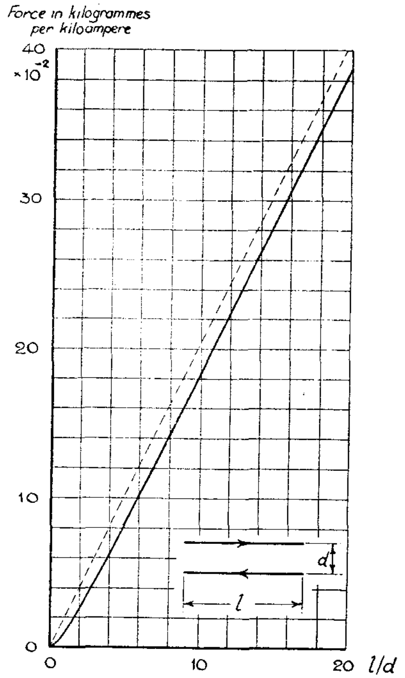


FIG. 71. MECHANICAL FORCE ON PARALLEL WIRES

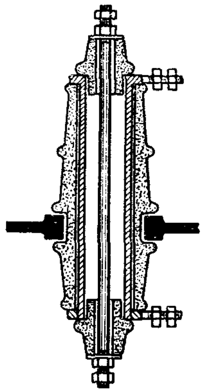


FIG. 72. LEAD-IN BUSHING WITH COAXIAL LEADS

Some firms, e.g. the A.S.E.A., eliminate entirely the force on the lead-in conductors by the arrangement shown diagrammatically in Fig. 72. One conductor is a rod passing axially down a tube which serves as the return lead. It is not difficult to show (see Appendix I) that even when the conductors are not coaxial there is no force tending to produce relative displacement; the tube is subjected to internal pressure and its material to hoop tension, and there is no difficulty in proportioning the tube to withstand the short-circuit stresses.

With parallel conductors vibration and possible resonance may greatly increase the stresses, as Grillet has pointed out. Treating the conductors inside the porcelain bushing as bars fixed at their ends, the natural frequency of such bars in cycles per second is easily shown to be*

$$f_n = (112/l^2)\sqrt{(E\kappa^2/\gamma_c)}$$

where l is the length in cm., E the modulus of elasticity for copper ($1{\cdot}15 \times 10^6$ kg. per cm.^{2}), κ is the radius of gyration of the section of the bar about an axis through its centre of figure and perpendicular to the plane of flexure, and γ_c is the density of copper ($8{\cdot}9 \times 10^{-3}$ kg. per cm^3). For example, consider two parallel bars 3 cm. broad and 0·8 cm. thick; then $\kappa^2 = 0{\cdot}8^2/12$; if $l = 55$ cm. the natural frequency of oscillation is found from the above formula to be 100 cycles per sec. Since the force due to the a.c. in the bars has two maxima per cycle of the current, if $f = 50$ cycles per sec. the bar will be set into resonance. Such a circumstance can only be avoided by modifying the size of the bar until resonance does not occur and by preventing forced oscillation of any kind by firmly binding the two lead-in conductors together.

22. **The Bar-type current transformer.** The simplest type of current transformer has a ring core upon which the secondary turns are toroidally wound; the primary consists of a single conductor passed centrally through the opening of the ring. The primary and secondary are easily insulated from one another up to the highest voltages by the use of paper or porcelain tubes, condenser bushings, or by other means. The symmetrical arrangement of the parts results in a compact, robust, and mechanically strong construction; there are no electrodynamic forces tending to produce relative displacement of the windings. Moreover, it is easily possible to provide such a cross-section of primary conductor that excessive primary currents produce negligible heating. The insulation is dry and

---

* S. Timoshenko, *Vibration Problems in Engineering*, p. 233 (1928). Also for the classical treatment of the vibration of bars, see Lord Rayleigh, *Theory of Sound*, vol. 1.

practically fireproof. Consequently, this construction may be regarded as ideal from the standpoint of thermal and mechanical security under primary short-circuit conditions, since its strength in both these respects is almost unlimited.

We have seen earlier in this book that the ratio error and phase-angle of a simple current transformer will be small only if the exciting ampere-turns for the core are a small fraction of the total primary ampere-turns, and it has been shown that this necessitates a core of large section and short length of magnetic path together with a value of primary ampere-turns not less than 1 000 when high accuracy is demanded of the transformer. Since, in the type under consideration, there is only one primary turn, the ampere-turns become equal to the primary current; hence high accuracy can be obtained with a single primary conductor only with large primary currents. It is easy to obtain the low accuracy of ratio suitable for relay operation with any value of primary current, large or small. With a silicon-iron core and a rated output of about 15 VA. high accuracy within the limits of Class 0·5 cannot be easily secured with less than 500 amperes or of Class 1·0 with currents below 300 amperes. Bar-type transformers are, therefore, unsuited for the operation of watt- and watt-hour meters below these primary current limits, unless auxiliary means are provided

for the reduction of their ratio and phase defects at these lower currents, between 50 and 500 amperes, usual in modern high voltage systems. Again, at high voltages the thickness of insulation between the primary conductor and the secondary becomes considerable, thereby greatly increasing the mean perimeter of the ring core, the exciting ampere-turns required by the iron and the ratio error and phase-angle of the transformer.* The simplicity of the single-conductor or bar-type transformer, the ease with which it can be insulated for high voltages and its ideal short-circuit-proofness make it most desirable for general adoption wherever possible in modern networks. Considerable attention has been given in recent years to the perfecting of numerous compensating methods by which high accuracy, suitable for metering, can be secured at currents as low as 100 amperes; many of these are discussed in Sections 4, 5, 6, and 8 of this chapter. The desired properties may be most simply obtained by the use of a nickel-iron core, as described in Section 9; further, by combining one of these compensating devices with a nickel-iron core, metering accuracy has been attained down to 50 amperes.

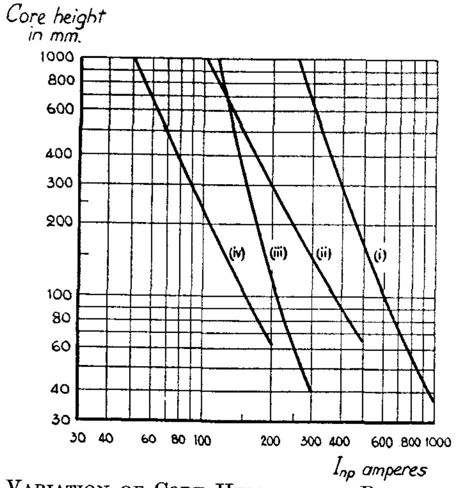


FIG. 73. VARIATION OF CORE HEIGHT WITH PRIMARY CURRENT RATING IN BAR-TYPE TRANSFORMER

(i) Si-Fe core; (ii) Si-Fe core with additional magnetization; (iii) Ni-Fe core; (iv) Ni-Fe core with additional magnetization

In this connection some tests made by Reiche† may be cited. Using rings of the same diameter, transformers were made of silicon-iron and of nickel-iron plates built up into cores of various heights, with or without additional magnetization. With a constant non-reactive burden of 15 VA. the primary current was determined for which the transformer retained the accuracy of Class 0·5; the results are plotted in double-logarithmic co-ordinates in Fig. 73, showing in a striking way the advantage of nickel-iron over silicon-iron in regard to the bulk of the transformer core. It is to be noted that to secure high accuracy with low primary currents, even using nickel-iron and auxiliary magnetization, the core becomes very high; e.g. with $I_{np} = 50$ amperes the core is nearly a metre high and will be both heavy and expensive, but is practically feasible if required.

Billig‡ has pointed out that there are instances where it is desired to obtain, with a small number of primary ampere-turns, a higher accuracy than can conveniently be got from a silicon-iron core and yet to maintain this accuracy over an unusually long range of current, an

---

* W. B. Buchanan, "Overload limitations of high voltage single-turn-primary current transformers," *Univ. of Toronto Eng. Res. Bull.*, No. 1, pp. 191–239 (1919). H. Neugebauer, "Stromwandler für Schutzsysteme," *Siemens Zeits.*, vol. 11, pp. 147–151, 192–198 (1931). "Accuracy of measurement and short-circuit reliability of current transformers," *A.S.E.A. Journal*, vol. 7, pp. 55–56 (1930).

† W. Reiche, "Die Verbesserung des Stabstromwandlers für kleine Primärströme," *Elekt. Zeits.*, vol. 53, pp. 961–965 (1932).

‡ E. Billig, "Normale Stromwandler mit Mischkernen," *E.u.M.*, vol. 52, pp. 199–203 (1934). See also A. Kussmann, loc. cit. on p. 103.

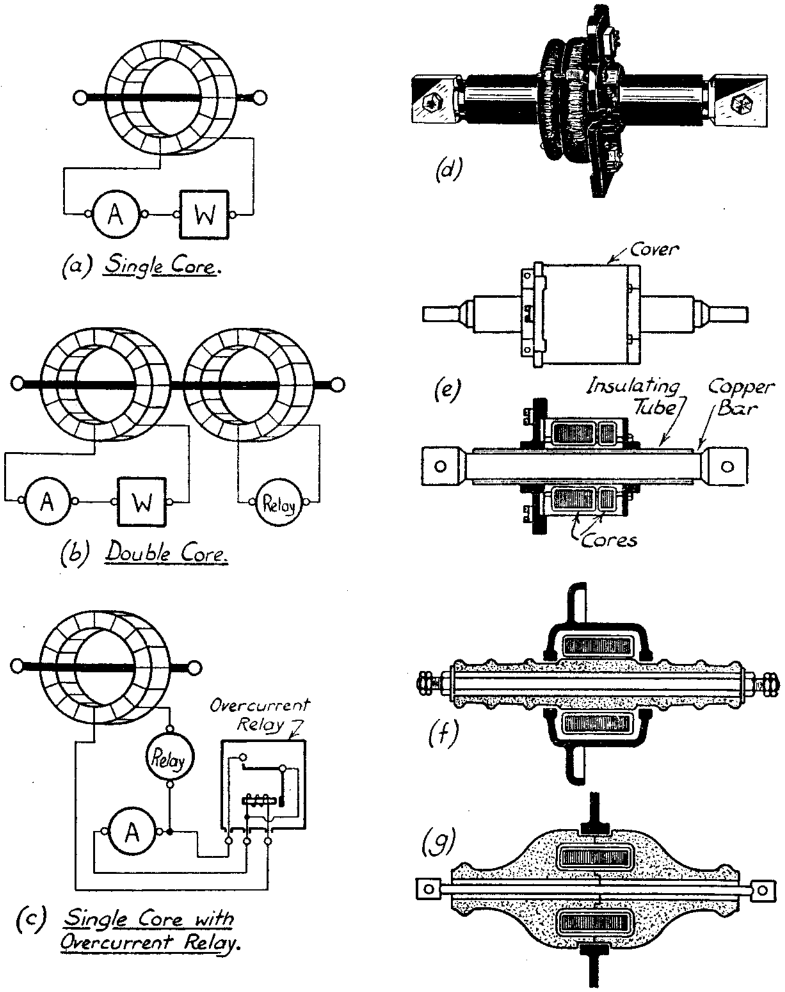
advantage which the high saturation density of silicon-iron would provide. He suggests combining the properties of nickel-iron (high accuracy, low saturation density) with those of silicon-iron (lower accuracy, high saturation density) by constructing a transformer with a "mixed core," consisting of a suitably selected combination of both materials. His tests show that it is easy by this means to secure an accuracy of ratio and phase-angle little inferior to that obtainable from a nickel-iron core, and to maintain this accuracy up to a "break-away" point almost as good as would be given by a silicon-iron core.

In the bar-type transformer proper the primary conductor forms an integral part of the construction and consists of a copper bar of circular section proportioned to satisfy the conditions of the short-circuit thermal rating, this bar passing through the toroidally-wound ring core, as shown diagrammatically in Fig. 74 (*a*). Such transformers are frequently made with two independent cores having different characteristics, one core operating measuring instruments and the other actuating relays, etc., as in Fig. 74 (*b*). To obtain adequate accuracy it is now common practice to make the measuring core of nickel-iron plates, while silicon-iron is used for the relay core. Since nickel-iron saturates at about 9 000 lines per sq. cm. and silicon-iron at about 17 000 lines per sq. cm. a small nickel-iron core will, in consequence of its early saturation, give considerable protection to the instruments against damage on overload while giving high accuracy of measurement under normal conditions. It is cheaper to use a double-core transformer of this type than two separate transformers. The Siemens & Halske Co. obtain measurement and relay service from a single core by the device shown in Fig. 74 (*c*). Up to a predetermined value of the secondary current the ammeter and relay are supplied in series; when this current is reached an over-current relays hort-circuits the ammeter while the relay continues to operate alone.

Fig. 74 (*d*) shows a 200/5 bar-type transformer with two cores manufactured by Siemens & Halske,* the sheet metal cover being removed to show the internal construction. Fig. 74 (*e*) shows the arrangement of the transformer in a more

---

* Numerous examples are given in the following papers and elsewhere: C. Schrader, "Kurzschlusssichere Stromwandler," *Elekt. Zeits.*, vol. 43, pp 1478–1482 (1922); E. Zopf, "Kurzschlusssichere Stromwandler," *Elekt. Betrieb*, vol. 22, pp. 27–28 (1924); C. Lampis, "Riduttori di corrente a scopo protettivo," *L'Elettro.*, vol. 12, pp. 665–672 (1925); A. M. Wiggins, "Ring-type current transformers," *Elect. J.*, vol. 24, pp. 621–625 (1927); A. Kutzer, "Über Einleiterstromwandler," *Elekt. Zeits.*, vol. 49, pp. 316–318 (1928); K. Sachs, "Progress in Brown-Boveri design during 1932," *B.B. Rev.*, vol. 20, pp. 29–31 (1933).



(*a*) Single Core.

(*b*) Double Core.

(*c*) Single Core with Overcurrent Relay.

FIG. 74. THE BAR-TYPE CURRENT TRANSFORMER

diagrammatic way. At higher voltages longer insulating tubes are necessary, as in the example of Fig. 75 (*a*) constructed by the Metropolitan-Vickers Co. for 35 kV. Alternatively the insulation may be a condenser bushing with intersheaths. Other firms* insulate the bar with porcelain, accommodating

---

* G. Keinath, "Porzellanisolierte Stromwandler," *Arch. f. tech. Mess.*, Z286–1, July (1933). Figs. 74 (*f*) and (*g*) are adapted from diagrams in this article.

the cores on the outside of a one-piece bushing, as in Fig. 74 (*f*) or within the body of a two-piece insulator in the way shown by Fig. 74 (*g*). Fig. 75 (*f*) illustrates a 35 kV transformer with porcelain insulation, the cores being enclosed in a cast-iron box suitable for outdoor use.

Fig. 75 (*b*) is an interesting example of a transformer provided with three cores upon a common primary bar, one for relays, the second for meters, and the third for differential protective gear. The ratio is 150/5, the circuit voltage 10 kV; the output is 30 VA for errors within Class 1 for the measuring core and Class 3 for the other cores.

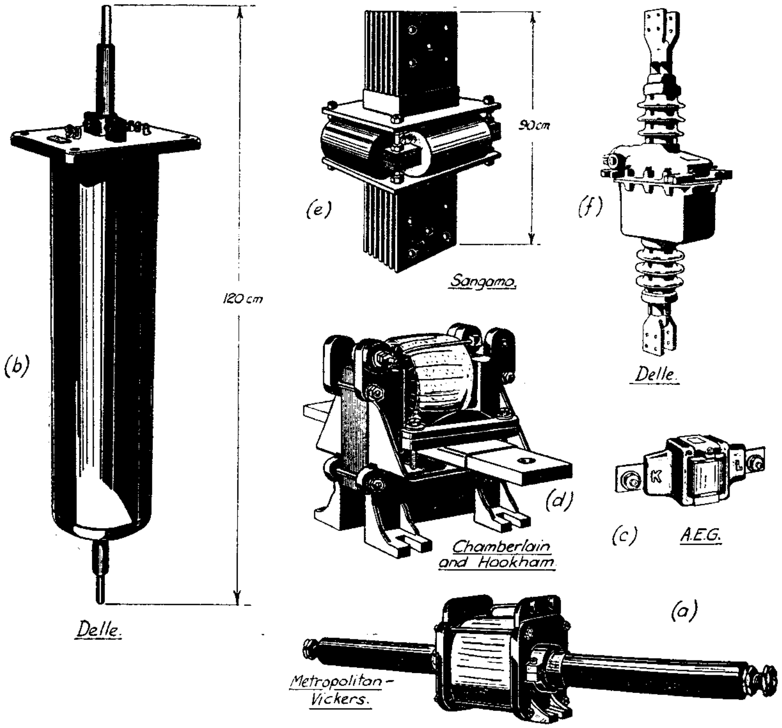


FIG. 75. TYPES OF BAR-TYPE CURRENT TRANSFORMERS WITH FIXED PRIMARY CONDUCTOR

The bar of circular section is usually used for currents up to about 5 000 amperes; above this value it is often more convenient to use a bar of rectangular section, or several such bars in parallel, passing through a core with a rectangular opening. Fig. 75 (*c*) shows a type of transformer for direct insertion in a bus-bar for currents from 100 to 3 000 amperes; the core is clamped between moulded insulating caps and two secondary coils are provided, one on each side of the bar. A great advantage of the circular ring core is that it is quite immune from the influence of stray magnetic fields set up by neighbouring conductors, since it is completely overwound with its secondary coil. This advantage is not nearly so easily secured with the rectangular core used with rectangular-section bars for large currents and the accuracy of transformers so constructed may be seriously impaired by stray field effects. Fig. 75 (*d*) shows an example of a non-astatic transformer with only a single secondary coil upon the upper yoke, the primary bar being designed for currents up to 2 000 amperes and insulated for 2 kV to earth. Fig. 75 (*e*) illustrates a transformer for 30 000 amperes with four secondary coils giving approximate astaticism: at 60 cycles per sec. and a rated burden of 25 VA with $\cos \phi = 0{\cdot}8$, K_c varied from 1·0025 to 0·999 between 10 per cent and 100 per cent of full-load, β changing from $+ 9$ min. to $- 2$ min.

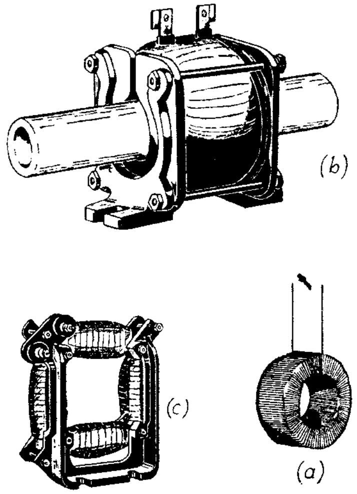


FIG. 76. BAR-TYPE CURRENT TRANSFORMERS FOR USE WITH CABLE OR BUS-BAR PRIMARY

Toroidal ring cores are frequently used to slip over a conductor or cable, thereby forming a bar-type transformer without the primary as an integral part of the construction. A core such as Fig. 76 (*a*) (Ferranti) may either be mounted upon the insulator of a switch or a transformer or assembled upon the primary conductor within the oil-tank.* For mounting in the run of a conductor or cable, suitable insulating bushings of porcelain, paper, bakelite or other material may be

---

* For a consideration of the properties of bushing type transformers, see P. Bary, "La protection différentielle des lignes et des transformateurs de puissance du moyen des transformateurs de bornes," *Rev. Gén. de l'Él.*, vol. 18, pp. 565–572 (1925). K. Maekawa, Y. Noritomi and G. Takeuchi, "The bushing type current transformer," *Res. Elect. Lab. Tokyo*, No. 280, pp. 1–29 (Feb., 1930). C. Bresson, "Montage sur les bornes des interrupteurs de transformateurs de mesure," *Rev. Gén. de l'Él.*, vol. 29, pp. 227–234 (1931).

provided, as in Fig. 76 (*b*) (Metropolitan-Vickers). For assembly over bus-bars the rectangular core with four secondary coils is usually preferred, as shown by Fig. 76 (*c*) (Ferranti). This example is rated for 60 VA up to 15 000 amperes and, in order to avoid very high voltages if the secondary is opened, has a secondary current of 50 amperes; this is stepped down to the standard 5 amperes by means of an intermediate 50/5 transformer. An interesting example of a special design* of the transformer to build round very large bus-bars is shown in Fig. 77. The secondary consists of a closed chain of units in series, each of which is designed for a current of 5 amperes with a primary current of 2 000 amperes; the number of units is chosen to correspond with the total primary current. The transformer illustrated is used to measure a current of 40 000 amperes supplied to a carbide furnace, and consists of 20 units assembled on site round the primary conductors, which are omitted from the diagram. Error curves are shown. It was found that these were practically unaffected by stray fields; a return conductor 20 cm. distant had no influence upon the results, showing that the arrangement is almost astatic. It is to be noted that the curves are of an abnormal shape, due largely to the effects of the magnetic leakage (see p. 134).

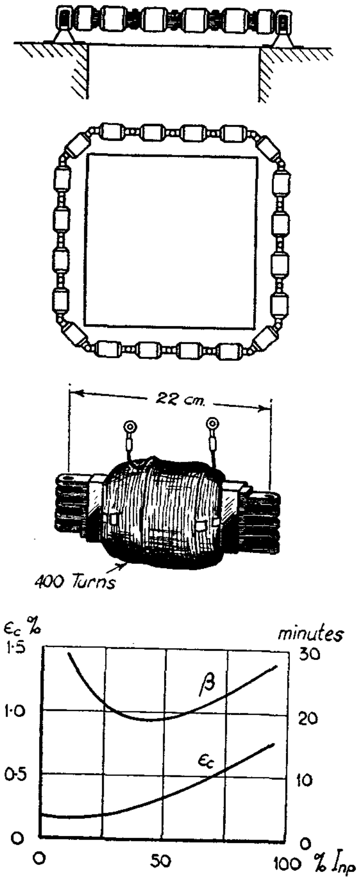


Fig. 77. Keinath's "Chain" Current Transformer (S. & H.)

The modern high-voltage cable is a triumph of successful insulation design; by using such a cable as the primary conductor of a bar-type transformer the insulation of the cable takes the place of the bushing, etc., normally interposed between the primary and the ring core with its secondary winding. Two types of cable-primary transformers have been developed in Germany. The first, due to Rottsieper,* is made by the A.E.G. and takes the form of a cable terminal box of silicon-aluminium alloy designed to accommodate the ring core, as shown in Fig. 78 for a 30 kV unit; the diagram indicates clearly the general principle of construction. To facilitate the testing of ratio and phase-angle before and after installation, an auxiliary primary, consisting of a copper tube, is provided. A full range of transformers for primary currents from 100 to 500 amperes and voltages from 30 to 100 kilovolts is available; the accuracy is within the limits of Class 1, but by the use of auxiliary magnetization provided by auxiliary cores it is possible to reach the accuracy of Class 0·5. It is claimed that the cable terminal transformer is lighter, less bulky, and cheaper than through-bushing or oil-filled types.

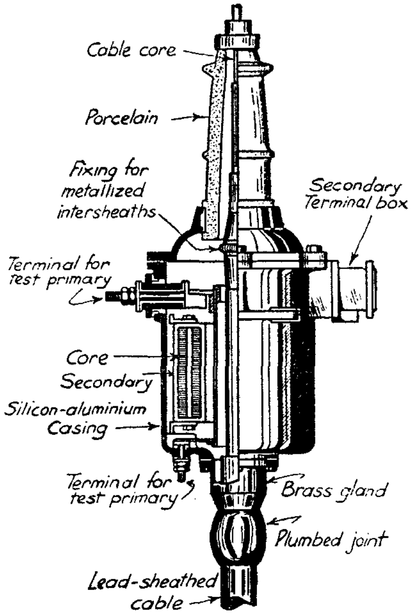


Fig. 78. Cable Terminal-box Current Transformer (A.E.G.)

The second type of cable-primary transformer is manufactured by Koch & Sterzel A.G. of Dresden, and is shown in Fig. 79. It is designed for assembly upon the spare length of cable passing from the cable-duct to the cable terminal, the method of construction being as follows: The divided base plate and

---

* G. Keinath, "Der Kettenstromwandler, ein Wandler für höchste Stromstärken," *Elekt. Zeits.*, vol. 41, pp. 788–790 (1920).

* K. Rottsieper, "Kabel-Endverschluss-Stromwandler für hohe Spannungen," *A.E.G. Mitt.*, part 4, pp. 161–164 (1928); *Elekt. Wirts.*, vol. 27, pp. 555–558 (1928); *E.u.M.*, vol. 46, pp. 963–964 (1928). "Schaltungen zur Prüfung von Kabelendverschluss-Stromwandlern und angeschlossenen Netzschutzeinrichtungen," *A.E.G. Mitt.*, part 8, pp. 289–291 (1932). "Kabelendverschluss-Stromwandler mit Fehlerkompensation (D.R.P. Angemeldet)," *A.E.G. Mitt.*, part 2, pp. 38–40 (1933).

insulating tube are first fixed round the cable to form a foundation for the secondary coils and cores. Each secondary coil consists of two separate parts wound upon formers subtending a central angle of 150°; the two coil-sectors are placed round the cable to leave an angular aperture of 60° as shown by the dotted lines in the plan view. Each core plate is a ring split along one radius so that the plate can be opened and bent into position within the coil-sectors; care is taken to "break joint" and when the whole space is filled the coils are turned to be symmetrically situated about the horizontal line in the plan. The process is repeated for the second, third, etc., cores, the windings being finally put in series. After adding the upper divided plate the transformer is covered with a split insulating tube and a sheet-iron casing. By using a large area, divided

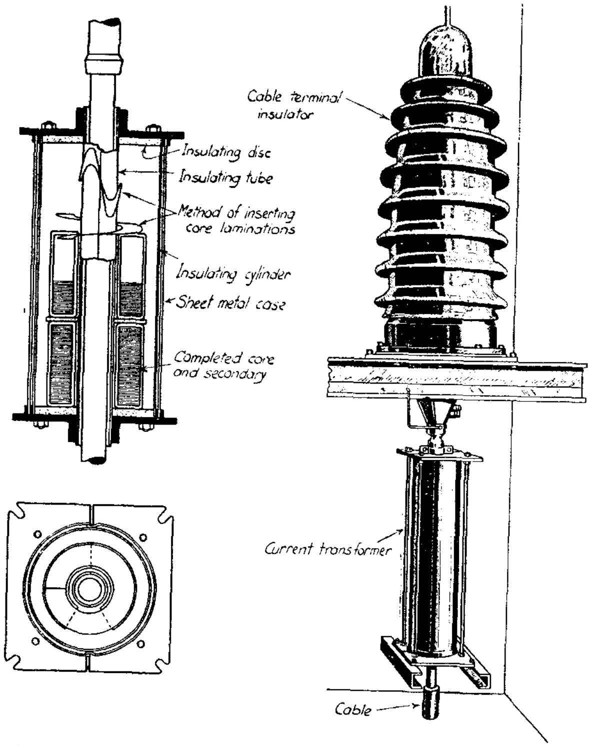


FIG. 79. CURRENT TRANSFORMER FOR ASSEMBLY AROUND A CABLE (KOCH & STERZEL)

for convenience of construction between one to five elements, high accuracy and considerable output can be obtained over a current range from 100 to 1 000 amperes.[*] For example, a five-element core gives Class 1 accuracy for 100, 230, 500 and 1 000 amperes with rated burdens of 10, 30, 120 and 400 volt-amperes respectively; the transformer weighs 60 kg. and has an overall height of 77 cm.

It is appropriate to mention here a further modification of the bar-type transformer, namely, the *split-core* or *clip-on* type of current transformer used in conjunction with a portable ammeter to check the current in feeders, fuses, etc., during inspection routine. The ring core is divided into two parts, one attached to each of the limbs of a pair of tongs which are normally kept closed by a strong spring. Pressure on the handles of the tongs opens the core, which may then be passed over any suitably insulated conductor and clipped thereon while readings are taken. A typical design is illustrated in Fig. 80 (Compagnie des Compteurs), having the abutting faces of the core shaped to give a good joint; the apparatus is insulated for 800 volts. Similar transformers are made by many manufacturers, both British and foreign, and are adapted to higher voltages by the provision of long insulated handles. These transformers are calibrated to work at a single frequency with a given ammeter; the faces of the core must make good contact and, to this end, must be kept free from dust and dirt. A much improved split-core transformer has been patented by Mr. Shotter and is manufactured by Messrs. Elliott Bros. This has a rectangular core with the secondary wound upon inwardly projecting poles, the magnetic circuit containing a permanent air-gap so that the presence of dirt has no effect on the accuracy; moreover, the length of the gap can be adjusted by means of a graduated screw to vary the range of the transformer. The rated primary current ranges from 50 to 1 000 amperes; the rated secondary current is only 50 milliamperes, enabling long leads to be used without affecting the accuracy of calibration.

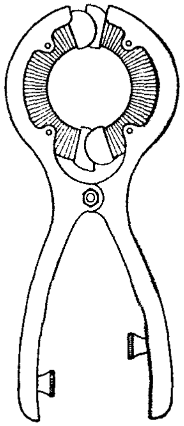


FIG. 80. CLIP-ON CURRENT TRANSFORMER (COMPAGNIE DES COMPTEURS)

23. Short-circuit-proof current transformers with multi-turn primary windings. No current transformer is, in the strictest sense, absolutely proof against the thermal and mechanical effects of an unlimited primary current, but we have seen in the preceding section that the bar-type transformer approaches the ideal in this respect, since its construction renders it almost perfectly secure under the severest short-circuit conditions encountered in practice. At high

---

[*] See Reiche, loc. cit. *ante* on p. 173.

voltages the rated primary currents are often much less than a few hundred amperes, and it has been shown that the attainment of a sufficiently small ratio error and phase-angle in a bar-type transformer presents considerable difficulty, especially for the lowest currents, below 150 amperes. Nor is it easy to obtain from low-current bar-type transformers a reasonably large secondary output unless heavy and bulky cores are used. The desired high accuracy and large output can be secured, and the size of the transformer reduced, by the use of a wound primary with several turns, thereby increasing the primary ampere-turns, but this tends to sacrifice the inherently robust character of the bar construction and makes it much more difficult to attain a high degree of dynamical security. Much attention has recently been given, especially in Germany,* to the design of a short-circuit-proof current transformer with a multi-turn primary; it is the purpose of this section to examine some of the most successful solutions of the problem.

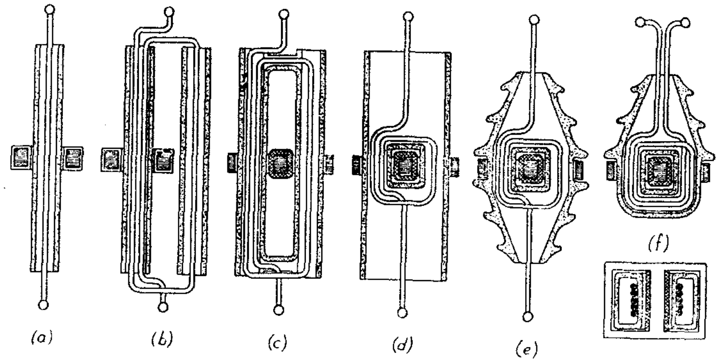


FIG. 81. EVOLUTION OF WOUND PRIMARY FROM BAR PRIMARY

Fig. 81 (*a*) shows in diagrammatic section a simple bar-type transformer consisting of a ring core with its toroidal winding, mounted upon a high voltage bushing. The most obvious way to develop from this a transformer with a wound primary is to provide a second bushing to accommodate the turns, as in Fig. 81 (*b*); a single toroid may be provided, but it is more usual to have two cores, one on each insulator, giving different accuracies and outputs if so desired. Alternatively, the bushings may pass through the "windows" of a shell-type core with the secondary on the middle limb, as in Fig. 81 (*c*) * It will be appreciated that the primary turn, especially at high voltages, is necessarily long, making it wasteful of copper and mechanically weak; considerable bracing is required to produce any reasonable security against deformation under short-circuit forces. Moreover, the long coil has a relatively large self-inductance, increasing the susceptibility to transient voltage rises (see Section 26).

The insulators may be of porcelain or of paper, the former alone being suitable for outdoor transformers. Paper bushings for the highest voltages are constructed on the condenser terminal plan.† Fig. 82 (*a*) shows a porcelain-insulated transformer for 7 kV to earth and a primary current not exceeding 300 amperes, with one ring core (B.T.H.); the insulators are provided with a copper lining connected to the primary coil, in order to eliminate silent discharge and corona, the production of acid action and consequent deterioration of the insulation. The overall length is about 52 cm. Fig. 82 (*b*) shows an example with brown-glazed porcelain insulation for a working voltage of 100 kV (Koch & Sterzel), the primary current being from 20 up to 800 amperes and the rated output 120 VA at 50 cycles with an accuracy within the limits of Class 1. It will be noted that the insulators are braced at the open ends by metal caps, within which the primary turns are enclosed, these acting as terminals. The transformer is 288 cm. long. A further example, Fig. 82 (*c*), is a very large transformer nearly 4 metres high constructed by the Sachsenwerk A.G. for the super-power station at Böhlen; this transformer has paper condenser bushings for a working voltage of 135 kV, its current ratio being 400/20.

The A.E.G. has recently developed‡ a series of short-circuit-proof transformers, for working voltages of 11·5 and 23 kV (42 and 64 kV test voltage) and rated primary currents from 5 to 400 amperes, which have interesting features. Fig. 83 illustrates a 23 kV transformer for an output of 30 VA (Class 1) or 15 VA (Class 0·5). The primary winding is porcelain-insulated, and the insulators are braced by strong cast-iron terminal caps. The secondary is wound upon a shell-type

---

* G. Keinath, "Über die Anforderung an Stromwandler in Kraftwerken," *Elekt. Wirts.*, vol. 30, pp. 60–68 (1931). "Über den Bau moderner Stromwandler," *idem.*, vol. 30, pp. 85–90, 195–199 (1931). "Kurzschlussfeste Stromwandler für Hochspannungsanschlüsse mit kleinem Nennstrom," *Arch. f. tech. Mess.*, Z250–1 (1932). Also see *Elekt. Zeits.*, vol. 42, pp. 905–910 (1921).

* For a discussion of the influence of different numbers of conductors in the two windows upon the magnetic leakage and accuracy of the transformer, see p. 137.

† The first application of a condenser terminal to a wound-type bushing transformer is in *German Patent* No. 229 920 of 4th Dec., 1909 (Siemens & Halske).

‡ G. Friedlaender and K. Wethmüller, "Über die Kurzschlussfestigkeit des Mehrleiterwandlers," *A.E.G. Mitt.*, No. 8, pp. 491–495 (1931). "Mehrleiter-Durchführung-Stromwandler mit hoher Kurzschlusssicherheit," *idem.*, No. 4, pp. 156–157 (1932); also see *Arch. f. tech. Mess.*, Z284–2 (Nov., 1932), and *A.E.G. Mitt.*, No. 3, pp. 85–86 (1934). A somewhat similar type made by the Sachsenwerk A.G. is described by W. Riegel, "Ein und Mehrleiter Stromwandler mit Porzellan Durchführungen," *Elekt. Zeits.*, vol. 52, p. 286 (1931).

core; two cores can be provided, both for measuring purposes or one for this purpose and one of lower accuracy for relay operation. The transformer is compensated by the method of Figs. 43 and 55. A thermal security of 100 and dynamic security of 250–300 can easily be guaranteed, i.e. the transformer will withstand a sustained short-circuit current of 100 times the rated value for one second without thermal damage and a first current amplitude 250–300 times that of the normal current without mechanical deformation. It is possible greatly to increase these figures, say ten times or more, by suitable means. Since there is neither compound, paper, nor oil in the transformer it is absolutely fire-proof. The design illustrated is for use as a through bushing, but it can be easily adapted for insertion in a bus-bar or other horizontal conductor. In this form a one-piece porcelain is used, identical above the core with Fig. 83 but completed below as a U-tube. The core, windings and the U-shaped part of the insulator are contained in a light metal case.

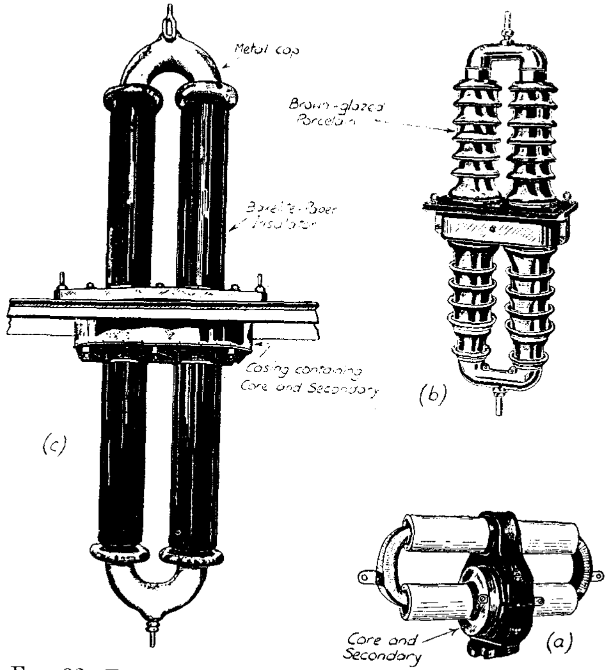


FIG. 82. TYPES OF CURRENT TRANSFORMERS WITH WOUND PRIMARY AND THROUGH BUSHINGS

The length of turn in the shell-type, two-bushing transformer of Fig. 81 (*c*) can be greatly reduced, and the short-circuit-proofness thereby increased, by the simple artifice of Fig. 81 (*d*), in which the insulator consists of a single bushing intersected at right angles to its axis by an insulating tube surrounding the secondary coil. Such an arrangement is readily constructed in ribbed porcelain, as in Fig. 81 (*e*), the ribs resulting in a further reduction in overall height for a given voltage. Current transformers on this "cross-hole" principle are made by Messrs. Koch & Sterzel* for primary currents from 5 to 800 amperes up to a test voltage of 100 kV; as it is difficult to make satisfactory one-piece porcelains of the desired type to withstand higher test voltages, cascade connection of two elements is resorted to up to 180 kV and of three elements for 250 kV (see Section 25). Fig. 84 shows a cross-hole transformer of the through-bushing type in part section, and indicates clearly the method of construction; Fig. 85 gives, in a similar way, an illustration of the supporting-insulator type, also shown diagrammatically in Fig. 81 (*f*). The insulators are metallized upon the inner surface and the completed transformer is filled with graphite and sand, forming a fireproof conducting filling which eliminates internal corona from the primary winding. This latter is made of flexible cable to facilitate the insertion of the turns through the confined space within the insulator. As would be inferred from the excellent mechanical design, this transformer is in the highest possible degree short-circuit-proof.

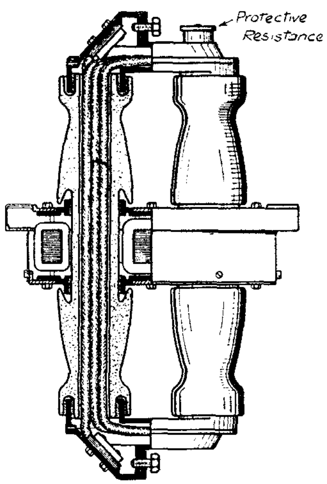


FIG. 83. SHORT-CIRCUIT-PROOF CURRENT TRANSFORMER (A.E.G.)

From the mechanical point of view a circular primary coil is least liable to deformation; several types of short-circuit-proof transformers embodying this principle are in existence. Of these the "crossed coil" transformer due to Keinath,† manufactured by the Siemens & Halske A.G., is of particular

---

* D.R.P. 325 495, 11th June, 1919. F. J. Fischer, "Stromwandler," *K.u.S. Mitt.*, No. 12, pp. 1–35 Sept. (1927). W. Reiche, "Die Anpassung des Querloch-Stromwandlers an schwere Kutzschlussbedingungen," *E.u.M.*, vol. 46, pp. 1006–1009 (1928). "Schwierige Porzellankörper für den elektrotechnischen Apparatebau," *Elekt. Zeits.*, vol. 52, pp. 278–279 (1931).

† D.R.P., Nos. 394 552 (4th Dec., 1920), 413 254 (Aug. 30, 1923), 430 317 (12th Dec., 1923). G. Keinath, "Neue Richtlinien für den Bau elektrischer

interest; it is shown both diagrammatically and in outside view in Fig. 86. The secondary winding is a toroid wound upon a ring core; the circular primary turns are threaded through the opening and are overwound with insulation until the hole is filled. A presspahn shield protects the more lightly insulated upper portion of the primary coil. This construction is mechanically ideal and lends itself readily for use at very high voltages. For the lower voltages porcelain-insulated types have been designed by the Compagnie des Compteurs and by the Sachsenwork A.G. The former design is shown in Fig. 87 (*a*); from this sectional diagram it will be seen that the primary conductor is threaded through a curved porcelain tube which gives to the winding the desired circular form. Similar in principle, but rather different in construction, is the *Reifenstromwandler* (literally, "tyre current transformer") of the Sachsenwerk,* shown in external view in Fig. 87 (*b*). Tests on a

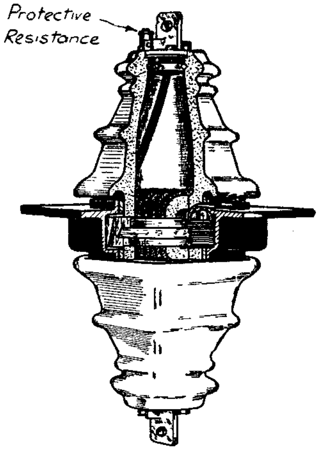


FIG. 84. THE "CROSS-HOLE" CURRENT TRANSFORMER; THROUGH TYPE (KOCH AND STERZEL)

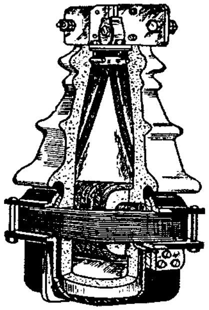


FIG. 85. THE "CROSS-HOLE" CURRENT TRANSFORMER; SUPPORT TYPE (KOCH AND STERZEL)

Messgeräte," *Zeits. des V.D.I.*, vol. 72, pp. 1784–1790 (1928); "Die Entwicklung der Stromwandler für Hochspannung," *Messtechnik*, vol. 4, pp. 287–292 (1928); "Messwandler. Allgemeines. Stromwandler. Spannungswandler," *Bull. Schw. Elekt. Verein*, vol. 24, pp. 93–97 (1933). Keinath's other general papers, previously cited, may also be advantageously consulted.

* "Reifen-Stromwandler," *Elekt. Zeits.*, vol. 53, p. 219 (1932). A. L. Müller, "Kurzschlussversuche an einem Reifen-Stromwandler," *E.u.M.*, vol. 51, p. 138 (1933).

transformer of this type for 20 kV, 30/5, 15 VA, Class 0·5, show that it safely withstood for 10 cycles a sustained short-circuit current of an amplitude 319 times that of the normal rated current, clearly demonstrating its mechanical security. A further interesting design for use up to 35 kV is that due to the Ateliers des Constructions Électriques de Delle, shown in Fig. 87 (*c*). The porcelain body takes a form resembling a disc suspension insulator, the circular primary turns passing through a channel in it; the core is of horseshoe form and carries two secondary coils, one within the porcelain and one on the lower yoke.

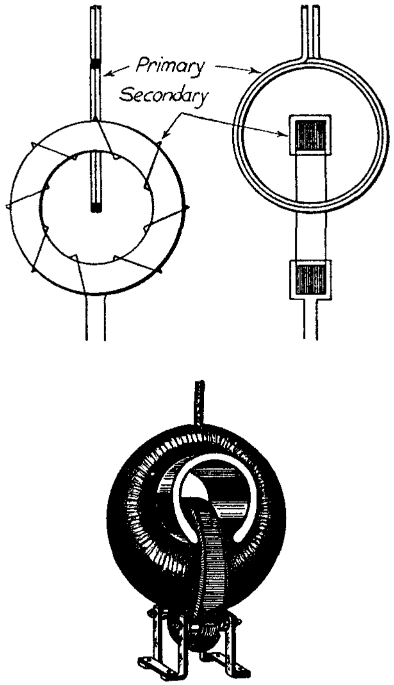


FIG. 86. THE "CROSSED-COIL" CURRENT TRANSFORMER (S. & H.)

A further group of transformers uses both primary and secondary coils in toroidal form, a typical example shown in Fig. 88 (*a*) being due to the firm of Walter in Paris. The core with the secondary is mounted on a porcelain bobbin, the whole being fixed with compound inside a supporting insulator which carries the toroidal primary; this insulator is, in turn,