shown in Fig. 235 (a). The secondary current of the two-stage transformer passes through a resistance of 0·4 ohm, giving a drop of 2 volts at 5 amperes; the small corrective current from the tertiary winding on the auxiliary core flows in a second 0·4 ohm resistor joined in series with the first. The volt-drop across the two resistors together is thereby corrected for ratio error and phase-angle, and is impressed on R which, as before,

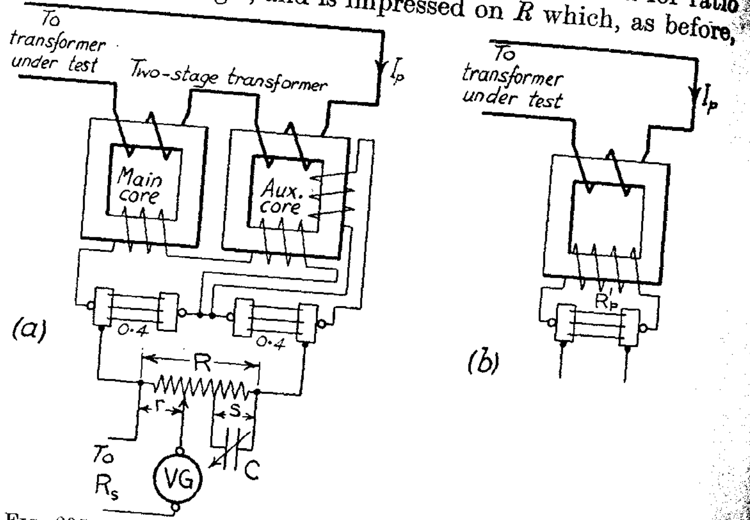


FIG. 235. ARRANGEMENT OF SCHERING AND ALBERTI'S METHOD FOR LARGE CURRENTS

has a value of 200 ohms and carries 10 milliamperes. The rest of the circuit is exactly as in Fig. 233 (a).

A suitable ring-type two-stage transformer with secondary and tertiary resistors is made by Siemens & Halske A.G.; it is provided with primary windings giving ranges of 50, 100, 250 and 500 amperes, and with the aid of a cable primary threaded through the central hole enables ranges of 1 000, 1 500 and 3 000 amperes to be attained.

The use of nickel-iron cores in transformers has completely revolutionized the problem of constructing a standard transformer with negligible errors, as has been discussed on p. 119, so that for large currents the simple arrangement shown in Fig. 235 (b) has been adopted at the National Physical Laboratory as the result of careful investigations made by Arnold and Spilsbury.* The standard transformers have been described

* R. S. J. Spilsbury and A. H. M. Arnold, "Some accessory apparatus for precise measurement of alternating current," *Journal I.E.E.*, vol. 68, pp. 889–897 (1930). Also see Arnold, loc. cit. on p. 470 (1930).

in pp. 120 and 136; they have ring cores of nickel-iron alloy and the primary winding consists of a suitable number of loops of cable passed through the central hole. Various numbers of primary turns enable primary currents down to 200 amperes to be provided for, below which air-cooled resistors directly connected in the primary circuit are used; see p. 349. It is found that for nearly all practical purposes the ratio error and phase-angle may be entirely neglected, except in the most precise work, and are independent of the transformer ratio; careful tests have shown that the transformer characteristics are quite stable and are uninfluenced by ageing, mechanical shock, and d.c. magnetization. With currents over 1 000 amperes it is necessary to keep the standard transformer at least two yards from that under test, to avoid trouble due to the influence of stray magnetic fields on the core.

(i) *Residuals.* In its original form, and with the simple theory given on p. 470, Schering and Alberti's method enables the ratio to be given to 0·1 per cent and the value of β to within a few minutes. Recent improvements in the design of current transformers render it necessary to obtain the ratio to 0·01 per cent and β to 0·1 minute; Arnold has shown (loc. cit.) that by sacrificing something of the direct-reading characteristics of the method this degree of precision can be attained (i) by retaining terms originally neglected and (ii) by taking into account the residual errors in the network, principally those in R_p and R_s, no term being neglected that would introduce an error greater than 3 parts in 100 000. Taking the general equations given for positive values of β on p. 469 and for negative values of β on p. 470, write $R_p + j\omega l_p$ and $R_s + j\omega l_s$ in place of R_p and R_s respectively, where l_p and l_s are the residual inductances of the primary and secondary resistors; it should be noted that one or both may be negative if capacitance effects in the resistors are preponderant over their magnetic stray fields. The resulting expressions for $\boldsymbol{i}_p/\boldsymbol{i}_s$ can be readily simplified to be valid within the desired degree of accuracy by neglecting the following terms in comparison with unity in expanding the operators—

(i) Powers of $\omega l_p/R_p$, $\omega l_s/R_s$ and R_p/R above the first and products of two or more such terms.

(ii) Powers of the preceding terms as above when multiplied by powers of ωCs or ωCp higher than the first.

(iii) Powers of ωCs or ωCp higher than the third.

The somewhat lengthy algebra will be omitted here and leads to the following conclusions—

Positive β. When β is positive, so that the reversed vector of $\boldsymbol{I}_s$ leads on I_p, as is most commonly the case in practice, it can be shown that if $s = 136{\cdot}1$ ohms in Fig. 233 (a)

$$K_c = \frac{R_s}{R_p}\cdot\frac{R}{r}\left[1 + \frac{R_p}{R}\right](1 - 0{\cdot}449\theta^2)$$

and $$\tan\beta = \frac{s}{R}\theta + \frac{\omega l_p}{R_p} - \frac{\omega l_s}{R_s} - \frac{s}{R}\cdot\frac{R_p}{R}\theta,$$
where $$\theta = \omega Cs \text{ and } s/R = 136\cdot1/200 = 0\cdot6805$$
The simple formulae of Schering and Alberti are
$$K_c \doteqdot (R_s/R_p)\,(R/r) \text{ and } \tan\beta \doteqdot \omega Cs^2/R \doteqdot (s/R)\theta.$$
In the corrected expression for the ratio the first bracket is a factor allowing for the shunting effect of R on R_p, i.e. for the current drawn by R from I_p: the maximum value of this correction is 0·2 per cent with $R_p = 0\cdot4$ ohm when testing a transformer of 5/5 ratio; for higher ratios its value is proportionately less since R_p becomes smaller. The second bracket allows for the shunting effect of the condenser on the potential divider R. Since s and R are constants, θ is proportional to the nominal value of β found by the simple formula; hence the error in the determination of ratio, consequent upon the condenser shunt, is $-44\cdot9\theta^2$ per cent, i.e. proportional to the square of the radian measure of the phase-angle. The first term in the corrected expression for $\tan\beta$ is the simple Schering and Alberti formula; the fourth term is usually negligible. The second and third terms are tangents of the phase-angles of the primary and secondary resistors; if the residuals are of the same sign and such that the resistors have equal time-constants the error due to these terms will vanish.

Negative β. When β is negative, so that the reversed vector of I_s lags on I_p, as may occur when a transformer works on a heavily inductive burden, it is possible in the same way to show that if $p = 72\cdot8$ ohms in Fig. 234,
$$K_c = \frac{R_s}{R_p}\cdot\frac{R}{r}\left[1 + \frac{R_p}{R}\right](1 + 0\cdot202\phi^2)$$
and $$\tan\beta = -\left(1 - \frac{p}{R}\right)\phi + \frac{\omega l_p}{R_p} - \frac{\omega l_s}{R_s} - \frac{p}{R}\cdot\frac{R_p}{R}\phi,$$
where $$\phi = \omega Cp \text{ and } p/R = 72\cdot8/200 = 0\cdot364.$$
The simple formulae, neglecting the residual effects, are
$$K_c \doteqdot \frac{R_s}{R_p}\cdot\frac{R}{r} \text{ and } \tan\beta = -\omega Cp + \frac{\omega Cp^2}{R} = -\left(1 - \frac{p}{R}\right)\phi.$$
Interpretations of the several correction terms may be given in exactly the same way as before.

The complete determination of the residuals requires some care, and Arnold has described fully the processes employed. It is important to determine the zero error of the ratio scale on the slide-wire. The resistance of R_s, nominally of value 0·1 ohm, is measured on the Kelvin double bridge to 1 part in 100 000; for primary currents ≯ 200 amperes R_p is a resistor with 2-volt drop and its resistance can be found in the same way to 1 in 10 000. Above 200 amperes the arrangement of Fig. 235 (*b*) is used, the resistor R_p', nominally 0·4 ohm, being measured by the Kelvin bridge while the ratio of the standard transformer is found with a burden of 12·5 volt-amperes. Nickel-iron-cored transformers are available at the N.P.L. with ratio constant to 1 in 10 000 from full load current to 1/20 load. The residuals of R_p and R_s may be measured to 1/20 minute by Hartshorn's* double a.c. bridge; if a transformer

* See B. Hague, *A.C. Bridge Methods*, 3rd edition, pp. 279–282, (1932.)

and R_p' are used instead of R_p the phase angle of the transformer must also be taken into account, this being important in high precision work. Arnold's paper should be consulted for full details.

(ii) *Inductive interference.* Schering and Alberti's method, containing a condenser standard, is intrinsically less liable to inductive interference troubles than are methods containing self- or mutual-inductance standards. It is, however, essential to take all the usual precautions to prevent direct magnetic action between the detector circuit and the primary and secondary currents. Thus the primary and secondary leads should be twin wire, as also should those joining up the various parts of the detector bridge, i.e. no open loops should be permitted; the bridge should be at least a yard from the primary circuit and from the transformer under test. The resistors R_p and R_s should be of astatic construction so that their residual phase-angle is not affected by stray fields.

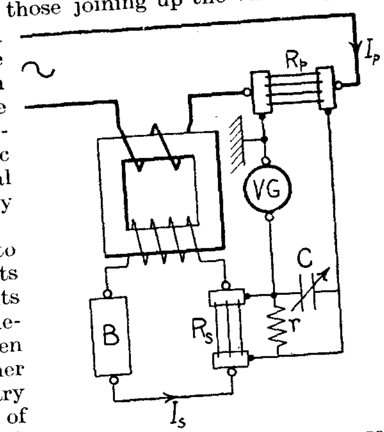


FIG. 236. ARNOLD'S MODIFICATION FOR VERY SMALL PHASE-ANGLES

(iii) *Earthing.* It is necessary to avoid leakage and capacity effects tending to circulate stray currents from the primary through the detector. The supply should be taken from a well-insulated transformer with the mid-point of its primary winding earthed; the insulation of the bridge set-up should be as high as possible, the whole arrangement being kept at some distance from the supply transformer. It will be found necessary to earth one terminal of R_p or R_s; provided the galvanometer is kept in the lead going to the sliding contact the terminal earthed is of little importance. The terminal of R_s shown earthed in Fig. 233 (*a*) is preferable, since this ensures the galvanometer coming to zero potential when the balance is secured.

When testing transformers with very small angles Arnold has shown that the potential-divider arrangement of Schering and Alberti results in loss of sensitivity; it may therefore, be omitted as shown in Fig. 236. The resistors R_p and R_s each drop about 2 volts, balance being secured by regulation of C and r, neither of which need be known with high precision. By this means it is possible to detect changes of β as small as 0·01 minute at $\frac{1}{5}$ of rated current. Taking account of the residuals and proceeding to the same order of small quantities as before gives
$$K_c \doteqdot (R_s/R_p)\;[(1 - (R_s/r)]$$
and $$\tan\beta \doteqdot \omega CR_s + (\omega l_p/R_p) - (\omega l_s/R_s)$$

An alternative method of using a condenser to effect phase compensation has been introduced by Biffi,* developed and improved by Place† and others in the Research Dept. of the Metropolitan-Vickers Co.; the circuit is shown in Fig. 237. The voltage drops across R_p and R_s are approximately equal, and of the order of 0·5 to 1 volt. Balance is secured by adjustment of r_2 and C, this arrangement sufficing for leading or positive values of β. If β is lagging or negative it is necessary

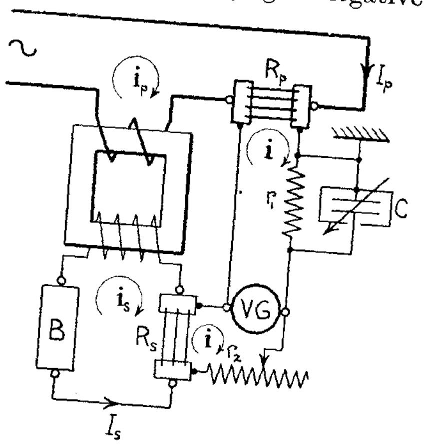


FIG. 237. PLACE'S METHOD

to fix r_2 and put C across it, balancing by variation of C and r_1. Since I_s is not greater than 5 amperes, R_s can be fixed at 0·1 ohm; the choice of R_p will be referred to later, but will not exceed 0·1 ohm (for a 5/5 transformer) and will usually be much less. The values of r_1 and r_2 are of the order of 100 to 400 ohms, according to circumstances; the resistance of the potential leads should be included in the values of r_1 and r_2, but may in most cases be neglected.

Omitting for the present any residuals the theory can be easily worked out. If the detector is without current then

$$R_s(\boldsymbol{i}_s - \boldsymbol{i}) - r_2\boldsymbol{i} = 0,$$

and

$$R_p(\boldsymbol{i}_p - \boldsymbol{i}) - \frac{r_1(1 - j\omega Cr_1)}{1 + \omega^2C^2r_1^2}\boldsymbol{i} = 0$$

* E. Biffi, "Il doppio ponte con corrente alternata," *L'Elettro.*, vol. 10, pp. 508–512 (1923).

† W. J. Place, "A method of measuring the ratio and phase-angle of a current transformer," *Journal I.E.E.*, vol. 66, pp. 657–662 (1928).

Eliminating $\boldsymbol{i}$,

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = \frac{R_s}{R_p} \cdot \frac{[R_p(1 + \omega^2C^2r_1^2) + r_1] - j\omega Cr_1^2}{(R_s + r_2)(1 + \omega^2C^2r_1^2)}$$

In order to obtain balance the connections to R_s must be made in such a way that the drops over R_s and R_p tend to circulate currents of equal magnitudes in opposite directions through the detector. The current ratio is

$$K_c = \frac{R_s[R_p^2(1 + \omega^2C^2r_1^2) + 2r_1R_p + r_1^2]^{\frac{1}{2}}}{R_p(R_s + r_2)(1 + \omega^2C^2r_1^2)^{\frac{1}{2}}},$$

while the angle by which the reversed secondary current leads on the primary is given by

$$\tan\beta = \frac{\omega Cr_1^2}{R_p(1 + \omega^2C^2r_1^2) + r_1}.$$

To a sufficiently high degree of approximation

$$K_c \doteqdot \frac{R_s(R_p + r_1)}{R_p(R_s + r_2)(1 + \omega^2C^2r_1^2)^{\frac{1}{2}}}$$

and $\tan\beta \doteqdot \omega Cr_1$

Residual effects in r_1 and r_2 are negligible, but it is necessary to account for those in R_p and R_s. Writing $R_p + j\omega l_p$ and $R_s + j\omega l_s$ in the preceding analysis, and again neglecting very small quantities, it is easy to show that the ratio formula is unaffected by residuals while the angle expression becomes

$$\tan\beta \doteqdot \omega Cr_1 + (\omega l_p/R_p) - (\omega l_s/R_s).$$

The third term is usually negligible, but the second may amount to as much as 12 minutes and cannot usually be omitted.

When β is an angle of lag, balance being effected as described by putting C in parallel with r_2, the corresponding formulae are

$$K_c \doteqdot \frac{R_s(R_p + r_1)(1 + \omega^2C^2r_2^2)}{R_p(R_s + r_2)}$$

and $\tan\beta \doteqdot -\omega Cr_2 + (\omega l_p/R_p) - (\omega l_s/R_s)$

It is necessary, as in all methods, to use bifilar connections in all meshes of the network so that loops, with consequent chance of inductive interference, are avoided. Moreover, it is essential to preserve good insulation, with the network at as small a potential above earth as possible. It appears in practice to be best to earth the terminal of R_p shown in the diagram; this has the additional advantage of permitting the use of a shielded condenser with the shield joined to one set of plates and to earth, a condition in which most precision condensers are calibrated.

For the majority of tests C may be a 1 μF three-decade mica condenser, to which an additional fixed 1 μF may be added if β becomes very large. Place shows that with $R_s =$ 0·1 ohm only five values of R_p are needed to cover a range of I_p from 5 to 5 000 amperes; he does not give any particulars of his resistors, but the higher current values could with advantage be accommodated by the use of a single resistor with a nickel-iron cored transformer, thus reducing the number of resistors to four.* The values given by Place are shown in the table.

I_p in amperes	5 to 20	25 to 40	50 to 150	200 to 400, 800 and 1 200	500 to 600, 1 000, 1 500 to 5 000
R_p in ohms	0·1	0·025	0·01	0·0025	0·001

9. **Bridge methods with two mutual inductances. Compensation by resistance in primary circuit.** The methods to be described in this and in the following section are similar to some that have been discussed in earlier sections, except that the two four-terminal resistors used to compare the primary and secondary currents are replaced by two mutual inductors; consequently, the phase-angle compensation must be effected by some auxiliary resistance in one or other of the primary or secondary circuits. These two-mutual-inductance methods have an advantage not possessed by any hitherto described, that there is no direct connection between the primary and

* In the most recent development of the apparatus at the Metropolitan-Vickers works primary resistors are replaced entirely by a standard ring-type transformer with a 0·1 ohm resistor in its secondary circuit, the range of primary current being from 10 to 6 000 amperes; see, A. M. Armour, "Direct reading testing equipment for current transformers," *M.V. Gaz.*, vol. 14, pp. 355–359 (1934). The transformer has a ring core of mumetal strip with a uniform toroidal secondary. A wound primary is provided, consisting of sixty similar coils of ten turns, each occupying one-sixtieth of the circumference of the ring. By means of a system of plugs these coils can be grouped in a variety of different ways; each coil carries 10 amperes, so that with all in parallel currents up to 600 amperes are provided for. Above this value the primary consists of one or more loops of cable through the central hole; a single loop gives the 6 000 amperes range. A further series of ranges can be secured by combining cable loops with various settings of the plugs.

secondary sides of the test circuit; consequently, errors due to stray capacitance and leakage currents flowing from the primary supply to the detector may be quite avoided. Inductive interference troubles are, however, very much more pronounced than before, rendering necessary the use of perfectly astatic mutual inductors.

Barbagelata* has described the arrangement shown in Fig. 238 (*a*), in which the two inductors M_p and M_s have approximately equal secondary voltages set in vector opposition upon

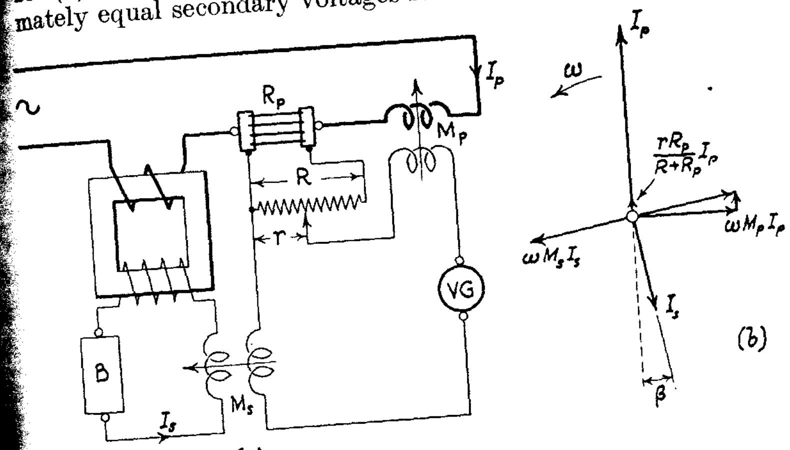


FIG. 238. BARBAGELATA'S BRIDGE METHOD

the detector. The primary winding of M_p must be capable of carrying the primary current I_p; the primary of M_s carries a maximum current of 5 amperes. One of the inductors should be variable, more conveniently M_s. The required in-phase component of voltage is derived from a four-terminal resistor R_p shunted by a potential divider R; balance is obtained by variation of the tapping r and M_s, with or without adjustment of M_p, the vector relations shown in Fig. 238 (*b*) holding for null indication. From the diagram

$$\omega M_s I_s \sin\beta = [rR_p/(R+R_p)]I_p,$$

$$\omega M_s I_s \cos\beta = \omega M_p I_p,$$

from which $$K_c = (M_s/M_p)\cos\beta \doteqdot M_s/M_p$$

and $$\tan\beta = \frac{rR_p}{\omega M_p(R+R_p)} \doteqdot \frac{r}{R}\cdot\frac{R_p}{\omega M_p}$$

* A. Barbagelata, loc. cit. on p. 433 (1921).

The method is the mutual inductance analogue of Campbell's method, Fig. 229 (*a*).

10. **Bridge methods with two mutual inductances. Compensation by resistance in secondary circuit.** In a method, suggested by Sharp and Crawford* but considerably developed by Fortescue† at the Westinghouse Co., two mutual inductors are used to compare the primary and secondary currents, as in the preceding method, but the phase-angle compensation is effected more conveniently by resistance in the secondary

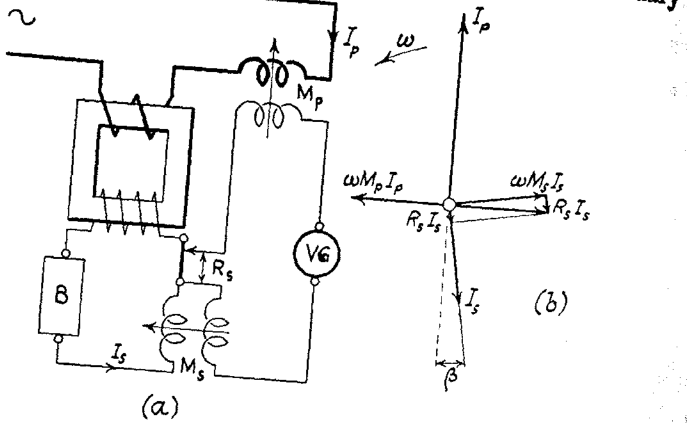


FIG. 239. FORTESCUE'S BRIDGE METHOD

circuit, as shown in Fig. 239 (*a*). The method is the analogue of Fig. 231 (*a*).

By adjusting M_s (or M_p) and R_s balance will be secured when the vector relations of Fig. 239 (*b*) are satisfied; that is,

$$\omega M_p I_p \sin\beta = R_s I_s,$$

$$\omega M_p I_p \cos\beta = \omega M_s I_s,$$

so that

$$K_c = M_s/(M_p \cos\beta) = M_s/M_p$$

and

$$\tan\beta = R_s/\omega M_s.$$

Residual errors in this method are very slight, since it is much easier to construct perfect mutual inductances—i.e.

* C. H. Sharp and W. W. Crawford, loc. cit. on p. 463 (1911).

† C. L. Fortescue, "The calibration of current transformers by means of mutual inductance," *Proc. Amer. I.E.E.*, vol. 34, pp. 1199–1215 (1915). An almost identical method has been described by F. Neri, "Sull'uso delle induzioni mutue per il controllo dei trasformatori di corrente," *L'Elettro.*, vol. 21, pp. 69–74 (1934).

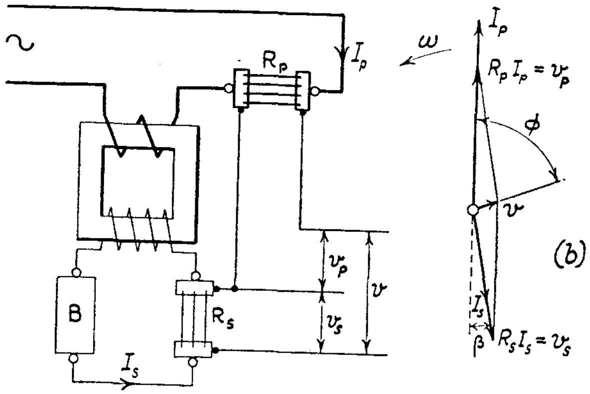
those in which the secondary voltage and the primary current are in exact quadrature—than it is to make non-reactive low resistances; consequently, any error due to residuals influencing the measurement of β are negligible when inductors are substituted for four-terminal resistors. The secondary burden is small, being of the order of 4 VA, while at the same time a large voltage can be impressed on the galvanometer circuit, with resulting high sensitiveness; the voltage at the secondaries of the inductors is about 4 volts. Again, as has already been mentioned, since there is no connection between the primary and secondary circuits, capacitance and leakage effects in the detector circuit are quite negligible. The principal trouble arises from the inductive interference set up by the stray fields of the inductors, and it is essential to construct them in perfectly astatic fashion so that they can neither influence one another nor be affected by the transformer under test. As usual, the wiring of all circuit elements should be bifilar in order to avoid any inductive loops.

Fortescue, loc. cit., has overcome stray-field troubles by designing toroidal ring-wound inductors, in which the windings are uniformly distributed over the periphery of accurately-machined marble rings of circular cross-section. M_s consists of three such toroidal elements connected in series, each of which has tapped windings so that the inductance can be regulated in five steps from zero to a maximum value of 3·0222 millihenrys. M_p consists of a single toroid of 0·8265 millihenrys maximum inductance. Full particulars of the complete set-up are given in his paper, with a detailed discussion of its uses in practice. The method is very flexible in its application, and a range of primary currents up to 5 000 amperes is covered.

11. **The use of alternating current potentiometers.** An alternating current potentiometer, in one of the forms described on p. 380, used in conjunction with suitable non-reactive four-terminal resistors, is an accurate and convenient instrument for the measurement of ratio error and phase-angle.* Referring to Fig. 240 (*a*) R_p and R_s are arranged to give approximately equal drops of about 0·5 volt, so that the secondary burden is small, being about 2 to 3 volt-amperes; for a 50/5 transformer the resistors may be about 0·01 and 0·1 ohm respectively.

* C. L. Dawes, *Proc. Amer. I.E.E.*, vol. 34, pp. 927–940 (1915); D. C. Gall, "Testing transformers by the alternating current potentiometer," *Elecn.*, vol. 83, pp. 603–604 (1920); A. C. Jolley, "Some tests on modern current transformers," *Journal Sci. Insts.*, vol. 3, pp. 43–50 (1925); C. V. Drysdale, "The testing of current transformers," *Journal Sci. Insts.*, vol. 3, pp. 57–58 (1925); T. Spooner, "Some applications of the a.c. potentiometer," *Journal Sci. Insts.*, vol. 3, pp. 214–221 (1926).

They should be non-reactive, and in precise work their resi-duals should be known. Measurements may be made of $v_p = R_p I_p$, $v_s = R_s I_s$ and β directly; since β is a small angle and therefore not readable with very great precision, it is



(a)

FIG. 240. THE USE OF AN A.C. POTENTIOMETER

better to measure v and ϕ, the angle between v_p and v, in its stead. Then from Fig. 240 (b),

$$v_s \sin \beta = v \sin \phi,$$

giving with the preceding measurements,

$$K_c = (v_p/v_s)\,(R_s/R_p)$$

and $$\sin \beta = (v/v_s) \sin \phi.$$

The method is capable of high accuracy and is quick to use. Its principal defect is that it makes use of an expensive instru-ment that is not always available.

12. **The use of thermal detectors.** It is worth while to notice that limited use has been made of thermal detectors in measure-ments of current transformer errors. The methods that have been suggested differ in no essential particular from those already described, except that a thermal detector replaces the dynamometer or vibration galvanometer.

Robinson* has suggested the use of two thermo-couples acting in opposition through a d.c. galvanometer, the couples being heated by currents proportional to I_p and I_s, as shown in Fig.

* L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 28, p. 1005 (1910).

41 (a). By adjusting r and s until the galvanometer is unde-ected the voltages of the couples can be made equal; con-quently the temperatures of the couples are equal and the

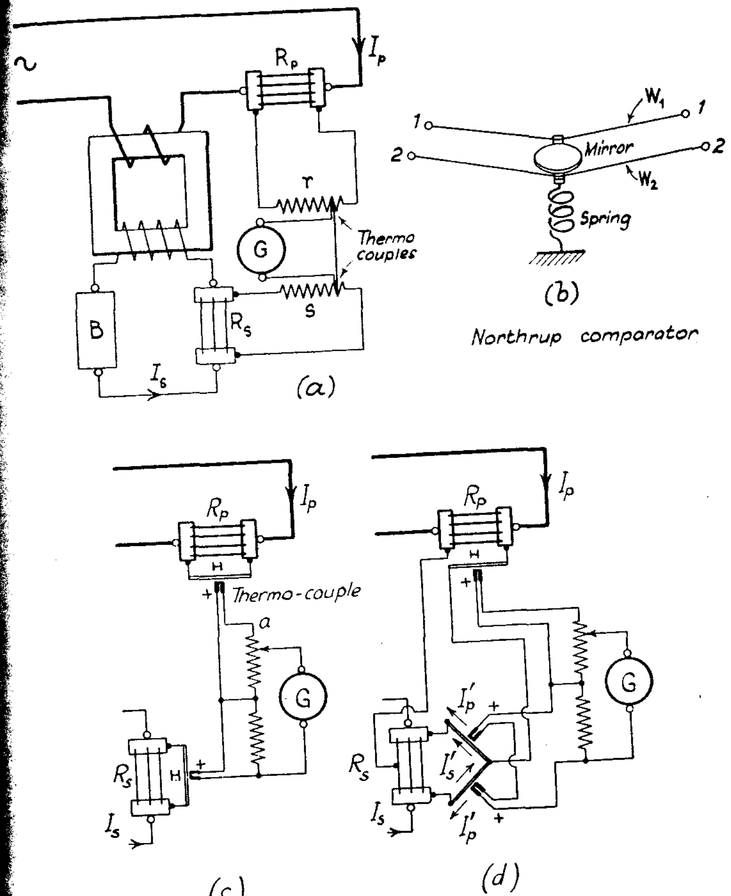


FIG. 241. THERMAL METHODS

rates at which heat is developed in r and s must be the same, that is

$$r\left(\frac{R_p}{r + R_p}\right)^2 I_p^2 = s\left(\frac{R_s}{s + R_s}\right)^2 I_s^2$$

so that $$K_c = \frac{R_s}{R_p} \cdot \frac{(r + R_p)}{(s + R_s)} \cdot \sqrt{\frac{s}{r}}.$$

A different principle is used in the Northrup comparator, shown in Fig. 241 (*b*), which is a differential hot-wire voltmeter; its use in transformer testing was also suggested by Robinson. The instrument consists of two wires W_1 and W_2 attached respectively to fixed terminals 1, 1, 2, 2, an insulating bridge at their middle carrying a mirror. A spring pulls the bridge from below and keeps the wires taut. The wire W_1 is connected in parallel with R_p and W_2 with R_s; if the volt-drops in these resistors are unequal and the wires are of equal resistance, they will carry different currents and will therefore heat unequally, causing the mirror to tilt. By adjusting R_p or R_s until there is no tilt the heating of the two wires will be made the same. The two resistors are then switched into auxiliary d.c. circuits in which the direct currents are adjusted until the comparator is again undeflected. Then the ratio of the two direct currents is the same as that of the alternating currents; the direct current ratio can be found by measuring the volt-drops over the two resistors by means of an ordinary d.c. potentiometer. Agnew* found it possible to obtain agreement to 1 part in 8 000 between the ratio found by this method and that determined by a dynamometer method. The instrument is subject to the usual defect of time-lag common to all hot-wire apparatus and can only be used with certainty in work where steady conditions are maintained.

The use of thermo-couples to measure both ratio and phase-angle has recently been described by Bruckman and Reichert,† in a portable high-precision testing set made by P. J. Kipp & Zonen of Delft. Fig. 241 (*c*) shows the essential parts of the arrangement for ratio tests. The resistors R_p, R_s are such that $R_s/R_p \fallingdotseq K_{nc}$ and each is shunted by the heaters H, H, of similar thermo-couples which are connected in opposition. When the transformer has no ratio error the couples give equal currents and G will read zero when the slider is at a; when there is a ratio error, however, the currents supplied by the couples are unequal and the slider must be moved to restore G to zero, the amount of displacement being a measure of the ratio error. To find the phase-angle, the connections of Fig. 241 (*d*) are employed. Two heaters are joined across R_s and carry a current I_s' proportional to I_s; a current I_p', proportional to I_p and nearly equal to I_s', is passed into the secondary heaters in such a way that one of them carries the sum and the other the difference of I_p' and I_s'. The voltages of the secondary couples are put in opposition so that their momentary resultant voltage is proportional to

$$(i_p' + i_s')^2 - (i_p' - i_s')^2 = 4\, i_p' i_s',$$

* P. G. Agnew, "A study of the current transformer with particular reference to iron loss," *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911).

† H. W. L. Bruckman and W. J. Reichert, "Ein einfaches Gerät zur Untersuchung von Messwandlern," *Elekt. Zeits.*, vol. 56, pp. 479–481 (1935); *Journal Sci. Insts.*, vol. 12, pp. 171–172 (1935).

their average voltage over a period to $I_p'I_s'\cos\beta$. The tage of the primary couple is proportional to $I_p'^2$ and is osed to that of the secondary couples. Balance is obtained operating the slider, its movement measuring the phase-angle To obtain greater accuracy for small phase-angles the rent in the primary heater is given a considerable phase-placement by means of an auxiliary condenser or choking il not shown in the diagram.

CHAPTER XXI

RELATIVE DEFLECTIONAL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Two-ammeter method.** The ratios of two transformers of the same nominal ratio can be easily compared* by the use of two ammeters, as shown in Fig. 242. In this diagram S is the standard transformer, for which the ratio-secondary current characteristic is known; X is the unknown transformer, its nominal ratio being equal to that of S; A_1 and A_2 are two similar ammeters, the calibrations of which need not be accurately known. These ammeters can be inserted at will into the secondary circuits of S and X by operating the throw-over switches S_1 and S_2. With S_1 to the left and S_2 to the right, thereby inserting A_1 in the secondary of S and A_2 in that of X, let the secondary currents of S and X be I_s and I_s' respectively. Let the readings of A_1 and A_2 be I_1 and I_2': then if k_1 and k_2 are the correction factors for these points on the ammeter scales,

$$I_s = k_1 I_1 \text{ and } I_s' = k_2 I_2'$$

By throwing S_1 to the right and S_2 to the left the ammeters are interchanged with respect to the transformers; if the ammeters are similar, the secondary currents will be practicall unaltered. Let the new reading of A_1, now in the secondar of X, be I_1', while that of A_2, now in the secondary of S, h become I_2; then very nearly

$$I_s = k_2 I_2 \text{ and } I_s' = k_1 I_1'$$

From these two sets of observations,

$$I_s^2 = k_1 k_2 I_1 I_2 \text{ and } I_s'^2 = k_1 k_2 I_1' I_2'$$

The ratios of S and X are

$$K_c = I_p/I_s \text{ and } K_{cx} = I_p/I_s' = K_c I_s/I_s'.$$

Substituting for I_s and I_s',

$$K_{cx} = [\sqrt{(I_1 I_2/I_1' I_2')}]K_c$$

* F. B. Silsbee, *Trans. Amer. I.E.E.*, vol. 43, pp. 282–294 (1924).

hich eliminates the calibrations of the two instruments. Care ust be taken when interchanging the ammeters that the condary circuits are not opened; for this purpose the switches own in the diagram should be utilized to short-circuit the condaries before operating the switches S_1 and S_2.

The method gives only the ratio of the transformer and is sy and quick for tests on site where a moderate accuracy is fficient; the precision falls off considerably at low loads.

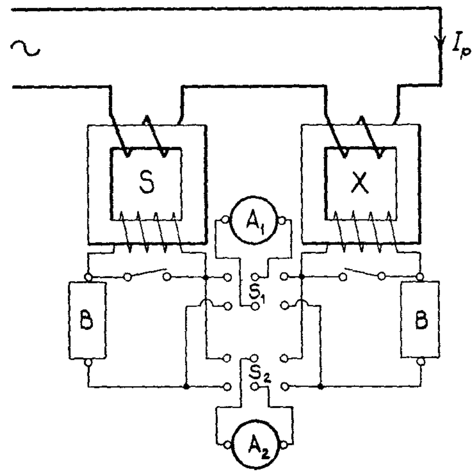


FIG. 242. TWO-AMMETER RELATIVE METHOD

The method can be readily modified to give greater precision, especially at low loads, and at the same time to determine the phase-angle error, by substituting for the ammeters the current coils of two dynamometers, the voltage coils of which are excited in parallel from a phase-shifter. The phase-angle can be found by adjusting the phase-shifter until each dynamometer in turn gives a zero reading, the angle through which the phase of the auxiliary supply must be moved being equal to the phase difference between I_s and I_s'. The magnitudes of the currents, and hence the ratio of the unknown in terms of the standard, can be obtained by turning the phase-shifter through $\pi/2$ from each of the zero-reading positions, each dynamometer coming in turn to a maximum reading proportional to the respective currents. Calibration errors in the dynamometers can be eliminated by repeating observations with the current coils interchanged in the secondary circuits, just as has been described for ammeters.

2. **Two-dynamometer methods.** There is a number of ways of comparing two transformers with the aid of two dynamometers, some of the more useful of these being now noticed. In the method of Makower and Wust* two similar transformers of the same nominal ratio are connected back-to-back, the combination being tested as a single unit by the dynamometer method of p. 429. It is then assumed that the ratio of either transformer is equal to the square root of the effective ratio

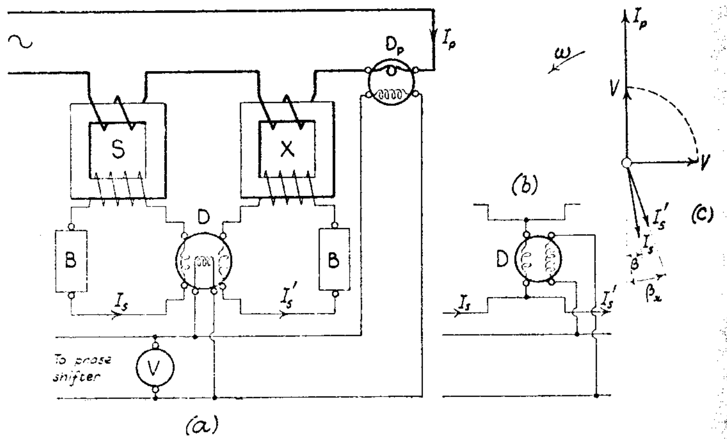


FIG. 243. BARBAGELATA'S TWO-DYNAMOMETER RELATIVE METHOD

of the combination and that the phase-angle of each is one-half that of the combination. This assumption cannot be regarded as generally true; quite frequently the standard and unknown transformers have very different characteristics, so that this method can only be regarded as a rough test, unless the transformers should prove by chance to be nearly identical.

In the method shown in Fig. 243 (a)† the common primary current of the two transformers flows in the current coil of the dynamometer D_p. The secondary currents, I_s of the standard and I_s' of the unknown, flow through separate current coils of a dynamometer D; these may be two current coils of the single-element wattmeter with a common voltage coil; or they may be two current elements of a double wattmeter, in which

* A. J. Makower and A. Wust, *Elecn.*, vol. 79, pp. 581–582 (1917).

† A. Barbagelata, loc. cit. on p. 433 (1921). See also E. C. Westcott, "Differentially wound watt-hour meter for testing current transformers," *Elec. World*, vol. 76, p. 433 (1920), for a similar method used for routine tests on the turns adjustment of transformers.

case the two voltage coils corresponding thereto will be excited in parallel from a common supply. In either case the currents are arranged to flow in directions producing opposite torques. Alternatively, the currents may be superposed in opposition in the single current coil of a simple dynamometer, as in Fig. 243 (b). The voltage coils of D_p and D are excited at voltage V from a phase-shifter; they should be of equal or of very low time-constants, or be alternatively supplied in series, when it will be necessary to measure the common current flowing in them instead of the voltage.

The phase-shifter is adjusted until D_p reads zero, indicating that the currents in its two coils are in quadrature; assuming a non-reactive volt-circuit, V and I_p will be in quadrature as shown in Fig. 243 (c), and the reading of D will be W_1. The phase-shifter is now adjusted through $\pi/2$, so that D_p attains its maximum reading since V and I_p will be in phase; the corresponding reading of D will be W_2. From the vector diagram it is easily seen that

$$W_1 = VI_s \cos[(\pi/2) - \beta] - VI_s' \cos[(\pi/2) - \beta_x] \text{ watts,}$$
$$W_2 = VI_s \cos(\pi - \beta) - VI_s' \cos(\pi - \beta_x), \text{ watts;}$$

whence

$$W_1 = -V[I_s' \sin\beta_x - I_s \sin\beta] = -VI_p[(\sin\beta_x/K_{cx}) - (\sin\beta/K_c)]$$
$$W_2 = V[I_s' \cos\beta_x - I_s \cos\beta] = VI_p[(\cos\beta_x/K_{cx}) - (\cos\beta/K_c)]$$

Now β and β_x are small; therefore

$$(1/K_{cx}) - (1/K_c) \equiv (\varepsilon_{cx} - \varepsilon_c)/K_{nc} \doteqdot W_2/VI_p$$

and

$$\beta_x - \beta \doteqdot -(W_1K_c/VI_p)$$

where K_c, K_{cx} are the ratios of standard and unknown, ε_c, ε_{cx} their fractional errors and β, β_x are their phase-angles.* The value of I_p is calculated from V and the observed maximum reading of D_p. It is assumed in deducing the above expressions that β_x exceeds β; W_1 will therefore be negative in this instance.

Silsbee† has described a method, somewhat similar to the preceding, in which the phase-adjustment of the voltage circuit

* This method has been used by H. W. Price and C. K. Duff, "Method of measuring ratio and phase-angle," *Univ. Toronto Eng. Res., Bull.*, No. 2, pp. 191–201 (1921), to compare the primary and secondary currents of a 1 : 1 ratio transformer, the former taking the place of I_s and the latter of I_s' with $K_c = 1$ and $\beta = 0$.

† F. B. Silsbee, "A method of testing current transformers," *Bull. Bur. Stds.*, vol. 14, pp. 317–329 (1919). The original suggestion appears to be due to W. A. Folger in 1916; his paper in the Proceedings of the Pennsylvania Electrical Association is not readily available in Great Britain. The present author has not succeeded in locating it in any technical library in London. See also Silsbee, loc. cit. on p. 488 (1924).

supply is regulated by means of a dynamometer D_s in the secondary circuit of the standard transformer, while the secondary currents of the two transformers are superposed in the current coil of the dynamometer D, as shown in Fig. 244 (a). The auxiliary voltage V, indicated by the voltmeter V, is supplied by a phase-shifter* first in quadrature with I_s and then in phase with it, these conditions resulting in a zero and a maximum reading of D_s respectively. Referring to

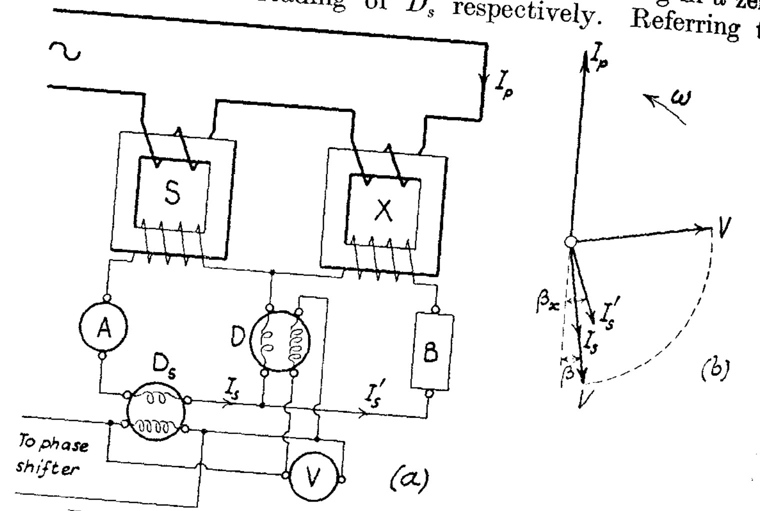


FIG. 244. SILSBEE'S TWO-DYNAMOMETER RELATIVE METHOD

Fig. 244 (b), let the corresponding readings of D in watts be W_1 and W_2; then

$$W_1 = -VI_s' \cos[(\pi/2) + \beta - \beta_x] = -VI_s' \sin(\beta_x - \beta),$$
$$W_2 = V[I_s - I_s' \cos(\beta_x - \beta)];$$

from which

$$\frac{K_{cx}}{K_c} \doteq 1 - (\varepsilon_{cx} - \varepsilon_c) \doteq \frac{1}{1 - (W_2/VI_s)} \doteq 1 + \frac{W_2}{VI_s}$$

$$\tan(\beta_x - \beta) = \frac{-W_1/VI_s}{1 - (W_2/VI_s)} \doteq -\frac{W_1}{VI_s} \doteq \beta_x - \beta.$$

The current I_s may be read on the ammeter A but is more

* For an alternative way of taking readings, using simple phase-shifting apparatus or a three-phase supply, see G. W. Stubbings, "Ratio and phase-error tests on current transformers," *Elec. Rev.*, vol. 94, pp. 604–606 (1924). See also F. A. Kartak, *Elec. World*, vol. 75, pp. 1368–1370 (1920).

accurately determined, especially at low loads, from the maximum reading of D_s and the known voltage V. As before, it is assumed that the ratio and phase-angle of X are greater than those of S, as is usually the case. To determine this point, an additional resistance of about an ohm should be added to the burden of X. If the ratio and phase-angle of X were originally greater than those of S the readings W_1 and W_2 will be increased; if these readings should be diminished or even reversed then the errors of X were originally less than those of S.

The principal limitation of the method is the sensitiveness of the detector D, the current coil of which must have a sufficiently low impedance; a 1 ampere wattmeter may give adequate sensitivity for most purposes, but much better results can be secured by the use of a specially designed instrument. Greater sensitiveness can be obtained by reducing the working forces, which is not desirable in an instrument intended for workshop use; alternatively, the current-coil may be given a large number of turns or the ampere-turns of the voltage-coil may be made large. The first of these alternatives cannot be pressed too far since it increases the impedance operator z_D of the detector. The total burden on the secondary of X will be

$$B_x = \frac{B\,\boldsymbol{i}_s' - z_D(\boldsymbol{i}_s - \boldsymbol{i}_s')}{\boldsymbol{i}_s'} = B - \frac{\Delta \boldsymbol{i}}{\boldsymbol{i}_s'} z_D$$

where $\Delta\boldsymbol{i}$ is the current in the detector current-coil. Hence the actual burden differs from the attached burden B by an amount dependent on z_D and on $\Delta\boldsymbol{i}/\boldsymbol{i}_s'$, so that it varies with the load current and cannot be compensated. Care must be taken, therefore, to make z_D small enough to cause a negligible error. The second of these alternatives will, if pushed too far, result in an error due to the excessive e.m.f. of mutual induction between the voltage- and current-coils of the instrument; to overcome this trouble Silsbee suggests the use of zero-pattern dynamometers where the moving element is always restored to a position of zero mutual inductance with respect to the fixed element. In deflectional instruments all that can be done is to keep the mutual inductance small enough, consistent with the production of adequate sensitiveness.

Spilsbury* has designed a special instrument, made by Elliott Bros., on the lines of a substandard central-zero wattmeter scaled in amperes

* R. S. J. Spilsbury, "An instrument for workshop tests of current transformers," *Journal Sci. Insts.*, vol. 1, pp. 273–278 (1924).

at the rated value of the auxiliary voltage. There are 50 divisions on each side of zero, each with an average length of 1·2 mm., and the constant of the scale is 0·005 amperes per division. The impedance of the current circuit is 0·68 ohm at 50 cycles per sec., giving a negligible effect on the burden of X. The current range is $+ 0·25$ to $- 0·25$ amperes and the voltage range 55 and 110 volts, the voltage circuit current being

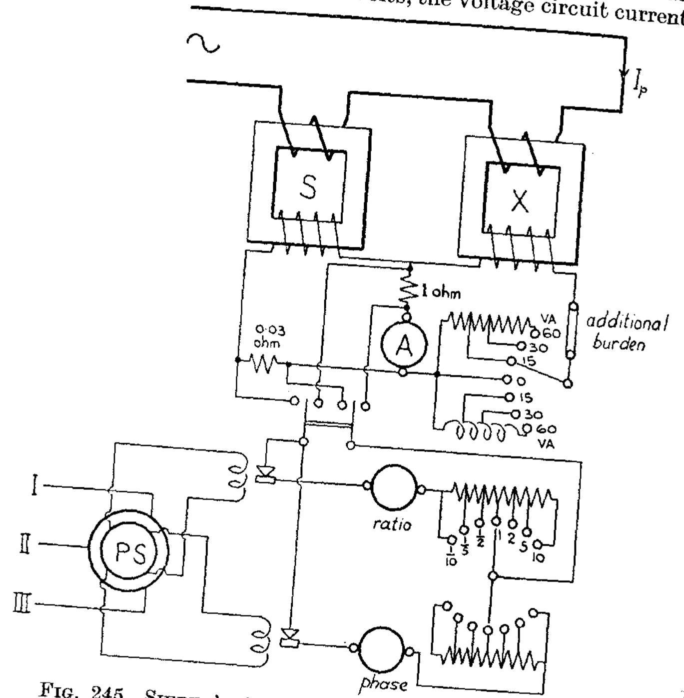


FIG. 245. SIEBER'S ARRANGEMENT OF SILSBEE'S METHOD

0·075 amperes. Mutual inductance error at full scale reading is quite negligible. The instrument enables ratio differences of 5 per cent and angle differences of 3 degrees to be determined, the results agreeing excellently with values obtained by absolute methods. V is a substandard voltmeter of 55 or 110 volt range; D_s is a 5 ampere, 55 or 110 volt wattmeter; A is convenient but not necessary as I_s is more accurately found by the reading of D_s. Care must be taken to check the polarity of the transformers by observing the deflection W_2 of D for about per cent of full load current; if it is very small the polarity is correct.

Siemens & Halske A.G., manufacture a portable testing set described by Sieber* and illustrated in Fig. 245. The ingenious feature of the apparatus is the provision of two detectors, one to give the ratio error and the other the phase-angle. These detectors consist of d.c. pointer

* O. Sieber, "Eine neue tragbare Stromwandler-Prüfeinrichtung," *Siemens Zeits.*, vol. 9, pp. 845–850 (1929).

galvanometers supplied with current by small mechanical rectifiers of the vibrating reed type, see p. 400. The rectifiers are shown diagrammatically at the left of the figure. Their excitation is taken from two phases in quadrature on the secondary of a phase-shifting transformer, the primary of which is excited by a three-phase supply. Burdens, both non-inductive and inductive, are provided for X, and a removable link provides for additional external burdens. The ammeter A is scaled to 10 amperes and serves to check that the transformers have been connected with correct polarity, this being shown by approximately zero reading. The switch is thrown to the left, thus connecting the detectors across the 0·03 ohm resistor in the secondary of S; by adjustment of the phase-shifter the reading of the "phase" detector can be brought to zero, showing that its excitation is in quadrature with I_s while that of the "ratio" detector is automatically in phase therewith. Throwing the switch to the right puts the detectors across the 1 ohm resistor carrying the difference current $i_s - i_s'$; the ratio error and phase-angle are read off the appropriate detector scales, a range of $\pm$ 3 per cent of ratio error and $\pm$ 2 degrees of angle being provided. The detectors are provided with a ratio switch enabling full sensitivity to be maintained at 1/10, 1/5, 1/2, 1, 2, 5 and 10 times full load, the last three factors being used in tests of relay transformers under the V.D.E. rules. The entire equipment, including phase-shifter, detectors and rectifiers, resistors, burdens, sensitivity switch and ammeter, is contained in a single case, only the standard transformer being additional. A nine-point test requires only seven minutes to carry out, and an accuracy of $\pm$ 0·1 per cent in ratio and $\pm$ 5 minutes in phase-angle is attainable.

Masa* has designed a testing set which is a simple modification of Silsbee's method. The dynamometer D in Fig. 244 is replaced by a double differential instrument, one of the current coils of which carries I_s while the other carries I_s'. The voltage coils of D and of D_s are joined in series and are supplied with a common current I from the phase-shifter. With D reading zero, by putting I in quadrature with I_s, the deflection of D is d_1; let k be the constant of D, then

$$d_1 = - kII_s' \cos[(\pi/2) + \beta - \beta_x] = - kII_s' \sin(\beta_x - \beta)$$

Changing the phase of I until D_s gives its maximum reading d and the deflection of D becomes d_2,

$$d_2 = kI[I_s - I_s' \cos(\beta_x - \beta)] \text{ and } d = k_s II_s$$

if k_s is the constant for D_s. If ε_c and ε_{cx} are the ratio errors of S and X in per cent it is not difficult to show that

$$\varepsilon_{cx} - \varepsilon_c \doteqdot - 100k_s d_2/kd$$

and $$\tan(\beta_x - \beta) = \frac{d_1}{d_2 - (kd/k_s)} = - \frac{k_s d_1}{kd\{1 + [(\varepsilon_{cx} - \varepsilon_c)/100]\}}$$

The apparatus is portable and the various parts are designed to be direct reading. The difference of ratio error is readable to 0·05 per cent and of phase-angle to 2 minutes down to 2 per cent of rated current.

* P. Masa, "Un nouveau dispositif pour l'étalonnage des transformateurs de mesure," *Rev. Gen. de l'El.*, vol. 32, pp. 727–731 (1932).

3. **Single-dynamometer method.** A simple modification of the preceding methods has been introduced by Crothers,* enabling a single dynamometer to be used, as shown in Fig. 246 (*a*). Assuming the volt-coil of D to have negligible reactance and neglecting the shunting effect of the high resistance

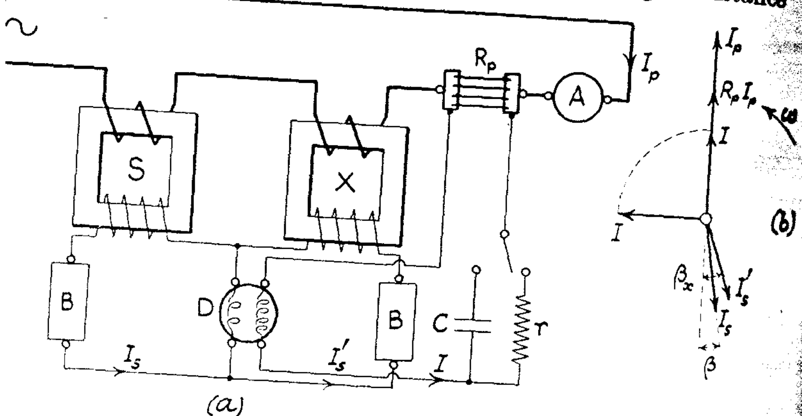


FIG. 246. CROTHERS'S SINGLE-DYNAMOMETER RELATIVE METHOD

voltage circuit on R_p, the reading of D in watts will be very closely

$$W_1 = R_p I_p \; [I_s \cos(\pi - \beta) - I_s' \cos(\pi - \beta_x)]$$
$$= R_p I_p \; [I_s' \cos \beta_x - I_s \cos \beta],$$

as Fig. 246 (*b*) shows. From this expression

$$(1/K_{cx}) - (1/K_c) = (\varepsilon_{cx} - \varepsilon_c)/K_{nc} \doteqdot W_1/R_p I_p^2$$

The method so far resembles Barbagelata's procedure and gives the ratio error of X; Crothers does not state how the phase-angle is to be found. Comparing Fig. 246 (*a*) with Fig. 217 (*a*) showing Drysdale's absolute method for a 1:1 ratio transformer, it will be seen that there is considerable similarity. In Drysdale's method the approximately equal primary and secondary currents of a 1 : 1 transformer are superposed in the dynamometer coil, while in the present method the superposed nearly equal currents are obtained from two separate transformer secondaries. This comparison suggests that the phase-angle may be found by substituting for the swamp resistance

* H. M. Crothers, "Field testing of instrument transformers," *Elec. World*, vol. 74, pp. 119–121 (1919).

of the voltage circuit of D a condenser of equal reactance; the current I is thereby advanced in phase by $\pi/2$ relative to I_p and the new reading of D will be

$$W_2 = R_p I_p \; \{[I_s \cos\,[(3\pi/2) - \beta] - I_s' \cos\,[(3\pi/2 - \beta_x)]\}$$
$$= -\,R_p I_p \; [I_s \sin \beta - I_s' \sin \beta_x]$$

from which

$$\beta_x - \beta \doteqdot W_2 K_c / R_p I_p^2$$

The method is suggested as very convenient for testing transformers on site, since the amount of necessary apparatus is reduced to a minimum and is of a readily portable nature.

4. **Watt-hour meter methods.** In Section 1 of the present chapter it has been pointed out that the ratio error and phase-angle of a current transformer can be compared with the corresponding characteristics of a standard transformer by the aid of two interchanged, separately-excited dynamometers. Agnew* has shown that since the differences in ratio and phase-angle of the two transformers are usually small it is not possible by this method to attain high precision. If, however, the dynamometers be replaced by watt-hour meters considerable precision can be attained, since the effects of ratio error and phase-angle in the transformers supplying such instruments are cumulative over an interval of time.

Referring to Fig. 247, the meters *a* and *b* are of similar type and should first be adjusted to work correctly on full current at unity and at zero power-factor. It is a great convenience if the ratio of gearing can be arranged so that 10 revolutions of the disc correspond with one division on the first dial; the disc may also be marked off with equally-spaced divisions to enable fractions of a revolution to be observed if desired. It is an advantage if the meters can be speeded up above their normal running speed, to enable the time occupied by a test to be reduced as much as possible; this can be arranged by shunting the flux of the brake magnets by the addition of small soft-iron bridge pieces.

Observations are made of the number of revolutions made by each meter disc (i) with S_1 to the left and S_2 to the right,

* P. G. Agnew, "A watt-hour meter method of testing instrument transformers," *Bull. Bur. Stds.*, vol. 11, pp. 347–357 (1915); also F. A. Kartak, loc. cit. (1920); A. Barbagelata, loc. cit. (1921); F. B. Silsbee, loc. cit. (1924). An identical process is described by A. Alemany, "Note sur une méthode pratique de vérification des transformateurs de mesure," *Rev. Gén. de l'Él.*, vol. 18, pp. 515–518 (1925).

and (ii) with S_1 to the right and S_2 to the left at a known phase-displacement between I_p and the auxiliary voltage of the phase-shifter. Care should be taken not to open-circuit the secondaries when operating the switches; short-circuiting switches, as shown in Fig. 242, may be advantageously added. Let a_s, b_x be the numbers of revolutions with arrangement (i) and a_x, b_s the corresponding revolutions with (ii) each in the

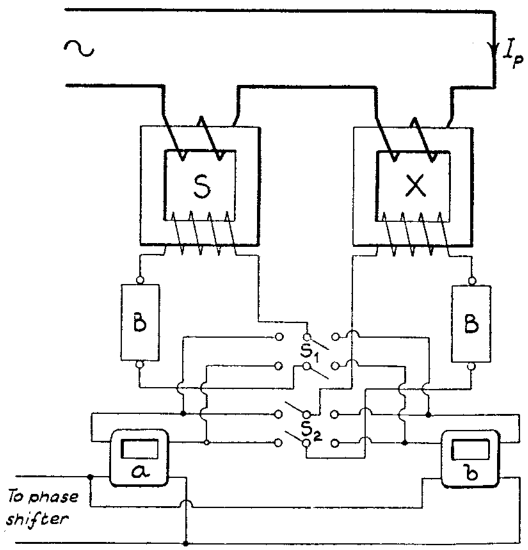


FIG. 247. AGNEW'S WATT-HOUR METER RELATIVE METHOD

same total time. If the meters are designed so that the dial constant k, the nominal watt-hours per revolution, is the same for each, let m_a, m_b be the rates of the two meters, i.e. the ratio of recorded watt-hours to the true watt-hours. In general, if a meter is connected to a circuit of power-factor $\cos\phi$ through a current transformer of ratio K_c, nominal ratio K_{nc} and phase angle β the watt-hours in the circuit will be

$$\frac{K_c \cos\phi}{K_{nc}\cos(\phi-\beta)}\cdot\frac{kr}{m}=\frac{K_c kr}{K_{nc}\, m\cos\beta\,(1+\tan\phi\,.\,\tan\beta)}$$

for r revolutions, as on p. 300. Applying this to conditions (i) and (ii) successively, since K_{nc} is the same for S and X

$$\frac{K_c k a_s}{m_a\cos\beta\,(1+\tan\phi\,.\,\tan\beta)}=\frac{K_{cx} k b_x}{m_b\cos\beta_x\,(1+\tan\phi\,.\,\tan\beta_x)},$$

$$\frac{K_{cx} k a_x}{m_a\cos\beta_x\,(1+\tan\phi\,.\,\tan\beta_x)}=\frac{K_c k b_s}{m_b\cos\beta\,(1+\tan\phi\,.\,\tan\beta)},$$

for any power-factor. Now $\cos\beta \doteq \cos\beta_x \doteq 1$ since the angles are small, so that very closely,

$$\frac{K_c a_s}{m_a(1+\tan\phi\,.\,\tan\beta)}=\frac{K_{cx} b_x}{m_b(1+\tan\phi\,.\,\tan\beta_x)},$$

$$\frac{K_{cx} a_x}{m_a(1+\tan\phi\,.\,\tan\beta_x)}=\frac{K_c b_s}{m_b(1+\tan\phi\,.\,\tan\beta)}.$$

With $\cos\phi = 1$, i.e. I_p in phase with the auxiliary voltage, the formulae reduce to

$$K_c a_s/m_a = K_{cx} b_x/m_b$$

$$K_{cx} a_x/m_a = K_c b_s/m_b$$

from which
$$\frac{K_{cx}}{K_c}=\sqrt{\left[\frac{a_s}{a_x}\cdot\frac{b_s}{b_x}\right]}\doteq 1-(\varepsilon_{cx}-\varepsilon_c).$$

The meters are now run on a low power-factor, with $\cos\phi = 0{\cdot}5$ or less; let a_s', b_x', a_x', b_s' be the new readings under conditions (i) and (ii), then

$$\frac{a_s'}{a_x'}\cdot\frac{K_c}{K_{cx}}\cdot\frac{(1+\tan\phi\,.\,\tan\beta_x)}{(1+\tan\phi\,.\,\tan\beta)}\doteq\frac{b_x'}{b_s'}\cdot\frac{K_{cx}}{K_c}\cdot\frac{(1+\tan\phi\,.\,\tan\beta)}{(1+\tan\phi\,.\,\tan\beta_x)}$$

or
$$\frac{(1+\tan\phi\,.\,\tan\beta_x)^2}{(1+\tan\phi\,.\,\tan\beta)^2}\doteq\frac{a_x' b_x'}{a_s' b_s'}\cdot\frac{K_{cx}^2}{K_c^2}.$$

Remembering that $\tan\beta$ and $\tan\beta_x$ are small this may be written,

$$1+2\tan\phi\,(\tan\beta_x-\tan\beta)\doteq\frac{a_x'\,b_x'}{a_s'\,b_s'}\cdot\frac{K_{cx}^2}{K_c^2},$$

whence
$$\tan\beta_x-\tan\beta\doteq-\frac{1}{2\tan\phi}\left[1-\frac{a_x' b_x'}{a_s' b_s'}\cdot\frac{a_s b_s}{a_x b_x}\right].$$

The signs in these expressions are correct for both I_s and I_s' leading on $-I_p$ and with $\beta_x > \beta$. To test which transformer has the greater errors add a further non-inductive burden to the secondary of X; if the difference is increased then X had originally greater errors than S.

By taking a sufficiently great number of revolutions ample accuracy can be attained in all commercial tests; the ratio can be found to 0·03 per cent and the angle within 1 or 2 minutes. The main objection to the method is, however, the considerable time taken by a test. The readings are independent of fluctuations in the line current and no specialized apparatus is required to obtain them; consequently, the method is admirably adapted to tests of transformers on site. For this purpose, a simplified procedure has been suggested by Crothers* and by Craighead and Weller.† The last-named investigators reduce the time of a test by a carefully-planned procedure and the use of a tabular form of calculation; a six-point check takes seven hours. The standard transformer should be carefully calibrated and not differ by more than 5 per cent in ratio or 1 degree in angle from the unknown. The meters must be well adjusted and their difference in rate must be in the same direction for $\cos\phi = 1$ as for $\cos\phi = 0{\cdot}5$; they must not "creep." The following results show the order of agreement obtained on a 10/5 transformer at 60 cycles per sec., the burden being 0·396 ohm and 510 microhenries, corresponding with 11 volt-amperes at 0·9 power-factor.

Primary Current	Absolute Method		Watt-hour Meter Method	
	Ratio Factor F_{cx}	β_x	Ratio Factor F_{cx}	β_x
1	1·0017	+ 29 min.	1·0010	+ 34 min.
2	0·9997	21	0·9996	23
4	0·9977	14	0·9977	18
6	0·9971	12	0·9969	13
8	0·9967	10		
10	0·9959	9	0·9961	12

An absolute method for testing a 1 : 1 ratio transformer has been introduced by Knopp and is described on p. 439. Comparing Fig. 219 (a) with Fig. 247, it will be seen that Knopp's method is equivalent to testing by Agnew's method an unknown transformer (the secondary current) with a perfect 1 : 1 transformer (the primary current). Assuming the meters to be interchanged to eliminate differences of calibration, write $K_s = 1$, $\beta = 0$, $K_{cx} = K_c$ and $\beta_x = \beta$, where K_c and β are the ratio and phase-angle for the transformer; then

$$K_c = \sqrt{(a_p b_p / a_s b_s)}$$

and
$$\tan\beta = -\frac{1}{2\tan\phi}\left[1 - \frac{a_s' b_s'}{a_p' b_p'} \cdot \frac{a_p b_p}{a_s b_s}\right]$$

where a_p, b_p, a_p', b_p'; a_s, b_s, a_s', b_s' are the readings of meters a and b in the primary and secondary of the transformer.

* H. M. Crothers, "Field testing of instrument transformers," *Elec. World*, vol. 74, pp. 119–121 (1919).

† J. R. Craighead and C. T. Weller, "Watt-hour meter method of testing current transformers for ratio and phase-angle," *Gen. Elec. Rev.*, vol. 24, pp. 642–651 (1923).

he chief defect of Agnew's method is the time required making a test; Slavik* has overcome this objection an ingenious device shown in Fig. 248. Double-element tt-hour meters are used, each provided with two voltage ls adjusted to be in time-quadrature; consequently, ether the meters are working at high or at low power-factor

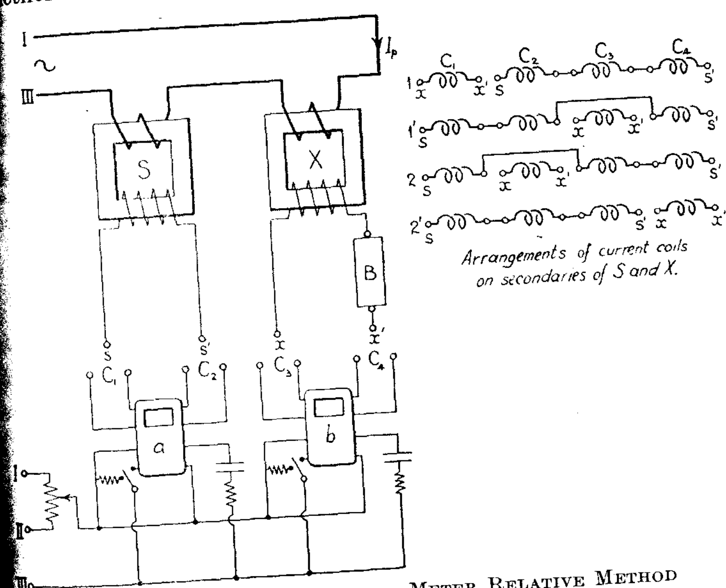


FIG. 248. SLAVIK'S WATT-HOUR METER RELATIVE METHOD

he meter discs are subjected to full torque and will run quickly, hus saving time over Agnew's method in which the low ower-factor condition causes the meters to run very slowly.

The quadrature voltage coils are permanently connected cross the auxiliary supply, the necessary quadrature condition eing realized with the aid of condensers and series resistances s shown. The in-phase voltage coils may either be connected o the auxiliary supply or be replaced by equivalent ballast esistances by throwing the switches to the right or to the left. The current coils C_1, C_2, C_3 and C_4 may be connected to the secondaries of the standard S and the unknown X, in the equences shown to the right of the diagram, by operating a small drum-type controller. The auxiliary supply for the

* J. Slavik, "Ein neues Verfahren zur Bestimmung der Fehlergrössen bei Messwandlern," *Elekt. Zeits.*, vol. 50, pp. 1360–1362 (1929).

voltage circuits is derived from phase III of a three-phase system and a potential divider connected across phases I and II, this providing a phase-adjustment of $\pm 120°$; the primary current I_p is supplied by phases I and III.

The current coils are arranged in Sequence 1, the ballast resistance of meter b being in use while the remaining three voltage circuits are active. The phase of the auxiliary voltage is regulated until meter b is at rest and meter a runs at full speed. Since b operates on its quadrature system only, the auxiliary voltage must now be in phase with the secondary current I_s of the standard transformer S. This preliminary adjustment being accomplished the ballast resistance is removed, so that all four voltage coils are in use, and the meters are set to zero. Two sets of observations are now taken of the numbers of revolutions performed by the meter discs when the current coils are grouped in Sequences 1 and 1′ respectively. Let a_x, b_x be the revolutions of meters a and b when connected to X; a_s, b_s the corresponding readings when connected to S, all readings being taken in a common time t. Let k_a, k_b be the revolutions per watt-hour for meters a and b; then it is easy to see that in Sequence 1,

$$a_x = tk_a V(I_s' \cos\delta + I_s \sin 0),$$
$$b_s = tk_b V(I_s \cos\delta + I_s \sin 0),$$

where $\delta = \beta_x - \beta$ and V is the auxiliary voltage; from these

$$k_a I_s'/k_b I_s = a_x/b_s$$

In a similar way, if a_s and b_x be the readings in Sequence 1′,

$$k_b I_s'/k_a I_s = b_x/a_s$$

Eliminating the meter constants gives

$$I_s'/I_s = \sqrt{(a_x b_x/a_s b_s)}$$

or $$K_{cx}/K_c = \sqrt{(a_s b_s/a_x b_x)} \doteqdot 1 - (\varepsilon_{cx} - \varepsilon_c).$$

With current coils in Sequences 2 and 2′ two further sets of readings are taken. In Sequence 2

$$a_x' = tk_a V(I_s \cos 0 + I_s' \sin\delta),$$
$$b_s' = tk_b V(I_s \cos 0 + I_s \sin 0),$$

or $$k_a (I_s + I_s' \sin\delta)/k_b I_s = a_x'/b_s'.$$

With the coils in Sequence 2′ gives in a similar manner

$$k_b (I_s + I_s' \sin\delta)/k_a I_s = b_x'/a_s'$$

iminating the instrument constants

$$\sin(\beta_x - \beta) = \frac{I_s}{I_s'}\left[\frac{\sqrt{(a_x'b_x')} - \sqrt{(a_s'b_s')}}{\sqrt{(a_s'b_s')}}\right],$$

om which an exact formula can be developed by substituting r I_s/I_s' from the tests in Sequences 1 and 1′. As this ratio is ery nearly unity and the sine of δ nearly equal to its tangent

$$\tan(\beta_x - \beta) \doteqdot \frac{\sqrt{(a_x'b_x')} - \sqrt{(a_s'b_s')}}{\sqrt{(a_s'b_s')}}.$$

High accuracy is attainable, with a small amount of readily portable apparatus, and a considerable saving of time is claimed over Agnew's method with single-element meters.

The ratio formula can be written in the form

$$\frac{K_c - K_{cx}}{K_{cx}} = \frac{\sqrt{(a_x b_x)} - \sqrt{(a_s b_s)}}{\sqrt{(a_s b_s)}},$$

which is precisely similar to that for the phase-angle and can be still further simplified. In practice $a_x \doteqdot b_x$, and $a_s \doteqdot b_s$, with similar relations for the primed letters. Replacing the geometric means by arithmetic means

$$\frac{K_c - K_{cx}}{K_{cx}} \doteqdot \frac{(a_x + b_x) - (a_s + b_s)}{(a_s + b_s)} \text{ and}$$

$$\tan(\beta_x - \beta) \doteqdot \beta_x - \beta \doteqdot \frac{(a_x' + b_x') - (a_s' + b_s')}{(a_s' + b_s')}$$

In a numerical example $a_x = 30{\cdot}38_8$, $b_x = 30{\cdot}27_5$, $a_s = 30{\cdot}37_4$ and $b_s = 30{\cdot}42_0$; hence, $a_x + b_x = 60{\cdot}66_3$ and $a_s + b_s = 60{\cdot}79_4$. Then, $(K_c - K_{cx})/K_{cx} = (60{\cdot}66_3 - 60{\cdot}79_4)/60{\cdot}79_4 = -0{\cdot}0022$ and $K_{cx} = 1{\cdot}0022 K_c$. Similarly, $a_x' = 30{\cdot}38_0$, $b_x' = 30{\cdot}34_1$, $a_s' = 30{\cdot}55_3$ and $b_s' = 30{\cdot}67_2$; hence $a_x' + b_x' = 61{\cdot}72_1$ and $a_s' + b_s' = 61{\cdot}22_5$. Then,

$$\beta_x - \beta \doteqdot (61{\cdot}72_1 - 61{\cdot}22_5)/61{\cdot}22_5 = 0{\cdot}0081 \text{ radian} = 28 \text{ minutes.}$$

CHAPTER XXII

RELATIVE NULL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Shotter's double-dynamometer method.** G. F. Shotter* has recently described a null method for the comparison of two transformers, shown in Fig. 249, in which a double dynamometer or two-element wattmeter is used; since the coils

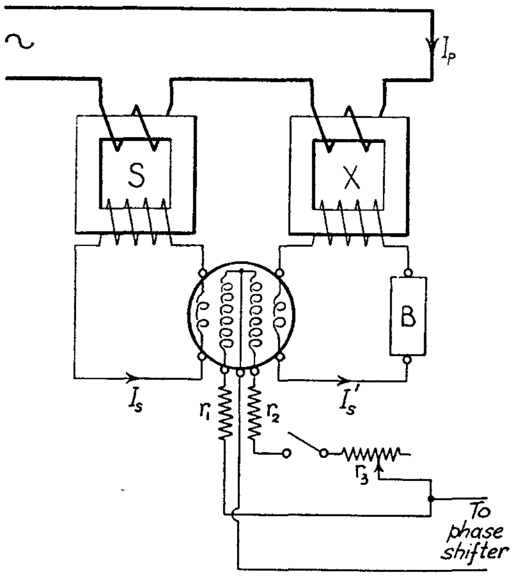


FIG. 249. SHOTTER'S DOUBLE-DYNAMOMETER METHOD

of this instrument carry current at the null condition, the method is of the null-reading type, see p. 449. As will be seen from the diagram, I_s flows in one current coil and I_s' in the other; the voltage coils are supplied in parallel from a phase shifter. The dynamometer may be a 5 ampere, 100 volt double wattmeter, in which it is possible to increase the sensitivity 5 to 10 times by applying a momentary over-voltage to the voltage coils; with a special instrument on the lines of that

* G. F. Shotter, "A new null method of testing instrument transformers and its application," *Journal I.E.E.*, vol. 68, pp. 873–888 (1930).

designed by Spilsbury (see p. 493) very high sensitiveness can be obtained, so that it becomes possible to measure the ratio to 1 part in 25 000 at full load.

As a preliminary it is necessary to ensure that the torques exerted by the two elements are equal and opposite under conditions of zero ratio or phase-angle differences. To effect this the current coils are joined in series with an auxiliary voltage to send an alternating current I through them; the voltage coils are supplied at voltage V by the phase-shifter, V being set in phase with I. If the torques are in opposition, as can be arranged by proper choice of polarity, and r_3 is set to zero, balance can be effected by adjusting r_1 or r_2 when

$$\frac{V}{\sqrt{(r_1^2 + \omega^2 l_1^2)}} I \cos \alpha_1 = \frac{V}{\sqrt{(r_2^2 + \omega^2 l_2^2)}} I \cos \alpha_2$$

if l_1 and l_2 are the volt-coil inductances and α_1 and α_2 are their phase-angles. If $l_1 = l_2$ and $\alpha_1 = \alpha_2$, then $r_1 = r_2 = r$.

The connections of Fig. 249 are then resumed and the torques are set in opposition. Assuming that the standard transformer has no errors, let ϕ be the phase-angle between V and I_p, i.e. between V and I_s; adjust r_3 until the dynamometer is undeflected, then

$$\frac{V}{r_1} I_s \cos \phi = \frac{V}{r_2 + r_3} I_s' \cos (\phi - \beta_x)$$

neglecting the volt-coil reactances. If, further, $r_1 = r_2 = r$

$$\frac{I_s}{r} \cos \phi = \frac{I_s'}{r + r_3} \cos (\phi - \beta_x)$$

Note that r_3 may be negative, i.e. a reduction of the volt circuit resistance, and that the negative sign before β_x corresponds with β_x leading.

Tests at two values of ϕ will enable the errors of X relative to S to be found. To determine these, open the key and adjust the phase-shifter to make the instrument read zero; then V is in quadrature with I_s (i.e. with I_p since S is assumed to be without phase-angle error). Turning the phase through $\pi/2$ from this position puts V and I_s in phase with one another, i.e. $\phi = 0$; then if r_3 gives balance

$$I_p/I_s' = (I_p/I_s)\ [r/(r + r_3)]$$

since β_x is small, or

$$K_{cx}/K_c = r/(r + r_3) \doteq 1 - (\varepsilon_{cx} - \varepsilon_c)$$

If the phase-angle be set at some value ϕ and balance is obtained by altering r_3 to a value r_4 then

$$\frac{\cos \phi}{\cos (\phi - \beta_x)} = \frac{r}{r + r_4} \cdot \frac{I'_s}{I_s} = \frac{r}{r + r_4} \cdot \frac{r + r_3}{r} = \frac{r + r_3}{r + r_4}$$

This ratio is the phase-angle correction factor when the transformer X is used to supply a wattmeter in a circuit of power-factor $\cos \phi$. If $\phi = 60°$ then

$$(\sqrt{3}) \sin \beta_x = (r_4 - r_3)/(r + r_3)$$

The method has been developed by the North Metropolitan Electric Power Supply Co. for use in routine testing of transformers. For this purpose the resistor r_3 is made up in decade form and can be arranged to read directly the ratio error and phase-angle of the transformer under test. The errors of the standard transformer can be allowed for on the dials of a second resistance in series with r_3. The apparatus is quick to use, and accurate results can be obtained by unskilled operators, since all errors and corrections can be automatically compensated. The original paper should be consulted for full details of the actual apparatus.

The range of ratios specified in the B.S.I. Specification No. 81—1927 up to a primary current of 2 000 amperes can be covered by two standard transformers. The first is of the hole type with an auxiliary wound primary; with one turn of cable through the hole the ratio is 1 200/5, increasing numbers of turns dealing with 600, 400, 300, 200, 150, 100, 80, 60, 50 and 40 amperes; to avoid large numbers of turns of cable the wound primary is used for 30, 20, 15, 10 and 5 amperes. The second transformer is of simple hole type, one turn giving 2 000/5 while greater numbers of turns give 1 000, 500 and 250 amperes. Currents of 800 and 1 500 amperes are dealt with by using an ingenious artifice. For 800 amperes the 600/5 ratio is used with the impedance of the voltage element on the S side of the dynamometer increased in the proportion of 8/6; for 1 500 amperes the 1 200/5 ratio is used with the impedance increased by the factor 15/12.

2. **Barbagelata's bridge method.** An interesting method shown in Fig. 250 (a), has been introduced by Barbagelata;* since a vibration galvanometer is used this is a null-current or true bridge method. The method has the advantage that it can be used to compare the characteristics of two transformers of unequal ratio, so that a fixed-ratio standard can be used to test others of any desired ratio. Suppose S to have a greater ratio than X; then I_sR_1 is made greater than $I_s'R_2$ approximately in proportion to the ratios. Balance is attained by adjustment of the tapping r on the voltage divider R shunting R_1 and of the mutual inductor M; then the drop of voltage

* A. Barbagelata, loc. cit. on p. 433 (1921).

in r and the e.m.f. of mutual induction exactly balances R_2I_s' as shown in the vector diagram of Fig. 250 (b), from which

$$R_2I_s' \cos (\beta_x - \beta) = [rR_1/(R + R_1)]I_s,$$

and

$$R_2I_s' \sin (\beta_x - \beta) = \omega MI_s.$$

The first gives

$$\frac{I_p}{I_s'} = \frac{R_2}{R_1} \cdot \frac{R + R_1}{r} \cdot \frac{I_p}{I_s} \cos (\beta_x - \beta)$$

or

$$\frac{K_{cx}}{K_c} \doteq \frac{R_2}{R_1} \cdot \frac{R + R_1}{r} \doteq \frac{R_2}{R_1} \cdot \frac{R}{r} \doteq 1 - (\varepsilon_{cx} - \varepsilon_c),$$

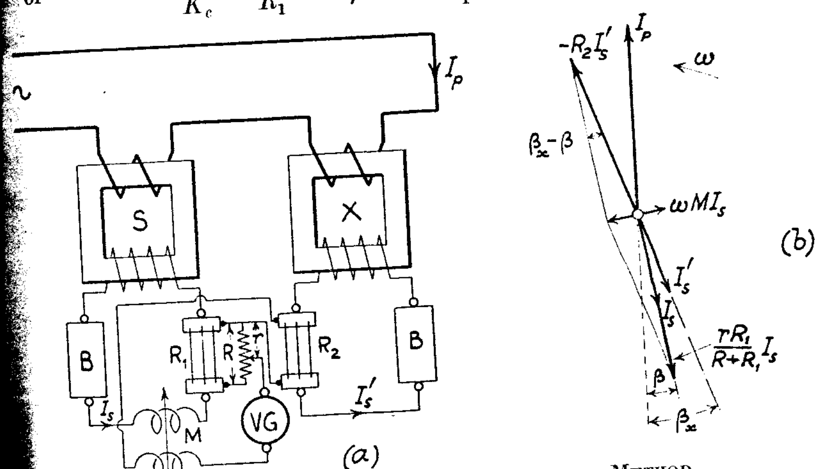


FIG. 250. BARBAGELATA'S RELATIVE BRIDGE METHOD

since $\beta_x - \beta$ is very small and R_1 can be neglected in comparison with R. Dividing the expressions,

$$\tan (\beta_x - \beta) = (\omega M/rR_1)(R + R_1) \doteq \omega MR/rR_1.$$

The method is easy to operate and of wide range; care must be taken to avoid inductive interference effects by systematic arrangement of the leads in bifilar lay-out and the use of an astatic inductometer.

A somewhat similar method due to de la Gorce* avoids the interference trouble by omitting M. The four-terminal resistors R_1 and R_2 are both provided with voltage-dividers, that shunting R_2 having the

* P. de la Gorce, "L'étalonnement des transformateurs de mesure au Laboratoire Central de l'Électricité," *Bull. Soc. Franç. des Elecns.*, vol. 5, 4th series, pp. 659–665 (1925).

adjustable tapping, while that across R_1 has a fixed tapping. Phase compensation is secured by shunting one of the sections of the R potential divider with a condenser of capacitance C. The arrangement is used at the Laboratoire Central for the standardization of current transformers with primary currents in excess of 100 amperes. The standard transformer is of toroidal design with a uniformly-wound secondary and a bar-wound primary arranged for series-parallel connection to cover various values of primary current.

3. **Silsbee's bridge method.** The deflectional method of Silsbee shown in Fig. 244 lends itself readily to null operation*

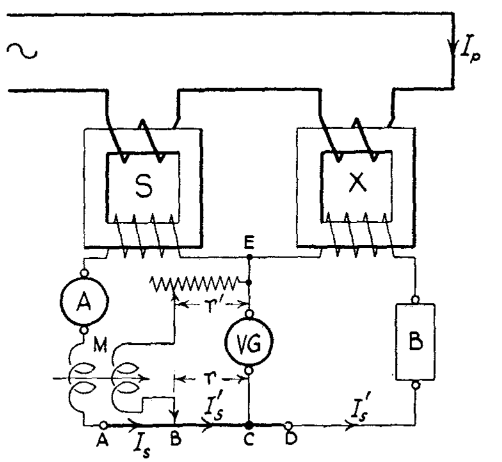


FIG. 251. SILSBEE'S RELATIVE BRIDGE METHOD

by the modification shown in Fig. 251. In this diagram AD is a slide wire of 0·2 ohm resistance; r' is a resistance of 30 ohms adjustable in steps of 2 ohms each; M is a variable mutual inductor of about 600 michrohenrys maximum value. By varying M and r, the portion of the slide wire between the point of attachment of the galvanometer, C, and the slider B, it is possible to bring the vibration galvanometer to zero; hence E and C are at the same potential. The vector difference between the secondary currents I_s' and I_s is diverted from the detector and passes through the path EB. If the total resistance of this path is R ohms and the inductance of the secondary winding of M is L, let $\boldsymbol{i}_s'$ and $\boldsymbol{i}_s$ be the harmonic vectors of the secondary mesh currents; then equating the potential difference

* F. B. Silsbee, "A method of testing current transformers," *Bull. Bur. Stds.*, vol. 14, pp. 317-329 (1919); also see F. B. Silsbee, loc. cit. *ante* (1924).

between E and C to zero and supposing B to be to the left of C gives

$$(R + j\omega L)(\boldsymbol{i}_s' - \boldsymbol{i}_s) + r\boldsymbol{i}_s' + j\omega M\boldsymbol{i}_s = 0,$$

that is $(R + r + j\omega L)\boldsymbol{i}_s' = [R + j\omega(L - M)]\boldsymbol{i}_s$,

from which

$$\frac{\boldsymbol{i}_s}{\boldsymbol{i}_s'} = \frac{R + r + j\omega L}{R + j\omega(L - M)} = \frac{1 + (r/R) + j\omega(L/R)}{1 + j\omega[(L/R) - (M/R)]}$$

Now for brevity write

$$a = r/R,\ b = -\omega M/R,\ c = \omega L/R$$

and remember that all these quantities are small; hence to the second order of small quantities,

$$\frac{\boldsymbol{i}_s}{\boldsymbol{i}_s'} = \frac{1 + a + jc}{1 + j(b + c)} \doteq (1 + a - b^2 - bc) - j(b + ab + ac)$$

In terms of the primary current the secondary current vectors can be written (p. xxiv),

$$\boldsymbol{i}_s = (I_p/K_c)\varepsilon^{-j(\pi-\beta)}\boldsymbol{1} \text{ and } \boldsymbol{i}_s' = (I_p/K_{cx})\varepsilon^{-j(\pi-\beta_x)}\boldsymbol{1}$$

where $\boldsymbol{1}$ is a unit vector along $\boldsymbol{i}_p$. From these

$$\boldsymbol{i}_s/\boldsymbol{i}_s' = (K_{cx}/K_c)\,\varepsilon^{-j(\beta_x-\beta)}$$
$$= (K_{cx}/K_c)[\cos(\beta_x - \beta) - j\sin(\beta_x - \beta)].$$

Comparing this expression with the preceding,

$$(K_{cx}/K_c)\cos(\beta_x - \beta) = 1 + a - b^2 - bc$$
$$(K_{cx}/K_c)\sin(\beta_x - \beta) = b + ab + ac$$

Solving for ratio and phase-angle, and again neglecting terms of higher order than the second,

$$K_{cx}/K_c = 1 + a - (b^2/2) - bc$$
$$\tan(\beta_x - \beta) = b + ac.$$

By the usual convention, if the coils of M are parallel and wound in the same direction, as shown in the diagram, M in the present circuit is actually a negative quantity. Hence, $b + ac$ is essentially positive, i.e. β_x must exceed β. Substituting for a, b and c and remembering that M is negative gives the numerical relations

$$K_{cx} = [1 + (r/R) - (\omega^2 M^2/2R^2) - (\omega^2 ML/R^2)]K_c$$

and

$$\tan(\beta_x - \beta) = (\omega M/R) + (\omega Lr/R^2).$$

If it should be necessary to move B to the right of C for balance the expressions can easily be shown to be

$$K_{cx} = [1 - (r/R) + (r^2/R^2) - (\omega^2 M^2/2R^2) - (\omega^2 ML/R^2)]K_c$$
$$\tan(\beta_x - \beta) = (\omega M/R) - (\omega Lr/R^2) - (\omega Mr/R^2).$$

These four expressions hold good if $\beta_x > \beta$. By arranging for S to have always the smaller ratio, CD may be omitted entirely. To find which transformer has the greater phase-angle, if B is to the left of C, so that S supplies the greater current and has the smaller ratio, add resistance to the secondary of X. Then the lead of I_s' on I_p reversed will increase; if M is increased to restore balance, then I'_s was originally leading on $-I_p$ by more than I_s. As a rule, unless the two transformers are of widely different characteristics, none but the first order terms need be taken into account; then very approximately, with B to the left,

$$K_{cx} \doteqdot [1 + (r/R)]K_c$$
$$\tan(\beta_x - \beta) \doteqdot \omega M/R.$$

With B to the right r is counted as negative, and if $\beta_x < \beta$ M is to be reversed also. If R be fixed, it follows that the slide wire can be calibrated directly in ratio error and the mutual inductor scale in phase-angle, both reckoned relatively to the standard transformer; by this means the apparatus can be made direct reading, and the second order terms can be taken into account, when important, as added corrections to the scale readings of r and M.

A portable self-contained testing set is manufactured by the Leeds & Northrup Company of Philadelphia, Pa., and is described in Bulletin No. 715—1927. The scale of r is calibrated from 95 per cent to 105 per cent, with 100 divisions, each of which represents 0·1 per cent; this scale gives the ratio of the transformer X in per cent of the ratio of S. The mutual inductor has two scales, one for 25 and the other for 60 cycles per second reading to ± 3 degrees. These scales are calibrated directly in minutes, one-half of each scale being marked with white lines and the other half with red lines; the white markings correspond with positive values of $\beta_x - \beta$, while the red markings give the negative values. If errors of 0·1 per cent in ratio can be permitted no correction need be applied to the ratio scale until the phase-angle exceeds 90 minutes leading. If errors of 5 minutes in angle are allowable no correction is necessary until the ratio departs from 100 per cent by more than 3 per cent. A chart is provided to enable these corrections to be applied when necessary. An ammeter is provided, as in Sieber's apparatus of Fig. 245, to enable the polarity and correctness of connection of the transformers to be tested. The inductometer and resistors are mounted together in a case containing the detector, this being an a.c. galvanometer of the separately-excited dynamometer

ype. A phase-shifting transformer is provided for the purpose of xciting the detector, enabling the ratio and phase-angle settings to e independently made at two positions of the phase-shifter in quadrature. The only remaining auxiliary is the standard transformer, which may be of any desired type.

4. **Stubbings's bridge method.** In Silsbee's method of Fig. 251 the by-pass path carrying the difference current forms a definite part of the total burden with which the transformers are

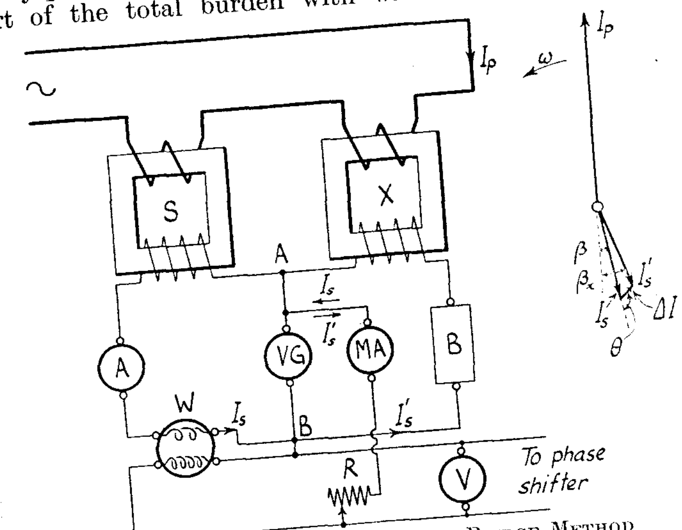


FIG. 252. STUBBINGS'S RELATIVE BRIDGE METHOD

loaded. Stubbings has described a method in which an ingenious device is employed, having the effect of reducing the common impedance to zero, the connections being shown in Fig. 252. The voltage supplied by the phase-shifter is first regulated to make the wattmeter W read zero; then the phase-shifter voltage is in phase with I_s and gives a datum from which angles can be reckoned. The current supplied by the phase-shifter is now regulated in magnitude and phase until VG indicates zero. Since A and B are now at the same potential the path of the difference current must be through the phase-shifter, as indicated by the double arrows, and occasions no ohmic drop since it is urged through this path by the voltage of the phase-shifter. Thus the path of the difference current has zero effective impedance. The difference current ΔI is

* G. W. Stubbings, "Testing current transformers," *El. Rev.*, vol. 112, pp. 480–481 (1933); *Elec. Times*, vol. 87, p. 498 (1935).

read by the milliammeter and its angle θ relative to I_s is observed from the displacement of the phase-shifter rotor from the first setting. The balance conditions are, for $I_s > I_s'$ and $\beta_x > \beta$,

$$I_s' \cos(\beta_x - \beta) + \Delta I \,.\, \cos(\pi - \theta) = I_s,$$

$$\Delta I \,.\, \sin(\pi - \theta) = I_s' \sin(\beta_x - \beta);$$

from these,

$$(1/K_{cx}) - (1/K_c) = (\varepsilon_{cx} - \varepsilon_c)/K_{nc} \doteqdot (\Delta I/I_p) \cos\theta$$

and

$$\tan(\beta_x - \beta) = \Delta I \sin\theta/(I_s - \Delta I \cos\theta)$$

where I_s is read by the ammeter A and I_p is for all practical purposes computed from I_s and the nominal ratio of the standard transformer.

5. **Arnold's bridge method.** Although the Silsbee bridge method is satisfactory for ordinary tests it is not nearly sensitive enough for tests on the best modern transformers; for this purpose Arnold* has described a method in use at the National Physical Laboratory, the circuit being shown in Fig. 253. The resistance R, which carries the difference current, consists of three four-terminal resistors in series, having values of 0·01, 0·1 and 1 ohm respectively; plugs are provided whereby any two can be short-circuited as desired. The burden B consists of a resistor variable in steps of 0·1 ohm from 1·6 ohm (40 VA at 5 amperes) down to 0·1 ohm, with a final step of 0·03 ohm (0·75 VA) consisting of the leads to the unknown transformer. In addition there is a resistor of 0·0004 ohm which can be used in place of R during preliminary tests to determine correctness of polarity of the transformers. The ammeter A has two ranges, one from 0·1 to 1·5, the other up to 6·5 amperes; the instrument has a mumetal movement and enables the secondary current to be set to 0·1 per cent of full scale value down to 1/20 load. A 5/5 current transformer insulates the measuring circuit from the secondary of S; its errors are small, about 2 parts in 10 000 for ratio and 2 min of phase-angle at 50 cycles per sec., and have a negligible effect on the accuracy of measurement. The potentiometer consists of an astatic mutual inductor, M, with a range of $\pm$ 2·4 μH and a resistor r giving $\pm$ 500 microhms. This resistor is a constantan tube with a coaxial return conductor; one potential lead is fixed at the middle of the length of the tube and the other lead is a sliding contact upon its surface.

* A. H. M. Arnold, "Current transformer testing," *Journal I.E.E.*, vol. 74, pp. 424–437 (1934). The apparatus is made by H. Tinsley & Co.

The procedure is as follows: The measuring circuit leads are disconnected from the potential terminals of R and are joined together. With full current flowing in the transformers, the leads, etc., are adjusted in position until, when r and M are set to zero, there is no deflection of the vibration galvanometer VG; inductive interference is then zero. Resuming the original connections with R replaced by the 0·0004 ohm resistor, a check is made of the polarity; if this is correct, balance

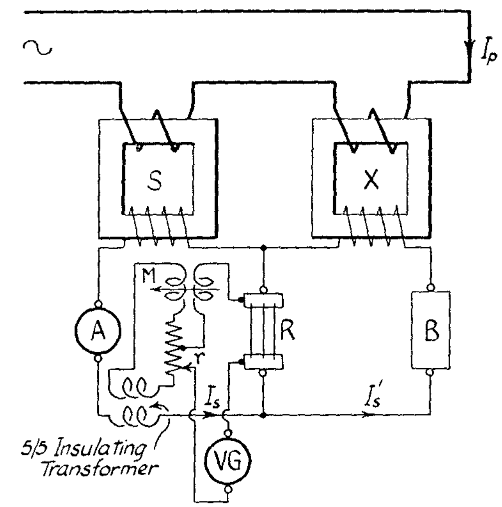


FIG. 253. ARNOLD'S RELATIVE BRIDGE METHOD

should occur when r reads about + 400 microhms. Balance is then obtained with R = 0·01, 0·1 or 1 ohm, according to the sensitivity and range desired. With 1 ohm the reading of r is proportional to the difference between the ratios of the transformers and can be read to 1 part in 10^6 for differences up to 0·05 per cent. The reading of M is proportional to the difference of phase-angle and can be read to 0·01 minute for differences up to 2·5 minutes. With R set at 0·1 or 0·01, the ratio difference is extended to 0·5 per cent or 5 per cent and the angle difference to 25 minutes or 250 minutes respectively, in both cases with reduced accuracy.

Neglecting residuals in R and impurity in M the theory is easily developed. The secondary current vectors are (p. xxiv).

$$\boldsymbol{i}_s = -(1/K_c)(\cos\beta + j\sin\beta)\boldsymbol{i}_p \doteqdot -(1/K_c)(1 + j\beta)\boldsymbol{i}_p,$$

$$\boldsymbol{i}_s' = -(1/K_{cx})(\cos\beta_x + j\sin\beta_x)\boldsymbol{i}_p \doteqdot -(1/K_{cx})(1 + j\beta_x)\boldsymbol{i}_p;$$

or $\quad i_s' \risingdotseq \dfrac{K_c}{K_{cx}} \dfrac{(1 + j\beta_x)}{(1 + j\beta)} i_s \risingdotseq \dfrac{K_c}{K_{cx}} [1 + j(\beta_x - \beta)] i_s.$

For balance,

$$R(i_s - i_s') - r i_s + j\omega M i_s = 0,$$

or $\quad R\{1 - (K_c/K_{cx}) [1 + j(\beta_x - \beta)]\} + (j\omega M - r) \risingdotseq 0.$

Equating components,

$$1 - (K_c/K_{cx}) \risingdotseq (\varepsilon_{cx} - \varepsilon_c) \risingdotseq r/R$$
$$(K_c/K_{cx})(\beta_x - \beta) \risingdotseq \omega M/R.$$

If F_c, F_{cx} are the ratio factors, i.e. the quotient of the true ratios by the nominal ratios, then since these factors are nearly unity and the nominal ratios of S and X are the same we can

write $\quad F_{cx} - F_c \risingdotseq r/R$

and $\quad \beta_x - \beta \risingdotseq \omega M/R.$

Three standard transformers are used and cover the entire range of primary current from 5 to 12 000 amperes. All have mumetal ring cores. The first transformer covers ratios from 5/5 to 80/5; the second from 60/5 to 1 000/5, and the third above 1 000/5. The first two transformers have toroidal primary and secondary windings, both of which are tapped to give a variety of ratios. The third transformer has a toroidal secondary and an inserted cable primary; this transformer has a leakage shield and has been described on p. 136. Full details of the transformers and their calibration are given in Arnold's paper.

6. **Hohle's bridge method.** Hohle* has designed a portable testing set on a principle resembling Arnold's method with omission of the insulating transformer; the circuit is shown in Fig. 254. Like Arnold's method this bridge has the advantage over Silsbee's circuit, that at balance the secondary of the mutual inductor carries no current, so that its resistance and self inductance do not enter into the balance conditions.

When r lies to the left of B the balance equation is

$$R(i_s - i_s') + j\omega M i_s + r i_s = 0$$

* W. Hohle, "Eine tragbare Stromwandler-Prüfeinrichtung hoher Genauigkeit," *Arch. f. Elekt.*, vol. 27, pp. 849–855 (1933); "Eine tragbare Messwandler-Prüfeinrichtung hoher Genauigkeit," *Phys. Zeits.*, vol. 35, pp. 844–851 (1934), describes apparatus suitable for testing current and voltage transformers by the same principle.

Substituting for i_s and i_s' from p. 513 it is easy to show that

$$K_{cx}/K_c \risingdotseq 1 - (\varepsilon_{cx} - \varepsilon_c) \risingdotseq R/(R + r)$$

and

$$\beta_x - \beta \risingdotseq \omega M/R,$$

which necessitates $K_{cx} < K_c$. If $K_{cx} > K_c$ the balance value of r will lie to the right of B, the conditions being

$$R(i_s - i_s') + j\omega M i_s - r i_s' = 0,$$

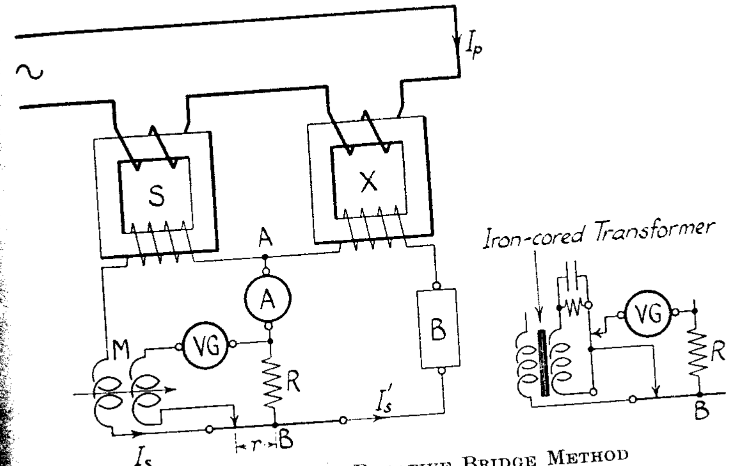


Fig. 254. Hohle's Relative Bridge Method

which reduces to

$$K_{cx}/K_c \risingdotseq 1 - (\varepsilon_{cx} - \varepsilon_c) \risingdotseq (R + r)/R,$$
$$\beta_x - \beta = \omega M/(R + r).$$

Since M in the diagram is conventionally negative these equations hold for $\beta_x < \beta$. M must be reversed if $\beta_x > \beta$.

In the actual apparatus R is normally 1 ohm, but a tapping at 0·2 ohm enables the range to be increased five-fold. The normal range covers a $\pm$ 1 per cent difference of ratio and + 73 to − 15 minutes difference of phase-angle between X and S. The mutual inductor is replaced by the arrangement shown at the right-hand side of Fig. 254, consisting of an iron-cored transformer with a slide wire in its secondary circuit; exact quadrature between the primary current and the secondary voltage is secured by the shunted condenser. This device is much less bulky than an air-core inductor and is practically free from leakage field troubles. The entire apparatus, exclusive of the transformer S and the burden B, is contained in a portable case 46 cm. × 32 cm. × 15 cm.

7. **The use of alternating current potentiometers.** Two transformers of any ratio, equal or unequal, can be compared by including a suitable four-terminal resistor in each secondary

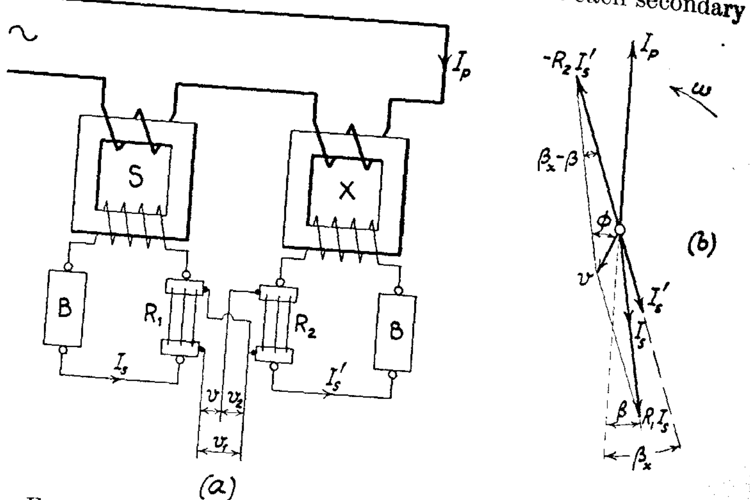


FIG. 255. USE OF A.C. POTENTIOMETER FOR RELATIVE TESTS

circuit, as in Fig. 255 (*a*), and measuring by any type of alternating current potentiometer the voltages v_1 and v_2 across these resistors and their vector difference v, as well as the phase displacement ϕ between v and v_1. Then, as the vector diagram of Fig. 255 (*b*) shows,

$$K_{cx} = (R_2/R_1)\,(v_1/v_2)K_c$$

and also $$v_2 \sin(\beta_x - \beta) = v \sin\phi$$

whence $$(\beta_x - \beta) \doteqdot (v/v_2)\sin\phi.$$

The method is very convenient when a suitable potentiometer is available, and is capable of very high precision.

* D. C. Gall, *Elecn.*, vol. 83, pp. 603–604 (1920).

CHAPTER XXIII

CHOICE OF METHOD FOR MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Introductory.** Chapters XVIII to XXII contain a classification of the numerous methods that are available for the testing of the ratio error and phase-angle of current transformers, grouped according to the nature of the apparatus used in the test-circuit. While this is a satisfactory procedure from the technical point of view, it is not quite so useful to an engineer who wishes to find out what methods are most suitable for making measurements under specified practical conditions. Quite a number of the methods described are of purely historical interest; others, though good in themselves within certain limitations, have since been superseded by methods of superior accuracy, wider applicability and greater practical value. It is the object of the present chapter, therefore, to assist the reader to select, from the large number available, those methods which experience has shown to be best adapted to modern requirements.

The methods required in testing practice fall into four main classes, viz. (i) the methods of high precision suitable for use in the laboratory or test-room; (ii) the methods used for routine testing in the works; (iii) the methods suitable for making tests on site without removing the transformer; and (iv) the methods used to check whether the performance of a transformer remains unchanged or not, without undertaking a lengthy test of its complete behaviour. The first class makes use of all the resources available in a well-equipped laboratory, such as highly sensitive instruments, refined experimental procedure and all auxiliary aids to the attainment of high precision; the methods used are almost all of the absolute type described in Chapters XIX and XX, but the excellent characteristics of nickel-iron cored standard transformers have recently led to certain relative methods being promoted to the first class. The second class of methods demands somewhat less accuracy than the preceding, with instruments of a more robust character capable of working successfully under the more severe conditions of the works test, and with special insistence upon speed and convenience in operation; both absolute and relative methods are available. Methods in the third class use apparatus which must be simple and readily portable; the observations are usually made with pivoted pointer instruments or with calibrated watt-hour meters, and the methods are almost always relative. The fourth class is not really a measurement of ratio error and phase-angle at all; it usually consists of some test of the magnetic condition of the core, from which the condition of the errors may be inferred.

In the absolute methods the vector difference between approximately equal volt-drops at the terminals of two standard resistors, one in the primary and the other in the secondary circuit of the transformer, is measured by some suitable instrument or bridge device. In the relative methods the vector difference between the approximately equal secondary currents of the transformer under test and of a standard transformer of the same nominal ratio is measured in a similar way. It is not often realized that the measurement of the difference voltage or current, as the case may be, need not be made with very high accuracy. For example, an error of 0·5 per cent measured to an accuracy of 2 per cent is equivalent to the measurement of the quantity upon which the error is computed to an accuracy of 0·01 per cent. Nevertheless, the errors of the standard resistors or of the standard transformer must be determined to the full accuracy to which the ratio error and phase-angle of the test transformer are required.

2. **Methods for laboratory tests.** For laboratory work of the highest precision two methods have been much used, namely Agnew and Fitch's method of Fig. 213 (*b*) and the Electrometer methods of Fig. 220. Both have been used f[or] some of the most accurate of the earlier investigations into th[e] properties of current transformers. The first method w[as] developed at the Bureau of Standards; forms of the electr[o-] meter method are in use at the Reichsanstalt and at th[e] National Physical Laboratory. In spite of their high accura[cy] and other excellent qualities both methods suffer from th[e] disadvantage of being too slow and troublesome for ordina[ry] use. The first requires a delicate reflecting dynamometer, a[nd] the second a sensitive electrometer of special constructio[n]; both instruments necessitate a technique much too elabora[te] for their satisfactory use in any but the most favourab[le] conditions, such as can only be found in a great nation[al] standardizing laboratory. It is noteworthy that even the[n] these methods are now largely superseded by others litt[le]

[in]ferior in accuracy and of much greater convenience. For [hi]gh-class laboratory work there is no doubt that only three [m]ethods can claim the combination of high precision with [pr]actical convenience, viz. the absolute method of Schering [an]d Alberti (Fig. 233), the absolute method of the Bureau of [S]tandards (Figs. 231 and 232), and Arnold's relative method [(]Fig. 253). All are null methods and are based on the well-[k]nown a.c. bridge technique.

It is probable that the most convenient method for general [l]aboratory purposes is Schering and Alberti's method, especially [i]n the form developed by Arnold at the National Physical Laboratory (see p. 474). A very high order of precision is attainable over an enormous range of primary current values. The apparatus required is simple and easily obtained in any well-equipped test-room, and in Arnold's modification all due allowance is made for residuals, inductive interference, etc. The method is quick to use and the experimental procedure is simple.

Sharp's method developed at the Bureau of Standards for routine high-precision work is also very suitable for general adoption. The adjustments are again simple and tests can be quickly made; residual errors are slight, but since the test-circuit contains a mutual inductor there is more susceptibility to stray-field interference than in the preceding method. Methods of reducing this trouble have been discussed on p. 466, and will be further mentioned in Chapter XXIV; with these precautions the method is satisfactory and has found favour in many laboratories.

Both the preceding methods require the use of primary and secondary standard resistors with low residuals, and for a wide range of primary currents a number of resistors will be required, with consequent expense. Arnold's high-precision relative method eliminates resistors and compares the unknown transformer with a mumetal-cored standard transformer; an enormous range can be covered with only two or three multi-range transformers and the method has many advantages in practice. These are, notably, much smaller residual errors due to the standard transformer than can be obtained with resistors; greater stability of the residuals; ample sensitivity at low loads, which is hard to obtain with resistor methods unless the secondary resistor exceeds 0·1 ohm; the minimum burden imposed in the relative method is about 0·75 VA while it is about 3·25 VA with resistors; the relative method is much quicker to use, the equipment is cheaper and is more easily checked periodically than the resistor method. Many of these advantages can be transferred to the resistor method by replacing the primary resistor by a mumetal-cored transformer with a 5 ampere resistor in its secondary circuit, as has frequently been mentioned in the preceding chapters.

3. **Methods for routine tests.** The methods considered in Section 2 have for their object the standardizing of high-accuracy transformers, every precaution being taken to obtain the greatest possible precision. In the works it is necessary in the normal manufacturing routine to test considerable numbers of transformers intended for different purposes, ranging from the accurate transformers to be used for power and energy metering down to low-grade ammeter transformers. The methods used in routine testing must be capable of speedy working; it is preferable that they should be direct reading, with negligible or easily applied corrections, of adequate accuracy for the intended purpose and of wide range. A very considerable number of methods is in use in the various works testing departments; indeed, quite a large proportion of the methods described in preceding chapters has been devised by individual manufacturers for works use, and such methods are often conditioned by the nature of the apparatus available in given circumstances.

For testing substandard transformers and those intended for accurate metering, the simpler forms of the three methods recommended in Section 2 may be used with advantage. To these may be added Place's method of Fig. 237, and many other rather less convenient absolute methods. Some of the relative methods, such as Silsbee's (Figs. 244 and 251) or Shotter's (Fig. 249), are specially convenient when large numbers of transformers are to be tested. Such relative methods have th advantage that more robust detecting instruments can b used than is usually the case with the absolute methods. I many instances the sensitivity required falls within that whi can be obtained from good pivoted instruments.

4. **Methods for tests on site.** In many cases where a tra former has passed from the manufacturer's hands and cann be taken out of circuit to be returned to the works for testin it is necessary to test it on site. For this purpose the metho to be used must have an accuracy dependent upon the cla of transformer to be tested, the apparatus must be of robu construction and easily portable, and it is preferable that th

est should be made without interruption of the supply. The ast requirement can be met by temporarily cutting the transormer out of service by means of a jumper connected across s primary side, making the requisite test connections, and hen removing the jumper so that the transformer is excited y, and tested with, its load current. The method, so far as ossible, should be independent of fluctuations in the testing current.

It is usual to prefer relative methods for tests made on site, the method best meeting all the conditions being Agnew's watt-hour meter method of Fig. 247. Very high accuracy can be obtained but the time taken on a test is rather long, seven hours being quite usual; this can be greatly reduced by Slavik's modification of Fig. 248. Greater speed is attained in the non-integrating methods, such as those of Silsbee (Fig. 244) or Crother's (Fig. 246). The simplest method of all is that of Barbagelata (Fig. 214) since all the apparatus required is a suitable pair of dynamometers and two three-way switches.

5. **Testing protective transformers.** The behaviour of protective transformers is radically different from that of measuring transformers and their testing presents special problems. Transformers used with measuring instruments must have high accuracy over the range of current up to rated value; their performance with overload is usually unimportant. The purpose of protective transformers is to compare or balance currents in different parts of a network: consequently, absolute accuracy of ratio is not important provided that the transformers which are to work together vary in ratio in the same manner. Again, the performance of protective transformers is most important under overload conditions, even up to fifty or more times the rated current of the system, such as might occur with a heavy fault. The testing of protective transformers,* therefore, has for its object the comparison of their errors for similarity of variation up to the highest overloads with which they may have to work; ratio error is usually of more interest than phase-angle.

The tests are usually made by a relative method. Sometimes two transformers that are subsequently to work together are

* W. Wilson, "The testing of protective current transformers," *World Power*, vol. 15, pp. 371–374 (1931). The whole subject of the performance and testing of such transformers is fully treated by G. W. Stubbings, *Automatic Protection of A.C. Circuits*, Chaps. II and III (1934).

compared and specially marked for use, but this cannot be recommended as a process suitable for works routine. It is better to compare all transformers with a standard transformer within certain limits of tolerance. For this purpose the Silsbee method in one of its modifications (Figs. 244 or 251) or any other good relative method is suitable. The two-dynamometer methods are usually slower than the null methods, and speed is an important factor with the considerable heating due to large overload currents.

The relative methods are only suitable for tests at overloads when the characteristics of the standard transformer are known over the whole range of currents to be tested, and also when it is possible to obtain the very large currents required. This last is not always easy to do. When the current can be obtained, the ratio can often be obtained sufficiently accurately by measuring directly the primary and secondary currents by means of suitable ammeters. The primary ammeter can be operated from a current transformer; the secondary ammeter, being for a current greater than the rated 5 amperes, has a low impedance and will not much affect the burden. Bushing-type and other one-turn primary transformers can easily be tested under overload conditions without the use of large currents by winding on an auxiliary primary with a number of turns approximately equal to those of the secondary, giving a nominal unity ratio. Tests can then be made by any suitable absolute or relative method without undue heating, since the greatest current is now equal to the maximum overload current divided by the nominal ratio.

Since the errors of a current transformer are a function of its exciting current, Stubbings* has suggested that when the transformer is originally tested its open-circuit curve should also be taken. This is most easily done by measuring the current taken by the secondary winding and the voltage across it, the primary being open; ordinary pointer instruments give quite reliable results. It is reasonable to suppose that if the ratio error and phase-angle are unchanged the saturation curve will be also unaltered. It is only a matter of a few minutes to take one or two observations of points on the open-circuit curve to check whether any change has taken place;

* G. W. Stubbings, "Testing for the permanence of the ratio and phase errors of series-transformers," *Journal Sci. Insts.*, vol. 4, p. 207 (1927); "Approximate current transformer tests," *Elec. Rev.*, vol. 113, p. 154 (1933). Also see W. Wilson, loc. cit., and Edgcumbe and Ockenden, loc. cit. and p. 574 (1927).

only quite small currents are needed, even to reach the saturation region corresponding to overload operation. It is also suggested that the open-circuit test forms a very convenient and reliable method for checking the performance of batches of transformers of a given type, of which only one typical member need be completely tested for ratio error and phase-angle.

CHAPTER XXIV

PRACTICAL PRECAUTIONS

1. **Introductory.** The accuracy of any tests of ratio and phase-angle, whether made in the laboratory or on site, can only be guaranteed if certain practical precautions are observed. Many of these matters have been mentioned in the discussion of some of the methods described in the preceding chapters; there are, however, certain general questions common to all methods that may be conveniently treated here. These are (i) the effects of stray magnetic fields; (ii) the influence of capacitance and leakance currents; (iii) the constitution of test burdens, particularly with reference to leads; and (iv) the systematic process best suited to a given method.

2. **Stray magnetic field effects.** Stray magnetic fields of the test frequency may vitiate the results by their direct action upon the measuring apparatus. Such fields are usually due to the primary current, and their effects are more important with large primary currents. Several methods* are available for the reduction of their effects, and these will now be considered.

It is essential to arrange the primary circuit in such a way that the magnetic field at the position of the test apparatus is very small. To ensure this the primary leads carrying heavy currents must be laid out so that they contain loops of the smallest possible area; this is most easily done by making the outward and return conductors in the form of thin flat strips or slabs placed as close together as possible. The ideal construction is, of course, an outward conductor coaxially within a tube serving as a return conductor; this arrangement has no external field, but it is not easy to arrange terminal connections on such a system of leads. It is not often realized, however, how very closely the perfection of the coaxial arrangement may be imitated by subdividing the return into a number of conductors arranged parallel to and equally spaced around the central outward conductor and at equal distances from it. Russell shows (*Alternating Currents*, vol. I, p. 333) that with

* F. B. Silsbee, "Precautions against stray magnetic fields in measurements with large alternating currents," *Trans. Amer. I.E.E.*, vol. 48, pp. 1301–1305 (1929).

a central conductor surrounded symmetrically by n equally-spaced, parallel return conductors at a distance a from it, each return carrying an equal share of the total current i c.g.s. units, the magnetic field strength at a distance r from the central wire is approximately $2ia^n/r^{n+1}$ when r is much larger than a. The field is tangential to the radius vector joining the central wire to a return wire; and is also tangential, but in the opposite direction to the preceding, to the vector midway between two wires. Silsbee gives the following table for the value of the stray field at various distances from a conductor when the return is arranged in different ways; it is assumed that the current is 10 000 amperes, i.e. $i = 1\,000$ c.g.s. units and that $a = 10$ cm. The great efficacy of a "cage" of four wires is readily seen, as has been mentioned earlier on p. 466.

Arrangement of Return Conductor	Stray Field with r equal to			
	50 cm.	100 cm.	200 cm.	500 cm.
At very great distance	40	20	10	4
Single conductor	8	2	0·5	0·08
Two conductors, one on each side	1·6	0·2	0·025	0·0016
Four conductors at corners of square	0·064	0·002	0·0000625	0·00000064
Coaxial tube	0	0	0	0

Even with the most careful arrangement of the primary circuit it is difficult to ensure that the stray field is entirely absent. A further precaution consists, therefore, in arranging the test-circuit in such a way that the field cannot influence it appreciably. Potential leads from resistors, connections from the secondary windings to burdens or instruments, etc., should be twisted together. Instruments should be magnetically shielded when they operate on the moving-magnet principle. In those instruments which operate as the result of the interaction of two magnetic fields, one of which is much stronger than the other, the weaker field should be produced by the moving element. Thus separately-excited dynamometers should be excited by their fixed coils. Similarly, a moving-coil vibration galvanometer is to be preferred to one of the moving-iron type. The phase-angle of standard resistors may be appreciably affected by strong stray fields, and "closed"-type resistors are to be preferred to those of "open" type, since the method of construction which minimizes their residual errors also reduces their susceptibility to external magnetic influences. Self- and mutual-inductors should be of astatic construction; the com plete freedom from magnetic trouble experienced by condensers is a strong argument in favour of using capacitance standards in place of inductance standards whenever possible. Finally, the test circuit should be set up as far from the primary circuit as is convenient, certainly not less than a metre distant and preferably more.

It is usually fairly easy to test whether stray fields are exerting any appreciable effect on the measuring apparatus, and some slight adjustment in the layout can often be made in such a way that the effect is eliminated. A set of observations having been made, such changes are made in the connections as will reverse the direction of all currents except that flowing in the primary circuit; this can often be done by changing over the connections to the transformer secondary terminals. A second set of observations is then made. If the results are the same as before, stray-field effects have been eliminated. If they differ slightly, their mean value can be taken as the true result. If the results should be widely different, it is a sign that some serious source of error exists; this can be sought out and corrected by taking a pair of readings with reversed connections for each part of the apparatus in turn, until the offending element has been found.

3. **Capacitance and leakance effects. Earthing.** Earth capacitance and leakance currents may sometimes be troublesome, particularly in high-precision null methods. They usually pass into the test circuit from the high-voltage side of the power transformer supplying the primary current, and have an effect upon the balance similar to that in a.c. bridge technique.

Capacitance and leakance effects can be avoided by main taining a high insulation resistance between the primary and secondary windings of the power transformer and putting the test-circuit as far away from it as possible. It is also desirable though not always possible, to earth the mid-point of it primary winding.

One point of the test-circuit should also be earthed. It difficult to lay down any rule as to which point should b selected, but it is usual to make it one of the terminals of th detecting instrument so that this is entirely at zero potentia when balance occurs. If the test network contains a shielded condenser the shield should be on the earthed side. These questions have been discussed in specific instances on pp. 47 and 480.

Burdens. Secondary leads. The general question of bur has been discussed in Chapter XVI as regards standard tings, etc. When a transformer is to be tested for conformity a given class, the test burden must be adjusted to duplicate e rated number of ohms and power-factor specified in the andard rules; the test burden is composed of the testing pparatus and its leads, and is made up to the correct total alue by suitable coils and resistors. With a given test method imposing a known burden, it is easy to prepare a series of coils and resistors to give a range of standard burdens. In some cases it is required to test a transformer with the actual instrument and leads with which it will subsequently work; in such circumstances the burden of the testing apparatus should either be small enough to be neglected or the leads should be reduced to compensate for it. If this is not entirely possible an allowance can be made by testing before and after changing the leads by a known amount; then if a linear variation of ratio and phase-angle with lead resistance be assumed it is easy to get the required result by simple proportion.

Silsbee* has pointed out that it is a mistake to specify burdens of less than 2·5 VA in transformers which are to be subjected to precise test, since the burden imposed by the testing apparatus is seldom less than this figure, corresponding with an impedance of 0·1 ohm or more. Consequently, it is recommended by the Bureau of Standards that a non-reactive resistor of at least 0·1 ohm should be included in the rated secondary load of all transformers that are to be accurately tested. Such a resistor would have little effect on the errors of the transformer and would render the adjustment of test burdens a simple matter. For switchboard transformers and in other cases where a considerable length of secondary leads connect the transformer to its instrument, there is usually enough resistance in the leads to enable the testing apparatus to be inserted and allowed for as mentioned above.

5. **Testing procedure.** The adoption of a systematic procedure not only reduces the time and labour involved in making a test, but also enables possible sources of error to be located and corrected. It is a good plan before beginning a test to check the insulation resistance with a high-voltage megger. If this is satisfactory, the resistance of the secondary winding should be measured on a Wheatstone bridge, in order to see

* F. B. Silsbee, "Lead resistance for current transformers," *Elec. World*, vol. 81, pp. 1082–1083 (1923).

that it is not abnormally high and to detect breaks and b connections; the current used should be less than 10 mA avoid serious magnetization of the core. If the method testing is not one that determines the polarity of the tra former, this should next be checked by one of the proces described on p. 587. Thereafter the core should be thorough demagnetized (p. 125). In some cases, e.g. where a transforme has been subjected to a heavy transient current, it is necessary to test the transformer in the condition in which it is received; in this circumstance the resistance measurement, polarity test, and demagnetization should be omitted.

The appropriate burden must now be assembled and connected in the secondary circuit with the transformer in its place in the testing network. After a preliminary observation has confirmed the absence of stray field and other troubles, the frequency of supply should be maintained constant and the complete test carried out. It is usual to make measurements of the ratio and phase-angle for 0·1, 0·2, 0·4, 0·6, 0·8 and 1·0 times the rated primary current, since experience has shown that the characteristics vary sufficiently regularly to enable smooth curves to be drawn through the results obtained at these points in order to interpolate the values at intermediate currents. After the observations have been corrected for instrumental errors, residual effects in standards, etc., ε_c and β are computed, tabulated, and plotted. After independent checking of the calculation, the appropriate test certificate can be drawn up and all the test data is then filed for future reference.

PART 4

THE TESTING OF VOLTAGE TRANSFORMERS FOR RATIO AND PHASE-ANGLE

CHAPTER XXV

INTRODUCTORY

The general fundamental principles of voltage transformer testing have been introduced in Chapter VII at the conclusion of Part 1; the object of the present Part is to undertake a detailed discussion of the various methods used for the purpose.

As with current transformers the most important test on voltage transformers is to determine the ratio error and phase-angle; there is, however, an important difference between the two classes of transformers. A current transformer, even with a burden of specified impedance, operates with a variable primary voltage since it is actuated by the variable primary current of the supply network. A voltage transformer, on the other hand, operates at a constant, or approximately constant, primary voltage; its ratio error and phase-angle are, therefore, functions only of the amount and nature of the secondary burden, and of the frequency.* Methods for the performance of these tests are described and classified in Chapters XXVI to XXIX. Other practical matters, such as determination of terminal polarity, of the mechanical, thermal and dielectric strengths are discussed in Part 5, and will not be treated further here.

The *Indirect method* of testing voltage transformers has only occasionally been used, the most thorough investigation of the method being made by Iliovici.† Briefly, the method follows the procedure of the well-known open-circuit and short-circuit tests used to find the regulation of power transformers. The ratio on open-circuit can be found by the use of a suitable electrostatic voltmeter, or by a dynamometer. The

* Also to some extent of the primary voltage wave-shape; this is, however, usually nearly sine-shaped.

† A. Iliovici, "Methode d'essai des transformateurs de mesure de tension," *Bull. Soc. Int. des Elecns.*, vol. 6, 3rd series, pp. 155–187 (1916); *Lum. Elect.*, vol. 33, 2nd series, pp. 276–277 (1916).

magnitude and phase of the exciting-current are obtainable from ammeter, voltmeter and wattmeter readings taken on one of the windings with the other open. This test is usually made, for convenience, on the l.t. side. Resistances are found by the usual bridge method; reactances are determined by the short-circuit test, Iliovici obtaining adequate sensitivity by the use of a double dynamometer instrument. The results of both tests are then used in the usual expressions for the ratio and the phase-angle between primary and secondary voltages that can be obtained from the Kapp transformer diagram. Iliovici gives results taken from transformers having nominal ratios of 5 500/100, 10 000/100 and 30 000/100, and enters into a careful analytical and graphical discussion of the method. *Direct methods* are, however, so simple, and of such superior accuracy and convenience that they are now invariably employed in practice; the indirect method may, therefore, be regarded as superseded by them.

Barbagelata* has recently drawn attention to a modified indirect test specially adapted to high-voltage transformers. Treating the transformer and its case as a five-terminal network, Bottani has shown that a sufficient number of open-circuit and short-circuit tests can be made with various pairs of terminals to determine the characteristics of the transformer, taking into account the capacitances between the windings and the case.

As with current transformers, the direct methods for testing voltage transformers may be either *absolute* or *relative*. The absolute methods are all based upon the principle of comparing the secondary voltage with an almost equal portion of the primary voltage, the difference being measured in magnitude and phase. The comparison may be made by a deflectional method, using suitable dynamometers or electrometers separately excited by an auxiliary phase-shifting supply. Alternatively a null or bridge method can be devised by the inclusion of a suitable compensating circuit to balance out the vector difference concerned and the use of one of the usual detectors. The absolute deflectional methods receive treatment in Chapter XXVI and the null methods in Chapter XXVII. In practically all these methods the required fraction of the primary voltage is obtained from a resistance voltage-divider. Modern networks operate at such high voltages that the construction of suitable voltage-dividers constitutes, as has been pointed out

* A. Barbagelata, "Sur les essais des transformateurs de mesure à très haute tension," *Rev. Gén. de l'Él.*, vol. 34, p. 586 (1933).

in Chapter IX, an extremely difficult problem; the most up-to-date methods overcome these difficulties by the substitution of condenser voltage-dividers in which an essential feature is a high-voltage air-condenser, many types of which have been successfully developed for use in Schering bridge technique, see Chapter X.

It will be realized that all absolute methods essentially necessitate the connection of some part of the testing apparatus to the high-voltage side; this is entirely avoided in the relative methods where the errors of one transformer are compared with those of another by measurements made entirely upon their secondary sides. Since the secondary voltage does not exceed 110, the possibility of dangerous shock is avoided; the apparatus is compact and readily portable: and being a difference method adequate accuracy can usually be secured by use of pointer instruments. These advantages make the relative methods specially valuable for use in the works test and on site. The relative deflectional methods form the subject of Chapter XXVIII, and the relative null methods of Chapter XXIX.

It is worthy of notice that any 1 : 1 voltage transformer can be tested by an adaptation of the relative methods as well as by the absolute methods, as was pointed out in connection with current transformers on p. 409.

It has been shown in Chapter IV that the phase-angle γ is usually negative, i.e. V_s lags on $-V_p$, for all except highly-inductive burdens. In particular, γ is always negative for a non-reactive burden. Since the British Standard Specification No. 81—1927 requires tests to be made with a burden having unity power-factor, γ has been shown negative in the vector diagrams of Chapters XXVI to XXIX.

CHAPTER XXVI

ABSOLUTE DEFLECTIONAL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Two-voltmeter methods.** The ratio of a voltage transformer is most easily found by the use of two voltmeters,* as shown in Fig. 256 (*a*), one connected in parallel with the primary winding and the other with the secondary winding. The two windings are connected to an earthed common point. Then if V_p and V_s are the readings of the voltmeters the ratio is given by

$$K_v = V_p/V_s;$$

the two instruments must necessarily be calibrated.

The necessity for calibration may be removed by the adoption of the following artifice.† Two similar voltmeters are used, connected first as in Fig. 256 (*a*), the resistance in series with the primary voltmeter being adjusted until the reading v_p on it is about equal to the reading v_s of the instrument on the secondary side. The two voltmeters are then removed, connected in series, and a current I passed through them so that they give readings v_p', v_s' about equal to the preceding values. Then if R_p is the total resistance of the primary voltmeter and its series resistance, and R_s is the resistance of the secondary voltmeter,

$$V_p = R_p\,(v_p/v_p')I,\ \ V_s = R_s\,(v_s'/v_s)I;$$

whence,
$$K_v = (R_p/R_s)\,.\,(v_p/v_p')\,.\,(v_s'/v_s).$$

A further variant of the two-voltmeter method is shown in Fig. 256 (*b*). A single electrostatic voltmeter is connected successively across the secondary terminals and then across a portion r of a voltage-divider resistor R in parallel with the primary; r is adjusted until the reading v_p of the voltmeter is about the same as the reading v_s when it is put across the

* F. A. Kartak, loc. cit. on p. 424 (1920).

† F. A. Laws, *Electrical Measurements*, p. 584 (1917).

econdary, thereby rendering calibration unnecessary. Then ince

$$V_p = (R/r)v_p \text{ and } V_s = v_s,$$

hen
$$K_v = (R/r)\,(v_p/v_s).$$

The disadvantage of the two-voltmeter method is that it nly provides a measure of the ratio of the transformer. The ossible accuracy is, moreover, not very high unless reflecting oltmeters of high sensitivity can be used.

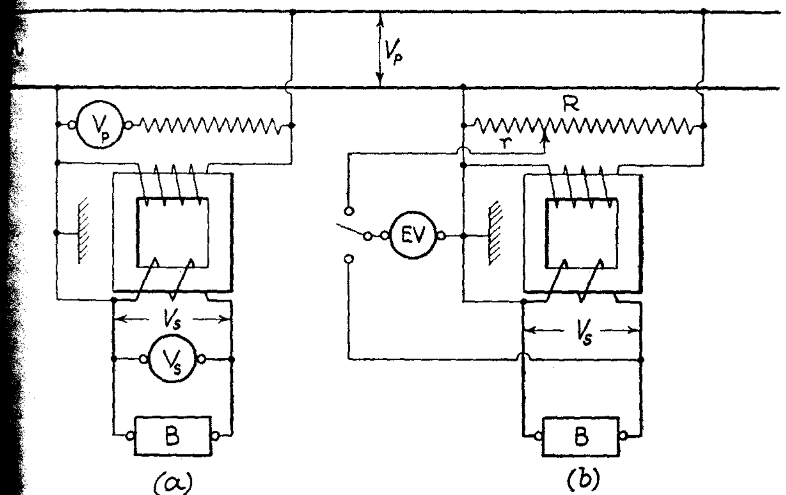


FIG. 256. TWO-VOLTMETER METHODS FOR RATIO TESTS

2. **Two-dynamometer methods.** The sensitivity of the two-oltmeter method can be increased and, at the same time, both tio error and phase-angle can be found, by substituting separ-tely-excited dynamometers for the ordinary types of a.c. oltmeter. Referring to Fig. 257 (*a*) the voltage-circuits of he two dynamometers D_p and D_s are connected respectively the primary and secondary terminals of the transformer. he fixed coils of the dynamometers are connected in series nd carry a current I supplied by a phase-shifter.* The phase-hifter is first regulated until D_p gives a maximum reading; hen if the volt-circuit of this dynamometer is non-reactive

* A. Barbagelata, loc. cit. on p. 433 (1921); F. A. Kartak, loc. cit. on p. 424 920). The process was used by L. T. Robinson, loc. cit. on p. 424 (1910), for easurement of γ. A full test-room set-up is described in his paper.

V_p and I will be in phase, as shown by Fig. 257 (b), and the reading in watts is

$$W_p = V_p I;$$

the corresponding reading of D_s, assuming for this also a non-reactive volt-circuit, will be

$$W_s = V_s I \cos(\pi - \gamma) = -V_s I \cos\gamma.$$

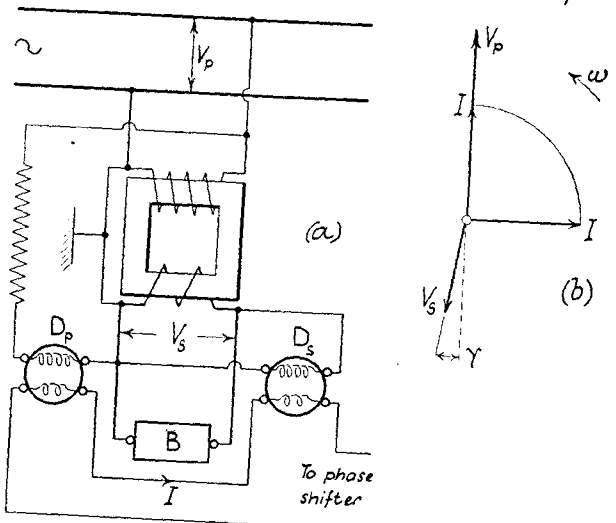


FIG. 257. SIMPLE TWO-DYNAMOMETER METHOD

The phase of I is now altered by $\pi/2$, thus making D_p read zero, so that D_s gives a new reading

$$W_s' = V_s I \cos[(3\pi/2) - \gamma] = -V_s I \sin\gamma.$$

Then the numerical value of the ratio is

$$K_v = W_p/(W_s^2 + W_s'^2)^{\frac{1}{2}} \doteqdot W_p/W_s,$$

since $\cos\gamma \doteqdot 1$; the phase-angle is given by

$$\tan\gamma = W_s'/W_s \doteqdot \gamma.$$

The method is capable of considerable precision and is of wide range. The main source of error lies in the supposition that the voltage circuits of the dynamometers are non-reactive; the theory is easily modified to take the reactances into account

as has been done on p. 427 for an analogous method of testing current transformers. Exactly as in that method it is possible to show that the error will be zero in two cases (a) if the volt-circuits of the two instruments have equal time-constants and (b) if their inductances are compensated by the use of the well-known shunted-condenser device commonly used in wattmeters.

The General Electric Co. of Schenectady measure the phase-angle by the use of two dynamometers in a manner similar to that described, one dynamometer being connected across the secondary of the transformer and the other across a tapping r on a voltage-divider R joined across the primary as in Fig. 256 (b). The ratio is first determined by arranging the volt-drop across r in opposition to V_s and applying the resultant to the volt-coil of a dynamometer voltmeter, the other coil of which is connected to V_s; r is adjusted to make the voltmeter read zero, so that $K_v = R/(r\cos\gamma)$. The connections are then rearranged by a suitable throw-over switch, D_p being across the tapping r as stated; adjusting the phase-shifter to make D_p read zero let W be the reading of D_s in watts, V the voltage across its volt coil and I the current common to the current coils of D_p and D_s; then $\gamma = \arcsin(W/VI)$.

A number of methods, essentially the same in principle, has been described in which a two-phase or a three-phase source of supply is utilized instead of the phase-shifter for exciting the dynamometers. One of the most important of these is the method of Agnew and Fitch* shown in Fig. 258 (a), and used by these experimenters in work of the highest precision. The transformer to be tested is supplied from the main phase of a two-phase alternator through a step-up transformer, a high resistor R being connected across its primary, as shown in the diagram. In series with R is the fixed coil of a dynamometer D_p. The volt-drop in an adjustable fraction r/R of the resistor is opposed to the secondary voltage of the transformer at the switch S_1. With S_1 to the left r is adjusted until D_p reads zero; then if the volt-coil of D_p is assumed to be non-reactive the resultant voltage v applied thereto is in quadrature with the current I' in its fixed coil. Neglecting the reactance of R, I' will be in phase with V_p; further, neglecting the effect of the volt-coil of D_p as a shunt on r, the resultant voltage v in Fig. 258 (b) will be the vector sum of V_s and $(r/R)V_p$. The switch S_1 is now thrown to the right, so that with S_2 also to the right v is applied to the volt-coil of the second dynamometer D, the current coil of which is supplied with current I from the second or quadrature phase of the alternator. Since I is in

* P. G. Agnew and T. T. Fitch, *Bull. Bur. Stds.*, vol. 6, p. 281 (1910).

phase with v the reading of D is a measure of v and the instrument can be calibrated as a voltmeter. Calibration under the conditions of test is easily effected by disconnecting the primary from the main phase, throwing S_2 to the left with S_1 to the right, thus applying a known voltage sI to the volt-coil of D while its current coil is still connected to the quadrature phase.

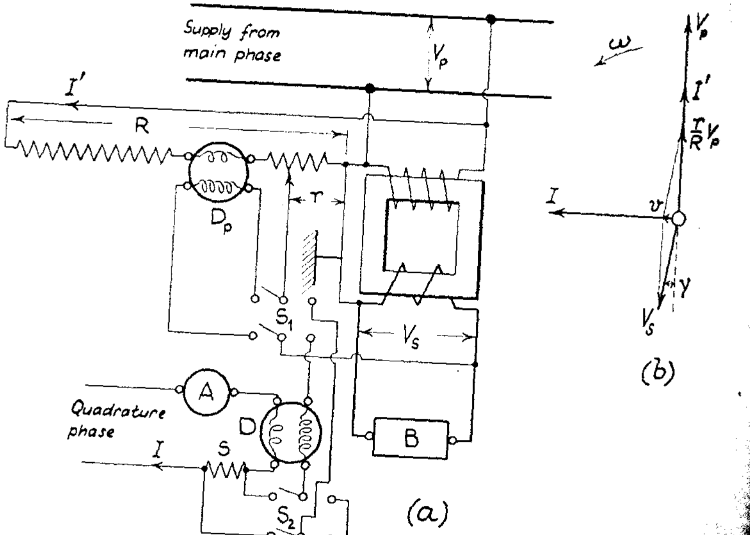


FIG. 258. AGNEW AND FITCH'S TWO-DYNAMOMETER METHOD

From the simple vector diagram,

$$V_s \cos\gamma = (r/R)V_p,$$
$$V_s \sin\gamma = v;$$

whence

$$K_v = (R/r)\cos\gamma \doteqdot R/r,$$
$$\sin\gamma = v/V_s \doteqdot \gamma.$$

The voltage V_s can be measured by a suitable voltmeter included in the secondary burden B.

Possible sources of error are due to residual inductance in r, $R-r$ and the voltage-circuit of D. The influence of the latter on both the ratio and phase-angle is generally quite negligible. The effect of residuals in r and $R-r$ on the ratio is less than 1 part in 50 000 in the most unfavourable case. Agnew and Fitch (loc. cit.) show that so far as the phase-angle is concerned

$$\gamma \doteqdot (v/V_s) + [(\omega L_r/r) - (\omega L_R/R)]$$

the bracketed correction being the difference between the phase-angles of r and R. Hence if these resistors have equal time-constants the residuals cause no error in the estimation of γ; it is possible to construct resistors giving a correction of less than 1 minute at ordinary supply frequencies. The reflecting dynamometers used by these investigators enabled a sensitivity of 1 part in 20 000 in ratio and 0·1 minute in phase-angle to be attained.

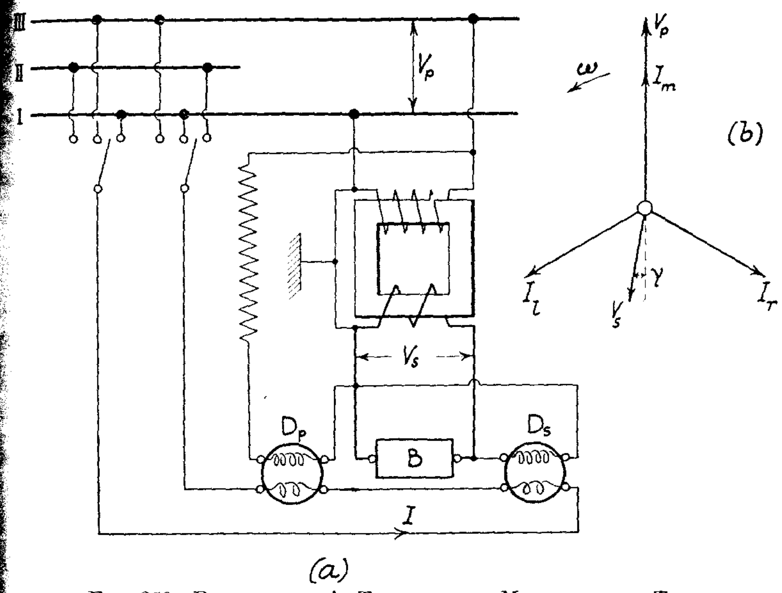


FIG. 259. BARBAGELATA'S THREE-PHASE METHOD WITH TWO DYNAMOMETERS

Of three-phase methods the most important is that introduced by Barbagelata* illustrated in Fig. 259 (*a*) and corresponding with the method for current transformers shown in Fig. 214 (*a*). The transformer under test is supplied from lines III and I of a three-phase system through a step-up transformer not shown in the diagram. The volt-coils of two dynamometers D_p and D_s are connected across the primary and secondary windings respectively; their current coils are joined in series and can be put at will across lines III and I, I and II, II and III by operating the three-point switches successively to the middle, right and left contacts. In each position of the switches equal currents, of magnitude I, will be passed through the current coils respectively in phase, 120° behind and 120°

* A. Barbagelata, loc. cit. on p. 433 (1921).

ahead of V_p as shown by the vectors I_m, I_r, and I_l in Fig. 259 (*b*). In each position the dynamometers are read, the ratios of the reading of D_p to that of D_s being, *a*, *b*, and *c* respectively. Then,

$$a = \frac{V_p I_m}{V_s I_m \cos(\pi - \gamma)} = \frac{V_p}{-V_s \cos\gamma}$$

$$b = \frac{V_p I_r \cos(2\pi/3)}{V_s I_r \cos[(5\pi/3) - \gamma]} = \frac{-V_p \cos(\pi/3)}{V_s \cos[(\pi/3) + \gamma]}$$

$$c = \frac{V_p I_l \cos(4\pi/3)}{V_s I_l \cos[(\pi/3) - \gamma]} = \frac{-V_p \cos(\pi/3)}{V_s \cos[(\pi/3) - \gamma]}.$$

Remembering that γ is small, it is not difficult to show that

$$K_v \doteqdot a \text{ numerically,}$$

and $$\sin\gamma \doteqdot (b - c)/2(\sqrt{3})a \doteqdot \gamma.$$

The method is subject to the same sources of error as the simple two-dynamometer method, of which it is an adaptation suitable for use when a phase-shifter or a two-phase supply cannot be utilized.

A further two-dynamometer method is that used by Rosa and Lloyd* in one of the earliest precise investigations of the properties of voltage transformers. The two dynamometers are combined in a special instrument, a double dynamometer voltmeter, resembling the well-known two-element polyphase wattmeter in construction. The instrument is of a reflecting pattern, the two volt-coils being mounted upon a common suspension with their planes at right angles; each volt-coil is enclosed within a fixed current coil, these also having perpendicular planes. To measure the ratio, one volt-coil is connected to the primary and the other to the secondary, each in series with its own fixed coil and an appropriate resistance. By arranging the torques to be in opposition, zero deflection can be secured by adjustment of the resistances; the ratio is then easily obtained from the resistance settings. The phase angle is found by using the two elements as a differential wattmeter. The apparatus gives results of high accuracy, but has not been generally adopted, presumably on account of its special character. The method has recently been reintroduced by Shotter in a way to be described for voltage transformers on p. 575, and already discussed in its application to current transformers on p. 504.

* E. B. Rosa and M. G. Lloyd, *Bull. Bur. Stds.*, vol. 6, pp. 1–30 (1910).

3. Single-dynamometer methods. A single dynamometer can be easily employed* to measure K_v and γ by using the potentiometer principle shown in Fig. 260 (*a*). The secondary voltage V_s and the fall of potential in the portion *r* of the high resistor *R* joined across V_p are set in opposition, the resultant voltage being applied to the voltage coil of the dynamometer *D*. The auxiliary current *I* in the current coil of *D* is first set in phase

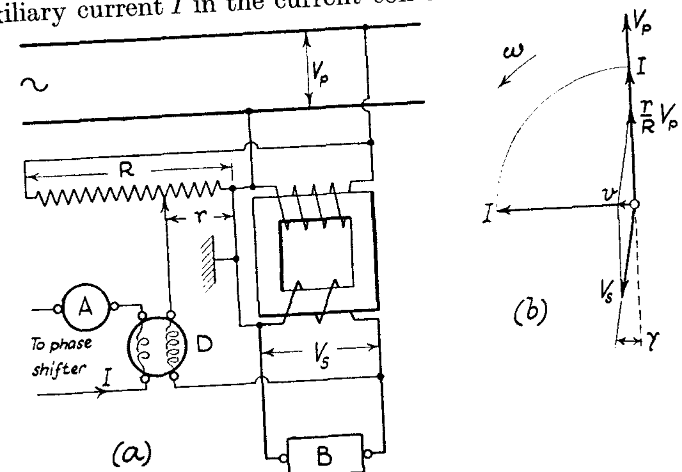


FIG. 260. SIMPLE SINGLE-DYNAMOMETER METHOD

with V_p and *r* is varied until *D* reads zero; then, as Fig. 260 (*b*) shows, the resultant *v* of V_s and $(r/R)V_p$ is in quadrature with V_p and *I*. The phase of *I* is then advanced by 90° and the reading of *D* is taken in watts; let *W* be this value, then

$$V_s \cos\gamma = (r/R)V_p,$$
$$W = vI = V_s I \sin\gamma.$$

From these equations

$$K_v = (R/r)\cos\gamma \doteqdot (R/r)$$
$$\sin\gamma = W/V_s I \doteqdot \gamma;$$

I is read on the ammeter *A*, while V_s is given by a voltmeter forming part of the burden *B*.

The current *I* can be supplied from a phase-shifter, as shown, or may alternatively be derived from the main and auxiliary phases of

* In the method due to L. T. Robinson, loc. cit. on p. 424 (1910), used at the G. E. Co. the process is used for ratio measurements, *I* being set in phase with V_s instead of V_p; C. H. Sharp and W. W. Crawford, loc. cit. on p. 412 (1911), use it for phase-angle determinations. The complete process here described is due to Barbagelata, loc. cit. on p. 433 (1921).

a two-phase supply. The original adjustment of I in phase with V_s can readily be made by exciting the volt-coil across a tapping, r' say, on R and regulating the phase of I to make D read zero, after which the phase is altered by a quarter period; the original connections are then assumed and the test proceeded with. In a portable, self-contained apparatus designed by Allocchio and Bacchini* for testing voltage transformers on site, the original adjustment of r is effected with the connections as shown in the diagram. The quadrature reading is obtained by replacing the volt-coil series resistance by a large inductance of equal ohmic reactance; the need for a portable phase-shifter is thus avoided.

In the modification due to Robinson I is set successively in phase and in quadrature with V_s; then

$$(r/R)V_p \cos \gamma = V_s,$$

$$W = vI = V_s I \tan \gamma,$$

so that

$$K_v = R/(r \cos \gamma) \doteqdot R/r$$

$$\tan \gamma = W/V_s I \doteqdot \gamma.$$

The main source of error is the fact that the voltage-coil of D draws a current from the voltage-divider R. This coil must, in the first place, be non-reactive; this can be easily arranged by use of the usual condenser compensation. Craighead† has shown that the error in calculating the ratio from the above formula, i.e. neglecting the shunting influence of the volt-coil circuit of D on the voltage-divider, can be neglected if the resistance of the detector circuit be about equal to $r(R - r)/R$. He further shows that if this value of volt-coil resistance be used the actual phase-angle is about double the value found by the use of the preceding formula. The simple method is, owing to this large correction, a poor one for phase-angle measurements.

Palm‡ avoids these errors by the following procedure. Applying the dynamometer volt-circuit directly to a portion r_s of R, the phase-shifter is adjusted to make D give its maximum reading, d_1 say; this is best done by reducing the reading to zero and then changing the phase by 90°. Then I and V_s are in phase. Resuming the connections of Fig. 260 (a) the reading of D is brought to zero by adjustment of r. Applying the volt-circuit directly to r the phase-shifter is regulated to

* "Apparecchio per la misure del rapporto e dell'angolo di fase dei trasformatori," *L'Elettro.*, vol. 9, pp. 344–346 (1922).

† J. R. Craighead, "Potential transformer testing. Note on the effect of the resistance of the detector circuit in determining the ratio of two alternating voltages, and the phase-angle between them, by the balance method," *Trans. Amer. I.E.E.*, vol. 31, pp. 1627–1633 (1912).

‡ A. Palm, "Prüfung von Messtransformatoren mit dem Spiegel-Elektrodynamometer," *Zeits. f. Inst.*, vol. 34, pp. 281–290 (1914).

make D again read zero; then I and V_p are in quadrature. Finally, returning to the connections of Fig. 260 (a), a reading d_2 is observed. Palm proves that

$$K_v = (R/r) \cos \gamma \doteqdot (R/r),$$

and

$$\tan \gamma = \frac{d_2}{d_1} \cdot \frac{R - r + \rho\,(R/r)}{R - r_s + w_s\,(R/r_s)},$$

where ρ is the resistance of the volt-coil itself and w_s that of the whole volt-coil circuit.

4. **Electrometer methods.** At the Reichsanstalt and at the National Physical Laboratory the quadrant electrometer has been developed for use in precision testing and takes the place of separately-excited dynamometers in methods for the testing of voltage transformers.

In the Reichsanstalt method* the needle and case of the instrument are joined together and earthed when the switch S in Fig. 261 (a) is thrown to the left. One pair of quadrants is joined to the unearthed secondary terminal; the other pair is connected to a tapping on a voltage-divider put across the primary terminals. The torques exerted by the primary and secondary quadrants upon the earthed needle are in opposite directions, so that the electrometer acts as a differential voltmeter. By regulating the position of the tapping until the reading of the instrument is zero, the ratio of the transformer is at once given by R/r. The phase-angle is obtained by throwing S to the right, and impressing a p.d. of 50 to 100 volts between the needle and case from a phase-shifter. The latter is regulated until the electrometer again reads zero, i.e. until the voltage between the needle and case is in quadrature with the resultant voltage v between the quadrants, see Fig. 261 (b). The angle is then read directly from the phase-shifter or, still better, computed from the readings of a dynamometer wattmeter D, ammeter A, and voltmeter V.

In the National Physical Laboratory method† the voltage ratio is obtained from the readings of two electrostatic voltmeters, one connected across the secondary and one across a tapping on the primary voltage-divider, as shown in Fig. 261 (c). These are reflecting instruments reading up to 130 volts with long scales divided to 0·01 volt, enabling the ratio

* E. Orlich, *Elekt. Zeits.*, vol. 30, pp. 435–439, 466–470 (1909); H. Schultze, "Die Untersuchung von Spannungstransformatoren mittels des Quadrantelektrometers," *Zeits. f. Inst.*, vol. 31, pp. 332–346 (1911).

† R. S. J. Spilsbury, *Beama J.*, vol. 6, pp. 505–513 (1920).

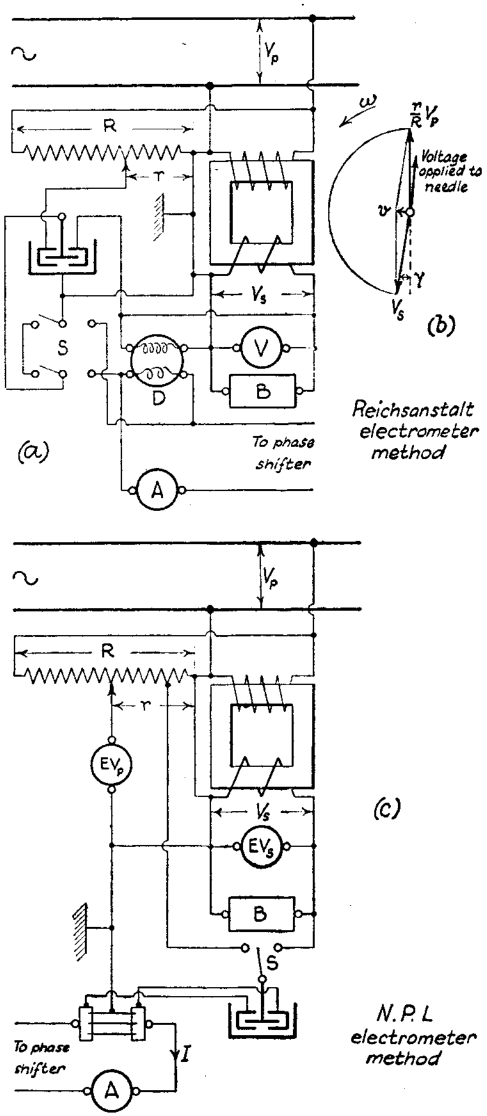


FIG. 261. ELECTROMETER METHODS

to be found within 0·02 per cent. The electrometer is used to determine the phase-angle by throwing S to the left and adjusting the phase-shifter until the instrument reads zero; then I and V_p are in quadrature. If S is now thrown to the right, so that the electrometer is used as an electrostatic wattmeter, the reading of the instrument is proportional to sin γ. The angle can be obtained within 2 minutes.

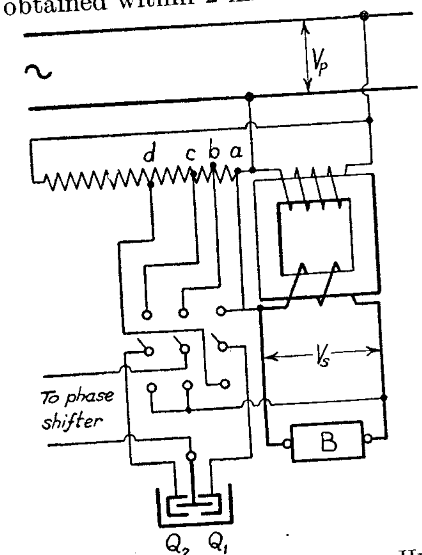


FIG. 262. NATIONAL PHYSICAL LABORATORY HIGH-PRECISION ELECTROMETER METHOD

The sensitivity of the preceding method has proved adequate for all ordinary purposes until recently, the improved standard of performance of voltage transformers rendering necessary a method of testing with higher precision. This is attained in the arrangement devised by Spilsbury* at the National Physical Laboratory, the essential connections being given in Fig. 262. The voltage divider in parallel with the primary of the transformer has the same nominal ratio as the transformer, the drop of potential over ad being 110 volts. Additional tappings are provided giving drops over ab of 1 volt and over ac of 2 volts respectively. The phase-shifter consists of a small two-phase alternator mounted on the same shaft as the source giving V_p and having the same number of poles; adjustment

* R. S. J. Spilsbury, "A precise electrometer method for voltage transformer testing," *Journal I.E.E.*, vol. 67, pp. 1143–1146 (1929).

of the phase is effected by movement of the field magnet. It is obvious, of course, that any other type of phase-shifter would be equally effective. The switch is put in the upper position, thus applying 2 volts to the quadrants, the phase-shifter being regulated to reduce the electrometer deflection to zero; the auxiliary voltage is then in quadrature with V_p. Throwing the switch to the lower position gives a deflection proportional to the phase-angle. The phase of the auxiliary voltage is then changed by 90°, so that it is in phase with V_p; the electrometer reading is now proportional to the ratio error of the transformer.

Under the condition of the ratio test let the voltage of the phase-shifter be $a \sin \omega t = a \sin \theta$, the voltage across the tapping ad be $F_v b \sin \theta$, and the transformer secondary voltage $b \sin (\theta - \gamma)$. The transformer has a ratio-factor $F_v = K_v/K_{nv}$, where K_{nv} is the nominal ratio, and a lagging phase-angle γ. Since the electrometer is used heterostatically, the force on the needle is proportional instantaneously to

$$[a \sin \theta + F_v b \sin \theta - b \sin (\theta - \gamma)]^2 - (a \sin \theta)^2.$$

The deflection d_r is proportional to the average force taken over a period; thus if λ is a constant

$$d_r = \frac{\lambda}{2\pi}\int_0^{2\pi} \{[a \sin \theta + F_v b \sin \theta - b \sin (\theta - \gamma)]^2 - (a \sin \theta)^2\} d\theta$$

$$= \lambda[\tfrac{1}{2}F_v{}^2 b^2 + \tfrac{1}{2}b^2 + abF_v - ab \cos \gamma - F_v b^2 \cos \gamma]$$

In a good transformer $\cos \gamma \doteqdot 1$ so to a first approximation

$$d_r = \tfrac{1}{2}\lambda b^2 (F_v - 1)^2 + ab\lambda (F_v - 1)$$

Again $(F_v - 1)^2$ is negligible in comparison with $(F_v - 1)$, and b^2 and ab are quantities of the same order; hence very closely

$$d_r = ab\lambda(F_v - 1).$$

As a percentage,

$$100(F_v - 1) = 100 d_r/(ab\lambda).$$

Under the condition of the phase-angle test let d_γ be the deflection, then

$$d_\gamma = \frac{\lambda}{2\pi}\int_0^{2\pi} \{[a \cos \theta + F_v b \sin \theta - b \sin (\theta - \gamma)]^2 - (a \cos \theta)^2\} d\theta$$

$$= \lambda[\tfrac{1}{2}F_v{}^2 b^2 + \tfrac{1}{2}b^2 + ab \sin \gamma - F_v b^2 \cos \gamma]$$

$$\doteqdot ab\lambda \sin \gamma + \tfrac{1}{2}\lambda b^2 (F_v - 1)^2$$

It is found that the second term is negligible, so that

$$\sin \gamma \doteqdot d_\gamma / ab\lambda \doteqdot \gamma.$$

The electrometer is of the type described by Paterson, Rayner and Kinnes; it is a reflecting instrument giving a deflection of 2 820 mm. for voltages of 100 on the needle and a p.d. of 2 volts between the quadrants. As the capacitance to earth of the wiring is considerable, the quadrants should be at earth potential if errors are to be avoided. This is effected by connecting a second voltage divider in parallel with the main divider and earthing its tapping point. All leads are electrostatically shielded and maintained at appropriate potentials by connection to the proper points on the auxiliary divider. With such precautions a sensitivity of 1 part in 100 000 for ratio and 0·05 minute for phase-angle is obtained for secondary voltages of 110.

To show the accuracy attainable, a 3 300/110 voltage transformer rated at 25 VA was tested at 50 cycles per sec. by the method just described and by the absolute method of Agnew and Silsbee (see p. 553); in addition the ratio was obtained by means of two electrostatic voltmeters, as on p. 532. In the first section of the table results are given for various non-inductive burdens; in the second section the same burdens were used, but in addition an inductive burden of 11 VA with power-factor of 0·24 (4 watt-hour meter voltage elements) was connected throughout. The agreement between the methods is excellent; similar results have been secured at 33 000 volts.

Volt-amperes Added Secondary Burden	True Ratio-Nominal Ratio			Phase-angle Minutes	
	Electro-meter Method	Agnew and Silsbee's Method	Two Voltmeter Method	Electro-meter Method	Agnew and Silsbee's Method
0	0·9973	0·9973	0·9975	+ 3·7	+ 3·7
6·05	0·9992	0·9992	0·9992	+ 1·9	+ 2·0
12·10	1·0011	1·0011	1·0010	− 0·2	+ 0·1
16·15	1·0024	1·0023	1·0021	− 0·9	− 1·0
24·20	1·0049	1·0048	1·0049	− 3·7	− 3·5
0	0·9997	0·9997	0·9998	+ 18·1	+ 18·3
6·05	1·0016	1·0016	1·0016	+ 16·4	+ 16·3
12·10	1·0035	1·0034	1·0036	+ 14·5	+ 14·6
16·15	1·0047	1·0047	1·0045	+ 13·5	+ 13·5
24·20	1·0071	1·0072	1·0073	+ 10·9	+ 11·0

CHAPTER XXVII

ABSOLUTE NULL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Introductory.** The absolute methods discussed in the preceding chapter are essentially deflectional, the ratio error and phase-angle being calculated from the readings of appropriate instruments. The methods now to be examined have as an essential feature the adjustment of the measuring circuit until a suitable detecting instrument gives a null indication, the ratio and phase-angle being found from the settings of the circuit components necessary to bring about this condition.

All absolute null methods for testing voltage transformers are based upon the potentiometer principle, in which the secondary voltage is set in opposition to an approximately equal portion of the primary voltage, derived from some form of voltage divider. The methods are divisible into two main classes. In the first class the detector is a dynamometer, its volt-coil being connected to measure the resultant of the two voltages while its current coil is excited from a phase-shifting device. Since the primary and secondary voltages are not in direct opposition, but differ therefrom by a small angle γ, the resultant is never zero and there is always a small current flowing in the voltage coil. The reading of the dynamometer can, however, be made zero by adjusting the phase of the auxiliary exciting current until it is in quadrature with the resultant voltage. Such a class of method is of the *null reading* type.

The second class contains the real null methods in which no current flows in the detector, usually a vibration galvanometer. This *null current* condition is secured by providing means for altering the phase of the current taken by the voltage divider relative to that of the secondary voltage, so that the phase-difference γ can be compensated; this can be done in a variety of ways to be considered in later sections of this chapter. These null current methods have much in common with bridge networks and are conveniently classed as such.

2. **Method with dynamometer and phase-shifter.** The single dynamometer method of Fig. 260 (*a*) can be used as a null method in the following way, illustrated by the vector diagrams

of Fig. 263 (*a*), (*b*) and (*c*). First with I in phase with V_p adjust r to secure a null reading; then v and I are in quadrature. Now successively set I in advance and behind V_p by equal angles θ in each case, adjusting r to values r' and r'' to give null readings; then v' and v'' are normal to the appropriate

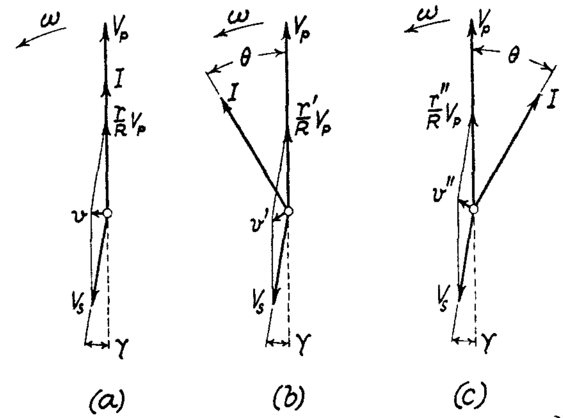


FIG. 263. VECTOR DIAGRAMS FOR FIG. 260 USED AS A NULL METHOD

vector of I. The three conditions are shown in Fig. 263 (*a*), (*b*) and (*c*) respectively, from the geometry of which

$$(r/R)V_p = V_s \cos\gamma,$$
$$(r'/R)V_p \cos\theta = V_s \cos(\theta + \gamma),$$
$$(r''/R)V_p \cos\theta = V_s \cos(\theta - \gamma).$$

From these equations it is easy to show that

$$K_v = (R/r)\cos\gamma \doteq (R/r),$$

and
$$\tan\gamma = (r'' - r')/2r \tan\theta.$$

If in particular $\theta = 45°$, so that $\tan\theta = 1$,

$$\tan\gamma = (r'' - r')2r.$$

3. **Bridge methods. Preliminary remarks.** In the true null or bridge methods the secondary voltage and an approximately equal portion of the primary voltage are set against one another on the detector circuit; their resultant, consequent upon the two voltages differing from true opposition by the phase-angle γ, is balanced out by some suitable device, so that the total voltage at the detector terminals is zero. Referring

to Fig. 264 (*a*), the principle of a number of methods is shown. Since in this simple circuit there is no allowance for the angle γ it will only be possible to obtain a minimum reading on the detector when r is adjusted by moving the contact along the resistance voltage divider R; this condition will occur when

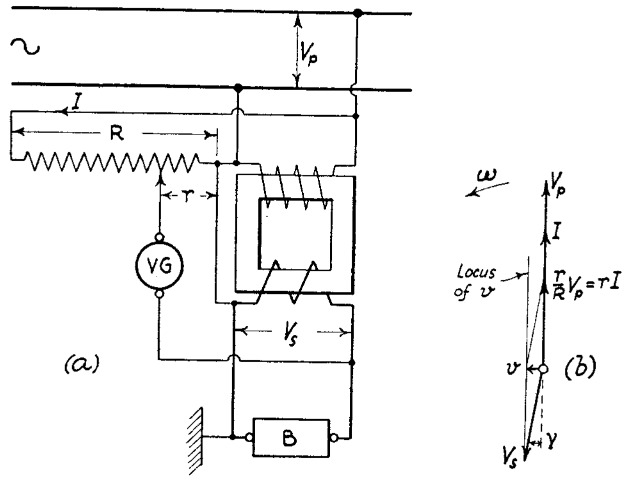


FIG. 264. PRINCIPLE OF THE BRIDGE METHOD WITH RESISTANCE VOLTAGE-DIVIDER

the resultant v of V_s and $(r/R)V_p$ is normal to V_v as shown in Fig. 264 (*b*). Then $V_s \cos\gamma = (r/R)V_p$, from which $K_v \doteqdot R/r$. To get null indication it is necessary that the current I should be displaced from V_p in the same direction as $-V_s$ is displaced from V_p, or to provide some arrangement on the secondary side having an equivalent effect. Numerous ways of effecting this balance adjustment have been developed by different workers,* some of these arrangements being discussed in the following sections.

It has been pointed out on p. 356 that it is a matter of some difficulty to construct resistance voltage-dividers for use at high voltages so that they shall be free from impurity effects. This has led some recent workers to devise methods in which a specially constructed, shielded air-condenser takes the place of the high resistor, since it is much easier to make a condenser nearly perfectly reactive than it is to arrange a high resistor

* Most of these devices were originally briefly suggested by C. H. Sharp and W. W. Crawford, loc. cit. on p. 412 (1911).

to be approximately non-reactive; these methods are considered later in this chapter.

4. **Bridge methods with resistance voltage-divider. Compensation by self-inductance.** When the phase-angle of the transformer is negative or lagging, the simple method of Fig.

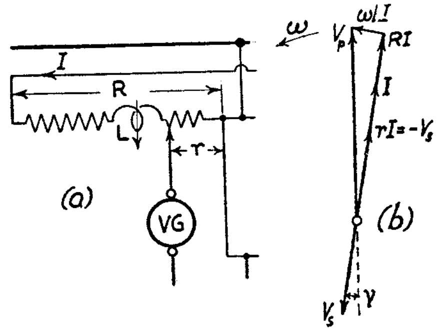


FIG. 265. BRIDGE METHOD WITH RESISTANCE VOLTAGE-DIVIDER AND SELF-INDUCTANCE COMPENSATION FOR γ

264 (*a*) can be compensated by the addition of a self-inductance L in series with R, as shown in Fig. 265 (*a*). Referring to the vector diagram, Fig. 265 (*b*), it is clear that balance will be secured when r and L are varied, then

$$rI = V_s \text{ numerically};$$

but $$I = V_p/\sqrt{(R^2 + \omega^2 L^2)} \doteqdot V_p/R$$

since ωL is generally small in comparison with R. Then

$$K_v \doteqdot R/r$$

and also $$\tan\gamma = -(\omega L/R) \doteqdot \gamma,$$

the negative sign denoting that V_s reversed lags on V_p. The utility of the method is limited to this condition. As in all methods where inductive apparatus is used, care must be taken to avoid error due to stray magnetic fields from the transformer acting upon L, which should preferably be astatic.

5. **Bridge methods with resistance voltage-divider. Compensation by mutual inductance.** The requisite compensation can also be secured by putting the primary of a mutual inductor in series with R, the secondary winding being in the galvanometer circuit, as shown in Fig. 266 (*a*). In the vector diagram of Fig. 266 (*b*) the current I will lag on V_p by a small angle α given by $\tan\alpha = \omega L_p/R$ where L_p is the inductance of the

primary of M, the resistance R being assumed free from impurity. Balance is secured by adjustment of r and M; the drop rI in r and the secondary voltage of the mutual inductor ωMI then give a resultant equal and opposite to V_s. Assuming γ to be lagging

$$V_s \sin(\gamma - \alpha) = \omega MI$$

$$V_s \cos(\gamma - \alpha) = rI$$

But $$I = V_p/\sqrt{(R^2 + \omega^2 L_p^2)} \doteqdot V_p/R$$

and also $$\cos(\gamma - \alpha) \doteqdot 1.$$

Therefore $$K_v \doteqdot (R/r)$$

$$\tan(\gamma - \alpha) = \omega M/r \doteqdot \gamma - \alpha.$$

Since M can be either positive or negative the method is applicable to transformers in which V_s reversed either leads or lags

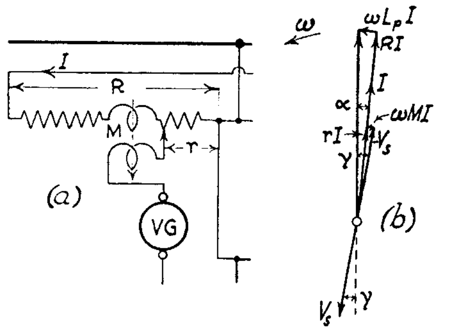


FIG. 266. BRIDGE METHOD WITH RESISTANCE VOLTAGE-DIVIDER AND MUTUAL INDUCTANCE COMPENSATION FOR γ

on V_p. The process is more flexible than that described in Section 4, but requires the same care to avoid stray-field troubles. Apparatus on this principle is made by H. Tinsley & Co.

6. **Bridge methods with resistance voltage-divider. Compensation by capacitance on primary side.** The insertion of an inductance L in series with the voltage-divider, as in Fig. 265 (*a*), enables lagging phase-angles to be compensated; it would appear then, that a suitably connected condenser would deal with leading or positive angles. A simple arrangement is shown in Fig. 267 (*a*). The impedance operator for the voltage-divider when a portion s is shunted by a condenser C is

$$z = R - s + \frac{s}{1 + j\omega Cs}$$

$$= R - s + \frac{s}{1 + \omega^2 C^2 s^2} - j\frac{\omega Cs^2}{1 + \omega^2 C^2 s^2}.$$

Now in practice $\omega^2 C^2 s^2$ is usually quite negligible in comparison with unity; hence

$$z \doteqdot R - j\omega Cs^2$$

to a high order of approximation, which should be compared with

$$z \doteqdot R + j\omega L$$

obtained for Fig. 265 (*a*). Hence the condenser has practically no effect on the resistance of the voltage-divider and gives the desired reversal of sign to the phase-compensation. Thus we can write

$$K_v \doteqdot R/r$$

$$\tan \gamma \doteqdot \omega Cs^2/R \doteqdot \gamma,$$

γ being an angle of lead. The approximate vector diagram is shown in Fig. 267 (*b*).

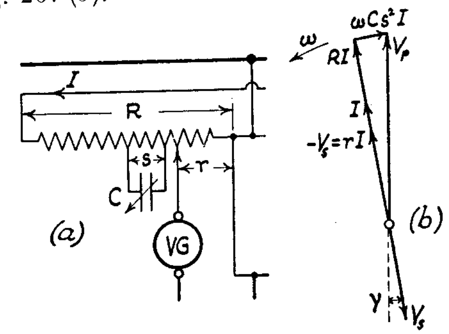


FIG. 267. BRIDGE METHOD WITH RESISTANCE VOLTAGE-DIVIDER AND CAPACITANCE COMPENSATION FOR γ

A condenser can be used to measure angles of lag by the device due to Barbagelata* shown in Fig. 268 (*a*). The voltage-divider consists of a section R' of high resistance in series with two equal sections s, s, upon the right-hand member of which the potential contact is applied, r being less than s. The condenser can be switched either across the left-hand or the right-hand section. With C on the left the method is exactly like Fig. 267 (*a*) and serves to measure leading angles. If C is

* A. Barbagelata, loc. cit. on p. 433 (1921).

transferred to the right the vector relationships of Fig. 268 (*b*) will hold. The common voltage across C and the right-hand s is the vector difference between V_p and $(R' + s)I$, I necessarily leading on V_p. The current in C leads on this p.d. by 90°, while the current in the right-hand s is in phase therewith;

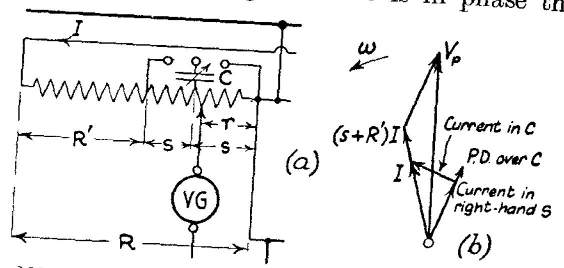


FIG. 268. BRIDGE METHOD WITH CAPACITANCE COMPENSATION FOR γ DUE TO BARBAGELATA

these two currents together must equal I. Hence, as the diagram of vectors shows, the current in the right-hand s and in the portion r upon which the potential contact is set lags on

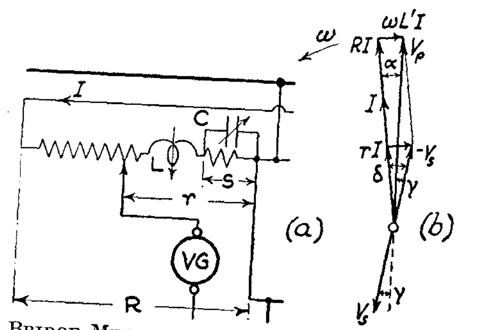


FIG. 269. BRIDGE METHOD WITH RESISTANCE VOLTAGE-DIVIDER AND COMPENSATION FOR γ BY SELF-INDUCTANCE AND CAPACITANCE

V_p, thus providing a means of compensating lagging values of γ. This artifice has much to recommend it since inductances are omitted and consequent inductive interference trouble is avoided. Then as before it is easily shown that

$$K_v \fallingdotseq R/r$$

and

$$\tan\gamma \fallingdotseq \pm(\omega Cs^2/R),$$

the positive sign being taken when the condenser is on the left and the negative when on the right.

7. **Bridge methods with resistance voltage-divider. Compensation by self-inductance and capacitance on primary side.** Agnew and Silsbee* have combined the inductance compensation of Fig. 265 (*a*) with the capacitance compensation of Fig. 267 (*a*) in order to allow for lagging and leading values of γ, the connections being shown in Fig. 269 (*a*). To a high degree of approximation the capacitance C shunted by s and the inductance L make the impedance operator of the voltage-divider

$$z \fallingdotseq R + j\omega(L - Cs^2) \fallingdotseq R + j\omega L'$$

Hence if $L > Cs^2$ balance can be obtained with lagging values of γ; while if $Cs^2 > L$ leading phase-angles can be accounted for. Balance is obtained by adjustment of r and L or C, the approximate vector diagram being given in Fig. 269 (*b*); from this

$$V_s \fallingdotseq [\sqrt{(r^2 + \omega^2 L'^2)}]I.$$
$$V_p \fallingdotseq [\sqrt{(R^2 + \omega^2 L'^2)}]I.$$

Neglecting $\omega L'$ in comparison with r and R gives

$$K_v \fallingdotseq R/r$$

Again, $\gamma = \delta - \alpha = \arctan(\omega L'/r) - \arctan(\omega L'/R)$

or since the angles are all small

$$\gamma \fallingdotseq \omega L'\,[(1/r) - (1/R)].$$

Since $r < R$ the bracket is always positive; the sign of L' is, therefore, determined by that of γ.

The method is the parallel in voltage transformer testing of the method of Schering and Alberti used for current transformers, as will readily be seen by reference to Fig. 233. When carefully set up it forms one of the most sensitive and accurate of the absolute methods; in a slightly modified form due to Spilsbury† it is in use at the National Physical Laboratory where results in close agreement with the standard electrometer method are obtained by attention to several practical details. The circuit is shown in Fig. 270. In this diagram the main voltage-divider, to which one terminal of the vibration galvanometer is attached, consists of a section r across which the p.d. is about 110 volts; a section s in parallel with a

* P. G. Agnew and F. B. Silsbee, "The testing of instrument transformers," *Trans. Amer. I.E.E.*, vol. 31, pp. 1635–1638 (1912).

† R. S. J. Spilsbury, *Journal I.E.E.*, vol. 67, pp. 1143–1146 (1929).

condenser C; a high resistor R', in series with a small inductor L of resistance R_L. With L of fixed value and C at zero the maximum desired lagging value of γ can be balanced; as C is increased γ is gradually reduced to zero, further increase in C balancing an angle of lead. As in a.c. bridge technique, it is desirable to have the terminals of the galvanometer at earth

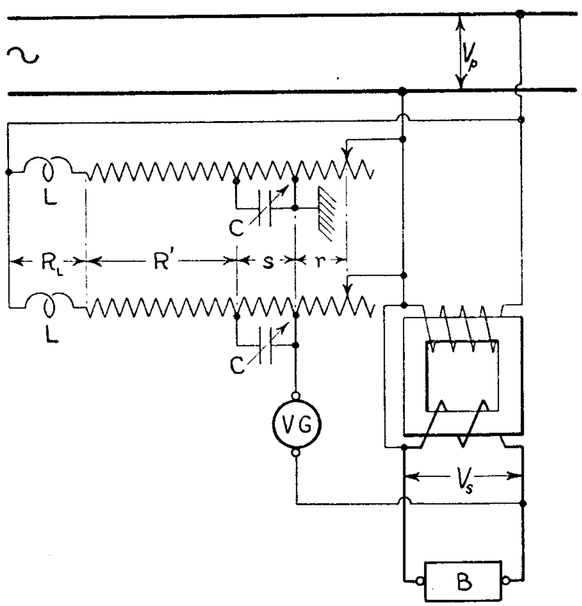


FIG. 270. NATIONAL PHYSICAL LABORATORY. MODIFICATION OF FIG. 269 WITH EARTHING DEVICE

potential when balance has been secured, in order to avoid errors due to earth admittance effects.* This can be done by the use of a Wagner earthing device consisting of a second voltage-divider, similar to the first, earthed at its tapping point. By setting r and C to the same value on both dividers the detector terminals are reduced to earth potential without directly earthing them. It is further desirable to place the galvanometer inside an earthed shield and to run its leads in earthed tubes. By taking these precautions against capacitance effects it is possible to attain a sensitivity of 1 part in 100 000 for ratio and 0·05 minute for phase-angle. With such high sensitivity it is permissible to include second order terms in the

* See *A.C. Bridge Methods*, pp. 358, 360–370.

expressions for K_v and γ; it can be shown that these are

$$K_v = \frac{1}{r}\left(r + \frac{s}{1 + \omega^2C^2s^2} + R' + R_L\right) + \frac{\{\omega L - [\omega C s^2/(1 + \omega^2C^2s^2)]\}^2}{r\{r + [s/(1 + \omega^2C^2s^2)] + R' + R_L\}}$$

in which the second term is usually quite negligible; and

$$\tan\gamma = \frac{\{\omega L - [\omega C s^2/(1 + \omega^2C^2s^2)]\}^2}{r + [s/(1 + \omega^2C^2s^2)] + R' + R_L} \doteq \gamma$$

On neglecting second-order terms these become $K_v \doteq R/r$ and $\tan\gamma = \omega L'/R$. An example of this modification is given on p. 545 in comparison with results obtained by the electrometer method.

8. Bridge methods with resistance voltage-divider. Compensation by capacitance on secondary side. The method illustrated in Fig. 271 is used by de la Gorce* at the Laboratoire Central d'Electricité. The phase-compensation is secured by a condenser C shunted across a portion r_1 of a resistance $r_1 + r_2$ joined to the secondary of the transformer. It is usual to make r_2 about the same as r_1, of the order of 5 000 to 10 000 ohms; the condenser is a few thousandths of a microfarad. The voltage-divider R is chosen to have a resistance of about 1 000 ohms per 30 volts of V_p. Balance is secured by variation of r and C. Let i and i_2 be the cyclic mesh currents with clockwise

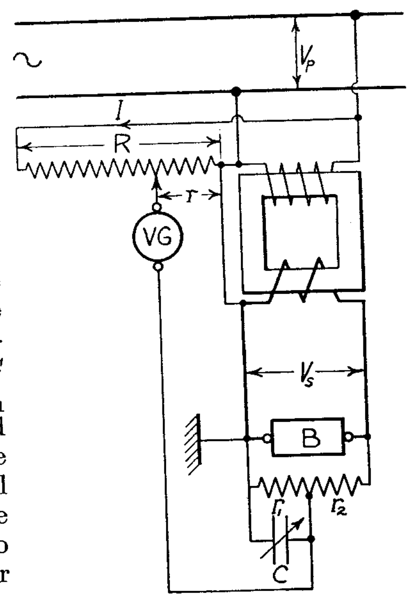


FIG. 271. DE LA GORCE'S BRIDGE METHOD WITH CAPACITANCE COMPENSATION FOR γ

* P. de la Gorce, "L'étalonnement des transformateurs de mesure au Laboratoire Central de l'Electricité," *Bull. Soc. Franç. des Elecns.*, vol. 5, 4th series, pp. 659–665 (1925). A modification in use at the N.P.L. replaces R by a condenser voltage-divider.

positive direction in R and r_2 respectively; $\boldsymbol{v}_p$ and $\boldsymbol{v}_s$ are the primary and secondary voltage vectors. Then at balance

$$r\boldsymbol{i} + z_1\boldsymbol{i}_2 = 0,$$

where $z_1 = r_1/(1 + j\omega Cr_1)$.

But $\boldsymbol{i} = \boldsymbol{v}_p/R$,

and $\boldsymbol{i}_2 = \boldsymbol{v}_s/z_2$,

where $$z_2 = r_2 + \frac{r_1}{1 + j\omega Cr_1} = \frac{(r_1 + r_2) + j\omega Cr_1r_2}{1 + j\omega Cr_1}.$$

Substituting

$$\frac{r}{R}\boldsymbol{v}_p = -\frac{r_1}{(r_1 + r_2) + j\omega Cr_1r_2}\boldsymbol{v}_s,$$

whence,

$$\frac{\boldsymbol{v}_p}{\boldsymbol{v}_s} = -\frac{R}{r}\cdot\frac{r_1}{(r_1+r_2)^2 + \omega^2C^2{r_1}^2{r_2}^2}[(r_1 + r_2) - j\omega Cr_1r_2] = -K_v\varepsilon^{j\gamma}$$

Thus, the ratio is

$$K_v = \frac{V_p}{V_s} = \frac{R}{r}\frac{r_1}{\sqrt{[(r_1 + r_2)^2 + \omega^2C^2{r_1}^2{r_2}^2]}} = \frac{R}{r}\cdot\frac{r_1}{(r_1 + r_2)}\cdot\cos\gamma$$
$$\doteqdot \frac{R}{r}\cdot\frac{r_1}{(r_1 + r_2)},$$

and the phase-angle is

$$\tan\gamma = \omega Cr_1r_2/(r_1 + r_2)$$

when $\boldsymbol{v}_p$ lags on $-\boldsymbol{v}_s$, i.e. when γ is positive.

When γ is negative it is necessary to shunt C across r_2 instead of r_1. By a similar process it is then easily proved that

$$\frac{\boldsymbol{v}_p}{\boldsymbol{v}_s} = -\frac{R}{r}\cdot\frac{r_1}{(r_1 + r_2)^2 + \omega^2C^2{r_1}^2{r_2}^2}(1 + j\omega Cr_2)\,[r_1 + r_2 - j\omega Cr_1r_2]$$
$$= -\frac{R}{r}\cdot\frac{r_1}{(r_1 + r_2)^2 + \omega^2C^2{r_1}^2{r_2}^2}[(r_1 + r_2 + \omega^2C^2{r_2}^2r_1) + j\omega C{r_2}^2]$$

From these,

$$\tan\gamma = \frac{-\omega C{r_2}^2}{r_1 + r_2 + \omega^2C^2{r_2}^2r_1} \doteqdot \frac{-\omega C{r_2}^2}{r_1 + r_2},$$

since the second order frequency term is usually negligible; the angle is now one of lag. Also

$$K_v = \frac{R}{r}\cdot\frac{r_1}{(r_1+r_2)^2+\omega^2C^2{r_1}^2{r_2}^2}\sqrt{[(r_1 + r_2 + \omega^2C^2{r_2}^2r_1)^2 + \omega^2C^2{r_2}^4]}$$
$$\doteqdot \frac{R}{r}\cdot\frac{r_1}{(r_1 + r_2)^2 + \omega^2C^2{r_1}^2{r_2}^2}\sqrt{[(r_1 + r_2)^2 + \omega^2C^2{r_2}^4]},$$

to the same order of approximation. The method gives good results in practice, but there is likely to be some error due to self-capacitance effects in the resistors, especially at high voltages.

A modification due to Vassillière-Arlhac* includes in series with r_2 the primary of a mutual inductor, C being shunted across r_1. The secondary of the mutual inductor is supplied with the vector resultant of the drop down r and the secondary voltage V_s of the transformer. Change in the sign of γ can be dealt with by reversing the mutual inductance polarity.

9. **Bridge methods with condenser voltage-divider. Compensation by mutual inductance.** It has been pointed out on p. 531 that resistance voltage-dividers cannot be used for accurate work at voltages exceeding about 30 kilovolts without greatly elaborating their construction. Their principal defect is the impurity introduced by distributed earth-capacitance of the resistance elements, necessitating elaborate systems of shielding and other auxiliary apparatus to keep the impurity errors down to a minimum. Moreover, such resistors absorb considerable power, with consequent heating and uncertainty of the resistance values. Experience with high-voltage air-condensers in other classes of measurements has shown that impurity effects therein are very slight (see p. 375); the degree of purity attainable with a condenser voltage-divider is, therefore, much higher than could be obtained from one of the resistance type. The apparatus is also much simpler, less costly, easier to operate, and there is practically no limit to the voltage for which such condenser voltage-dividers can be constructed.

These considerations have led Churcher† to devise the method shown in Fig. 272. The high- and low-voltage sides of the transformer are joined together so that the applied primary voltage acts in the same sense as the induced secondary voltage, as indicated by the positive signs; the junction is earthed. The condensers C_1 and C_2 form a condenser voltage-divider across the transformer windings, their junction being connected through a shunted vibration galvanometer, the secondary of a small mutual inductor M and one winding of an auxiliary mutual inductor m to the earthed point. The condenser C_1 is a standard air-condenser of accurately-known value and

* J. Vassillière-Arlhac, "Mesure des faibles angles de déphasage et nouveaux procédés pour les déterminer," *Bull. Soc. Franç. des Elecns.*, vol. 7, pp. 1330–1347 (1927).

† B. G. Churcher, "The measurement of the ratio and phase-displacement of high alternating voltages," *Journal I.E.E.*, vol. 65, pp. 430–439 (1927).

negligible losses designed to withstand the full primary voltage; the condenser used is of a cylindrical type with guard-rings on the l.t. electrode, having a capacitance of 100 $\mu\mu$F and capable of operation at 150 kilovolts (see p. 375). The condenser C_2 is capable of withstanding the secondary voltage. Since the ratio K_v is very approximately C_2/C_1 it follows that C_2 will be large in comparison with C_1. It usually consists of a 1 200 $\mu\mu$F. variable air condenser; for larger ratios fixed mica standards

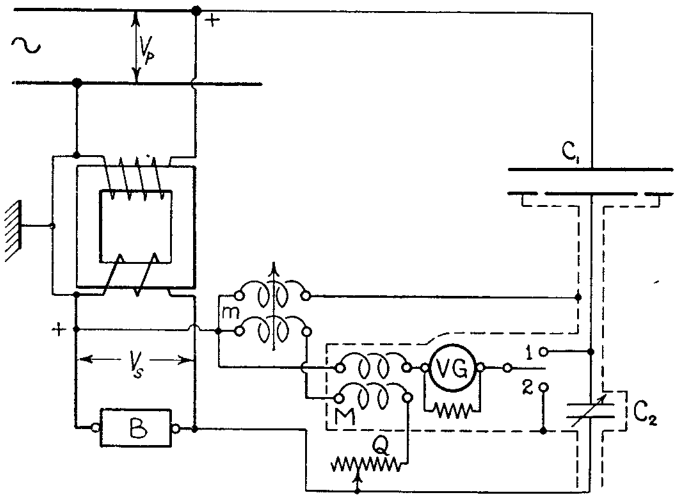


FIG. 272. CHURCHER'S BRIDGE METHOD WITH CONDENSER VOLTAGE-DIVIDER

are connected in parallel with this to bring the total to the desired value. On account of the angle γ between V_p and V_s reversed, balance cannot be obtained by regulation of C_2 only; the necessary phase-compensation is effected by the inductor M, the primary of which, in series with a resistance Q, is joined across the secondary terminals. Shields are provided for M, VG and its shunt, and C_2; the lead joining C_1 to C_2 is also shielded. All the shields are joined together through the secondary of m to the earthed point.

The procedure is as follows: Applying the voltage to the primary side and adjusting it to give the desired secondary voltage, the switch is put on contact 1, balance being secured by regulation of C_2 and M, varying Q also if necessary. It should be noted that Q is large in comparison with the reactance of the primary circuit of M. Now with the switch on contact 2

m is varied to give zero deflection. Replacing on contact 1, C_2 and M are adjusted to give a final balance. Since there is simultaneous balance in both positions of the switch, there can be no p.d. between galvanometer, shunt, lead from C_1 to C_2, and the inner electrode of C_1, which are all at the same potential, and the shield and guard-ring of C_1. Hence, although there is a stray capacitance between the guard-ring of C_1 and the inner electrode, and between the shields and the apparatus enclosed within them, this capacitance has no p.d. across it and can carry no current causing error in the balance setting. As a rule, this double balance is an unnecessary refinement; it is often quite sufficient to omit the earthing device and to earth the shields directly, since the p.d. between them and the apparatus shielded is never more than a few volts.

The theory of the balance condition is somewhat lengthy, whether worked out by the operational method or deduced from the geometry of the vector diagram, as is done by Churcher. The reader is referred to the original paper for the complete analytical details, it being sufficient for our present purpose to quote the results. Let R be the total resistance of the primary circuit of M, i.e. Q together with the resistances of the winding of M and m; also let L be the total self-inductance of the same part of the circuit. Then $\tan\theta = \omega L/R$ gives the phase-displacement of the current in the primary of M relative to V_s; it will be a very small angle since Q, and therefore R, is chosen large compared with ωL. Also, let δ be the loss angle of C_2, i.e. the defect from exact quadrature; this also will be a small angle. Then after some reduction it can be shown that

$$K_v = \frac{C_2}{C_1 \cos\gamma}\left[1 - \frac{\omega M}{R}\tan\theta\right] \doteqdot \frac{C_2}{C_1},$$

and also

$$\tan\gamma = \frac{1 + (C_1/C_2) + [\sin\delta\,(R/\omega M)]}{[\{[(R^2 + \omega^2 L^2)^{\frac{1}{2}}/\omega M \cos\theta - (\omega L/R)]^2 + 1\} - \{1 + \sin\delta\,(R/\omega M)\}^2]^{\frac{1}{2}} - \omega L C_1/R C_2}$$

If $\omega L/R$ does not exceed 0·05, the formula for γ becomes

$$\tan\gamma \doteqdot (\omega M/R)\ [1 + (C_1/C_2)] + \sin\delta$$

to a high order of approximation. Since so large a value of $\tan\theta$ is permissible it follows that impurity in M is quite negligible in practice. Further, since ωL is small in comparison with R the burden imposed on the secondary by the test-circuit is practically non-inductive and of a value equal to V_s^2/R volt-amperes.

It will be useful to give practical data for an actual test of the variation of ratio and angle with burden on a 200 VA, 66 000/110 transformer. As stated, C_1 was a 100 $\mu\mu$F cylindrical air-condenser. The condenser C_2 consists of a 0·04987 μF mica standard (power factor 0·0002), a 0·01005 mica standard (power factor 0·0006), and a variable air-condenser all in parallel; the nominal value of C_2 is thus 0·06 μF and sin δ does not exceed 0·00026. For M a fixed astatic inductor of 144·4 mH was used, L being 162·3 mH, and balance obtained by varying R, i.e. Q. The lowest value of R encountered was 3 800 ohms, so that the greatest value of tan θ is $\omega L/R = 0{\cdot}0134$, so that the approximate formulae are applicable. The detector is a vibration galvanometer giving a sensitivity of 20 mm. per microampere; no difficulty was experienced in obtaining consistent balances for ratio to 2 parts in 10 000. The load due to R is negligible while that due to the voltmeter forming part of the burden B is 2 VA.

The transformer had a very small phase-angle, being greatest at no-load, arctan 0·00107, and falling linearly to zero at about 120 VA, thereafter becoming negative; the maximum deviation from the line is about 0·00006. The ratio at no load is 603·60 and at full-load 604·95 the variation being linear with a mean deviation of not more than 2 parts in 10 000.

10. **Bridge method with condenser voltage divider. Compensation by mutual inductance and resistance.** Jimbo and Sakimura* have recently described the method illustrated in Fig. 273. The voltage divider consists of a 150 kV compressed-gas condenser C_1, having a capacitance of 0·0013 μF, in series with a mica decade condenser C_2 across the voltage V_p. C_2 is shunted by a resistance R consisting of a fixed resistance of 98, 100 or 102 ohms in series with a 4 ohm slide wire. Across V_s is connected a resistance S of 10 000 ohms in series with the primary of a fixed mutual inductor, having a self inductance of 307 mH. The secondary voltage of M is opposed through a vibration galvanometer to the drop down a fractio[n] of R. The value of M is chosen to suit the nominal ratio of the transformer; its value in mH is equal to one-tenth of the nominal ratio. The current in the primary of M is

$$\boldsymbol{i}_s = \boldsymbol{v}_s/(S + j\omega L),$$

and its secondary voltage will be $j\omega M\boldsymbol{i}_s$. The current entering the h.v. terminal of C_1 is

$$\boldsymbol{i} = \frac{\boldsymbol{v}_p}{(1/j\omega C_1) + [R/(1 + j\omega C_2 R)]}$$

* S. Jimbo and H. Sakimura, "A new method of testing potential transformers," *Res. Elect. Lab. Tokyo*, No. 363, pp. 1–42 (1934).

and the part of this flowing in R is

$$\boldsymbol{i}_R = \boldsymbol{i}/(1 + j\omega C_2 R) = j\omega C_1 \boldsymbol{v}_p/[1 + j\omega R(C_1 + C_2)]$$

For balance

$$r\boldsymbol{i}_R + j\omega M\boldsymbol{i}_s = 0$$

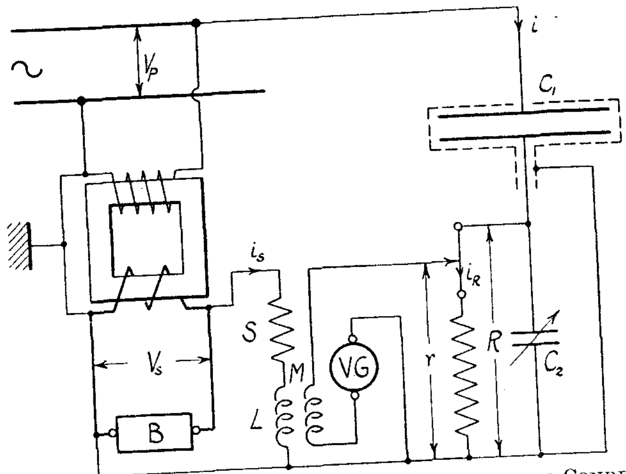


FIG. 273. JIMBO AND SAKIMURA'S BRIDGE METHOD WITH CONDENSER VOLTAGE-DIVIDER

which is equivalent to

$$\frac{\boldsymbol{v}_p}{\boldsymbol{v}_s} = -\frac{M}{C_1 r} \cdot \frac{1 + j\omega R(C_1 + C_2)}{S + j\omega L}.$$

Taking ωL as small compared with S and $\omega R(C_1 + C_2)$ with unity, it is easy to show that approximately

$$K_v \;.\; M/C_1 r S,$$

and

$$\tan\gamma \doteqdot \omega\,[L/S - R(C_1 + C_2)].$$

In the original paper more accurate expressions are worked out including second-order terms neglected above and residual errors in the condensers and resistors. Full details are also given of the calibration of each part of the apparatus and numerous tests are described.

11. Bridge method with condenser voltage-divider. Compensation by capacitance and resistance. Yoganandam* has described a method in which a high-voltage condenser is used, as shown in Fig. 274, the whole arrangement utilizing the Schering bridge equipment now available in all high-voltage laboratories. The windings of the transformer are joined in opposition, as indicated by the signs, the junction being earthed.

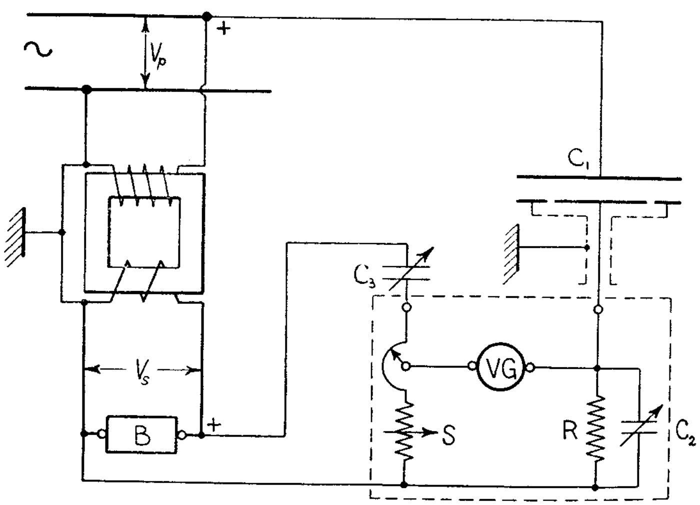


FIG. 274. YOGANANDAM'S BRIDGE METHOD WITH CONDENSER VOLTAGE-DIVIDER

The secondary is provided with the usual test burden B, in parallel with which is a branch composed of a good quality mica condenser C_3, a low-resistance slide wire and a variable resistance S. The voltage-divider across the primary side consists of the high-voltage condenser C_1 in series with the resistor R shunted by the condenser C_2. The condenser C_3 is set so that C_3/C_1 is about equal to the nominal ratio of the transformer, balance being obtained by adjusting S and C_2. As in the Schering bridge at 50 cycles per second, R is fixed at $1\,000/\pi$ ohms, and S will be about equal to it. Earthed shields are provided as shown in the diagram.

Let ρ be the resistance in series with C_3 when balance is obtained; ρ will be equal to the sum of the series loss-resistance

* G. Yoganandam, "A ready method of measuring the voltage ratio and phase-angle of high-voltage transformers," *Journal I.E.E.*, vol. 68, pp. 192–193 (1930).

of C_3 together with a portion of the slide wire. Let S denote the total of the variable decade resistor and the remaining part of the slide wire. The impedance operator for the branch across the voltage v_p will be

$$z_p = \frac{1}{j\omega C_1} + \frac{R}{1 + j\omega C_2 R} = \frac{1 + j\omega R(C_1 + C_2)}{j\omega C_1(1 + j\omega C_2 R)}.$$

It should be noted that any loss in C_2 can be represented by a shunt resistance; this in parallel with the resistor across which C_2 is connected constitutes the total R. As C_2 is often a small air condenser the losses can be regarded as zero so that R is simply the decade setting; even when C_2 contains mica standards the shunt loss-resistance is so enormous in comparison with the few hundred ohms in the decade resistor that its effect can be ignored. Similarly, the operator for the branch across the voltage v_s is

$$z_s = \rho + S + \frac{1}{j\omega C_3} = \frac{1 + j\omega C_3 (S + \rho)}{j\omega C_3}$$

When balance occurs the fall of potential through S is equal and opposite to that through R, i.e.

$$S(v_s/z_s) = -[R/(1 + j\omega C_2 R)](v_p/z_p)$$

or

$$(v_p/v_s) = -(S/R)(z_p/z_s)(1 + j\omega C_2 R).$$

Substituting and simplifying gives

$$\frac{v_p}{v_s} = -\frac{S}{R}\cdot\frac{C_3}{C_1}\cdot\frac{[1 + j\omega R(C_1 + C_2)]}{[1 + j\omega C_3 (S + \rho)]}.$$

The ratio is thus

$$K_v = \frac{V_p}{V_s} = \frac{S}{R}\cdot\frac{C_3}{C_1}\cdot\frac{[1 + \omega^2 R^2 (C_1 + C_2)^2]^{\frac{1}{2}}}{[1 + \omega^2 C_3^2 (S + \rho)^2]^{\frac{1}{2}}}$$

and the angle between v_p and v_s reversed is

$$\gamma = \arctan \omega C_3 (S + \rho) - \arctan \omega R(C_1 + C_2).$$

It may not be possible, when γ is a large lagging or negative angle to obtain balance with these connections. It is then necessary to join C_1 to S and C_3 to R, i.e. interchange the l.v. terminals of these condensers. Then taking z_p' and z_s' as the new operators,

$$z_p' = \sigma + S + (1/j\omega C_1) = [1 + j\omega C_1 (\sigma + S)]/j\omega C_1$$

and

$$z_s' = \frac{1}{j\omega C_3} + \tau + \frac{R}{1 + j\omega C_2 R}$$
$$= \frac{(1 - \omega^2 C_2 C_3 R\tau) + j\omega[R(C_2 + C_3) + \tau C_3]}{j\omega C_3 (1 + j\omega C_2 R)}$$

where σ is the portion of slide wire in series with C_1 and τ is the series loss-resistance of C_3; as a rule the terms involving τ may be neglected but will be retained here for completeness. For balance

$$S(\boldsymbol{v}_p/z_p') = -[R/(1 + j\omega C_2 R)]\,(\boldsymbol{v}_s/z_s')$$

or $$\boldsymbol{v}_p/\boldsymbol{v}_s = -(R/S)\,(z_p'/z_s')\;[1/(1 + j\omega C_2 R)]$$

Substituting and simplifying

$$\frac{\boldsymbol{v}_p}{\boldsymbol{v}_s} = -\frac{R}{S}\cdot\frac{C_3}{C_1}\cdot\frac{(1 + j\omega C_2 R)[1 + j\omega C_1 (S + \sigma)]}{\{(1 - \omega^2 C_2 C_3 R\tau) + j\omega[R(C_2 + C_3) + \tau C_3]\}},$$

from which

$$K_v = \frac{V_p}{V_s} = \frac{R}{S}\cdot\frac{C_3}{C_1}\cdot\frac{(1 + \omega^2 C_2^2 R^2)^{\frac{1}{2}}[1 + \omega^2 C_1^2 (S + \sigma)^2]^{\frac{1}{2}}}{\{(1 - \omega^2 C_2 C_3 R\tau)^2 + \omega^2[R(C_2 + C_3) + \tau C_3]^2\}^{\frac{1}{2}}},$$

and the angle between $\boldsymbol{v}_p$ and $-\boldsymbol{v}_s$ is

$$\gamma = \arctan\frac{\omega[R(C_2 + C_3) + \tau C_3]}{(1 - \omega^2 C_2 C_3 R\tau)}$$
$$- \arctan \omega C_2 R - \arctan \omega C_1 (S + \sigma).$$

Using $C_1 = 100\ \mu\mu$F, at 50 cycles per sec. and 110 kV it is easily possible to attain a sensitivity of 1 in 10 000 for ratio and 0·0001 radian for phase-angle.

12. **Bridge method with high-voltage condenser. Compensation by mutual inductance and capacitance.** In all the methods described in Sections 9 to 11 inclusive, where a condenser voltage-divider is employed, it is necessary to know the ratio of two condensers at least to the accuracy required in the measurement of the transformer ratio; the two condensers must also have negligible loss-angles. The h.v. condenser C_1 presents no serious difficulty. If it is an air-condenser the capacitance can be computed from its dimensions with very high accuracy; if it is a compressed gas condenser the capacitance can be measured in a variety of ways. In both cases the loss-angle is negligible. The accuracy of the methods is, therefore, thrown back upon the accuracy with which the other

much larger condenser can be measured and upon the magnitude of its loss-angle; with mica standards the corrections, especially in a phase-angle measurement, may be quite important. Dannatt* has avoided these difficulties by replacing the second condenser by a resistor as shown in Fig. 275. The standard air-condenser C_1 is joined in series with a variable resistor R of about 100 ohms across the primary voltage; the p.d. across R

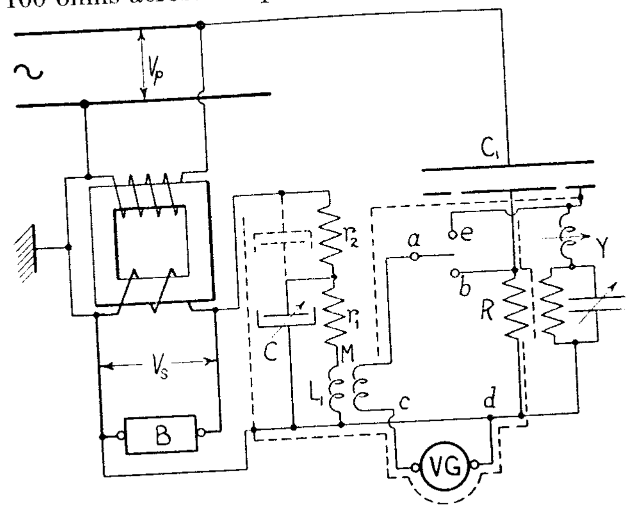


FIG. 275. DANNATT'S BRIDGE METHOD WITH HIGH-VOLTAGE CONDENSER

will thus lead on V_p by about a quarter-period. To obtain a similar voltage from the secondary side, two resistors r_1 and r_2 of about 4 000 ohms each are joined in series with one of the windings of a small mutual inductance M across the secondary voltage V_s; the current in L_1 is, therefore, nearly in phase with V_s and the voltage in the secondary of M is thus nearly in quadrature with V_s, i.e. with V_p. Precise compensation of phase-angle can be made by shunting a mica condenser C across r_1 or r_2 according to the sign of γ. The secondary voltage of M is opposed to the drop in R through a vibration galvanometer at cd, and balance is secured by varying R and C.

The guard-ring of C_1 is connected to a network Y consisting of an inductance in series with a condenser-resistance parallel group. This network is adjusted until the guard-circuit is at the

* C. Dannatt, "The testing of voltage transformers for ratio and phase angle," *M.V. Gaz.*, vol. 13, pp. 62–64 (1931).