high-voltage electrode is insulated from the gas vessel by a truncated porcelain cone; a paper tube mounted on the top of the vessel carries the h.v. terminal. The gas is again N or CO_2 at about 12 atmospheres. The overall height of the condenser is 186 cm., the floor space 70 cm. by 70 cm., and the weight 175 kg. (385 lb.). Careful tests on a condenser of this

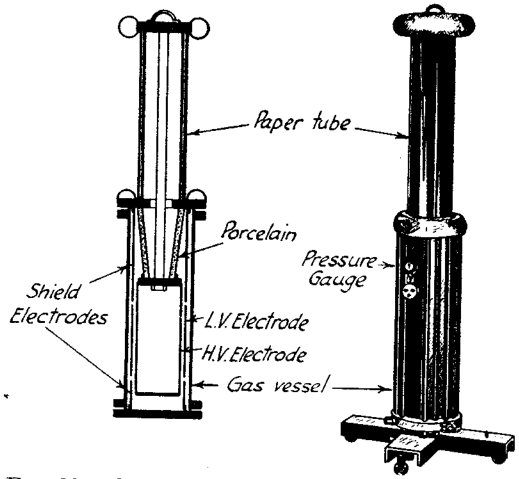

FIG. 195. COMPRESSED GAS CONDENSER FOR 140 kV

pattern have been made by Jimbo and Sakimura (loc. cit.), who show that the defect from quadrature is about $0{\cdot}8 \times 10^{-6}$ radians at 50 cycles per sec.; pressure gas condensers may, therefore, be regarded as perfect capacitances. This and the preceding condenser are made by Hartmann & Braun; somewhat similar types are constructed by the Metropolitan-Vickers Co. and by the Cambridge Instrument Co.

All compressed-gas condensers are tested to a considerably greater gas pressure than that with which they normally work. There is very little danger of explosion; with paper bushings such rare failures as have taken place are harmless.

It is hardly possible to compute the capacitance of a compressed-gas condenser with high accuracy from the dimensions of its electrodes, since the construction does not lend itself so readily to such precise location of the parts as is the case in air condensers; and in addition there is a small dependence of capacitance upon gas pressure. The capacitance is readily measured in terms of a standard air condenser by means of a Schering bridge operated at the rated voltage.

4. **Capacitance standards for low voltages.** Condensers are frequently used in the low-voltage portions of instrument transformer test circuits, usually to compensate for the phase-angle between the primary and secondary sides of the transformer. Such condensers must be accurate and permanent in value, of very low loss and phase-defect from exact quadrature between the p.d. applied to them and the current they carry; in a phrase, they should approximate to true standards of zero power-factor. These qualities are possessed in a high degree by well-made mica condensers, which, for ease of manipulation, are best arranged in plug- or switch-operated decades. A total capacitance of 1·11 μF subdivided into three decades of 1/10, 1/100 and 1/1 000 μF per step is very convenient in practice. In good mica condensers the phase-defect from quadrature may lie between 1 minute and 5 minutes, the smaller condensers having the higher angles; very good condensers may have defects one-third of these values. It is preferable that all condensers, especially those of small value, should be in shielded cases so that earth-capacitances can be made definite and suitably disposed in the network to cause little error in the measurements. Below 1/1 000 μF it is usual to use a shielded rotary variable air-condenser, giving an infinitely fine gradation of capacitance below the least mica unit. The general properties of mica and air condensers are fully treated in *A.C. Bridge Methods.*

CHAPTER XI

A.C. POTENTIOMETERS

1. **Introductory.** The d.c. potentiometer is an invaluable instrument for the accurate measurement of direct currents and voltages in all classes of testing, and its construction, principle and application will be familiar to the reader. Voltages up to about 1·5 volts are directly compared by the potentiometer with the voltage of a standard cell; higher voltages are measured with the aid of a potential divider or "volt-box," and currents by testing the volt-drop across a suitable four-terminal resistor through which the current to be measured is passed.

The convenience, accuracy and long range of a d.c. potentiometer have led numerous workers to design various types of a.c. potentiometer possessing similar qualities. Some of these are now only of historical interest, but others are commercial articles and are very widely used in general a.c. laboratory testing. Such an a.c. potentiometer must contain some transfer device by which the readings may be standardized in international volts by reference to a standard cell; the commonest devices in use are either dynamometer instruments or thermo-junctions.

A.C. potentiometers are very useful for testing instrument transformers, since they can be used to compare in magnitude and phase the primary and secondary voltages of a voltage transformer or the corresponding currents in a current transformer; the technique of the method is mentioned in Parts 3 and 4. It is not our intention to enter into a detailed discussion of a.c. potentiometers and their uses since this has been adequately done elsewhere;* we shall confine our attention to a few of the principal types, referring the reader to the authorities mentioned and to various original papers for further details.

Since a.c. quantities are vectors, an a.c. potentiometer must have means for measuring the size of a vector and for locating

* C. V. Drysdale, "Progress in the design and construction of electrical instruments," *Journal Sci. Insts.*, vol. 4, pp. 288–299 (1927); "Alternating current potentiometers and their applications," *Journal I.E.E.*, vol. 68, pp. 339–360 (1930). See also D. C. Gall, *Alternating current potentiometer measurements* (in the press), and Campbell and Childs, loc. cit. *ante*.

its position. The instruments are of two kinds: polar potentiometers, which measure the voltage in magnitude and phase in the form $r/\underline{\theta}$; and rectangular co-ordinate potentiometers, which measure the components, $x = r\cos\theta$ and $y = r\sin\theta$, of the voltage. Representative instruments of both kinds will now be considered.

2. **Polar potentiometers.** The best known polar potentiometer is that due to C. V. Drysdale,* manufactured by H.

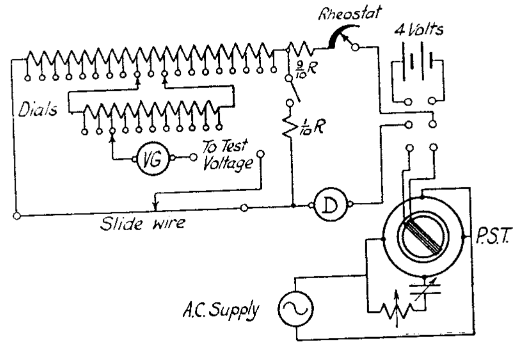

FIG. 196. DIAGRAM OF DRYSDALE'S A.C. POTENTIOMETER

Tinsley and Co., the general principle being illustrated in Fig. 196. The potentiometer has two dials on the well-known Kelvin-Varley slide principle. The first dial has eighteen coils of 2 ohms each, which with a standard current of 50 mA gives a p.d. of 0·1 volt per coil. Two brushes move over this dial, bridging two coils, and are connected to ten coils of 0·4 ohms each; the parallel combination has a resistance equal to that of one coil in the first dial and the ten steps thus drop 0·01 volt each. A slide wire of 0·23 ohm with a 115 division scale gives a drop of 0·001 volt per division; by shunting the potentiometer with a resistance $R/10$ and adding a resistance $9R/10$ in series with it to restore the total resistance again to R, where R is the potentiometer resistance, the current in the coils and slide wire is reduced to 5 mA and voltages down to 0·0001 can be measured on the wire.

The potentiometer is supplied from the secondary of a

* C. V. Drysdale, "The use of the potentiometer on alternate current circuits," *Proc. Phys. Soc.*, vol. 21, pp. 561–572 (1910); *Elecn.*, vol. 63, pp. 8–9 (1909). "The use of the alternate current potentiometer for measurements on telegraph and telephone circuits," *Elecn.*, vol. 71, pp. 687–690 (1913). "The alternate current potentiometer as a standard instrument," *Elecn.*, vol. 75, pp. 157–160 (1915).

phase-shifting transformer resembling a small, carefully designed two-phase induction regulator. The stator has a laminated core with two windings distributed sinusoidally, so that the air-gap field is as uniform as possible; the windings are displaced by 90° round the periphery. One winding is supplied directly from the 100 volt a.c. source which also feeds the circuit which is to be tested; the second winding is joined to the supply in series with a resistance and condenser, the values of which can be selected to make the currents taken by the windings equal in magnitude but differing in phase by a quarter-period, so that a uniform rotating field is set up in the air-gap. The rotor core is also provided with two windings, one of which is connected by flexible leads to the potentiometer; the second winding is joined to a circuit having a resistance and inductance equal to that of the potentiometer and thus ensures that the armature reaction of the rotor is the same in all positions. By means of a worm-wheel the rotor can be turned within the stator and the phase of the voltage applied to the potentiometer shifted by any desired angle, which can be read upon a scale. The voltage to be measured is balanced by setting the dials and rotating the phase-shifter, balance being indicated by a tuned vibration galvanometer.

By means of a change-over switch the phase-shifter is replaced by a 4-volt battery and the vibration galvanometer by a moving-coil galvanometer; by regulating the rheostats a sensitive balance can be secured against a standard cell. A sensitive dynamometer D should then indicate 50 mA, and the pointer can be brought to this mark on the scale of D by means of its zero adjuster. On changing over to the a.c. supply the rheostats are adjusted until D again reads 50 mA, thus standardizing the instrument ready for use.

3. **Rectangular co-ordinate potentiometers.** The earliest, and in many ways the simplest, co-ordinate potentiometer was introduced in 1910 by Larsen,* and is an application of the mutual inductance-resistance pair used a few years earlier by Campbell for the testing of impure mutual inductances. As shown in Fig. 197, a resistor and the primary of a variable mutual inductor are joined in series and supplied with an alternating current i (r.m.s. value I). By varying M and the position of the slider on R the test voltage may be balanced by

* A. Larsen, "Der komplexe Kompensator, ein Apparat zur messung von Wechselstromen durch Kompensation," *Elekt. Zeits.*, vol. 31, pp. 1039–1041 (1910); *Elecn.*, vol. 66, p. 736 (1911).

the voltage $(R + j\omega M)i$, which has the components RI and ωMI respectively. The current I must be maintained constant and accurately known; in addition the frequency must be known.

The disadvantage of Larsen's instrument is that the voltage calibration of M changes in proportion to the frequency. This

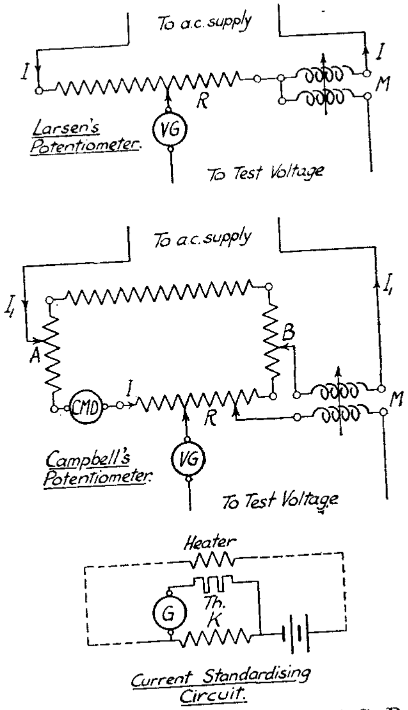

FIG. 197. DIAGRAMS OF CAMPBELL-LARSEN A.C. POTENTIOMETER

defect is overcome in the instrument designed by Campbell* and made by the Cambridge Instrument Co., by the simple device of arranging the current through the primary of the mutual inductor to be inversely proportional to the frequency; then ωI is constant and the e.m.f. proportional to M only. Fig. 197 shows the arrangement employed. The in-phase component is taken from the sliders tapping the resistor R, which is joined in series with a current-measuring device

* A. Campbell, "A new alternating current potentiometer of Larsen type," *Proc. Phys. Soc.*, vol. 41, pp. 94–97 (1928).

CMD described below; a loop shunt of total resistance $S+T$ completes the arrangement. Let S be the resistance of the upper portion of the loop between the sliders A and B, and let T be the resistance of the lower portion between the same points. Then at balance the measured voltage will be

$$v = Ri + j\omega M i_1$$

where $$i_1 = i(S+T)/S,$$

so that $$v = \{R + j\omega M[(S+T)/S]\}i$$

If the sliders A and B are now set so that $S/(S+T) = \omega/b$,

$$v = (R + jbM)i$$

which is independent of frequency; the constant b is chosen to make the inductor part read the same units as the resistor part. The current I is 10 mA and is standardized in the following way. A thermopile Th in Fig. 197 is joined in series with a galvanometer across a low resistance K; a heater-resistance and battery complete the circuit. The shunt K can be chosen so that with 10 mA in the heater the voltage of the thermopile balances the drop across K, as indicated by the galvanometer reading zero. The heater is now replaced by an equal resistance, to keep the direct current in K at the same value as before. By putting the heater in the a.c. circuit at CMD and adjusting the current through it until balance is again indicated, the a.c. is standardized at 10 mA and can be held at this value by keeping the galvanometer deflection to zero.

In the actual instrument the in-phase component is read from a potentiometer containing two dials and a circular slide wire; the range is up to 1·8 volts and can be read to 0·00001 volt. The same case contains the frequency-setting and current-standardizing devices, d.c. galvanometer, selector switches and rheostats. A reversing switch indicates automatically the quadrant in which the measured phase-angle lies. The quadrature component is read on a separate mutual inductometer with a scale of constant percentage accuracy; it gives a range up to 1 volt readable to about 10μV.

Other types of a.c. potentiometer based on the Larsen principle have been designed by other workers, notably by Geyger and by Gall, but in which the mutual inductance has a fixed value instead of being variable, the quadrature component being taken from a second set of dials similar to those used for the in-phase component.

The principle of Geyger's* instrument is shown by Fig. 198. A potentiometer connected direct to the a.c. supply gives the in-phase component; the quadrature component is derived from a second similar potentiometer coupled to the first by a fixed mutual inductance M. By connecting the mid-points of the two potentiometers it is possible to explore all four quadrants with the aid of the sliders. Since the secondary of the mutual inductor is loaded, the primary and secondary currents will not be in exact quadrature; by careful design of the inductor and the provision of compensating devices Geyger has succeeded in reducing the difference from quadrature to a negligible amount and in producing a reliable instrument, which is manufactured by Hartmann & Braun and widely used in Germany.

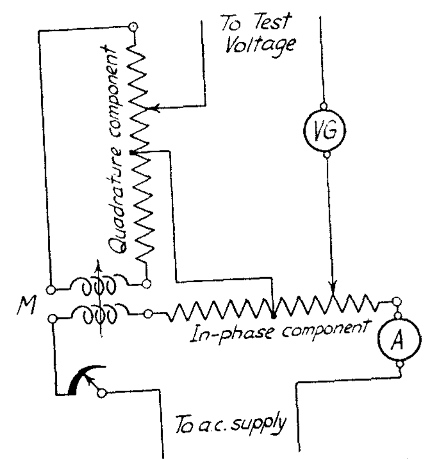

FIG. 198. DIAGRAM OF GEYGER'S A.C. POTENTIOMETER

Gall's† potentiometer is illustrated in principle by Fig. 199. Two separate units are used, supplied with currents in exact quadrature either from a two-phase alternator or from a single-phase supply with a phase-splitting device. The in-phase potentiometer is first standardized on d.c. with the aid of a

* W. Geyger, "Ein neuer Wechselstromkompensator," *Arch. f. Elekt.*, vol. 13, pp. 80–82 (1924); "Ein einfacher Wechselstromkompensator," *Elekt. Zeits.*, vol. 45, pp. 1348–1349 (1924); "Wechselstromkompensationsmessungen mit Lufttransformatoren," *Arch. f. Elekt.*, vol. 14, pp. 560–567 (1925); "Ueber die Verwendung Sekundärbelasteter Lufttransformatoren bei Wechselstromkompensationsmessungen," *ibid.*, vol. 15, pp. 174–189 (1925); "Messungen mit dem Schleifdraht-Wechselstromkompensator," *ibid.*, vol. 17, pp. 213–241 (1926); "Messung der höheren Harmonischen nach der Kompensationsmethode," *ibid.*, vol. 18, pp. 629–641 (1927); "Messung der Wechselstromkomponente von Gleichrichterströmen nach der Kompensationsmethode," *ibid.*, vol. 18, pp. 641–652 (1927); "Die Anwendung des Komplexen Wechselstromkompensators bei geoelektrischen Untersuchungen," *ibid.*, vol. 23, pp. 109–118 (1929); "Ein komplexer Wechselstromkompensator für höhere Spannungen," *ibid.*, vol. 23, pp. 447–458 (1930).

† D. C. Gall, "A new a.c. potentiometer," *Elecn.*, vol. 90, pp. 360–361 (1923); "A portable a.c. potentiometer," *Journal Sci. Insts.*, vol. 3, pp. 202–204 (1926). T. Spooner, "Some applications of the a.c. potentiometer," *ibid.*, vol. 3, pp. 214–221 (1926).

standard cell, a sensitive reflecting dynamometer *RD* indicating the potentiometer current. On switching over to a.c. the rheostats are adjusted so that the dynamometer reading is the same as before. To standardize the quadrature potentiometer it is coupled to the in-phase potentiometer by a mutual inductance, so that their voltages are then in opposition. The value of M is such that balance can be obtained with the correct primary current in it at a given frequency for some chosen

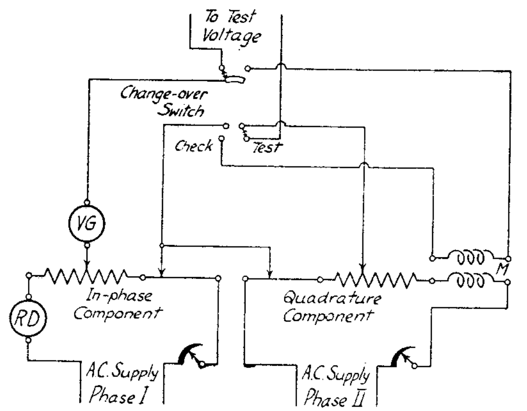

FIG. 199. DIAGRAM OF GALL'S A.C. POTENTIOMETER

point on the scale of the potentiometers, e.g. 0·5 volt at 50 cycles per second; the rheostats of the quadrature potentiometer are adjusted until this balance is secured, when the currents in the two units will be in quadrature and equal in magnitude.*

The two potentiometers are contained in a case, each consisting of a dial with eighteen coils of 2 ohms each and a circular slide wire of about 2·4 ohms subdivided to read 0·001 volt. Reversing switches are provided on each potentiometer so that the four quadrants may be tested. The case also contains all the necessary selector switches, terminals, the mutual inductance, etc. Messrs. Tinsley & Co. manufacture the instrument in its normal form and also in a portable pattern useful for track-circuit testing, geophysical work and the like.

* Several interesting modifications of Gall's potentiometer have been made in Japan; *see* M. Doté, "On the two-phase source for the co-ordinate potentiometer," *Journal I.E.E. Japan*, No. 465, pp. 434–439 (1927); M. Sase and T. Muto, "On a new a.c. potentiometer," *Journal I.E.E. Japan*, No. 466, pp. 489–500 (1927); *Proc. Physico-Math. Soc. Japan*, vol. 12, pp. 32–37 (1930).

CHAPTER XII

THE MAGNETIC POTENTIOMETER

1. Theory. The magnetic potentiometer has received some application in instrument transformer testing for the measurement of exciting ampere-turns and the determination of winding turn ratios. Consequently, a brief discussion of its theory and construction will be given here; its uses are explained on pp. 416 and 419. The instrument was introduced by Chattock* as far back as 1888 and independently discovered by Rogowski† in 1913; it is not nearly so well known as its usefulness warrants.

In Fig. 200 let A, B be two points in a magnetic field and let l be the length of any path joining them. At any point P let H be the component of magnetic force tangential to the path; then, by definition, the line integral of H from A to B is the difference of magnetic potential $\mathscr{M}$ between the points, i.e.

$$\mathscr{M} = \int_A^B H dl.$$

Now let the path be the axis of a tube of small but constant sectional area a, so that A and B lie in equal small plane areas; if $\overline{\mathscr{M}}$ is the average magnetic p.d. between these surfaces,

$$\overline{\mathscr{M}} = \frac{1}{a}\int \mathscr{M} da = \frac{1}{a}\iint H dl \, . \, da = \frac{1}{a}\int H dv$$

where dv is an element of volume of the tube. Let the tube be wound with a uniform helix with n turns per cm. and let the

* A. P. Chattock, "On a magnetic potentiometer," *Proc. Phys. Soc.*, vol. 9, pp. 23–26 (1888). A. Campbell and D. W. Dye, "The magnetic testing of bars of straight or curved form," *Journal I.E.E.*, vol. 54, pp. 35–46 (1916).

† W. Rogowski and W. Steinhaus, "Die Messung der magnetischen Spannung (Messung des Linienintegrals der magnetischen Feldstarke)," *Arch. f. Elekt.*, vol. 1, pp. 141–150 (1913); W. Rogowski, "Ueber einige Anwendungen des magnetischen Spannungsmessers," *ibid.*, vol. 1, pp. 511–527 (1913); F. Goltze, "Ueber die Verwendung des magnetischen Spannungsmesser bei der Prüfung der magnetischen Eigenschaften des Eisens," *ibid.*, vol. 2, pp. 303–313 (1914); W. Rogowski, "Die Messung des elektromagnetischen (Poyntingschen) Energieflusses (Messung des lokalen Eisenverlustes)," *ibid.*, vol. 4, pp. 279–293 (1916); W. Wolman, "Ueber ein Verfahren der Eisenprüfung mit dem magnetischen Spannungsmesser. Messung an Eiseneinkristallen," *ibid.*, vol. 19, pp. 385–404 (1928). L. and P. Lombardi, "Misura delle dissipazioni locali di energia entro una parte circoscritta del circuito magnetico," *L'Elettro*, vol. 16, pp. 142–146 (1929).

field vary with time; then the e.m.f. induced in the helix is, in e.m.u.,

$$e = -\frac{d}{dt}\iint Hda \, . \, ndl = -\frac{d}{dt} \, . \, n\int Hdv = -na\frac{d\mathcal{M}}{dt}$$

Thus the e.m.f. is proportional to the rate of change of magnetic p.d. between the ends of the helix, and to this alone if external inductive effects are guarded against by winding the helix

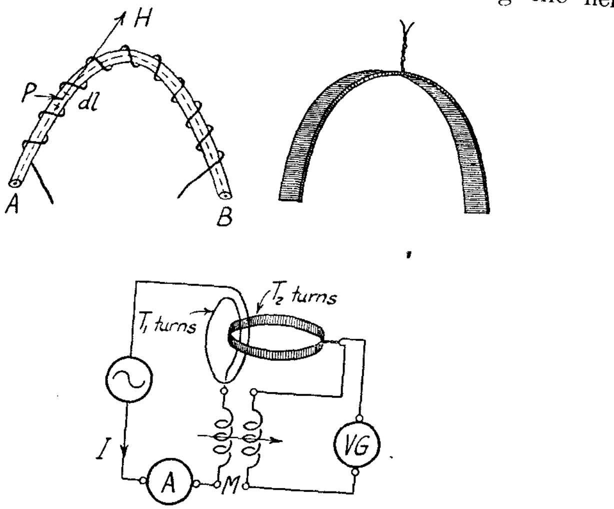

FIG. 200. THE MAGNETIC POTENTIOMETER

with an even number of layers of wire. If the change in $\mathcal{M}$ is due to alteration of a steady current from one value to another, or to the motion of permanent magnets, then by connecting the ends of the helix to a ballistic galvanometer a reading will be obtained which is proportional to the change that has been made in the magnetic p.d. between A and B. If the field is steady and the end B is suddenly moved from B to another point B' while A is fixed, then the galvanometer gives a reading proportional to the magnetic p.d. between B and B'. Such uses of the instrument were described by Chattock.

It is to Rogowski, however, that the application to alternating fields is due. By connecting the coil to a voltmeter or, better still, to some form of a.c. potentiometer device, the r.m.s. value of e can be measured. But $\mathcal{M}$ is, by definition, $4\pi/10$ times the ampere-turns impressed on the field between

A and B; thus the instrument can be used to explore the distribution of ampere-turns over the various parts of a magnetic circuit. It has been used by Rogowski and his associates in the investigation of leakage phenomena in magnetic testing apparatus (the Epstein square in particular) and in electrical machinery. In the same way, Alberti and Vieweg have used it to measure the exciting ampere-turns in a current transformer, their method being described on p. 416. To increase the sensitiveness these workers provided the instrument with an iron core; while this vitiates the simple theory it is quite easy to calibrate the instrument to take into account the saturation of the core.

2. **Construction and calibration.** Chattock's original instrument consisted of a piece of solid rubber cord 1 cm. in diameter and 37 cm. long uniformly wound with a double layer of insulated wire with $n = 20$ turns per cm. To ensure the cord remaining of constant section as the flexible potentiometer is moved in the field, the turns are separated by cotton thread. He suggested that a core of flexible gas tubing made of plaited and varnished canvas would be better; it could easily be wound in the lathe with great regularity by mounting the tube on a suitable mandrel.* Rogowski's potentiometer is a press-spahn strip 60 cm. long, 2·5 cm. wide, and 0·1 cm. thick wound with a double layer of 0·2 mm. diameter wire. This form is particularly suitable for work on transformers and similar magnetic circuits.

It is essential that the sectional area a and the number of turns per centimetre n be constant. Also, in order to avoid trouble due to stray fields the ends of the coil should be at the middle of the length of the potentiometer and there attached to twin flexible leads. The simplest way to secure this result is to start the winding at the middle and wind to one end; then to return in a second layer to the middle, proceed thence to the other end, and then return to the middle in a second layer.

The constant na can be easily found by the method shown in Fig. 200. A coil with T_1 turns and the primary of a variable mutual inductor are joined in series to an a.c. supply; the secondary of the mutual is connected in series with the magnetic potentiometer and a vibration galvanometer. By varying M it is possible to balance the mutual inductance between T_1

* For particulars of a number of rubber cord potentiometers see J. E. Shaw, "Design and behaviour of magnetic potentiometers," *Elec. Rev.*, vol. 102, pp. 678–680 (1928).

and the potentiometer when the latter is linked through the coil and its ends are brought into contact. Remembering that a uniform toroid makes no external field, the mutual inductance between T_1 and T_2 will be

$$4\pi(T_1/l)aT_2 10^{-9} \text{ henry.}$$

Equating this to the balancing value of M gives

$$na = T_2 a/l = 10^9 M/4\pi T_1.$$

The ampere-turns calibration is easily found; let the primary current be I. The ampere-turns with which the instrument is linked is IT_1 and the voltage induced in the potentiometer is ωMI so that the voltage per ampere-turn is $\omega M/T_1$ at the given frequency.

The use of the magnetic potentiometer to determine the number of turns in a coil, first by linking it through the unknown coil and then through a known coil joined in series, a balance being obtained in each case, will be further referred to on p. 419.

CHAPTER XIII

STANDARD TRANSFORMERS

1. General. In the routine testing of instrument transformers it is common practice to employ a relative method, the characteristics of the transformer under test being compared with those of a standard transformer by a differential process. The ratio error and phase-angle of the standard transformer are themselves determined by an absolute method and are frequently retested.

A transformer, whether of voltage or current type, that is to be used as a reference standard of ratio and phase-angle must be chosen for the permanence of these quantities and for the smallness of their magnitude. This necessitates great care in the construction of the transformer, particularly of its magnetic circuit. In addition the transformer should be carefully used, always with its proper secondary burden, and frequently re-calibrated; the core should never be magnetized by direct current in the windings and the secondary should not be opened when the primary is connected to the supply. It is found in practice that with proper attention to its choice and maintenance a standard transformer is a very permanent and reliable piece of apparatus.

Standard voltage transformers do not present any serious difficulties and are widely used in works testing of transformers up to about 20 kV. In order to cover a wide range with the minimum of apparatus, multi-range transformers are usually provided, generally in a portable form. Numerous examples are described on p. 236, to which the reader is referred. Current transformers, however, warrant further discussion.

2. Current transformers. Standard current transformers are required for two purposes, first, in relative methods to serve as reference standards and second, in absolute methods to eliminate the use of high-current resistors. For both purposes multi-range portable transformers are desirable and these usually take the form of ring cores with toroidal secondary windings, the primary being a cable looped through the ring a suitable number of times. Suitable transformers are made by many makers (see p. 143) and are very reliable, especially since the adoption of nickel-iron cores for high grade transformers.

For very large currents the ring-cored transformer with cable primary is exclusively used. At the National Physical Laboratory a transformer for 5 000/5 with one primary turn has an accuracy within 0·01 per cent for ratio and 0·2 minute for phase-angle, the core being of mumetal (see p. 119). With larger currents the position of the primary turns may have a serious influence on the characteristics. To avoid this trouble Arnold has devised a leakage shield and has applied it to 10 000/5 and 20 000/5 mumetal ring-cored transformers, giving a ratio error within 0·01 per cent and a phase-angle of about 0·5 minute for any position of the primary conductors (see p. 136). At the Bureau of Standards a silicon-iron cored transformer for 12 000/5 is in use; the ratio error is about 0·01 per cent but the phase-angle is from 2·5 to 8 minutes. To ensure absence of errors due to variations in the position of the primary conductors an elaborate fixed primary winding of twenty-four loops is provided, and these can be connected in various series-parallel combinations by manipulating seventy-two bolts (see p. 467). While effective for its purpose the arrangement is costly and complicated; Arnold's shield is a much simpler solution of the problem and its use in conjunction with the nickel-iron core enables phase-angles of about a tenth of the Bureau of Standards values to be obtained.

CHAPTER XIV

SOURCES OF SUPPLY

1. **Alternators and transformers.** For tests in the laboratory and test-room the best source of supply is an alternator direct-coupled to a d.c. shunt motor run from a storage battery. By this means the greatest flexibility of speed-control is possible and the frequency can be maintained very accurately constant,* an important consideration when using vibration galvanometers. In works testing the alternator is often driven by a synchronous motor fed from the local a.c. supply mains, but it is not so easy in this case to guarantee absolute constancy of speed and of the test frequency. The alternator should preferably be of polyphase construction with a winding capable of giving three-phase and two-phase supplies, thus giving a variety of single-phase voltages as well as the polyphase voltages often required by some of the methods of testing. Where a single standard method has been adopted, as in routine testing, it is sufficient for the alternator to give only the system of voltages required for the method. The wave-shape of the machine should be sinusoidal and the alternator should cover the range of frequency met with in practice, say 25 to 60 cycles per second. The output and voltage necessary cannot readily be specified and must depend upon the range of testing to be undertaken.

The supply is taken to the test-circuit through a suitable transformer. In voltage transformer testing this may conveniently be a voltage transformer similar to the one under test, but used in an inverted sense as a step-up transformer. Similarly, in current transformer testing the circuit is connected to the alternator through a step-down transformer giving 4 to 8 volts at its secondary terminals; an inverted current transformer often suffices. When the current transformer under test is the sole load on the alternator, care should be taken that there is no serious wave-distortion which would upset the characteristics of the transformer (see p. 141). If this is likely to occur, the alternator should deliver a considerably greater

* A sufficient amount of flywheel effect in the rotating parts is an important factor in maintaining constancy of speed.

output than that required for the test, consuming the excess in a series resistance so that the effect of the test transformer is swamped thereby. Regulation of the current is most conveniently achieved by use of the alternator field regulator in conjunction with resistors in the primary circuit of the step-down transformer; alternatively, if the test current is not too great, regulation can be effected by an adjustable iron-cored reactance and carbon resistances in the test circuit. The connection of the test circuit to the alternator, through a transformer, both in voltage and in current transformer testing, has the further advantages of isolating the test circuit from the alternator and in providing freedom for its appropriate earthing; this is an important factor in precise work where earth-capacitance effects may be troublesome. Individual instances are dealt with later in discussing the various test methods.

For tests of instrument transformers on site the supply must be taken from the available a.c. network, suitable transformers being interposed to bring the voltage to a value convenient for use in the test circuit. In current transformer testing it is often impossible to remove the transformers from their circuits; they must in such cases be tested while normally connected, using as the testing current that flowing to the load on the supply network.

2. **Phase-shifting devices.** In many methods it is necessary not only to supply the transformer to be tested but also to apply to certain parts of the test circuit auxiliary voltages bearing a known adjustable phase-relationship to the voltage of the main supply. The ideal device for this purpose is that frequently used for the testing of meters by the method of artificial loading, namely, a pair of similar alternators coupled directly to one another and to a common d.c. driving motor, Fig. 201. The stator of one alternator is mounted in bearings so that it may be moved around the axis of the machine by a worm gear. By this means the voltages induced in the armatures of the two alternators can be adjusted relatively in phase, without change of magnitude, by any desired electrical angle, the amount of which is at once obtainable from the observed relative angular displacement of their stator frames and the number of poles in the machines. This angle can be measured by a suitable scale and vernier attached to the movable stator, or for the precise measurement of small angular movements a mirror and scale device is easily applied. The worm gear can be operated by hand; where the machines are at some distance

from the test circuit a small motor drive provides a ready means of remote control. The double alternator is expensive

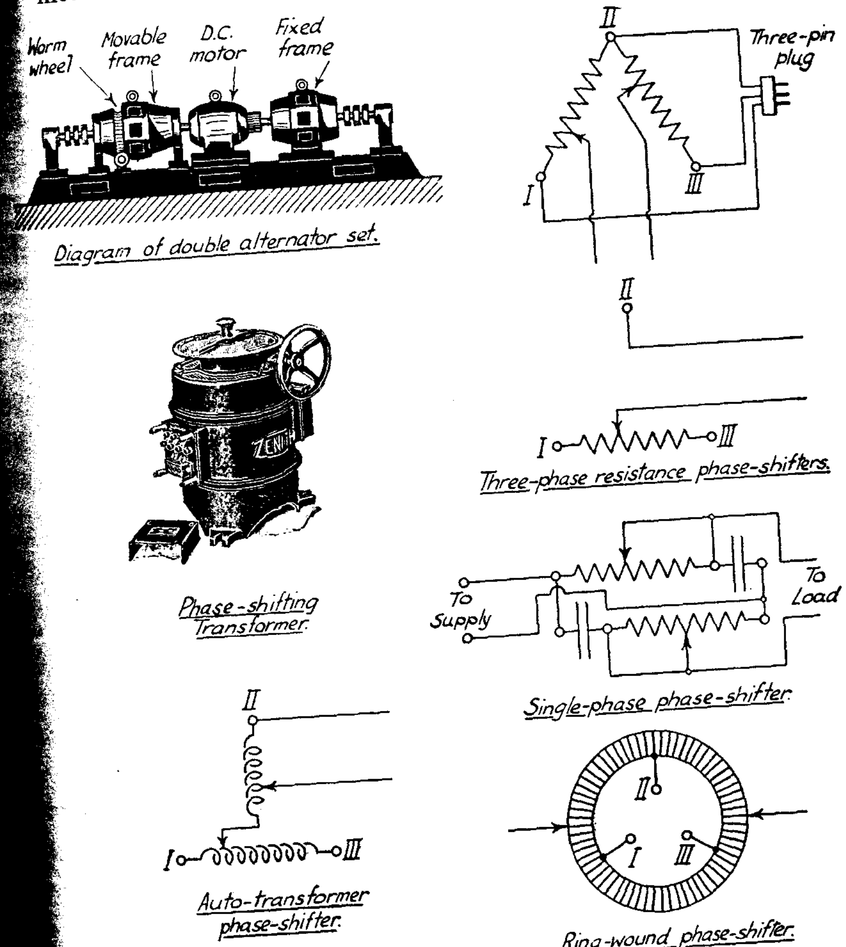

FIG. 201. PHASE-SHIFTING DEVICES

and not portable, but its flexibility makes it a most desirable feature of a well-equipped laboratory.

Another excellent method of obtaining an auxiliary voltage with any desired phase relationship is provided by the phase-shifting transformer perfected for purposes of a.c. testing by

Dr. Drysdale;* this apparatus is portable and can be used not only in the test-room but also on site. In principle it is identical with an induction regulator and resembles an induction motor in the arrangement of its parts. A somewhat similar, but considerably smaller device, is used in Drysdale's a.c. potentiometer (see p. 382). The stator is wound with a polyphase winding, which can be connected directly to the main source of supply if this be polyphase, or through a phase-splitting circuit for use with single-phase sources; the currents flowing in this winding set up a rotating field in the air-gap of the machine. The rotor is provided with a single-phase winding and can be moved by a worm gear about its axis in the stator bore. The phase of the voltage induced in the rotor winding by the field can then be adjusted to any position relative to the main supply without the magnitude of the voltage being altered; the angle is read upon a suitable scale. A sinusoidal secondary voltage is secured by careful distribution of the stator and rotor windings. The phase-shifting transformer is much cheaper than the double alternator and is widely used.

The example illustrated in Fig. 201 is manufactured by the Zenith Co. It is wound for a three-phase supply up to 460 volts and has an output of 600 volt-amperes, which is ample for all usual testing purposes.

Cases often arise, especially in testing on site, where a much simpler device than either of those mentioned above is quite satisfactory. If a three-phase supply is available, two slider resistances† connected as shown in Fig. 201 provide a ready means of varying the phase of the p.d. between the sliders without changing its magnitude. With the arrangement shown a total range of 120° can be obtained; this can be extended to 360° by changing the sequence of phases by manipulating the plug in its socket. An even simpler arrangement uses a single slider resistance joined across one pair of lines, the voltage being taken between the third line and the slider. The change of phase is 60°, but the magnitude of the voltage is not constant. To overcome this latter defect Sharp and Crawford‡ use two auto-transformers. These consist of toroidally-wound iron rings; the insulation is removed from the wires on the end surface and an arm pivoted at the centre of the ring makes

* C. V. Drysdale, "The use of a phase-shifting transformer for wattmeter and supply meter testing," *Elecn.*, vol. 62, pp. 341–343 (1909). Also see, S. Numakura and H. Fujiki, *Res. Elect. Lab. Tokyo*, No. 306, pp. 1–69 (1931).

† G. W. Stubbings, *Elec. Rev.*, vol. 94, pp. 604–606 (1924).

‡ C. H. Sharp and W. W. Crawford, *Trans. Amer. I.E.E.*, vol. 29, pp. 1517–1541 (1911).

the necessary sliding contact; the arms on the two rings are linked together in such a way as to keep the voltage constant while its phase is varied. Another device consists of an iron ring closely wound with a layer of insulated wire like a Gramme ring. The insulation is removed from the central portion of the wires on the outer circumference of the ring; at three points 120° apart on the inner surface tappings are taken to the three lines of a three-phase supply. An arm pivoted at the centre of the ring carries two brushes which touch the bared wires on the outer surface at diametrally opposite points; as the arm is rotated the phase of the voltage tapped off by the brushes is varied, while the magnitude remains constant.

With a single-phase supply one of the simplest devices* is the circuit shown in Fig. 201. The two condensers are equal and the two resistors are linked together so that they remain equal to one another. Provided the impedance of the apparatus joined to the output terminals is high compared with that of the condenser-resistor combination, the p.d. changes very little in magnitude and a phase-shift of about 120° can be obtained.

All the above devices provide for continuous variation of the phase of the voltage; in many cases, however, shifting the phase through a definite angle is quite adequate. This is most simply done with the aid of a polyphase supply, one phase of which feeds the main circuit while the auxiliary circuit is joined to one of the other phases. Numerous methods will be described in Parts 3 and 4 in which this artifice is used.

* C. R. Cosens, *Proc. Phys. Soc.*, vol. 46, p. 821 (1934).

CHAPTER XV

INSTRUMENTS

1. **Indicating Instruments.** In all methods of transformer testing, indicating instruments such as ammeters, voltmeters, wattmeters, etc., are required for the purpose of making suitable adjustments in the circuit. These should be of the laboratory portable type, of first grade or precision accuracy, according to the purpose for which they are intended, and must be carefully recalibrated at frequent intervals. Ammeters and voltmeters may be either of the dynamometer pattern or of the modern moving-iron design with a nickel-iron movement.

Some methods make use of calibrated watthour meters. These are of the normal construction, but usually require to be specially adjusted for this particular purpose. Their use is fully described on pp. 439 and 497.

2. **Detectors.** In most test methods, as already mentioned, the secondary current or voltage is opposed to a portion of the primary current or voltage. In the deflectional methods some form of sensitive detecting instrument is used to measure the resultant obtained from the opposition of the two quantities; the instrument usually favoured is either an electrometer or a dynamometer. In the null methods the resultant is balanced out by some auxiliary compensating circuit, a suitable instrument* being used to indicate when balance has been secured. For this purpose the instruments most commonly used are the separately-excited dynamometer, the d.c. galvanometer with rectifying device, and the vibration galvanometer. Of these some are phase-selective, i.e. they can be made to respond independently to resistance and to reactance adjustments of the balancing network.

2 (*a*). The Electrometer. A modified form of quadrant electrometer, specially designed to have high sensitivity and stability, is used in high-precision methods of testing instrument transformers both at the Reichsanstalt in Berlin† and at the National Physical Laboratory at Teddington.‡ The

* For a detailed discussion of various detectors the reader is referred to *A.C. Bridge Methods.*

† E. Orlich, *Elekt. Zeits.*, vol. 30, p. 435 (1909); also H. Schultze, *Zeits. f. Inst.*, vol. 27, p. 65 (1907).

‡ C. C. Paterson, E. H. Rayner and A. Kinnes, *Journal I.E.E.*, vol. 51, p. 294 (1913).

methods are discussed on pp. 440 and 541. Owing to its special construction and to the difficulties attending its set-up and use, the electrometer has not been much adopted for this purpose elsewhere than in these national standardizing laboratories.

2 (*b*). The Separately-excited Dynamometer. This instrument is the same in general construction as an ordinary dynamometer wattmeter with fixed current coils and a moving fine-wire system; indeed for some of the rather less refined tests a good wattmeter serves admirably. To attain sufficient sensitivity for laboratory use, however, the moving system is suspended and its deflections are observed by the usual mirror and scale.

If I_F be the current in the fixed coils of the instrument and I_M that in its moving coil, the torque on the latter is proportional to $I_F\, I_M \cos\phi$, if ϕ is the phase-displacement between these currents; with a uniform suspension the deflection is proportional to the torque. In using the dynamometer, I_F may be the current to be measured, while I_M is supplied from a phase-shifter; the choice will depend on the magnitudes involved, since the moving coil can only carry a small current. By adjusting the phase-shifter until the deflection is zero, I_F and I_M are put in quadrature; then when the phase is changed by 90° the resulting maximum deflection is a measure of I_M, assuming I_F to be constant. In order that I_M shall be in phase with the voltage applied to the moving coil it is necessary that the reactance of the coil be small in comparison with the resistance joined in series with it; or the reactance can be annulled by a shunted condenser in the way often adopted in precision wattmeters. When this adjustment is correct the dynamometer can be used to set an auxiliary *voltage* applied to the moving system in any desired phase relation to the current flowing in the fixed coils.

When used in null tests the moving coil is usually connected to the network and the fixed coil to the phase shifter. The balancing network must be adjusted until the dynamometer reads zero whether ϕ is 0° or 90°; this can only occur when either I_F or I_M is zero. During the balancing process the instrument will be sensitive to resistance changes at one phase position and to reactance changes when the phase is altered by 90°; hence the deflection of the instrument indicates independently the amount and direction of each adjustment to be made to attain balance.

2 (*c*). D.C. Galvanometer with Rectifier. This detector

consists of some type of rectifier used to rectify the current to be measured, after which it is passed through a d.c. galvanometer; by this means the advantage of the very high sensitiveness of the galvanometer is made available in a.c. testing.

In its simplest form the rectifier is a commutator driven synchronously with the a.c. supply, but it is apt to give rise to considerable trouble in practice owing to the difficulty of securing steadiness of the brush contacts on the commutator, to the development of thermal e.m.f., and so on. These troubles are to a large extent overcome in Sharp and Crawford's*

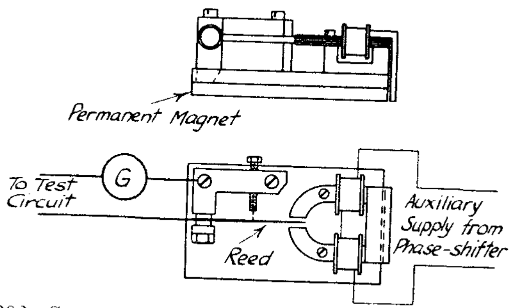

FIG. 202. SIEMENS & HALSKE VIBRATING TONGUE RECTIFIER

cam-operated rectifying key in which rubbing contacts are abolished. In mechanical rectifiers of these types the phase-selectivity is obtained by arranging for rectification to begin at different points in the cycle; this can be done by changing the setting of the brushes around the commutator or by moving the key relative to the cam, as the case may be.

A type of mechanical rectifier that has recently been applied to a very wide range of a.c. measurements,† especially in portable apparatus, is the vibrating tongue device illustrated in Fig. 202; its arrangement will be sufficiently clear from the diagram. An example of its use in transformer testing will be found on p. 495.

Of non-mechanical rectifiers the vacuum thermo-junction, the copper-copper oxide element and the thermionic valve may be cited. Applications of the former have been discussed by

* F. Bedell, *Journal Frank. Inst.*, vol. 176, p. 385 (1913); C. H. Sharp and W. W. Crawford, loc. cit.

† H. Pfannenmüller, "Überblick über die Messverfahren mit Gleichrichtern," *Arch. f. Elekt.*, vol. 28, pp. 356–384 (1934).

Pfannenmüller and others (loc. cit.). A very useful form of phase-selective valve galvanometer, having a sensitivity comparable with that of a vibration galvanometer and using only a pointer instrument, has recently been described by Cosens and is made by the Cambridge Instrument Co. So far only the thermo-junction has been applied to transformer testing, see p. 484.

2 (*d*). THE VIBRATION GALVANOMETER. The vibration galvanometer is the detector most frequently used in null methods of transformer testing. It is particularly well suited for this work since its sensitivity is very high at industrial frequencies. The vibration galvanometer is not phase-selective, nor does the deflection of the instrument give any indication to the operator of the nature and direction of adjustments to be made to secure balance. These disadvantages are much less important than might be supposed; a very little experience with the instrument and a systematic use of the circuit adjustments enables a balance to be rapidly obtained.

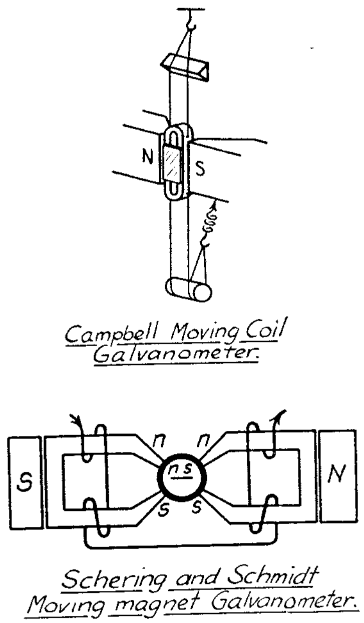

FIG. 203. DIAGRAMMATIC ARRANGEMENTS OF MOVING IRON AND MOVING COIL VIBRATION GALVANOMETERS

Briefly, a vibration galvanometer consists of a moving system which is set into forced vibration by the action upon it of the alternating current to be measured; high sensitivity is secured by tuning the moving system to be in resonance with the frequency of the current, when the resulting deflection will become a maximum. Two types of galvanometer are in general use, the moving-iron type originally introduced by Wien and the moving-coil type due to Campbell. The construction, theory and use of both types are fully discussed in Chap. III of *A.C. Bridge Methods*, to which the reader is referred.

Fig. 203 shows a diagrammatic plan of the Schering & Schmidt moving-iron instrument in which the moving system consists of a small piece of soft iron suspended in the polarizing field of an electromagnet, *NS*, excited by direct current. The alternating current flows in four small coils mounted on laminated U-shaped cores. The a.c. produces an alternating transverse distortion of the polarizing field, setting the moving part into oscillation. Tuning is effected by adjusting the strength of the polarizing field; this control can be effected from a distance, which is a great advantage in working with h.v. circuits. The a.c. windings and their cores are carried on an ebonite plate and an air-gap is left between the cores and the d.c. magnet; thus the a.c. system is very highly insulated and may safely attain quite high potentials. Damping of the moving iron can be adjusted by means of a movable copper block, which can be placed nearer to or farther from the "needle" as desired.

Fig. 203 also shows the essential principle of the Campbell moving-coil vibration galvanometer. The instrument consists of a small light coil of a few turns mounted upon a bifilar suspension which can be tuned to resonance by adjustment of its length and tension. The coil is in the field of a permanent magnet and is set into vibration when an alternating current is passed through it. The amplitude of the oscillation is observed by reflecting a beam of light to a distant scale from a small mirror carried by the coil. Since the inherent damping is very small the resonance curve is exceedingly sharp.

Since the vibration galvanometer is a tuned instrument the frequency of the a.c. supply must be steady if high sensitivity is to be maintained. In the laboratory this can easily be done by using an alternator driven by a motor supplied from a battery; in very precise work it may also be necessary to add some form of automatic speed regulator. In works testing it is often impossible to get a constant frequency supply, but the effect of speed fluctuations on the tuning can be much reduced if the damping of the galvanometer be increased. This broadens the peak of the resonance curve and makes the loss of sensitiveness with small changes of frequency much less important. As a rule the available sensitivity is ample for all practical purposes and is little changed by small additions to the damping. The desired increased damping is obtained in moving-coil instruments by a suitable shunt across the terminals; in moving-magnet galvanometers the damping is introduced internally by a copper block or shield.

CHAPTER XVI

BURDENS

1. General. The burden of an instrument transformer is the total external load connected to the secondary terminals; as has been shown on p. 141 the amount and nature of the burden have a great influence upon the ratio error and phase-angle of the transformer. Consequently, when transformers are tested they must be loaded with a burden equivalent to that with which they will be used under normal working conditions. All methods of testing involve the inclusion of some part of the testing apparatus in the secondary circuit. In some methods the burden so imposed is very small and can be neglected in comparison with the actual instrument which forms the normal load of the transformer. In other methods this is by no means the case and the transformer must then be tested with an artificial burden composed of resistors and inductors combined to give in conjunction with the testing apparatus an impedance equal to that of the burden with which the transformer is to operate. When estimating the burden on a transformer* it is necessary to take into account the leads connecting it to its secondary instrument; these leads are often quite long and represent an appreciable load (see p. 142) which may have the effect of greatly increasing the phase-error of the transformer.

2. **National standards.** The rated burden of a transformer is the total external load expressed either in volt-amperes or as an impedance in ohms when the secondary circuit supplies its rated current in the case of a current transformer, or its rated voltage in the case of a voltage transformer. With the rated burden connected to the transformer the ratio error and phase-angle must not exceed certain limits which are stated in the national specifications for transformers intended for different purposes; these limits have been fully discussed on p. 16 *et seq.* and need not be further mentioned here.

2 (*a*). CURRENT TRANSFORMERS. The International Electrotechnical Commission (I.E.C.) recommends that the rated burden with a 5 ampere rated secondary current should not be less than 5 VA for all grades of accuracy; this is equivalent to

* For a discussion of the choice of burdens see T. A. Hammond, "Calculation of instrument transformer burdens," *Gen. Elec. Rev.*, vol. 34, pp. 115–126 (1931).

an impedance of 0·2 ohm and it is further recommended that the power-factor be 0·8. In many cases the instrument with which a current transformer is loaded does not exceed 2 or 3 VA; by specifying a minimum of 5 VA something is allowed for connecting-leads and also for the fact that few test methods impose a burden of less than 5 VA. The I.E.C. further state that the limits of accuracy laid down for Class 0·5 and 1 shall apply also when the burden has any value between 25 and 100 per cent of its rated value. The following table gives particulars of the rated burdens as specified in the national standards of several countries.

Specification	Class	Purpose	Rated Burden VA.	Rated Burden Ohms	Power Factor of Burden	Limits of Error to hold for change of Burden between
Great Britain	A	Laboratory	7·5		1·0	
	B	Metering	5, 15		1·0	
	C	Switchboard	5, 15, 40		1·0	
	D	Relay / Ammeter	≯ 40		1·0	
Germany	0·2	High Precision	5, 15, 30	0·2, 0·6, 1·2	0·8	25% to 100%
	0·5	Laboratory	,, ,, ,,	,, ,, ,,	0·8	,,
	1	Metering	,, ,, ,,	,, ,, ,,	0·8	,,
	3	Ammeter	,, ,, ,,	,, ,, ,,	0·8	50% to 100%
	10	Relay	,, ,, ,,	,, ,, ,,	0·8	,,
France	All		from 15 VA.		not specified	
U.S.A.		Meter	2·5, 15, 50		0·9, 0·9, 0·5	

2 (*b*) VOLTAGE TRANSFORMERS. The I.E.C. recommendation for voltage transformers is a burden of not less than 10 VA at the rated secondary voltage, the power-factor being 0·8. The specifications of the various countries are as follows—

		Single-phase VA	*Three-phase VA/phase*	
Great Britain	Class A	10	—	$\cos\phi = 1$
	,, B	15, 50, 100, 200	25, 50, 100	
	,, C	,,	,,	
	,, D	—	—	
Germany	Class 0·2	5, 15, 30, 60 VA.		$\cos\phi = 0{\cdot}8$
	,, 0·5	15, 30, 60 VA.		
	,, 1	,,		
	,, 3	,,		

France. Single-phase, from 30 VA. Three-phase, from 30 VA/phase. $\cos\phi = 0{\cdot}5$ to 1.

U.S.A. 6 900 volts and under, 25, 50 and 200 VA. Over 6 900 volts, 200 VA.

3. **Typical instrument burdens.** The burden imposed by various types of instruments varies very widely in practice,* being least for indicating instruments and greatest for recording instruments, relays and trip coils. The following figures give typical values for different types, but considerable variation may be expected in instruments of different makes; for current circuits the rated current is 5 amperes for voltage circuits the rated voltage is 110 volts the frequency being 50 cycles per sec.

	VA	$\cos\phi$
Ammeter, Soft Iron	2 to 10	0·6
,, , Dynamometer	15	1
,, , Hot wire	1	1
Recording Ammeter, Soft iron	8 to 16	0·3 to 0·5
Voltmeter, Soft Iron	4 to 18	1
,, , Dynamometer	10	1
,, , Induction	10	0·9
Recording Voltmeter, Soft Iron	15 to 30	0·9 to 1
Frequency meter, Reed	3	0·6
Dynamometer wattmeter, current coil	3 to 5	0·7 to 1
,, ,, voltage coil	6 to 8	0·8 to 1
Power-factor meter, current coil	2 to 8	0·7 to 1
,, ,, ,, voltage coil	12	0·9
Watt-hour meter, current circuit	2	0·5
,, ,, voltage circuit	1 to 10	0·1 to 0·3
Relay, Overload	5 to 15	0·6
,, , Reverse-power, current coil	10 to 18	0·6
,, ,, ,, voltage coil	15	0·2

4. **Elementary principle of burden calculation.** The calculation of the constituents of a given burden follows the usual methods of a.c. theory, but it may be advantageous to set down briefly the principles upon which such calculations are made; detailed application to single-phase and polyphase burdens has been made by Hammond, loc. cit.

Consider first a current transformer of which the rated burden is VA volt-amperes at the rated secondary current I_{ns} and power-factor $\cos\phi$. Then

$$VA = I^2_{ns}Z,$$

which determines the total impedance Z of the burden. Its resistance and reactance are then found from

$$R = Z\cos\phi \text{ and } X = \sqrt{(Z^2 - R^2)} = Z\sin\phi\dagger$$

* For American practice see *Elect. J.*, vol. 17, p. 29 (1920). For more precise data than can be given here consult the manufacturers' catalogues.

† Note that X is positive for coils and negative for condensers.

It is then a relatively simple matter to arrange the resistance and reactance of the secondary burden to give these values. A number of air-core coils and slider resistors enable any desired burden to the readily imitated.

If several pieces of apparatus are joined in series, VA_1, VA_2, etc., being their volt-amperes and $\cos\phi_1$, $\cos\phi_2$, etc., their power factors, the resistance and reactance of each can be calculated as above. Then the total burden has an impedance.

$$Z = \sqrt{[(R_1 + R_2 + \ldots)^2 + (X_1 + X_2 + \ldots)^2]},$$

its power-factor is

$$\cos\phi = (R_1 + R_2 + \ldots)/Z$$

and the total volt-amperes

$$VA = I^2_{ns}Z = \sqrt{\{[I^2_{ns}(R_1 + R_2 + \ldots)]^2 + [I^2_{ns}(X_1 + X_2 + \ldots)]^2\}},$$

i.e. $VA = \sqrt{[(\textit{sum of active watts})^2 + (\textit{sum of reactive volt-amperes})^2]}$,

where the active watts are the components $VA_1 \cos\phi_1$, $VA_2 \cos\phi_2$, etc., and the reactive volt-amperes are $VA_1 \sin\phi_1$, $VA_2 \sin\phi_2$, etc.

In the case of a voltage transformer which is loaded at rated secondary voltage V_{ns}, the impedance is determined by

$$VA = V^2_{ns}/Z = V^2_{ns}Y$$

where Y is the admittance of the burden, the components* of which are the conductance G and susceptance B given by the relations

$$G = R/Z^2 \text{ and } B = -X/Z^2.$$

When several pieces of apparatus are joined in parallel across the secondary voltage V the total admittance is

$$Y = \sqrt{[(G_1 + G_2 + \ldots)^2 + (B_1 + B_2 + \ldots)^2]}$$

and the total volt-amperes are

$$VA = V^2_{ns}Y,$$

leading to the same rule as in the current transformer for the compounding of the volt-amperes of constituent burdens.

* Note that if $z = R + jX$ and $y = G + jB$, then since $y = 1/z$, $G + jB = 1/(R + jX) = (R - jX)/(R^2 + X^2) = (R - jX)/Z^2$, so that $G = R/Z^2$, and $B = -X/Z^2$, where $Y = \sqrt{(G^2 + B^2)} = 1/Z$. Note that B is negative for coils and positive for condensers.

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PART 3

THE TESTING OF CURRENT TRANSFORMERS FOR RATIO AND PHASE-ANGLE

CHAPTER XVII

INTRODUCTORY

IN Chapter VII, forming the conclusion to Part 1, a preliminary discussion of the testing of instrument transformer characteristics has been given; it is now our purpose to return to this subject with particular reference to current transformers, leaving voltage transformers for consideration in Part 4.

Of the various characteristics by far the most important are the variations of the ratio error and phase-angle with current, secondary burden and frequency.* For this purpose a considerable number of methods has been introduced; these are described and classified in the following pages. Other characteristics, such as the measurement of turns-ratio, exciting current, leakage reactance and resistance of windings, are particularly investigated with respect to the indirect method of ratio and phase-angle testing described in Chapter XVIII. Properties such as polarity of terminals, mechanical, thermal and dielectric strength of current transformers, and the like, are examined in Part 5 and will not be further referred to here.

The *Indirect Method*, in which the ratio error and phase-angle are computed from the results of open-circuit and short-circuit tests made on the current transformer, is treated in Chapter XVIII. *Direct Methods*, both *Absolute* and *Relative*, receive attention in Chapters XIX to XXII. Classification is made according to the arrangement of the apparatus used in them, a process that seems to lead to a greater measure of correlation and a reduction in the number of classes and sub-classes than is obtainable in other ways. It will be realized that absolute methods are based on the principle of testing the properties of a transformer in terms of calibrated impedance standards and instrument readings, while relative methods, as the name

* To some extent also with the primary current wave-shape.

implies, compare the characteristics of one transformer with those of another.

The *Absolute Deflectional Methods* of Chapter XIX utilize, with few exceptions, two main principles: (i) in which the primary and secondary currents are separately measured and the phase-angle between them computed from the readings of appropriate instruments; and (ii) in which drops of voltage at the potential terminals of four-terminal resistors inserted into the primary and secondary circuits are adjusted to approximate equality, their vector difference being measured by the readings of suitable instruments. The great majority of the methods are based on this opposition principle; in some cases the method becomes "semi-null" since a preliminary adjustment reduces the reading of one instrument to zero before the scales of the others are read. One or more of the instruments is a dynamometer, since the phase-selective property of this type makes it peculiarly valuable for the purpose. This being so, a phase-shifting device or a polyphase supply forms an essential feature of such dynamometer methods.

The *Absolute Null Methods* of Chapter XX are essentially opposition methods in which the vector resultant of two approximately equal voltages, proportional respectively to the primary and the secondary currents,* is balanced exactly by an auxiliary voltage, adjustable in magnitude and phase, balance being indicated by null indication of a suitable detector such as a separately-excited dynamometer or vibration galvanometer. The auxiliary voltage is, in some cases, derived from a phase-shifting transformer, but more frequently from a suitable auxiliary mesh of adjustable impedances; in this latter case the method becomes analogous to an a.c. bridge or to an a.c. potentiometer of limited range.

In Chapters XXI and XXII *Relative Methods*, respectively *Deflectional* and *Null*, are discussed. The deflectional methods are essentially opposition processes for comparing the secondary currents of an unknown and a standard transformer, the difference between them being measured by the readings of suitable instruments. The null methods are also opposition methods in which the difference between the two secondary currents (or volt-drops proportional thereto) is annulled by adjustable auxiliary circuits. Since in relative methods no measurements are made on the primary side, they are especially

* These voltages may be taken from the potential terminals of four-terminal resistors or from the secondaries of mutual inductors.

valuable for the testing of transformers on site where the primary winding may be at a dangerously high voltage above earth or may carry a very large current. In addition to this advantage of safety, the necessary apparatus is not bulky, since the secondary currents never exceed 5 amperes; the fact that they are difference methods enables high precision to be obtained with portable pointer instruments.

It is worthy of special note that the comparison of the two nearly equal secondary currents of a pair of similar transformers by a relative method is precisely similar to an absolute test on a 1 : 1 transformer. Conversely, it follows that any transformer with a ratio of 1 : 1 can be tested by any relative method, the primary current replacing the standard secondary current in the method; in addition all the customary absolute methods are available. This feature is specially useful in those transformers with cable primaries in which, by a suitable choice of primary turns, the ratio can be made unity for the purpose of the test.

It has been shown in Chapter II that the phase-angle β is usually positive, i.e., I_s leads on $-I_p$, for all values of power-factor usually encountered in practice. In particular, β is always positive for a non-reactive burden. Since the British Standard Specification No. 81—1927 requires tests to be made with a burden having unity power-factor, β has been shown positive in the vector diagrams of Chapters XVIII to XXII.

CHAPTER XVIII

THE INDIRECT METHOD FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Introductory.** It has been shown on p. 41 that the ratio K_c and the phase-angle β of a current transformer may be approximately expressed by the equations

$$K_c \doteqdot K_T + \frac{I_m \sin \phi_s + I_w \cos \phi_s}{I_s},$$

and

$$\tan \beta \doteqdot \frac{I_m \cos \phi_s - I_w \sin \phi_s}{K_T I_s},$$

where K_T is the turns ratio (secondary turns/primary turns), I_m and I_w are the magnetizing and loss components of the primary exciting current I_0, I_s is the secondary current, and ϕ_s is the phase displacement between I_s and the voltage induced in the secondary winding by the main core-flux. If these various quantities can be determined, by experiment or otherwise, K_c and β can be calculated, from the above expressions; this process constitutes an indirect method for the measurement of ratio and phase-angle, since it is based upon the determination of these other auxiliary quantities. It is, however, seldom used nowadays.

In early days the indirect method was the only one used in current transformer testing, following closely the well-known processes used for testing the regulation of power transformers; it was, however, soon superseded by simpler and more exact direct methods such as are described in Chapters XIX to XXII. Interest in the indirect method was revived a few years later when it became necessary to make tests on current transformers with very large primary currents. When primary currents of 2 000 amperes or more are to be dealt with the direct methods suffer from the practical difficulties inherent in the construction of four-terminal resistors or other standards to be inserted in the primary circuit; the use of the indirect method enables these troubles to be overcome, since only the very small exciting current is involved in the measurements upon which it is based. It should be remarked, however, that even for this particular application the indirect method has been supplanted by the use of direct methods in which the primary resistor or other standard is replaced by a high-ratio standard transformer of the nickel-iron cored type, carrying low-current resistance standards on its secondary side. Consequently, the indirect method

must now be regarded as primarily of historical and general interest rather than of immediate practical value. Some of the auxiliary measurements involved in it are, however, of interest in the analysis of the design of a transformer.

Briefly, the principle of the indirect method is as follows: By means of a suitable process I_m and I_w are found as functions of the induced secondary voltage. Assuming the resistance and the reactance of the secondary winding and its associated external burden to be known, it is possible to find ϕ_s and the impedance of the secondary circuit; hence the value of the induced secondary voltage required to circulate any desired secondary current I_s can be computed and the appropriate values of I_m and I_w taken from the test-curves. If the turns-ratio K_T be known, then K_c and β are at once calculable from the formulae on p. 410,* or by the use of Möllinger and Gewecke's diagram (p. 50).

The application of the indirect method involves, therefore, the following processes: (1) the experimental determination either of the magnitude and phase of the exciting current I_0, or of its components I_m and I_w; (ii) the evaluation of K_T, either from the design data or from an appropriate measurement; and (iii) the estimation of the impedance and phase-angle ϕ_s of the secondary circuit. Of these, (i) and (iii) correspond respectively with the well-known open-circuit and short-circuit tests commonly used in testing power transformers.

2. **Measurement of exciting current.** In the testing of power transformers the components of exciting current are deduced from the open-circuit test, in which the primary voltage, current and power are measured for a given o.c. secondary voltage. In the case of current transformers some modification of this procedure is necessary, since all the o.c. quantities are small and cannot be accurately measured with ordinary pointer instruments. The primary voltage and power are usually too small to be measured without the aid of reflecting dynamometers of some delicacy. Consequently, the modified o.c. test for a current transformer consists in a direct measurement

* For applications of the method reference should be made to the following: C. V. Drysdale, *Phil. Mag.*, 6th series, vol. 16, pp. 136–153 (1908); L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–1039 (1910); A. Barbagelata, *Atti dell'Assoc. Elett. Ital.*, vol. 14, pp. 639–654 (1910); C. H. Sharp and W. W. Crawford, *Trans. Amer. I.E.E.*, vol. 29, pp. 1517–1541 (1911); P. G. Agnew, *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911); A. Barbagelata, *L'Elettro.*, vol. 8, pp. 165–175 (1921).

of the components of I_0 by the use of a dynamometer or other suitable instrument and a phase-shifter; I_0 is seldom more than a few milliamperes. By this means the measurement of a small primary voltage and power is avoided. The secondary o.c. voltage is usually under 10 volts and can be measured by a high-resistance dynamometer or nickel-iron type voltmeter; still better, by a reflecting electrostatic instrument; best of all, by some potentiometer device.

K. L. Curtis* in 1907 deduced values of I_m and I_w from ballistic tests on a ring-type transformer, but these are far from being the correct values when the core is carrying an alternating flux, since no account is taken of the iron losses due to eddy currents or the fact that the dynamic hysteresis loss may be different from that estimated from a static hysteresis loop. E. L. Wilder† in 1904 measured approximately the exciting current with an ammeter, the secondary being open, and from these values and the o.c. secondary voltage was able to make an approximate calculation of the change of ratio with various resistance burdens. A. G. L. McNaughton‡ in 1915 used a similar but more precise form of the method by adopting the ordinary o.c. test with voltmeter, ammeter and wattmeter in the primary circuit, pointer instruments being employed.

The use of the separately-excited dynamometer in the open-circuit test seems to have been suggested first by Sharp and Crawford,§ being later improved by Agnew.‖ Referring to Fig. 204 the secondary voltage can be set to the desired value by means of a sensitive, high-resistance reflecting dynamometer voltmeter V; this instrument can be calibrated on d.c. by throwing the switch S_1 from the right to the left. The primary exciting current I_0 is passed through a suitable four-terminal resistor R_p. With S_3 up and S_2 to the left, the phase-shifter is adjusted until the reflecting dynamometer D reads zero, showing that the voltage across the primary winding and the current I in the fixed coils of D are in quadrature, i.e. I and the flux are in phase. By throwing S_2 to the right D will give a reading proportional to $II_0 \cos \xi$, i.e. to I_m the magnetizing component. The phase of I is now advanced through $\pi/2$ and

* K. L. Curtis, "The current transformer," *Trans. Amer. I.E.E.*, vol. 25, pp. 715–734 (1907).

† E. L. Wilder, "Operation of the series transformer," *Elect. Club J.*, vol. 1, pp. 451–455 (1904).

‡ A. G. L. McNaughton, "The current transformer," *Journal I.E.E.*, vol. 53, pp. 269–271 (1915).

§ C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, vol. 29, pp. 1517–1541, discussion pp. 1542–1551 (1911).

‖ P. G. Agnew, "A study of the current transformer with particular reference to iron loss," *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911).

the new reading of D is taken, this being proportional to $II_0 \sin \xi$, i.e. to I_w the loss component. A calibration of D with alternating current is readily made by throwing S_3 down so that the moving coils are excited by the p.d. rI across the

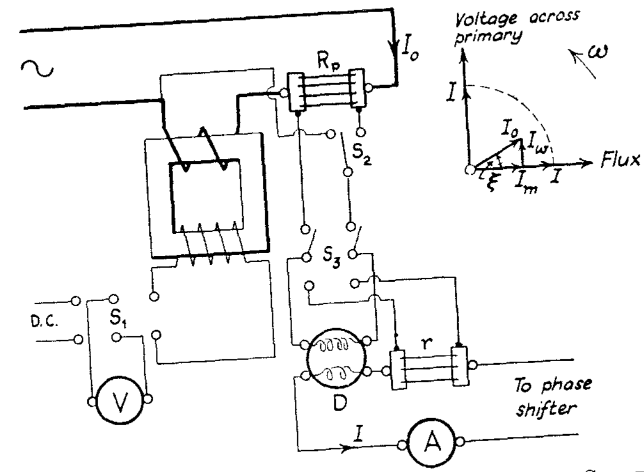


FIG. 204. AGNEW'S METHOD OF MEASURING EXCITING CURRENT

resistance r. If d_1, d_2 and d_3 are the three readings, and k is the dynamometer constant in watts per division,

$$R_p II_0 \cos \xi = kd_1,\ R_p II_0 \sin \xi = kd_2,\ rI^2 = kd_3;$$

whence

$$I_m = \frac{r}{R_p}\frac{d_1}{d_3} I \text{ and } I_w = \frac{r}{R_p}\frac{d_2}{d_3} I.$$

By this means curves of I_m and I_w as functions of the secondary voltage are easily obtained.

In a simple modification due to Barbagelata* the secondary voltage is found by a null method, using as the detector a separately-excited dynamometer with its well-known phase-selective properties. The circuit is similar to that shown in Fig. 205, described by Takatsu,† in which the balance detector is a vibration galvanometer VG instead of a dynamometer.

* A. Barbagelata, "Prova indiretta dei trasformatori di misura per forti intensità di corrente," *Atti dell'Assoc. Elett. Ital.*, vol. 14, pp. 639–654 (1910).

† K. Takatsu, "On current transformers," *Res. Elect. Lab. Tokyo*, No. 95, pp. 1–74 (June, 1921). (In English.)

The voltage V read on the voltmeter is adjusted until the volt-drop across the tapping r, namely, rV/R is numerically equal to the desired secondary voltage of the transformer. The primary current is regulated in magnitude and the phase of V is adjusted until on closing the key K the galvanometer remains undeflected; the secondary voltage is then equal and opposite to rV/R. The switch S is now closed; the resulting

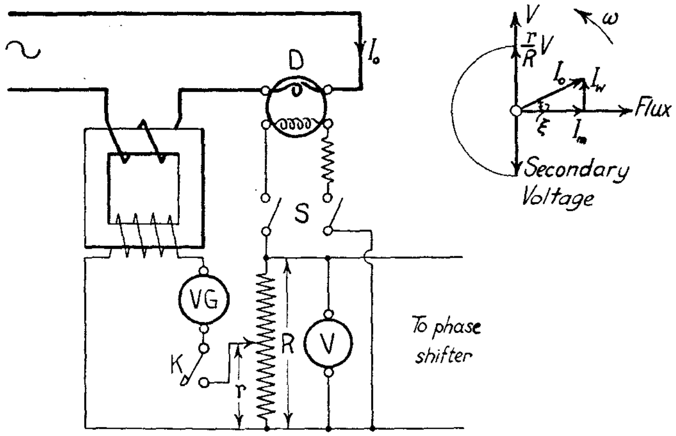


FIG. 205. TAKATSU'S METHOD OF MEASURING EXCITING CURRENT

reading of D will clearly be proportional to I_w. If W_w be the reading in watts, $I_w = W_w/V$. The key K is now opened and the phase of V changed by $\pi/2$, the new reading of D being W_m watts; then $I_m = W_m/V$. If the magnitude of the total exciting current I_0 is desired it can be readily calculated from its components, or it may be measured by opening K and regulating the phase-shifter until D gives a maximum reading, which will be proportional to I_0.

In another variant of this method the alternating-current potentiometer is used. Spooner* has shown that the Tinsley-Gall instrument (p. 486) is very convenient for the purpose, as shown in Fig. 206, though any other type of potentiometer can easily be adapted to the measurement. In Fig. 206 the instrument consists of two component potentiometers, A and B, supplied in quadrature from a phase-shifter PST. The switch S being thrown to the left and B set to zero, potentiometer A and the phase-shifter are regulated until the vibration

* T. Spooner, "Some applications of the a.c. potentiometer," *Journal Sci. Insts.*, vol. 3, pp. 214–221 (1926).

galvanometer VG indicates balance; then A gives the secondary o.c. voltage. With S to the right A and B are now adjusted to balance the drop R_pI_0 across the primary resistor R_p; then if V_A, V_B be the settings in volts,

$$I_w = V_A/R_p \text{ and } I_m = V_B/R_p$$

the loss and magnetizing components respectively.

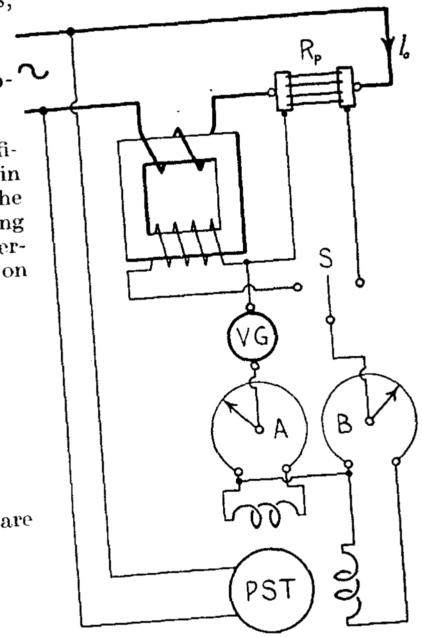


FIG. 206. SPOONER'S METHOD OF MEASURING EXCITING CURRENT

Gall* has made a slight modification which is useful not only in transformer testing but also for the more general problem of iron testing with a.c. Let the secondary power-factor be unity, then the formulas on p. 410 become

$$K_c \doteq K_T + (I_w/I_s) \text{ and}$$
$$\tan\beta \doteq I_m/K_TI_s$$

The iron loss is

$$p = E_pI_w = E_sI_w/K_T,$$

the reactive volt-amperes are

$$p_j = E_pI_m = E_sI_m/K_T,$$

and the secondary volt-amperes are

$$P_s = E_sI_s.$$

Then

$$K_c \doteq K_T[1 + (p/P_s)] \text{ and}$$
$$\tan\beta \doteq p_j/P_s$$

Now let I_0 and E_s be measured in terms of their components along and perpendicular to any datum direction. Let $\mathbf{1}$ be a unit datum vector, $\boldsymbol{i}_0$ and $\boldsymbol{e}_s$ the exciting current and secondary voltage vectors then

$$\boldsymbol{i}_0 = (a_1 + jb_1)\mathbf{1},\ \boldsymbol{e}_s = (a_2 + jb_2)\mathbf{1} \text{ and } \boldsymbol{e}_p = \boldsymbol{e}_s/K_T,$$

where a_1, a_2 are measured by A and b_1, b_2 by B. The power is the scalar product

$$p = \boldsymbol{e}_p \cdot \boldsymbol{i}_0 = (a_1a_2 + b_1b_2)/K_T,$$

the volt-amperes are the vector product

$$p_j = \boldsymbol{e}_p \times \boldsymbol{i}_0 = (a_1b_2 - b_1a_2)\boldsymbol{z}/K_T$$

where $\boldsymbol{z}$ is a vector of unit length in a direction normal to the plane containing $\boldsymbol{e}_p$ and $\boldsymbol{i}_0$. Using the magnitudes of these products gives for a transformer working at unity power factor

$$K_c \doteq K_T\left[1 + \frac{(a_1a_2 + b_1b_2)}{K_TP_s}\right] \text{ and } \tan\beta \doteq \frac{(a_1b_2 - b_1a_2)}{K_TP_s}.$$

* D. C. Gall, "Note on testing transformers," *Journal Sci. Insts.*, vol. 9, pp. 109–110 (1932). For a correction see *ibid.*, p. 362 (1932).

Alberti and Vieweg* have described a method based on an entirely different principle, using the magnetic potentiometer discussed on p. 387. The flexible magnetic potentiometer is linked through the primary and secondary windings of the transformer and its ends are brought into contact, as in Fig. 207. The instrument therefore measures the resultant magneto-motive force of the primary and secondary ampere-turns under

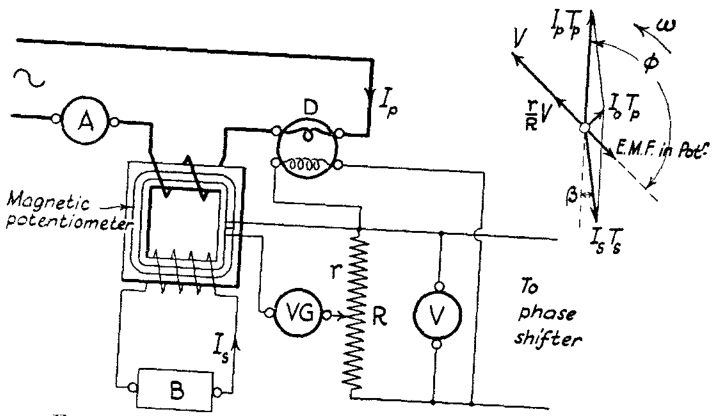


Fig. 207. Alberti and Vieweg's Method of Measuring Exciting Current

normal load conditions when the primary of the transformer is supplied with a current I_p and the secondary current I_s circulates through a given burden; this resultant is the exciting ampere-turns I_0T_p. To increase the sensitiveness, Alberti and Vieweg provide the magnetic potentiometer with an iron core. The voltage induced in the potentiometer is balanced by the adjustment of the fraction r/R on the resistance R and regulation of the phase-shifter until VG indicates zero. If V be the voltage across R the voltage in the magnetic potentiometer is rV/R and its phase ϕ relative to the primary current I_p is readily found from the readings of an ammeter and a dynamometer wattmeter D in the primary circuit. The voltage in the potentiometer is easily drawn in a vector diagram in relation

* E. Alberti and V. Vieweg, "Untersuchungen an Stromwandlern. Der Magnetisierungsstrom," *Arch. f. Elekt.*, vol. 2, pp. 208–216 (1914); V. Engelhardt, "Die Verwendung des magnetischen Spannungsmessers in der Kompensationsschaltung," *ibid.*, vol. 11, pp. 198–202 (1922); W. Geyger, "Mess-ungen mit dem Schleifdraht-Wechselstromkompensator," *ibid.*, vol. 17, pp. 213–214 (1926).

to the primary ampere-turns. Since the exciting ampere-turns are at right-angles to the voltage induced in the potentiometer the direction of I_0T_p is known; then if the secondary ampere-turns are known in amount I_sT_s, the magnitude of I_0T_p is readily found, and hence I_0. It will be noted that a knowledge of the primary and secondary turns is assumed. Results obtained by the method are in good agreement with those found by other means.

It is as well to point out here a precaution that may usefully be observed in all open-circuit tests. Although it is supposed that the normal core-flux is not exceeded, it is often difficult to ensure that this actually is the case when a transformer is put into a measuring circuit with its secondary opened. In order to guard against effects of accidental excessive magnetization it is best both before and after the test to subject the core to a thorough demagnetization, as on p. 125.

3. **Determination of the turns-ratio.** The turns-ratio, K_T, of a current transformer is, in some cases, obtainable from the design data in the possession of the manufacturer. It is not uncommon, however, either for the ratio to be unobtainable in this simple manner or, if obtainable, to be unreliable; the latter circumstance may arise in a transformer in which some adjustment of turns has been made on test so that the actual turns, in consequence, differ from those postulated in the design data of the transformer. In such cases K_T must be found by experiment.

Duff* has described a method for the measurement of K_T based on the following principle. If the primary and secondary windings are joined in series to act in the same sense upon the core, an alternating voltage applied to the combination will be divided between the windings in proportion to their numbers of turns, provided that the windings are without resistance or leakage reactance. In this ideal case the turns-ratio could be found by joining a resistance in parallel with the windings and finding a point thereon at the same potential as the junction of the windings, i.e. by a simple Maxwell inductance bridge method. It is further clear that this principle would hold exactly in an actual transformer if both the winding resistances and reactances were proportional to the turns, which is, unfortunately, not the case.

* C. K. Duff, "Determination of turns-ratio of current transformers," *Univ. of Toronto Eng. Res. Bull.*, No. 2, pp. 211–215 (1921). For a method of determining the number of turns in a winding before assembly see C. Dannatt, "Transformer coil winding. An accurate equipment for measuring the turns on coils," *Elecn.*, vol. 107, pp. 218–219 (1931).

In any actual transformer the resistances of the windings can be made approximately proportional to the numbers of turns by adding a compensating resistance in series with one winding, usually the primary. If R_p and R_s are the resistances of the primary and secondary windings, the amount to be added in series with the former will be $(R_s/K_T) - R_p$, which is very nearly $(R_s/K_{nc}) - R_p$ since the nominal or marked current ratio K_{nc} seldom differs more than 1 or 2 per cent from K_T. If this addition should prove negative, resistance of amount $K_{nc}R_p - R_s$ should instead be put into the secondary. By effecting this adjustment any error due to the resistances of the windings can be made very small.

The influence of the leakage reactances of the windings is important and cannot be corrected for in any such simple way as is suggested for their resistances. The path of the leakage field is largely in air, so that the leakage flux is practically proportional to and in phase with the current in the windings. If an alternating current galvanometer of the Stroud and Oates type, in which the field magnet is excited by a current in phase with the current in the transformer windings, is used as a balance detector, the instrument will be insensitive to voltages in quadrature with the current, i.e. to voltages induced by the leakage flux, and will have its greatest sensitiveness to voltages in phase with the current.

Referring first to Fig. 208 (*a*), one coil of a dynamometer or wattmeter D is joined in series with P and S, the other coil being supplied from a phase-shifter. With various applied voltages on P and S, read by the voltmeter V, the settings of the phase-shifter requisite to bring the reading of D to zero are noted; then the current in the transformer windings is in quadrature with that supplied by the phase-shifter, i.e. is in quadrature with the phase-shifter voltage, very nearly. The resistances r_p and r_s should be set to their expected values so that small adjustments only will be necessary later.

In Fig. 208 (*b*) the moving coil of the a.c. galvanometer is joined across A and B. The field magnet windings of the galvanometer are put in series with the current coil of D and with a variable resistance r and inductance l. With the same voltages and phase-shifter settings as before r and l are adjusted to make D read zero; then the current in the galvanometer field is in quadrature with the phase-shifter voltage and is consequently in phase with that in the transformer windings. The resistances r_p and r_s are then adjusted to make the galvanometer read zero; then $K_T = r_s/r_p$ when balance is indicated.

To illustrate the use of Duff's method a test was made on a 15 000 volt switchboard type current transformer having $K_{nc} = 75/5$ and a turns-ratio $K_T = 296/20 = 14{\cdot}8$. The value of $R_p = 0{\cdot}0085$ ohm and of $R_s = 0{\cdot}455$ ohm, so that additional resistance of $(0{\cdot}455/15) - 0{\cdot}0085 = 0{\cdot}0219$ ohm was added to the primary to make the resistance drop

approximately in the same ratio as the turns. At 60 cycles per sec. and voltages of 45, 60 and 75 the settings of r_s and r_p were 3005·9 ohms and 203·08 ohms, so that $r_s/r_p = 14{\cdot}80_2$, thus determining K_T to within 0·01 per cent.

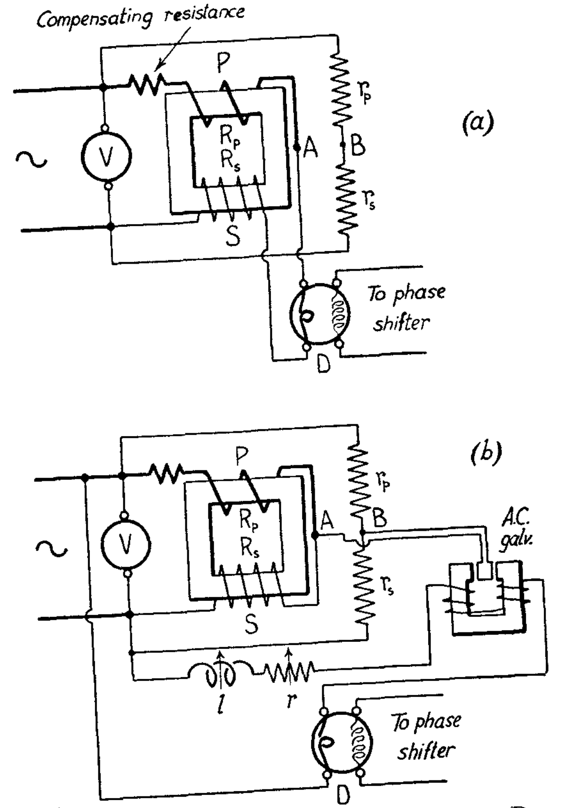


FIG. 208. DUFF'S METHOD OF FINDING TURNS RATIO

A much simpler method of finding K_T can be used if a magnetic potentiometer is available. One of the windings of the transformer is joined in series with a coil having a known number of turns, preferably of the same order as the turns in the windings, the other winding being left open. The magnetic potentiometer is linked in turn through the standard coil and the winding under test, the voltages induced in the instrument when an alternating current of constant amount flows through the coil and winding in series being proportional to the respective numbers of turns. These voltages can be conveniently compared by a null method using a resistance potential-divider and phase-shifter, with a vibration galvanometer as the balance detector, as in Fig. 207; balance is obtained first with the magnetic potentiometer linked with one coil and then with the other. A still better method is to use a mutual inductance balance, Fig. 200, in the way described on p. 388. It will be noted that these processes give the actual numbers of turns and not merely the turns-ratio.

4. **Estimation of impedance and phase-angle of secondary circuit.** The impedance of the secondary circuit is compounded of two portions, which may conveniently be considered separately. The first part consists of the impedance arising from the resistance and reactance of the external burden, consisting of the instruments, etc., with which the secondary is loaded, together with the appropriate connecting leads. The impedance and phase-angle of this external burden is not usually very difficult to measure, by the use of any of the standard laboratory methods. The second part of the secondary impedance arises from the resistance and the leakage reactance of the secondary winding itself; the resistance is easily accounted for but the leakage reactance presents some difficulty.

Some experimenters have escaped the difficulties by neglecting the leakage reactance of the secondary winding in comparison with the reactance of the external burden, a procedure which may be justified in some instances, e.g. in ring type secondaries, but which is, in general, inexact and may lead to considerable error. Others* endeavour to allow for the effect of the secondary winding by a calculation of the secondary leakage reactance from the measured dimensions of the transformer; this, however, is difficult to do with precision. Others again make a normal short-circuit test on the transformer measuring the current, voltage and watts supplied in the secondary circuit with the primary winding short-circuited and calculating therefrom the total effective impedance of the transformer at its secondary terminals. The problem now arises as to how this impedance shall be separated into the part contributed by the secondary winding, in which alone we are interested, and that due to the primary. Some workers† have assumed that half the measured reactance is attributable to the secondary, a procedure which careful tests on a wide

* For example K. L. Curtis, *Trans. Amer. I.E.E.*, vol. 25, pp. 715–734 (1907).

† For example A. G. L. McNaughton, *Journal I.E.E.*, vol. 53, pp. 269-27[illegible] (1915).

range of transformers show to be unjustifiably arbitrary, as shown on p. 129. In wound transformers with large primary current rating the secondary leakage reactance is much less than half the total secondary s.c. reactance but becomes more nearly half as the rating is reduced; it will be remembered that the indirect method is only valuable for large ratings, so that the common assumption is not applicable thereto. Reed* has shown that it is closely in agreement with practical observation to take the secondary leakage reactance X_{ws} as given by $X_{ws}/X_T = R_s/R_T$, where X_T, R_T are the measured reactance and resistance of the transformer observed on the secondary side with the primary short-circuited and R_s is the secondary resistance; the s.c. test should be made for various values of secondary current and the results plotted for use in the computations since X_{ws} may vary somewhat with I_s.†

While it must be confessed that precise determination of the impedance of the secondary winding is not very satisfactory, experience shows that it is often possible to estimate its value in any given case with sufficient practical accuracy; Z_s and ϕ_s being thus approximately obtained, the data for the indirect method are complete.

5. **Typical results of indirect method.** It is interesting to examine a typical test made by the indirect method and to compare it with the results obtained by a precision direct method. The following are taken from Agnew's‡ paper previously cited—

Nominal current ratio	= 40/5 = 8;
Turns ratio K_T	= 196/25 = 7·84;
Resistance of secondary winding	= 0·51 ohm;
Resistance of external burden	= 0·17 ohm;
Inductance of external burden	= 0·08 millihenry.

Using the method described on p. 412 the values of I_m and θ were measured at 25 and 60 cycles per sec. and are plotted in Fig. 209. From these data Agnew calculates the ratio and phase-angle of the transformer for various secondary currents; the leakage inductance of the ring type secondary winding is

* E. G. Reed, "Current transformer calculations," *Elect. J.*, vol. 23, pp. 67–71 (1926).

† For a direct measurement of leakage reactance see H. W. L. Brückmann and W. Engelenburg, "Eine direkte methode zur Bestimmung des Streufeldes eines Wandlers," *Elekt. Zeits.*, vol. 52, pp. 1171–1172 (1931); vol. 53, pp. 93–95 (1932). See also pp. 54 and 56.

‡ P. G. Agnew, loc. cit. on p. 412 (1911); also K. Takatsu, loc. cit. on p. 413 (1921).

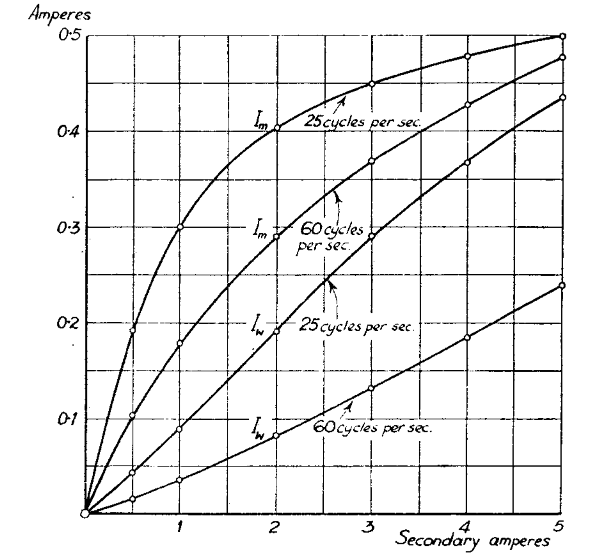


Fig. 209. Excitation Curves for Current Transformer

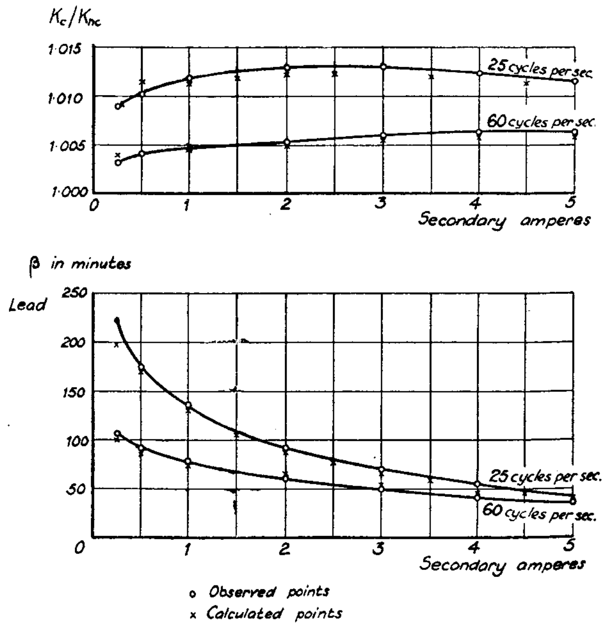


Fig. 210. Computed and Measured Characteristics for Same Transformer as Fig. 209

eglected. The results are shown by the crosses in Fig. 210 here the circles show the values obtained by an absolute ethod to be described later (see p. 431). It will be seen that he agreement between the indirect and the absolute methods s very satisfactory. Since the transformer has an abnormal atio curve the agreement is particularly gratifying; the peculiarities of this type of ratio curve have been discussed earlier on p. 120. Agnew and others, who have used the indirect method, give results on normal transformers in which the agreement is no less good; see p. 49 for example.

As we have seen, the principal difficulty in the method is that of determining the secondary leakage reactance; experience shows, however, that errors introduced on this account are usually small, especially if the external burden be fairly large and inductive, as is often the case in practice. In the case of bar-type transformers for very large currents, in which the secondary winding is wound toroidally upon a ring-shaped core, the secondary leakage reactance is almost negligible and the indirect method gives excellent results (see p. 131).

The indirect method may be used in an inverted sense for the determination of the leakage reactances and turns ratio of a transformer, as has been fully discussed on p. 54.

CHAPTER XIX

ABSOLUTE DEFLECTIONAL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Two-ammeter methods.** The ratio of a current transformer is most easily found by the use of two ammeters,* A_p and A_s, connected respectively in the primary and secondary circuits as shown in Fig. 211 (*a*). Then K_c is given by the ratio of the readings of the primary and secondary ammeters; no measure of the phase-angle is obtainable.

The accuracy of the method depends upon that of the two instruments and the precision with which simultaneous readings can be taken upon them. Moreover, the scales of a.c. ammeters are usually non-uniformly divided, the division being crowded at the lower readings; hence at low currents the accuracy falls off very considerably. At rated current the ratio may be determined with care to 0·5 per cent when the ammeter readings are situated upon the best parts of the scales; this is probably adequate for quick ratio tests, such as are frequently required when checking ammeter transformers on site. The accuracy at low loads usually cannot be improved by the substitution of a lower reading ammeter in the secondary circuit, since the burden imposed by the windings of an ammeter for a range of less than 5 amperes is quite considerable and would make the ratio of transformation quite different from the true value. The method is limited in the upward direction by the fact that self-contained a.c. ammeters for use in the primary circuit when the currents exceed 500 amperes are no[t] readily procurable.

The same principle applies to the use of shunted dyn[a]mometers, as suggested by Robinson† and shown in Fig. 211 (*b*). The current coils of D_p are arranged in the primary circuit a[nd] those of D_s in the secondary circuit; their voltage coils are co[n]nected in parallel with suitable four-terminal resistors R_p, an[d]

* See R. S. J. Spilsbury, *Beama J.*, vol. 6, pp. 505–513 (1920); F. A. Kart[...] *Elec. World*, vol. 75, pp. 1368–1370 (1920); F. B. Silsbee, *Trans. Amer. I.E.E.*, vol. 43, pp. 282–294 (1924).

† L. T. Robinson, "Electrical measurements on circuits requiring cur[rent] and potential transformers," *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–10[...] (1910).

R_s in the respective circuits. By using reflecting dynamometers it is possible to attain quite high accuracy; large primary currents can be dealt with by parallel grouping of the current coils of D_p and proper choice of R_p. Since the currents are proportional to the square-roots of the dynamometer readings, the accuracy is low at low loads, though some improvement may be obtained by series grouping of the current coils of D_s and by increasing the value of R_s. Care should be taken in selecting the secondary instrument that an excessive burden is not imposed. The test burden B in Figs. 211 (*a*) and (*b*) should be chosen to make, with the addition of the secondary instrument, a total burden equal to that with which the transformer will be loaded in service.

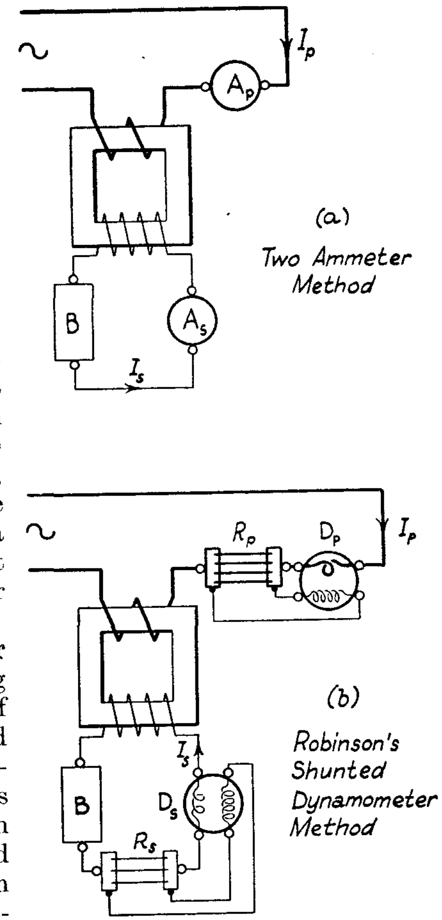


FIG. 211. TWO-AMMETER METHODS FOR RATIO TESTS

2. **Two-dynamometer methods.** The preceding methods give the ratio of the transformer but afford no indication of its phase-angle; a number of methods will now be described in which the ratio error and phase-angle are found from the readings of two dynamometers used as separately-excited ammeters. These methods require an auxiliary a.c. supply of variable phase, by means of which the voltage elements of the dynamometers may be excited; this supply may be derived either from a phase-shifting device actuated by the test circuit, or from an auxiliary phase of a two- or three-phase supply of which one phase is used to deliver the testing current.

The simplest arrangement is shown in Fig. 212 (*a*), where the primary of the transformer is connected to a suitable a.c. source in series with an ammeter A_p and the current coils of a dynamometer D_p of suitable range and sensitivity; the secondary is closed through an ammeter A_s and the current coils of a second dynamometer D_s, any desired make-up burden

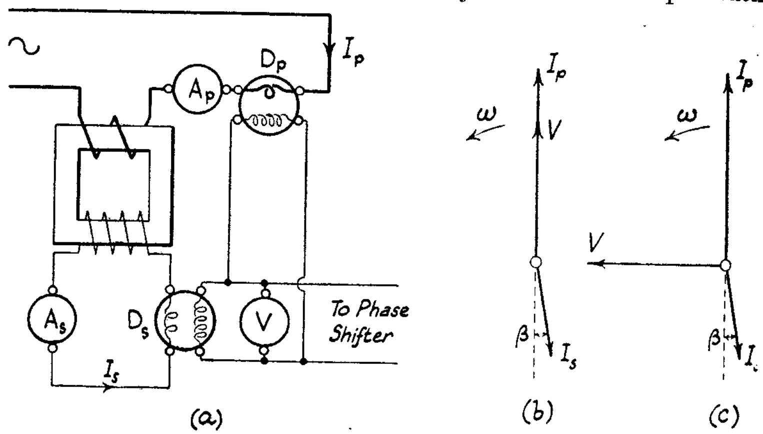


FIG. 212. SIMPLE TWO-DYNAMOMETER METHOD

being added if required. The voltage coils of D_p and D_s are joined in parallel across an auxiliary supply, the phase of which can be adjusted relatively to the source feeding the primary circuit. The auxiliary supply may be taken from a phase-shifting transformer or similar device excited by the primary source; or the primary and auxiliary sources may be two similar coupled alternators with means for altering their phase relationship. The ammeters A_p and A_s are convenient for setting the currents to the desired values, but only one is essential.

The process* is very simple and is illustrated by the vector diagrams of Fig. 212 (*b*) and (*c*). Assuming that the voltage coils have negligible reactance, so that the currents in them are in phase with the auxiliary voltage V applied by the phase-shifter and indicated by the voltmeter, the phase-shifter is

* L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 25, pp. 727–734 (1907), and vol. 28, pp. 1005–1039 (1910), uses the process for the determination of β, finding the ratio by the preceding shunted dynamometer method. The complete method is described by Kartak, loc. cit. (1920); Spilsbury, loc. cit. (1920); A. Barbagelata, *L'Elettro.*, vol. 8, pp. 165–175 (1921); and Silsbee, loc. cit. (1924); A. G. L. McNaughton, *Journal I.E.E.*, vol. 53, pp. 269–271 (1915), uses the process to find β but obtains the ratio from the ammeter readings.

adjusted until the reading of D_p is reduced to zero. Then V and I_p are in quadrature as shown in Fig. 212 (*c*), and the reading of D_s will be

$$W_s' = VI_s \cos[(3\pi/2) - \beta] = -VI_s \sin\beta \text{ watts.}$$

The phase of V is now turned through $\pi/2$ until the reading of D_p becomes a maximum, since V and I_p will be in phase. As can be seen from Fig. 212 (*b*), the readings of D_p and D_s will be

$$W_p = VI_p \text{ watts,}$$
$$W_s = VI_s \cos(\pi - \beta) = -VI_s \cos\beta \text{ watts.}$$

From these three readings

$$K_c = W_p/(W_s^2 + W_s'^2)^{\frac{1}{2}} \doteqdot W_p/W_s.$$

and

$$\tan\beta = W_s'/W_s \doteqdot \beta$$

which give the ratio and phase-angle of the transformer.

The method is simple and quick, an accuracy of 0·2 per cent being attainable at full load. The sensitivity falls only in proportion to the current, so that as a method for measuring the ratio it is to be preferred to the shunted dynamometer method of Fig. 211 (*b*) in which the sensitivity decreases in proportion to the square of the current.

The disadvantages of the method are: (i) The secondary load imposed by the dynamometer D_s may be considerable and for a 5 ampere instrument may be about 10 VA. (ii) The limit of primary current is about 200 amperes, which is the greatest current for which dynamometer instruments can be easily constructed without the use of auxiliary transformers. Moreover, unless the instrument D_p is provided with a number of coils which can be suitably grouped, it will be necessary to have a number of dynamometers in order to retain high sensitivity over a wide range of values of maximum primary current. (iii) The simple theory given above will not apply if the voltage coils of D_p and D_s are reactive.

The reactance error is easily determined. Let θ_p and θ_s be the angles of phase displacement between the currents in the voltage circuits of D_p and D_s and the common voltage V. Then with D_p reading a maximum it is easy to show that the readings in watts of D_p and D_s are

$$W_p = VI_p \cos\theta_p$$
$$W_s = VI_s \cos\theta_s \,.\, \cos(\pi - \beta + \theta_p - \theta_s) = -VI_s \cos\theta_s \cos(\beta + \theta_s - \theta_p)$$

and

$$W_p/W_s = I_p \cos\theta_p / I_s \cos\theta_s \,.\, \cos(\beta + \theta_s - \theta_p) \doteqdot I_p/I_s \doteqdot K_c,$$

since β, θ_p and θ_s are all small angles. If now the phase of V is altered until D_p reads zero, i.e. until V leads I_p by $(\pi/2) + \theta_p$, the reading of D_s becomes

$$W_s' = VI_s \cos\theta_s \cos[(3\pi/2) - \beta + \theta_p - \theta_s]$$
$$= -VI_s \cos\theta_s \,.\, \sin(\beta + \theta_s - \theta_p)$$

so that

$$W_s'/W_s = \tan(\beta + \theta_s - \theta_p).$$

Hence reactance in the voltage coils produces no appreciable error in K_c but may have a considerable effect on the value of β. The error can be made to vanish under two conditions: (i) if $\theta_s = \theta_p$, i.e. if the voltage circuits have equal time-constants; (ii) if $\theta_s = \theta_p = 0$, i.e. if the circuits are non-reactive. The reactance can be annulled by any of the well-known methods used in wattmeters, e.g. by the inclusion of a properly chosen shunted condenser in each voltage circuit. The error can be completely avoided, as suggested independently by Moore and Barbagelata,* by joining the voltage coils in series instead of in parallel and measuring the auxiliary current flowing commonly in them instead of the auxiliary voltage.

A useful variant of the method has been described by Isaacs† in which the two dynamometers are replaced by a Drysdale polyphase wattmeter. The current coils of one element of the wattmeter are grouped for 5 amperes and put into the secondary circuit; the current coils of the second element are arranged in the way appropriate to the magnitude of the primary current and are joined in the primary circuit. The voltage circuits of the elements may be connected in turn to the phase-shifter by the aid of a throw-over switch. With voltage applied to the secondary element the phase-shifter is adjusted to reduce the wattmeter reading to zero; throwing the voltage over to the primary element gives a reading W_p watts and $\sin\beta = W_p/VI_p$. This deflection is small, but it is possible to increase it by temporarily overloading the volt-coils with 5 or 10 times their normal current. The phase-shifter is now turned through $\pi/2$ and the reading $W_p' = VI_p \cos\beta$ noted; throwing the voltage on the secondary element gives $W_s = VI_s$ since V and I_s are now in phase. Hence the ratio is $I_p/I_s = K_c \doteq W_p'/W_s$.

It is sometimes suggested‡ that β may be found by adjustment of the phase-shifter so that first D_p and then D_s are successively reduced to zero reading; then β is the angle through which the phase has been rotated to obtain the two zero readings. Since β is small—seldom exceeding 3° and

* A. E. Moore, *Journal I.E.E.*, vol. 51, pp. 346–347 (1913); A. Barbagelata, *L'Elettro.*, vol. 8, pp. 165–175 (1921).

† R. G. Isaacs, "The testing of current transformers," *Journal Sci. Insts.*, vol. 4, pp. 75–78 (1926).

‡ A. E. Moore, loc. cit. (1913).

usually much less—this necessitates the use of some magnifying device to render more accurate the reading of the angle on the phase-shifter scale; this may be effected by optical means. Makower and Wust* have suggested a process whereby this magnification may be avoided. Neglecting the voltage circuit reactance let V be adjusted to be in advance of I_p by an angle ϕ; then the readings in watts of the two dynamometers will be

$$W_p = VI_p \cos\phi,$$
$$W_s = VI_s \cos(\pi - \beta + \phi) = -VI_s \cos(\phi - \beta);$$

from which

$$W_s/W_p = -(I_s/I_p)\ [\cos(\phi - \beta)/\cos\phi]$$
$$= -(1/K_c)\ [\cos\beta + \sin\beta \,.\, \tan\phi] = a.$$

Taking several values of ϕ let a be determined and a curve showing a as a function of ϕ be plotted. Then if a_1 and a_2 are values of a from the curve corresponding with angles ϕ_1 and ϕ_2 it is easy to show that

$$\tan\beta = (a_1 - a_2)/(a_2 \tan\phi_1 - a_1 \tan\phi_2),$$

from which β is found. Inserting this value in the expression for a_1 permits K_c to be calculated.

A number of important modifications of the two-dynamometer method, in which a polyphase source of supply takes the place of the phase-shifter, have been used and will now be considered.

Turning attention first to two-phase methods, that of Rosa and Lloyd† is shown in Fig. 213 (a). The dynamometers D_p and D_s are of the reflecting type, the voltage-circuit reactances having been compensated in the usual manner. The ratio of the transformer is found by throwing down the switches S_1 and S_2, thus using the dynamometer to measure the currents in the way shown in Fig. 211 (b). To find the angle β, the switches S_1 and S_2 are thrown up, thus connecting the voltage circuits of the two instruments in series across the second phase, which is in quadrature with the main phase supplying the transformer. The volt-coils carry a current in quadrature

* A. J. Makower, "Measurement of phase differences," *Elecn.*, vol. 58, p. 695 (1907). A. J. Makower and A. Wust, "Phase lag in current transformers," *Elecn.*, vol. 79, pp. 581–582, pp. 671 and 785 (1917).

† E. B. Rosa and M. G. Lloyd, "The determination of the ratio of transformation and of the phase relations in transformers," *Bull. Bur. Stds.*, vol. 6, pp. 1–30 (1910).

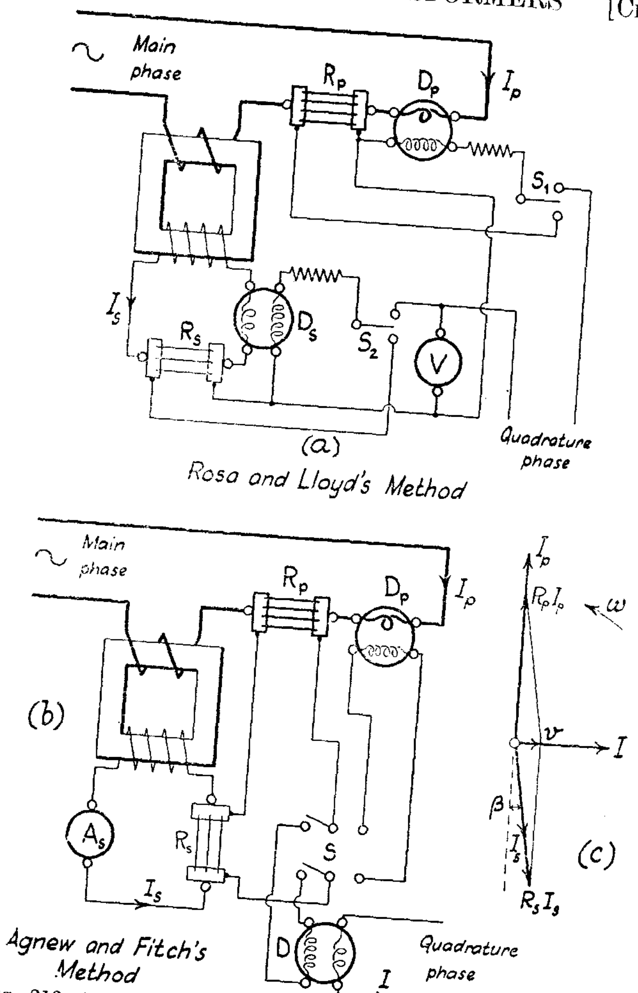


FIG. 213. BUREAU OF STANDARDS TWO-DYNAMOMETER METHODS

with I_p and D_p will cease to read; in consequence of the angl β, D_s will give a reading which may be written,

$$d = kI_sI \sin\beta = kI_s(V/r_s)\sin\beta,$$

where k is the dynamometer constant, I the current in th voltage coils, r_s the total resistance of the voltage circuit o D_s and V the voltage across that circuit, as read on the volt meter. If the switch S_2 is now thrown down and the resistance

f the voltage circuit of D_s, now working as a shunted dynamometer, is adjusted until the reading is again d we can write

$$d = kI_s[R_sI_s/(R_s + r_s')]$$

f r_s' is the new value of r_s. Eliminating k by division gives

$$\sin\beta = [R_sI_s/(R_s + r_s')] \,.\, (r_s/V) \doteqdot \beta$$

thus removing the necessity for calibrating D_s.

The windings of D_s used in the secondary circuit by Rosa and Lloyd had a fairly high resistance and imposed an excessive burden upon the transformer. This defect was overcome in the modification of Agnew and Fitch,* illustrated in Fig. 213 (*b*). This method is noteworthy as one of the earliest in which the principle of compensation is utilized. Referring to the diagram, R_p and R_s are four-terminal resistors chosen so that each gives a volt-drop of 0·1 to 0·4 volts at the rated current, i.e. R_s/R_p is made about equal to the nominal ratio of the transformer. The value of R_s must be adjustable and may consist of a four-terminal resistor shunted by a variable and much larger resistance, or alternatively may be a four-terminal slider resistor; R_s imposes a burden of 0·5 to 2 volt-amperes. R_p and R_s are connected together as shown so that the drops of voltage in them are in opposition; with the switch S to the right R_s is adjusted until D_p reads zero. The resultant voltage v, compounded of R_pI_p and R_sI_s in the way shown in Fig. 213 (*c*), is then in quadrature with I_p. Turning the switch to the left the reading of the dynamometer D is taken. The current I in the fixed coils of this instrument is supplied from the second phase and is thus in quadrature with I_p; the reading of D is therefore a measure of v. The dynamometer may either be a wattmeter calibrated in watts or the scale may be marked off in volts by applying known voltages to the moving coils while I is maintained constant in the fixed coils. From the geometry of the vector diagram,

$$R_sI_s\cos\beta = R_pI_p,$$
$$R_sI_s\sin\beta = v;$$

whence

$$K_c = \sqrt{\left[\frac{R_s^2}{R_p^2} - \frac{v^2}{R_p^2I_s^2}\right]} \doteqdot \frac{R_s}{R_p},$$

and

$$\tan\beta = v/R_pI_p \doteqdot \beta$$

* P. G. Agnew and T. T. Fitch, "The determination of the constants of instrument transformers," *Bull. Bur. Stds.*, vol. 6, pp. 281–299 (1910). P. G. Agnew, "A study of the current transformer with particular reference to iron loss," *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911). For another two-phase method, *see* Barbagelata, loc. cit. on p. 428 (1921).

The method of Agnew and Fitch is capable of considerable precision and was used by them in one of the earliest extensive investigations of the properties of current transformers; their dynamometers were of the reflecting pattern, enabling K_c to be found to within 0·005 per cent and β to the nearest 0·1 minute. The sensitivity of the compensation setting is proportional to the square of the current and was maintained

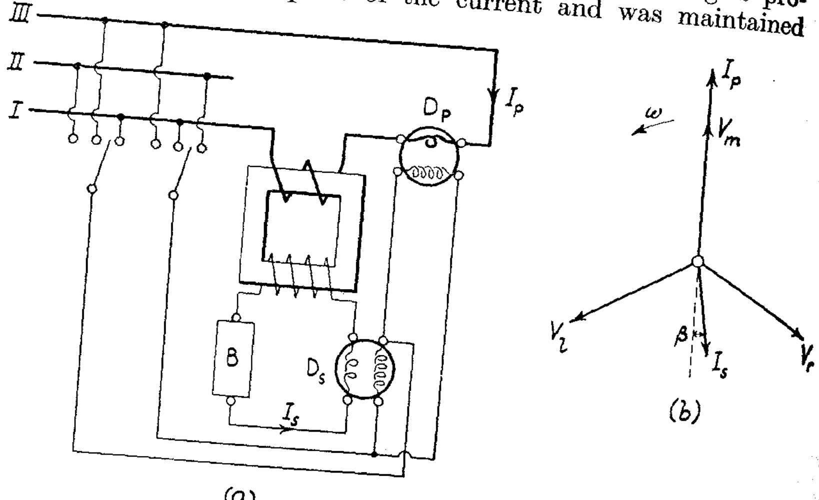


Fig. 214. Barbagelata's Three-phase Method with Two Dynamometers

at the degree of precision cited over a wide range of current values by the use of two-range dynamometers. The method is subject to certain errors, chiefly in the determination of β, arising from the reactance of the voltage coils of D and the residual reactances of R_p and R_s. These errors are investigated in detail in the original paper, where it is shown that the effect of volt-circuit reactance can be made negligible by the adoption of the usual methods of compensation used in wattmeters. The effects of the residuals on the measurement of β are shown to be of opposite sign; hence if R_p and R_s are both slightly inductive (or capacitive), residual errors will vanish when they have equal time-constants. As in other methods A_s serves for the adjustment of the desired secondary current and may be combined with any make-up burden to give a specified value.

Turning now to three-phase methods, the simplest is that

due to Barbagelata* shown in Fig. 214 (a). Assuming the volt-coils of D_p and D_s to be compensated for reactance, throwing the switch successively upon the middle, right- and left-hand contacts will apply the voltages between lines III and I, I and II, II and III to the voltage circuits in parallel. Let these three voltages be V_m, V_r and V_l in Fig. 214 (b), each of equal amplitude V and successively displaced by $2\pi/3$ in phase. Take readings of the two dynamometers in watts with the switch in the three positions and let a, b and c be the ratio of the reading of D_p to that of D_s in the three instances. Then noting that I_p is in phase with V_m

$$a = \frac{V_m I_p}{V_m I_s \cos(\pi - \beta)} = \frac{I_p}{-I_s \cos\beta},$$

$$b = \frac{V_r I_p \cos(4\pi/3)}{V_r I_s \cos[(\pi/3) - \beta]} = \frac{-I_p \cos(\pi/3)}{I_s \cos[(\pi/3) - \beta]},$$

$$c = \frac{V_l I_p \cos(2\pi/3)}{V_l I_s \cos[(5\pi/3) - \beta]} = \frac{-I_p \cos(\pi/3)}{I_s \cos[(\pi/3) + \beta]}.$$

Remembering that β is a small angle, it is easy to verify that

$$K_c = I_p/I_s \doteqdot -a,$$

and

$$\sin\beta \doteqdot (b - c)/2(\sqrt{3})a \doteqdot \beta.$$

A second method, due to Harned† is shown in Fig. 215 (a) and is intended for the testing of transformers on site with portable instruments and a three-phase supply. In the diagram D_p and D_s are wattmeters, the voltage circuit of the former being excited by the voltage between lines I and II. The switch S is first put to the right, exciting the voltage circuit of D_s across the secondary of the transformer, enabling the secondary volt-amperes to be adjusted to any desired value. With S to the left both dynamometers are excited from lines I and II and will give readings in watts, as shown by Fig. 215 (b),

$$W_p = VI_p \cos\theta \text{ and } W_s = VI_s \cos(\theta + \pi - \beta).$$

Eliminating θ, the unknown angle between the voltage V indicated by the voltmeter and the primary current I_p,

$$\beta = \pi + \text{arc}\cos(W_p/VI_p) - \text{arc}\cos(W_s/VI_s)$$

* A. Barbagelata, "Sulla prova dei trasformatori di misura," *L'Elettro.*, vol. 8, pp. 165–175 (1921); an obvious two-phase analogue is also described.

† M. L. Harned, "Operating characteristics of current transformers," *Elec. World*, vol. 67, pp. 869–872 (1916).

The ratio may be found roughly from the readings of the ammeters or more accurately as

$$K_c \doteqdot W_p/W_s.$$

3. **Single-dynamometer methods.** The use of a single dynamometer is among the earliest of the deflectional methods introduced for the measurement of the characteristics of

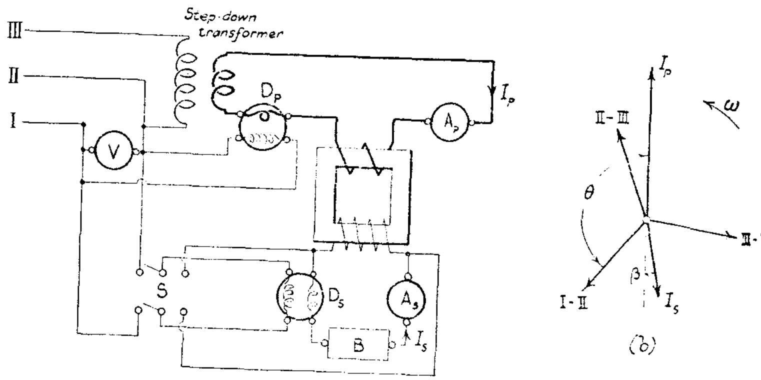


FIG. 215. HARNED'S THREE-PHASE METHOD WITH TWO DYNAMOMETERS

current transformers, being described at about the same time by Robinson in America and Drysdale in England.

The simple method used by Robinson* is shown in Fig 216 (*a*). The ratio is obtained by adjusting the phase-shifter to make the reading on D attain a maximum, first with S to the right and then to the left, I being maintained constant. These two maximum readings are a measure of I_s and I_p respectively. It is best to choose R_p and R_s to give approximately equal drops of voltage so that the readings are taken at about the same part of the scale, thus avoiding calibration troubles; then if W_s and W_p are readings in watts,

$$W_s = R_s I_s I \text{ and } W_p = R_p I_p I,$$

from which $\quad K_c = I_p/I_s = (R_s/R_p)\,(W_p/W_s)$

neglecting the shunting effect of the volt-circuit of D on the currents in R_p and R_s. To obtain the angle of the transformer, S is thrown to the left and the phase-shifter adjusted until D

* L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 25, pp. 727–734 (1907).

reads zero; then I is in quadrature with I_p, as shown in Fig. 216 (*b*). Then with S to the right the reading of D will be

$$W = R_s I_s[r/(r + R_s)] \,.\, I \cos[(3\pi/2) + \beta]$$
$$= [rR_s/(r + R_s)]\, II_s \sin\beta,$$

where r is the resistance of the voltage circuit of D shunting R_s; this voltage circuit is supposed to be non-reactive, though

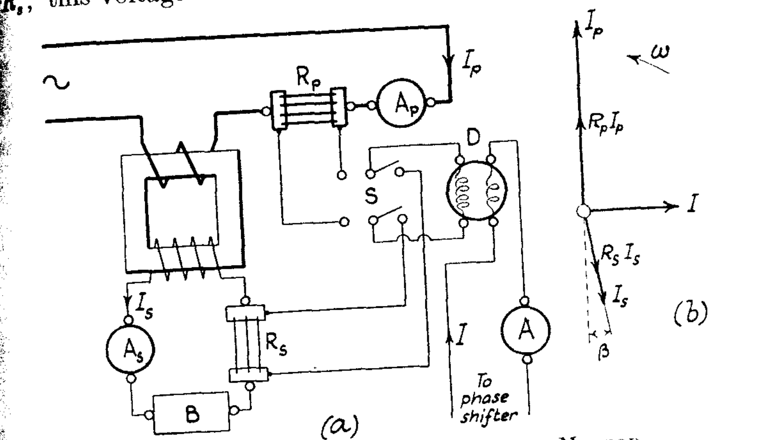


FIG. 216. SIMPLE SINGLE-DYNAMOMETER METHOD

any error due to reactance can easily be corrected. From this expression

$$\sin\beta = [(r + R_s)/rR_s]\,(W/II_s) \doteqdot W/R_s II_s$$

since r is usually large in comparison with R_s. The sensitivity is proportional to the current, since the dynamometer is separately excited. The burden imposed by R_s is slight; in a 125/5 transformer R_s may be 0·025 ohm and R_p 0·001 ohm so that the burden due to R_s is only 0·625 volt-amperes with rated current.

Rosa and Lloyd* have suggested that the angle may be obtained directly by adjusting the phase-shifter to make D read zero when connected to R_s and R_p successively, the change in the phase setting being equal to β. This may be read to 0·1° with the aid of a vernier or optical magnifier and the process is quick and often sufficiently accurate. The ratio is then found by taking the maximum readings of D as before, these being at once obtained by turning the phase-shifter through $\pi/2$

* E. B. Rosa and M. G. Lloyd, loc. cit. on p. 429 (1910).

from the settings just obtained. It may be noted in passing that the best way to obtain the maximum reading of a dynamometer instrument is first to adjust the phase of its excitation to reduce the reading to zero, since at that point the sensitiveness to small changes of phase is much greater than is to be expected near the maximum reading, and then to shift the phase from this position through $\pi/2$ radians.

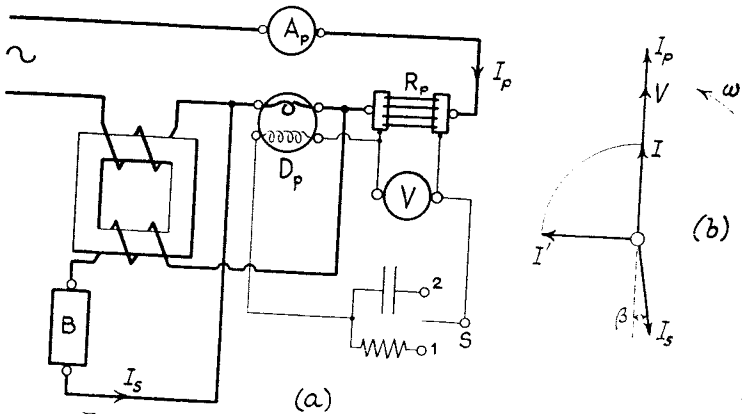


FIG. 217. DRYSDALE'S SINGLE DYNAMOMETER METHOD

Drysdale's method* is somewhat different from the preceding and was originally devised for the testing of transformers having a nominal ratio of unity. As shown in Fig. 217 (*a*) the primary and secondary currents are passed in opposition through the fixed or current coil of the dynamometer D_p. Alternatively the superposition may be effected by the use of a two-element dynamometer, passing I_p through the current coils of one element and I_s through those of the other; moreover this has the advantage that ratios other than unity are easily dealt with by choosing the turns in these coils so that their ampere-turns are approximately equal. Confining attention to the direct arrangement shown in the diagram, the vector relationships are shown in Fig. 217 (*b*). If V is the voltage indicated by the voltmeter, V will be in phase with the primary current I_p; with the switch S on contact 1 the current I in the

* C. V. Drysdale, "The measurement of phase differences," *Elecn.*, vol. 57, pp. 726–728, 783–784 (1906); "Some measurements on phase displacement in resistances and transformers," *Elecn.*, vol. 58, pp. 160–161, 199–201 (1907); "The use of shunts and transformers with alternate current measuring instruments," *Phil. Mag.*, 6th series, vol. 16, pp. 136–153 (1908).

voltage coil of D_p will be in phase with V, neglecting the reactance* of the voltage circuit in comparison with its resistance, and the dynamometer will read

$$W = VI_p + VI_s \cos(\pi - \beta) = VI_p - VI_s \cos\beta \text{ watts.}$$

With S on contact 2 the resistance originally in series with the volt-coil is replaced by a condenser of such magnitude† that

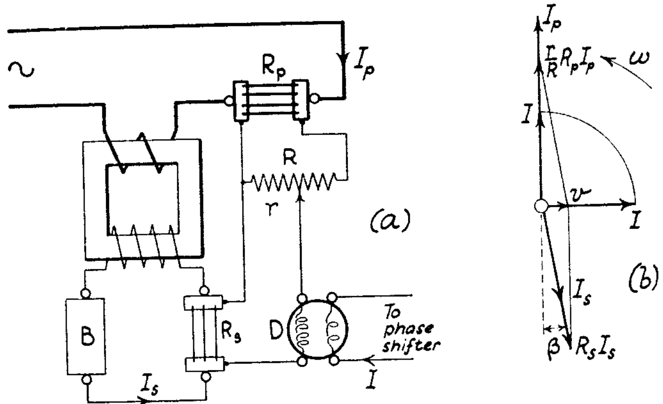


FIG. 218. BARBAGELATA'S SINGLE DYNAMOMETER METHOD

the current I' in it leads on V by $\pi/2$ and is equal to I in amount. The insertion of the condenser has the same effect on D_p as retaining the non-reactive voltage circuit and advancing the phase of V by $\pi/2$. The new reading of D_p will be

$$W' = VI_s \cos[(3\pi/2) - \beta] = -VI_s \sin\beta$$

From these equations

$$K_c = I_p/I_s = VI_p \cos\beta/(VI_p - W) \doteq VI_p/(VI_p - W)$$

and also

$$\tan\beta = W'/(W - VI_p)$$

* For a discussion of the effect of volt coil reactance and an alternative theory of the method see C. V. Drysdale and A. C. Jolley, *Electrical Measuring Instruments*, vol. 2, p. 293 (1924).

† It is easy to show that this condition requires $\omega l_p = r_p$ and $1/\omega C = R + 2r_p = R + 2\omega l_p$, where l_p and r_p are the inductance and resistance of the volt-coil itself, R the resistance removed, and C the condenser replacing it. Now the above theory supposes ωl_p to be negligible in comparison with the total volt-circuit resistance, i.e. $\omega l_p = r_p \ll R$; thus neglecting these small quantities, the value of the condenser is given very closely by $1/\omega C = R$, where R is given a sufficiently large value. Thus it is sufficient to replace the external resistance by a condenser of equal reactance without occasioning appreciable error in the phase advancement, provided that the volt-coil resistance and reactance are small compared with R.

Instead of using a voltmeter, V may be found from the known value of R_p and observed values of I_p.

A method resembling in some respects the two-dynamometer method of Agnew and Fitch (see Fig. 213 (*b*)) has been introduced by Barbagelata* and is illustrated in Fig. 218 (*a*). The auxiliary current I in the current-coils of the dynamometer is first set in phase with I_p and r is then adjusted until the reading of D is zero. The voltage v applied to the voltage circuit is the resultant of (r/R), R_pI_p, and R_sI_s, and must be normal to I and I_p. If the phase of I be changed by $\pi/2$, so that it comes into phase with v, the reading of D will be

$$W = vI = R_sI_sI \sin\beta \text{ watts.}$$

Also $\quad R_sI_s \cos\beta = (r/R)\, R_pI_p$

so that $\quad K_c = I_p/I_s = (R/r)\,(R_s/R_p) \cos\beta \doteqdot (R/r)\,(R_s/R_p);$

and $\quad \sin\beta = W/R_sI_sI,$

or $\quad \tan\beta = (W/I)\,(r/R)\, R_pI_p.$

Hence I and I_p or I_s must be measured. The method is readily adapted to null operation, see p. 450.

In a long and detailed paper Palm† has described a method similar to that of Barbagelata, but making use of a more complex system of observations in order to allow for the shunting effect of the voltage circuit on the currents in R_p and R_s, and for other sources of slight error; the original should be consulted for details.

A method of low precision has been described by Dawes‡ in which the phase-angle of a transformer may be measured under workshop conditions by the use of ordinary portable instruments. The voltage circuit of a wattmeter is excited from a constant voltage a.c. supply in series with an inductance or a condenser to simulate the effect of a low power-factor. Readings of the wattmeter are taken (i) when the current coils carry a known part of the primary current, the remainder passing in shunt through a resistance; and (ii) when the current coils are connected in the secondary circuit and carry a current of the same magnitude as before. Assuming the part of the primary current passed through the wattmeter in (i) to be in phase with

* A. Barbagelata, loc. cit. on p. 433 (1921).

† A. Palm, "Prüfung von Messtransformatoren mit dem Spiegel-Elektrodynamometer," *Zeits. f. Inst.*, vol. 34, pp. 281–290 (1914).

‡ C. L. Dawes, "The phase-angle of current transformers," *Proc. Amer. I.E.E.*, vol. 34, pp. 927–940 (1915).

the primary current and also that the primary current is in phase with the voltage exciting the wattmeter, it is clear that any difference between the two readings of the wattmeter will be due to the phase-displacement introduced by the transformer; hence β can be calculated. Even if these assumptions are not fulfilled it is easy to apply the requisite corrections. Tests on a 50/5 ampere transformer at full load showed $\beta = 28$ min. by a precision method using an a.c. potentiometer. The wattmeter method gave 24, 28 and 30 minutes in three tests with different values of inductance or condenser in the voltage circuit.

4. **Watt-hour meter method.** The method now to be described was devised by Knopp* for the standardization of multi-range transformers without the use of laboratory apparatus. The transformers tested by this worker had subdivided primary and secondary windings, the coils of which were grouped to make the nominal ratio equal to unity, the transformers being tested in that condition. It is well-known that the ratio and phase-angle of such multi-range transformers are practically independent of the grouping of the coils, provided that care is taken in the construction of the transformer; hence after testing at unity ratio the windings may be re-arranged as desired for use and the results obtained can be assumed to be correct with any ratio, to a high order of accuracy.

The principle of the method is very simple and is illustrated by Fig. 219 (*a*). Two standard a.c. meters are adjusted so that one runs about 10 per cent faster than the other; the speed-ratio of the meters is first found by supplying the current elements in series with normal current and the volt-coils in parallel at rated voltage; the volt-coils are supplied from a phase-shifter, which is adjusted to operate the meters at unity power-factor. The connections shown in the diagram are then assumed, the current element of one meter being put into the primary circuit of the transformer to be tested while that of the other meter is joined into the secondary circuit; the volt-coils are supplied as shown from the phase shifter. Adjusting the latter so that the meters are working at an apparent power-factor of unity, the speed-ratio is again determined; the change in speed-ratio of the meters is a measure of the transformer ratio error. The speed-ratio is

* O. Knopp, "The commercial standardization of instrument transformers," *Elec. World*, vol. 67, pp. 92–93 (1916).

again found when the apparent power-factor has been adjusted to about 0·5, and from these observations β can be easily calculated.* Observation of speed-ratio is most easily made by mounting the meters one above the other so that both discs may be simultaneously viewed in a mirror; the speed-ratio is then easily deduced from the observed coincidences of marked spots on the meter discs. The theory of the method

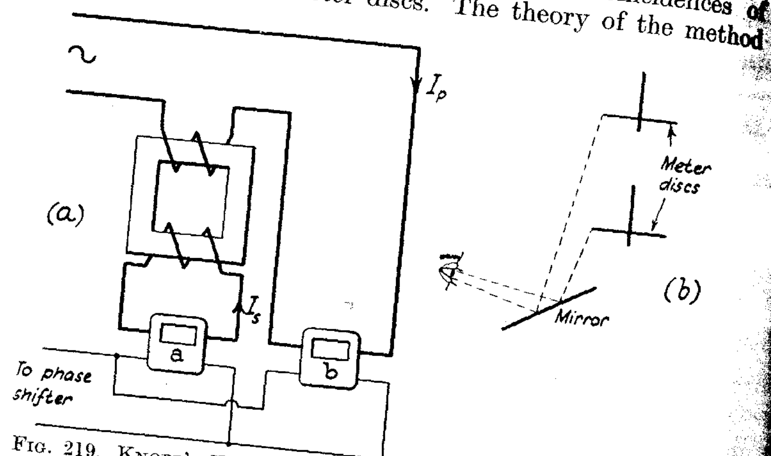


FIG. 219. KNOPP'S WATT-HOUR METER METHOD FOR UNITY RATIO TRANSFORMERS

will not be treated here but will be found on p. 497 as a special case of Agnew's watt-hour method for the comparison of two transformers.

5. **Electrometer methods.** It is appropriate here to examine certain methods, closely related to those already described, in which the measuring instrument is a reflecting quadrant electrometer. These methods have assumed a considerable practical importance, since they have been for some years established at the Physikalische Technische Reichsanstalt and at the National Physical Laboratory as standard processes for absolute testing of instrument transformers. Electrostatic measurement of alternating quantities has been developed to a high degree of perfection in both these institutions and the advantages of electrostatic instruments for such a purpose are too well-known to be repeated here. The design and construction of a suitable electrometer and the technique of its satisfactory

* See also H. M. Crothers, "Field testing of instrument transformers," *Elec. World*, vol. 75, pp. 319–320 (1920); F. A. Kartak, loc. cit. on p. 424 (1920).

use are matters of a highly specialized character which have been fully worked out in these national laboratories; the interested reader should refer to the original papers on the subject for a complete treatment of these problems and for their solution. On account of the special nature of the apparatus it is hardly likely that electrometer methods would be set up in any other place than a large standardizing laboratory; for this reason the following discussion will be brief, the reader desiring further information being referred to the literature that will be quoted and to the useful bibliographies contained therein.

Electrostatic measurements of small phase-displacements were made by Drysdale* as early as 1907 and by other workers at rather later dates. Most of this early work was done with the aid of the Kelvin quadrant electrometer, but it was not until Schultze and Orlich developed and modified the Kelvin instrument then in use that such measurements could be made with ease and certainty. Schultze† in 1907 described a reflecting electrometer specially designed for use at the Reichsanstalt; the National Physical Laboratory instrument, based on Schultze's design, was first described‡ in 1913. These electrometers were made to overcome the difficulties inherent in the use of the Kelvin instrument, to possess greater electric stability, ease of operation and higher sensitiveness, together with special adaptability to the needs of a technical laboratory.

All the electrometer methods for current transformer testing are based on the principle of using an electrometer instead of a dynamometer to compare the voltages over two four-terminal resistors, one in the primary and one in the secondary circuit. The advantages of the electrometer over the electrodynamometer are its independence of the influence of stray magnetic fields and its higher sensitiveness. In the Reichsanstalt method due to Orlich,§ shown in simplified form in Fig. 220 (*a*), R_p and R_s are four-terminal resistors, the former shunted by a resistor R of 100 ohms. For a 100/5 transformer R_p is 0·01 ohm and R_s is 0·1 ohm. The drop of voltage over the portion r of the resistor R is in approximate opposition to the drop over R_s, their resultant being applied to the quadrants of the electrometer. An auxiliary voltage of 50 to 100 volts is maintained between the needle and case of the electrometer

* C. V. Drysdale, "Some measurements on phase displacement in resistances and transformers," *Elecn.*, vol. 58, pp. 160–161, 199–201 (1907).
† H. Schultze, *Zeits. f. Inst.*, vol. 27, p. 65 (1907).
‡ C. C. Paterson, E. H. Rayner, A. Kinnes, *Journal I.E.E.*, vol. 51, pp. 294–330 (1913).
§ E. Orlich, "Über die Anwendung des Quadranten-Elektrometers zu Wechselstrommessungen," *Elekt. Zeits.*, vol. 30, pp. 435–439, 466–470 (1909).

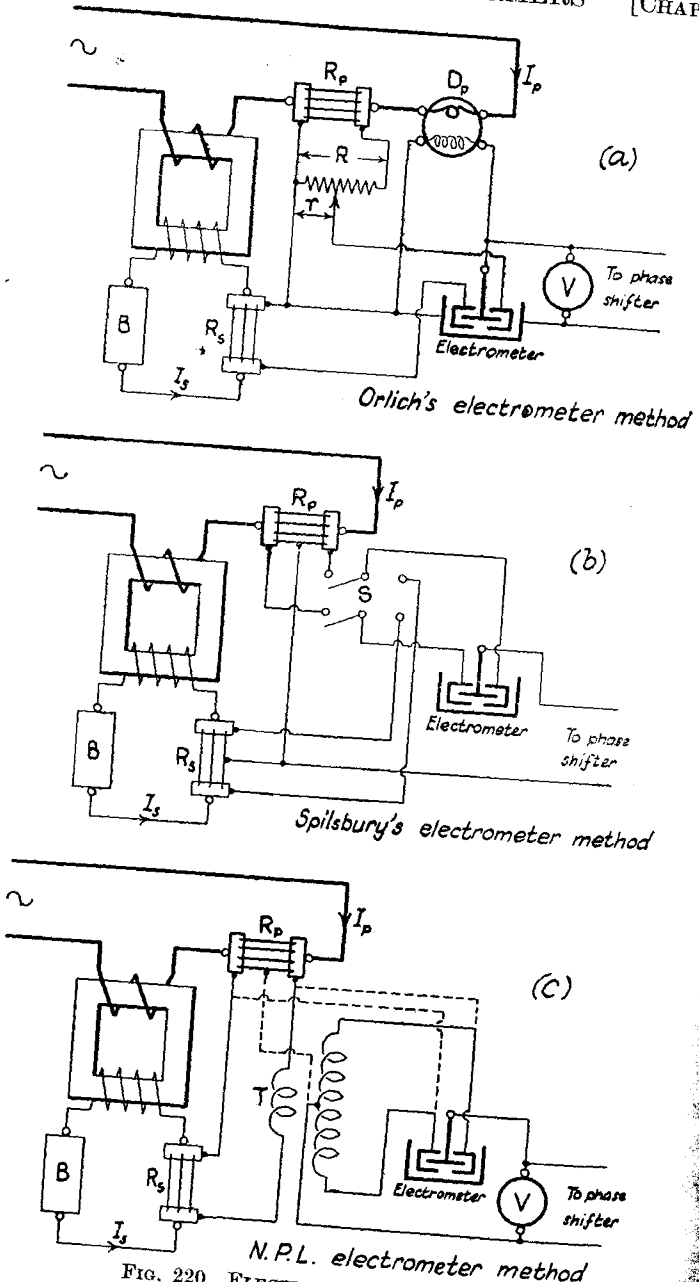


FIG. 220. ELECTROMETER METHODS

by means of a phase-shifter. The dynamometer D_p facilitates the setting of the phase relation of the voltage V read on the voltmeter, relative to the primary current. V is first set in phase with I_p and r varied until the electrometer reads zero; then the resultant voltage applied to the quadrants is in quadrature with V and

$$R_sI_s \cos\beta = [rR_p/(R + R_p)]I_p$$

so that

$$K_c = \frac{I_p}{I_s} = \frac{R_s}{r}\left(1 + \frac{R}{R_p}\right)\cos\beta \doteq \frac{R_s}{R_p}\cdot\frac{R}{r}\left(1 + \frac{R_p}{R}\right)$$

The phase of V is then changed by $\pi/2$ and the deflection of the electrometer is observed; this deflection is very closely proportional to β. This method is the electrostatic parallel to Barbagelata's single-dynamometer method of Fig. 218. A precision of 1 part in 10 000 in ratio and 0·1 minute in angle is claimed for the method.

Spilsbury* has described the method shown in Fig. 220 (*b*), which it is instructive to compare with the single-dynamometer method of Fig. 216 (*a*). A potential difference of about 100 volts is maintained by means of a phase-shifter between the needle of the electrometer and the mid-points of R_p and R_s. The phase-shifter is adjusted until the reading of the electrometer is a maximum, first with the switch S to the left and then to the right; these readings d_p, d_s are proportional to R_pI_p and R_sI_s so that the ratio is

$$K_c = (d_p/d_s)\,(R_s/R_p)$$

The phase of the auxiliary supply is now regulated until when S is to the left the reading of the electrometer is zero; then I_p and the voltage on the needle are in quadrature. If S is now thrown to the right, the resulting reading is proportional to β.

In order to get sufficient sensitivity in Spilsbury's method it is necessary to apply a reasonably high potential difference to the quadrants of the electrometer, i.e. the drops in R_p and R_s must be sufficiently large. The case of R_p presents no difficulty but R_s may impose a fairly large secondary burden, usually about 10 VA. This disadvantage is overcome in the modification† of Spilsbury's method adopted at the National Physical Laboratory, the connections for which are shown in

* R. S. J. Spilsbury, *Beama J.*, vol. 6, pp. 505–513 (1920).
† R. S. J. Spilsbury, "A new method of testing current transformers," *Elecn.*, vol. 86, pp. 296–297 (1921).

Fig. 220 (c). Here R_pI_p and R_sI_s are made about equal and their small vector resultant is impressed on the quadrants through a 100 to 1 step-up transformer T; a potential difference of 100 volts is applied by the phase-shifter between the needle and the mid-point of the secondary of T. Alternatively, by operating a switch not shown in the diagram, the connections are altered so that the drop across R_p is impressed on the quadrants while the auxiliary voltage is applied between the needle and the mid-point of R_p: the connections to R_s and T are removed. This alternative circuit is indicated by the dotted lines. With the dotted connections the phase-shifter is adjusted to make the electrometer read zero, the auxiliary voltage being then in quadrature with I_p; reverting to the full-line connections the electrometer will give a reading nearly proportional to β. Again assuming the dotted connections the phase-shifter is turned through $\pi/2$, so that the reading of the electrometer is a maximum and will be proportional to I_p; with the full-line connections a third reading is taken. Then if d_1, d_2, d_3 be the readings

$$d_1 = 100\,kVR_sI_s \sin\beta,$$
$$d_2 = kVR_pI_p,$$
$$d_3 = 100\,kV(R_sI_s\cos\beta - R_pI_p),$$

where V is the auxiliary voltage, k the electrometer constant and 100 the ratio of T. From these

$$K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p}\,\frac{100d_2}{d_3 - 100d_2}\cos\beta \doteq \frac{R_s}{R_p}\,\frac{100d_2}{d_3 - 100d_2},$$

and

$$\tan\beta = d_1/(d_3 - 100d_2) \doteq \beta.$$

R_s is about 0·4 ohm (maximum burden 2 VA) and causes 280 cm. deflection on the electrometer scale for 1 per cent error in ratio or for 30 minutes of angle; the ratio can be found to within 0·1 per cent and the angle to the nearest minute. Possible sources of error are (i) phase and ratio imperfections in T; (ii) magnetizing current taken by T modifying the drops in R_p and R_s; (iii) effect of stray fields on T; (iv) residual inductances of R_p and R_s. It is shown that in general the errors arising from these factors are negligible.

6. **M.M.F. balance methods.** The two methods now to be described introduce an entirely new principle, namely, that of passing the primary and secondary currents through the

indings of an auxiliary transformer in which the primary nd secondary ampere-turns are approximately balanced and heir ratio compared with the aid of a dynamometer detector.

In Baker's method* the primary and secondary currents are compared by passing them through separate windings arranged upon a laminated iron "test-ring" and finding the resultant magneto-motive force that they set up. Referring to Fig. 221 (a) the m.m.f. of the primary and secondary currents are arranged to act in approximate opposition round the test-ring,

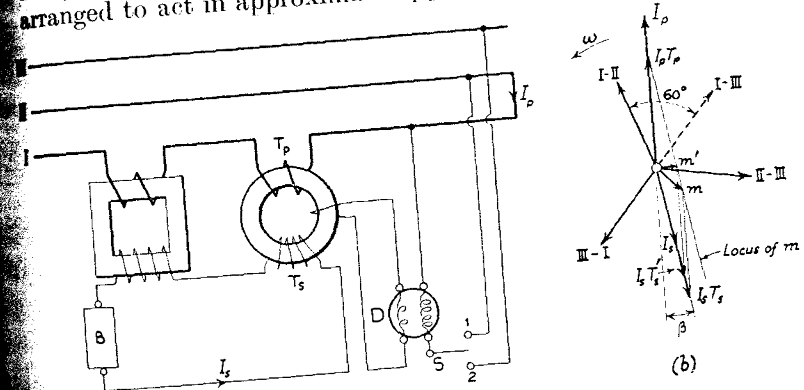


FIG. 221. BAKER'S TEST-RING METHOD

the ratio of the turns T_p and T_s in the windings being chosen so that I_pT_p is nearly equal to I_sT_s; a small flux is set up in the ring by the resultant m.m.f. and this flux is linked with a tertiary winding connected to one of the coils of a dynamometer D. The other coil of D can be excited at will by the voltages between lines I and II or I and III of a three-phase supply. In the vector diagram, Fig. 221 (b), the voltages between the pairs of lines are shown in relation to the primary and secondary currents of the transformer. The resultant ampere-turns on the ring will be m, this being responsible for the flux in the ring and for the voltage applied to the dynamometer coil. A reading of D is taken with S first on contact 1 and then on 2; these readings are proportional to the components of m along the directions of the voltages I–II and I–III respectively, i.e. along two axes separated by 60°. These two observations are

* H. S. Baker, "Current ratio and phase-angle tests of series transformers," Elec. World, vol. 57, pp. 234–235 (1911); "Current transformer ratio and phase-error by test ring method," Proc. Amer. I.E.E., vol. 37, pp. 1173–1183 (1918). See also F. B. Silsbee, loc. cit. on p. 424 (1924).

repeated for two or three other known values of T_s. Since I_pT_p is constant and I_sT_s is in the direction of I_s it follows that the locus of m is a straight line parallel to I_s through the extremity of I_pT_p. Since the values of I_pT_p, I_sT_s and the components of m are known, the locus can be set out on a sheet of paper and the diagram drawn. From this locus it is easy to interpolate the value of T_s, say T_s', that would make m perpendicular to I_p; then from the geometry of the diagram

$$I_sT_s' \cos\beta = I_pT_p,$$
$$m' = I_pT_p \tan\beta = I_sT_s' \sin\beta,$$

where m' is the value of m measured from the locus when normal to I_p. From these,

$$K_c = I_p/I_s = (T_s'/T_p)\cos\beta \fallingdotseq T_s'/T_p,$$

and

$$\tan\beta = m'/I_pT_p \fallingdotseq \beta,$$

or

$$\sin\beta = m'/I_sT_s' \fallingdotseq \beta.$$

The method gives results in excellent agreement with those obtained by other methods, it is very flexible in its application to a wide range of currents with a small amount of apparatus, and is very useful for the testing of transformers on site.

In Baker's apparatus the ring is built up of laminations 8 in. inside diameter to a section of 2 in. × 2 in.; the flux density in it is about 100 lines per sq. in. The dynamometer tertiary winding consists of 196 turns wound uniformly over the ring. The secondary winding consists of four coils of 40 turns each, two coils of 20 turns each and one coil of 10 turns, all being uniformly distributed; in addition a further distributed coil of 20 turns tapped at every turn is provided. Any desired value of T_s from 1 to 230 turns can be arranged. The primary consists of a number of U-shaped copper loops spaced out uniformly round the ring and grouped in series, series-parallel or parallel arrangement according to the value of the primary current. The dynamometer is preferably of the zero pattern so that readings are always taken in a position of constant reactance; the winding connected to the ring should be of low resistance. This serves to minimize the burden imposed on the test transformer by keeping the e.m.f. required in the tertiary winding of the ring down to a low value.

Another method of this type has recently been introduced by Bruges,* the circuit being shown in Fig. 222. The primary and secondary windings of the transformer to be tested are connected in series with those of a *balancing transformer* of

* W. E. Bruges, "A method of testing current transformers," *Journal I.E.E.*, vol. 68, pp. 305-307 (1930); also A. G. Powell, *Elec. Rev.*, vol. 113, p. 448 (1933).

pproximately the same ratio. A phase-shifter is connected o the secondary terminals of the balancing transformer, or o a tapping, or to a separate winding as shown in the diagram. The phase and magnitude of the current I_{ps} supplied by the phase-shifter are adjusted until the flux in the balancing transformer is zero; this is determined by means of a suitable detector, such as a sensitive milliammeter, joined to a fourth winding. The balancing transformer is preferably of ring form, as in Baker's

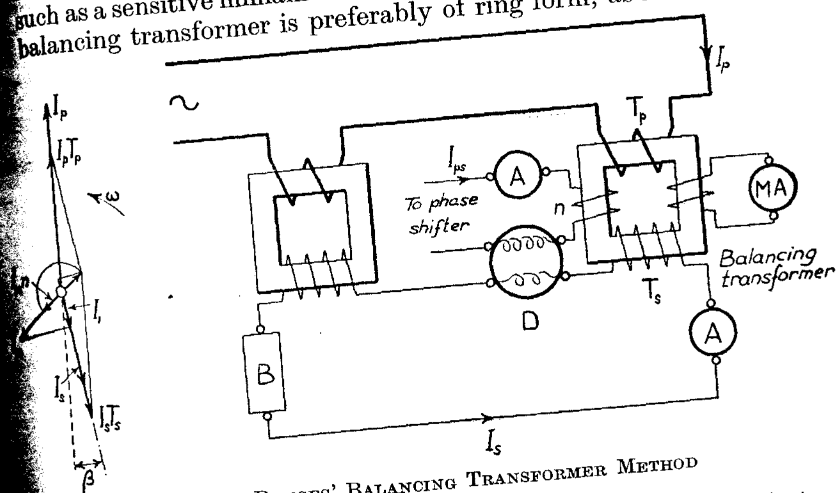


FIG. 222. BRUGES' BALANCING TRANSFORMER METHOD

method, with its several windings uniformly distributed so that magnetic leakage is reduced to a minimum. Ammeters give the values of I_s and I_{ps}, while the reading of the dynamometer D is proportional to the component of I_{ps} which is in phase with I_s. Let this component be denoted by I_1. Referring to the vector diagram, the resultant of the primary and secondary ampere-turns, I_pT_p and I_sT_s, is balanced out by the ampere-turns $I_{ps}n$, n being the number of turns supplied with current by the phase-shifter. Then

$$I_pT_p \cos\beta = I_sT_s + I_1 n,$$

and

$$I_pT_p \sin\beta = n\sqrt{(I_{ps}^2 - I_1^2)},$$

from which

$$K_c = \frac{I_p}{I_s} = \left[\frac{T_s}{T_p} + \frac{I_1}{I_s}\cdot\frac{n}{T_p}\right]\frac{1}{\cos\beta} \fallingdotseq \frac{T_s}{T_p} + \frac{I_1}{I_s}\cdot\frac{n}{T_p}$$

and

$$\tan\beta = \frac{n\sqrt{(I_{ps}^2 - I_1^2)}}{I_sT_s + I_1 n}.$$

Since the total ampere-turns acting on the core of the balancing transformer is zero, the core carries no flux; hence the secondary winding of the ring is a non-inductive burden on the secondary of the transformer under test.

7. **Use of phase-meter.** Among the less usual methods of measuring the phase-angle of a current transformer may be mentioned the use of a phase-meter. For this purpose the ordinary type, with three equal voltage coils mounted on a common spindle and supplied from a three-phase network while the fixed current coil is connected in one line, is not suitable on account of its low sensitiveness for small angular phase-displacements. To secure sufficient magnification Gifford* has made a phase-meter in which the voltage coils are provided with unequal numbers of turns so that their rotating field is elliptic instead of circular. The current coil reacts with the major axis of the elliptic field and produces a torque which, by making the field of suitable ratio of major to minor strength, can be made large for small angular phase differences. The current coil is switched successively from the primary to the secondary circuit of the transformer and measures directly the phase-difference between the currents therein. Though capable of good results and economy of time in making a test, the instrument has not come into regular use; no measure of the transformer ratio is provided.

8. **Use of the oscillograph.** The oscillograph has occasionally been used† to find the ratio and phase-angle of a current transformer by taking simultaneous oscillograms of the primary and secondary currents. The ratio can be found in this way with some certainty, but it is almost impossible to measure the phase-angle upon an oscillogram with any accuracy, on account of the smallness of the angle and the relatively broad lines by which the wave-form is traced out. While the method is of interest, it is of little practical use, since the results are of very low accuracy; the oscillograph cannot be considered for this purpose an instrument of precision.

* R. D. Gifford, "A method of determining the phase-angle of current and potential transformers," *Elecn.*, vol. 75, pp. 166–167 (1915).

† See for example, E. Bennett, "A milliampere current transformer," *Proc. Amer. I.E.E.*, vol. 33, pp. 625–639 (1914).

CHAPTER XX

ABSOLUTE NULL METHODS FOR THE MEASUREMENT OF RATIO AND PHASE-ANGLE ERRORS

1. **Introductory.** The methods discussed in the preceding chapter have the common characteristic that the ratio error and phase-angle are deduced from the readings of certain instruments of which one, at least, is a separately-excited dynamometer (or an electrometer used in an analogous way) combined with an auxiliary a.c. supply, of which the phase can be varied relatively to that providing the current for the transformer under test. Initially, it is necessary in some cases to reduce the reading of one instrument to zero, but the essential point is that all the methods are *deflectional*, the desired quantities being computed from instrument readings.

In the methods now to be examined the distinguishing feature is that the reading of a detecting instrument is brought to zero by appropriate circuit adjustments; they may, therefore, be described as *null* methods. The ratio error and phase-angle are then calculated from the circuit parameters appropriate to the null condition and not from instrument deflections.

The null methods are of two main types. In the first class the detector is a separately-excited dynamometer which, though giving a zero reading, carries a current in its detecting coil; such a method is properly a *null-reading* method, this condition being effected by the aid of resistance adjustments in the detector circuit and regulation of the phase-shifter supplying the excitation for the dynamometer. In the second, and much more numerous, class, the detector is usually a vibration galvanometer in which a null reading essentially means zero current through the instrument; such a method is a *null-current* method. This second class is conveniently divided into two sub-classes. In the first of these the null current condition is attained by the use of resistance adjustments and a phase-shifter operated in conjunction with an auxiliary dynamometer. In the second sub-class the null condition is obtained by the adjustment of resistances and reactances in the detector circuit, no dynamometer or phase-shifter being necessary; these methods may appropriately be termed *bridge methods* since they closely resemble in principle and technique the well-known alternating current bridge methods. It may be remarked that the bridge methods are the most numerous and also the most important in modern testing practice.

This preliminary general classification will be of assistance in following the detailed descriptions of the various methods treated in this chapter.

2. **Methods with one or two dynamometers and phase-shifter.** A few methods have been devised in which the ratio error and

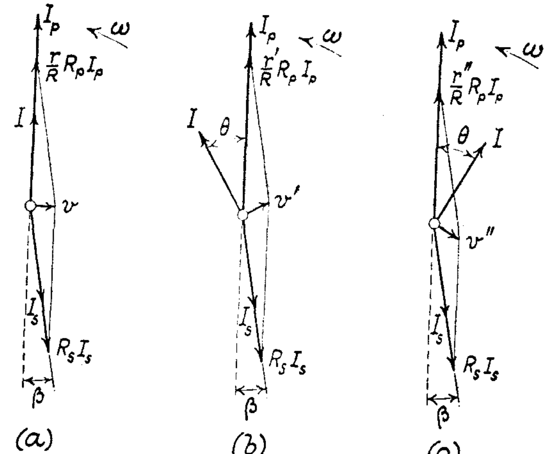


FIG. 223. VECTOR DIAGRAMS FOR FIG. 218 USED AS A NULL METHOD

phase-angle are found from the conditions which give a zero reading upon one or more dynamometers connected in the measuring circuit. These methods are rather slow to use, since there is usually a number of adjustments to be made; although they are now superseded by more direct processes they have a considerable technical and historical interest. In all the methods the zero reading is attained by adjusting the currents in the two elements of the dynamometer to be in quadrature; they are, therefore, null-reading methods since the coils of the instrument carry current even in the null condition.

Barbagelata has described, in the paper frequently cited, a deflectional single-dynamometer method, illustrated in Fig. 218 (a) and described on p. 438, that readily lends itself to null operation. Referring to the circuit diagram, I is first adjusted to be in phase with I_p and r is varied until D reads

ro, so that the resultant voltage v acting on the voltage rcuit—which is assumed to be non-reactive—is in quadrature ith I as in Fig. 223 (a). The phase of I is then shifted through n angle θ in advance of I_p, r being altered to a value r' until he resultant voltage v' is normal to the advanced I and D gain reads zero, as shown by Fig. 223 (b). Finally, I is retarded y an angle θ on I_p, r being changed to r'' so that v'' and I are once more in quadrature and null reading is secured, as in Fig. 223 (c). From the geometry of these vector diagrams,

$$(r/R)\,R_pI_p = R_sI_s\cos\beta,$$
$$(r'/R)\,R_pI_p\cos\theta = R_sI_s\cos(\theta-\beta),$$
$$(r''/R)\,R_pI_p\cos\theta = R_sI_s\cos(\theta+\beta).$$

From the first,

$$K_c = I_p/I_s = (R/r)\,(R_s/R_p)\cos\beta \doteq (R/r)\,(R_s/R_p).$$

Subtracting the third from the second and dividing by the first gives

$$\tan\beta = (r'-r'')/2r\tan\theta \doteq \beta.$$

In particular, if $\theta = 45°$,

$$\tan\beta = (r'-r'')/2r \doteq \beta.$$

A second method, resembling the deflectional two-dynamometer method of Agnew and Fitch shown in Fig. 213 (b), is due to Laws* and is illustrated in Fig. 224 (a). It differs from the method just described in that the balancing adjustments are made in the secondary circuit instead of in the primary, while two dynamometers are used, one as a detector and the other to show when the phase of the auxiliary supply has been adjusted to the desired position. The purpose of D_2 is to enable I to be set in phase with I_p, and so that this setting can be made with the greatest precision a condenser is put into the volt-circuit of this dynamometer to shift the phase of the current therein through a quarter-period relative to the voltage applied to that circuit. The desired adjustment can then be secured with a zero reading of D_2 instead of at a maximum reading, with resulting increase in precision. When the switch S is in position 1 the voltage applied to the voltage circuit of D_2 is rI and the current in the voltage-coil leads by nearly

* F. A. Laws, "Determination of constants of instrument transformers," *Elec. World*, vol. 55, pp. 223–224 (1910).

$\pi/2$ thereon. The slight lag of the current in the current-co relative to rI can be adjusted by the shunt s until the current

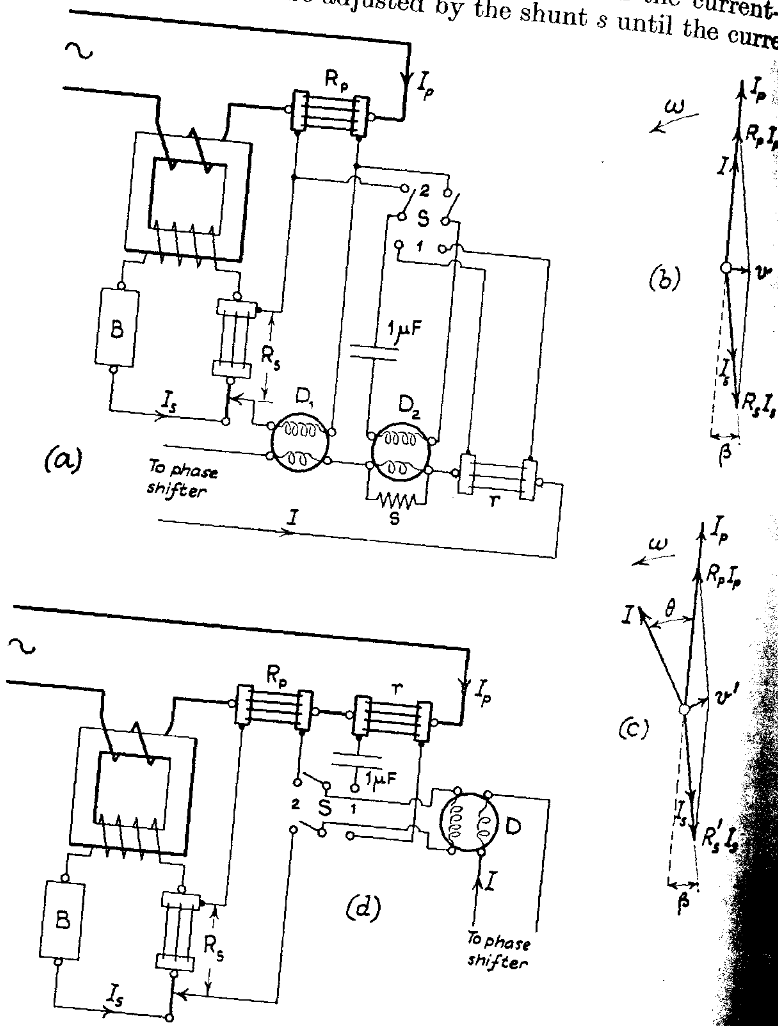


FIG. 224. LAWS' NULL DYNAMOMETER METHODS

in the voltage and current coils of this dynamometer are in quadrature and the instrument reads zero. The switch S is then put into the position 2, so that the voltage applied to the voltage circuit of D. is R_pI_p, the phase-shifter being regulated

educe the deflection to zero; then I and I_p are in phase. s now possible by regulation of R_s—which includes a low-stance slide-wire or consists of a four-terminal resistor nted by a plug box—to reduce the deflection of D_1 to zero, vector relations being shown in Fig. 224 (b). From this

$$R_pI_p = R_sI_s \cos\beta,$$

d

$$K_c = (R_s/R_p)\cos\beta \doteqdot R_s/R_p.$$

ws does not state how to find β; this can readily be done advancing the phase of I through a known angle θ and adjusting R_s to a value R_s' to make the reading of D_1 again ro, as indicated by the vectors of Fig. 224 (c). Then,

$$R_pI_p \cos\theta = R_s'I_s \cos(\theta - \beta).$$

From the two relationships

$$\tan\beta = (R_s - R_s')/R_s' \tan\theta \doteqdot \beta$$

r for $\theta = 45°$,

$$\tan\beta = (R_s - R_s')/R_s' \doteqdot \beta.$$

Since four separate adjustments are required the method is slow o use; Laws has described the following modification in which greater speed of working can be attained, one dynamometer only being used. Referring to Fig. 224 (d), the switch S s put to the right and the phase-shifter adjusted to make D ead zero, so that I is in phase with I_p. With S to the left R_s s adjusted to restore D to zero; then as before

$$R_pI_p = R_sI_s \cos\beta.$$

The current I is then advanced by θ and R_s altered to give zero deflection, so that

$$R_pI_p \cos\theta = R_s'I_s \cos(\theta - \beta).$$

Thus only three settings are needed. The dynamometer should be of a sensitive reflecting pattern; Laws has constructed a special instrument of the string-galvanometer type, details of the construction of which are given in his paper.

3. **Method with dynamometer, vibration galvanometer, and phase-shifter.** In the method now to be described the detector is a vibration galvanometer, in which the null condition occurs with zero current in the instrument. This condition is attained by resistance adjustments combined with the use of an auxiliary phase-shifter and a dynamometer. The method is the

sole representative of the first sub-class of the null-curr methods.

The circuit in Fig. 225 (*a*) shows the arrangement, du de la Gorce,* used in the Laboratoire Central de l'Electricit In principle, the resultant of R_pI_p and R_sI_s is balanced in magnitude and phase by the voltage drop through a resistance s carrying a current I, balance being indicated by zero deflec- tion of a vibration galvanometer VG. Neglecting the reactance of the voltage circuit of D_p it is clear that the current therein

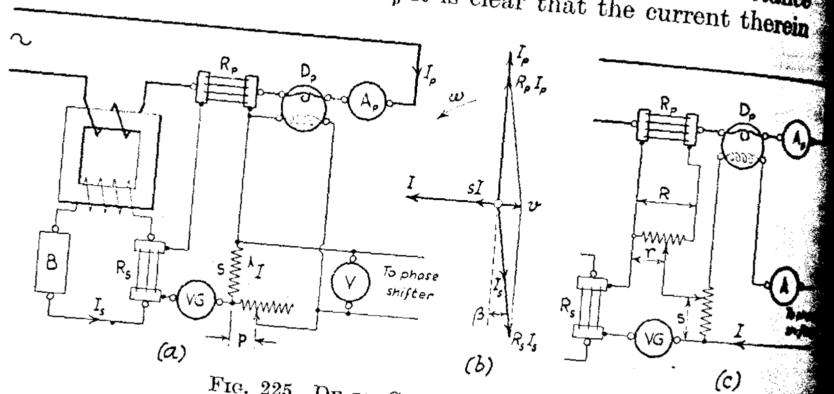


Fig. 225. De le Gorce's Null Method

will be in phase with I; if now the phase-shifter is adjusted until D_p reads zero, I and I_p will be in quadrature, as shown in Fig. 225 (*b*). By altering either R_p or R_s and s or p until the vibration galvanometer is undeflected, then v, the resultant of R_pI_p and R_sI_s, will be equal and opposite to sI so that

$$R_sI_s \cos\beta = R_pI_p,$$
$$R_sI_s \sin\beta = sI.$$

If V be the reading of the voltmeter, then $I = V/(s + p)$ an

$$K_c = I_p/I_s = (R_s/R_p)\cos\beta \doteqdot R_s/R_p,$$

while

$$\tan\beta = Vs/R_pI_p\,(s + p) \doteqdot \beta.$$

A simple modification, shown in Fig. 225 (*c*), avoids th use of variable low resistors by the artifice of shunting R_p wit a resistance box R provided with a travelling contact by mea

* P. de la Gorce, "Note sur l'essai des transformateurs de mesure d courant au Laboratoire Central de l'Electricité," *Lum. Élect.*, vol. 34, p. 30 (1916); *Bull. Soc. Int. des Élecns.*, vol. 6, pp. 299–307, (1916); "Phase la in current transformers," *Elecn.*, vol. 78, pp. 463–465 (1917).

which a tapping r/R can be obtained; the resistance s has imilar sliding contact. Barbagelata* has shown that any r due to voltage-circuit reactance may be eliminated by asuring I instead of V. Balance is secured by adjustments of phase-shifter, r and s until D_p and VG simultaneously read o; then

$$R_sI_s \cos\beta = [rR_p/(R + R_p)]I_p$$
$$R_sI_s \sin\beta = sI,$$

om which

$$K_c = \frac{I_p}{I_s} = \frac{R + R_p}{r}\cdot\frac{R_s}{R_p}\cos\beta \doteqdot \frac{R + R_p}{r}\cdot\frac{R_s}{R_p} \doteqdot \frac{R}{r}\cdot\frac{R_s}{R_p};$$

and also

$$\tan\beta = \frac{s}{R_p}\cdot\frac{R + R_p}{r}\cdot\frac{I}{I_p} \doteqdot \frac{s}{R_p}\cdot\frac{R}{r}\cdot\frac{I}{I_p},$$

since R is usually large in comparison with R_p. Note that in the first circuit the drops in R_p and R_s are nearly equal, whereas in the modification the drop in R_p must exceed that in R_s in approximately the ratio R/r. The method is easy to manipulate and enables K_c to be found within 0·1 per cent and β to within one minute of arc.

4. **Bridge methods. Preliminary remarks.** We come now to the most important sub-class of the null-current methods, in which two approximately equal and opposite voltages are impressed upon the detector circuit, the voltages being propor- tional to the primary and secondary currents respectively; their small resultant is balanced out by the introduction of a suitable auxiliary voltage provided by the potential drop across an appropriate impedance inserted in the detector network.

The principle of all methods in this class is easily understood by reference to Fig. 226, which shows an arrangement that is very often employed. Four-terminal resistors R_p and R_s are inserted in the primary and secondary circuits respectively, one or other of the resistors being adjustable. The connections to the potential terminals are made in such a way that the volt-drops are in opposition, the resultant being impressed on a vibration galvanometer. If I_p and I_s were exactly in oppo- sition of phase it would be possible to reduce the galvanometer reading to zero by adjustment of the resistors until $R_pI_p = R_sI_s$. Since, however, I_s is in defect of opposition by the

* Barbagelata, loc. cit. on p. 426 (1921).

small angle β it is clear that such a resistance adjustment would only succeed in reducing the galvanometer reading to a minimum value. In order to secure a true balance it is essential to introduce by means of some suitable reactive device an electromotive force in quadrature with one of the resistance drops so that the angle β can be compensated and a null indication obtained on the detector. This phase-compensation can be obtained in a variety of different ways, some of greater practical interest than others, the desired quadrature component being provided by self inductance, mutual inductance, or capacitance.

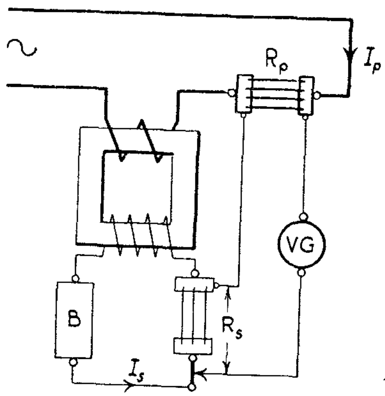


FIG. 226. PRINCIPLE OF THE BRIDGE METHOD

In another typical arrangement the voltages proportional to the primary and secondary currents are obtained by inserting two mutual inductors in place of the two four-terminal resistors shown in Fig. 226, the secondary voltages of the mutuals being approximately equal and opposite. The phase-compensation is effected in this case by the insertion of resistance in either the primary or the secondary circuit of the transformer, the drop of voltage in the resistance being combined with the resultant voltage obtained from the secondaries of the mutual inductors to give zero voltage across the terminals of the galvanometer. It is clear that this arrangement is identical with the preceding, but with all phase relations altered by a quarter-period.

All these methods may be regarded as analogous to alternating current bridges, adapted to the special purpose of measuring the ratio of two currents and the small phase difference between them, since balance is secured by means of a suitable network of properly adjusted impedances with two points bridged by a vibration galvanometer. We shall, accordingly, refer to them as bridge methods and examine their various characteristics in the following Sections.

It is necessary to refer back to the conventions established on p. 35 for the positive direction to be given to currents in parts of a network. In working out the balance conditions

r these bridge circuits we shall use Maxwell's principle of yclic mesh currents, and the positive sense of such currents ound any mesh in the network will be taken as clockwise. t will be understood, however, that the positive direction ound a mesh may not represent the instantaneous sense of he current actually flowing in that mesh. Such instantaneous irections, which are important in settling the relative polarity that must be satisfied by different parts of the network for

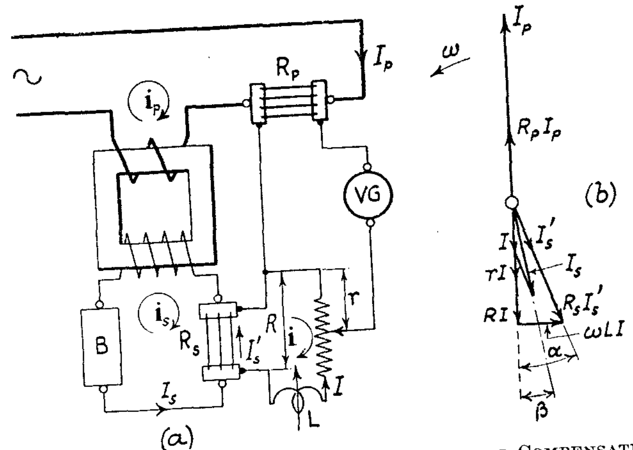


FIG. 227. BRIDGE METHOD WITH SELF-INDUCTANCE COMPENSATION FOR β (SHARP)

balance to be possible, are indicated by an arrow head upon the conductors in question.

5. **Bridge methods with two resistances. Compensation by self inductance.** The requisite phase compensation of the simple bridge method of Fig. 226 can be effected by inserting* a small variable self-inductor as shown in Fig. 227 (*a*), balance being attained by successive adjustment of r and L until VG remains undeflected. The resistor R conveniently consists in part of a slide wire, and inclusive of the small resistance of the variable self-inductor totals 100 ohms. Assuming balance to have been attained, the voltage R_pI_p is equal and opposite to the value of rI; hence I is in opposition to I_p as shown in Fig. 227 (*b*). The secondary current I_s is compounded of I_s', the current in R_s, and I. The voltage drop R_sI_s' over R_s is equal to that over the compensating circuit, the components being RI

* C. H. Sharp, *Trans. Amer. I.E.E.*, vol. 28, pp. 1040–1052 (1910).

and ωLI respectively in phase and in quadrature with I. From the geometry of the current triangle

$$I_s \cos\beta = I + I_s' \cos\alpha,$$
$$I_s \sin\beta = I_s' \sin\alpha;$$

while the balance condition makes

$$R_p I_p = rI \text{ numerically.}$$

But $\cos\alpha = RI/R_s I_s'$ and $\sin\alpha = \omega LI/R_s I_s'$, so that

$$I_s \cos\beta = I[1 + (R/R_s)],$$

and

$$I_s \sin\beta = (\omega L/R_s)I.$$

Using the balance condition with the first of these gives

$$K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p}\cdot\frac{r}{(R+R_s)}\cdot\cos\beta \doteq \frac{R_s}{R_p}\cdot\frac{r}{(R+R_s)} \doteq \frac{R_s}{R_p}\cdot\frac{r}{R};$$

while division gives

$$\tan\beta = \omega L/(R + R_s) \doteq \omega L/R.$$

These conditions are more concisely deduced from the vector equations, $R_s(\boldsymbol{i}_s - \boldsymbol{i}) = (R + j\omega L)\boldsymbol{i}$, $R_p\boldsymbol{i}_p + r\boldsymbol{i} = 0$, which are equivalent to the vector diagram. Eliminating $\boldsymbol{i}$ and then taking the quotient of the currents gives

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = -\frac{R_s}{R_p}\cdot\frac{r}{(R+R_s)+j\omega L}$$
$$= -\frac{R_s}{R_p}\cdot\frac{r}{(R+R_s)^2+\omega^2L^2}[(R+R_s)-j\omega L] = -K_c\varepsilon^{-j\beta},$$

from p. xxiv. The ratio of the current amplitudes is

$$K_c = \frac{R_s}{R_p}\cdot\frac{r}{[(R+R_s)^2+\omega^2L^2]^{\frac{1}{2}}} \equiv \frac{R_s}{R_p}\cdot\frac{r}{(R+R_s)}\cdot\cos\beta$$

and the primary current lags relatively to the secondary reversed by an angle $\arctan[\omega L/(R + R_s)] = \beta$.

It should be observed that for balance it is necessary to have R_pI_p less than R_sI_s; hence the secondary resistance R_s must be chosen to give a drop of voltage in excess of that over the primary resistance, and there may thus be some danger of adding an excessive secondary burden. This defect may be overcome by the artifice introduced by de la Gorce* in which

* P. de la Gorce, "L'étalonnement des transformateurs de mesure au Laboratoire Central de l'Électricité," *Bull. Soc. Franç. des Elecns.*, vol. 5, 4th series, pp. 659–665 (1925).

both R_s and R_p are shunted and have approximately equal drops of about 1 volt. Balance is secured by adjustment of r_b and L, the currents at balance being shown in Fig. 228. Using vector notation the balance condition is

$$r\boldsymbol{i}' + r_a\boldsymbol{i}_1 = 0.$$

But the currents $\boldsymbol{i}'$ and $\boldsymbol{i}_1$ are given by

$$\boldsymbol{i}' = \frac{R_s}{R_s + r + r_L + j\omega L}\boldsymbol{i}_s \text{ and } \boldsymbol{i}_1 = \frac{R_p}{R_p + r_a + r_b}\boldsymbol{i}_p,$$

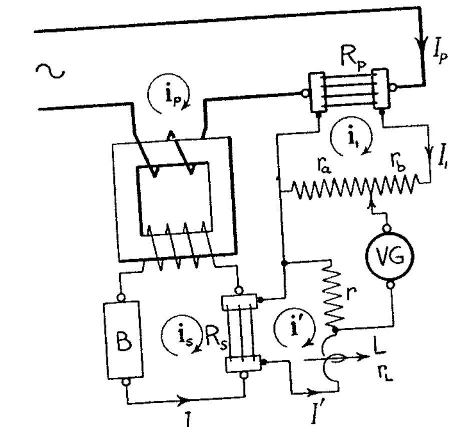


FIG. 228. BRIDGE METHOD WITH SELF-INDUCTANCE COMPENSATION FOR β (DE LA GORCE)

which on substitution in the balance condition and rationalization gives

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = -\frac{R_s}{R_p}\cdot\frac{r}{r_a}\cdot\frac{R_p + r_a + r_b}{(R_s + r + r_L)^2 + \omega^2L^2}[(R_s + r + r_L) - j\omega L];$$

from which

$$\tan\beta = \frac{\omega L}{R_s + r + r_L} \doteq \frac{\omega L}{r + r_L}$$

and

$$K_c = \frac{R_s}{R_p}\cdot\frac{r}{r_a}\cdot\frac{R_p + r_a + r_b}{[(R_s + r + r_L)^2 + \omega^2L^2]^{\frac{1}{2}}}$$
$$= \frac{R_s}{R_p}\cdot\frac{r}{r_a}\cdot\frac{R_p + r_a + r_b}{R_s + r + r_L}\cdot\cos\beta$$

Remembering that β is small, and neglecting R_p, R_s and r_L

$$K_c \doteq \frac{R_s}{R_p} \cdot \frac{r_a + r_b}{r_a}.$$

These self-inductance methods suffer from the defect of susceptibility to stray fields, though this may be minimized by careful arrangement of the circuit and by the use of an astatic self-inductor. The principal defect of the use of self-inductance is that it is only possible to secure balance when β

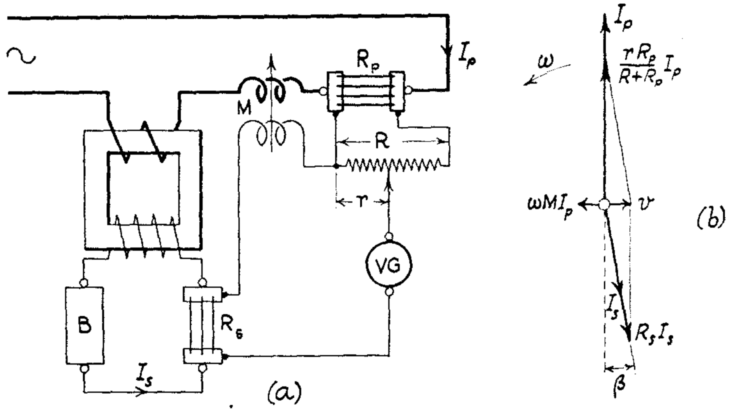


FIG. 229. CAMPBELL'S BRIDGE METHOD WITH MUTUAL INDUCTANCE COMPENSATION FOR β

is a leading or positive angle; if β is a lagging angle it is necessary to replace L by a condenser C and ωL by $-1/\omega C$ in the preceding circuits and analysis.

6. Bridge methods with two resistances. Compensation by mutual inductance between primary and detector circuits. Mutual inductances have the advantage over self inductances that they may be varied down to zero and, if desired, reversed; consequently, phase compensation by means of mutual inductance enables both positive and negative values of β to be measured.

Campbell* has described a method in which mutual inductance links the primary with the detector circuit, as shown in Fig. 229 (a). R_p is a four-terminal low-resistor shunted by a slide-wire of resistance R; balance is secured by adjustment of r

* A. Campbell, "On the use of mutual inductometers," *Proc. Phys. Soc.*, vol. 22, pp. 207–219 (1910). Also see A. Barbagelata, loc. cit. on p. 433 (1921).

and M successively. From the vector diagram of Fig. 229 (b) it is clear that v is balanced out by ωMI_p giving numerically

$$R_sI_s \cos\beta = [rR_p/(R + R_p)]\, I_p$$
$$R_sI_s \sin\beta = \omega MI_p;$$

from which,

$$K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p} \cdot \frac{R + R_p}{r} \cos\beta \doteq \frac{R_s}{R_p} \cdot \frac{R}{r},$$

and

$$\tan\beta = \frac{\omega M(R + R_p)}{rR_p} \doteq \frac{\omega MR}{rR_p},$$

since R_p is small compared with R.

The practical disadvantage of this method is that the primary of the mutual inductor must be capable of carrying the rated primary current of the transformer; hence its utility is limited, since it is not easy to construct standard inductors to carry large currents.

7. Bridge methods with two resistances. Compensation by mutual inductance between secondary and detector circuits. The disadvantage of the preceding method is easily overcome by inserting the mutual inductor into the secondary circuit; its primary winding then never carries more than 5 amperes and a compact piece of apparatus can be readily constructed. The use of mutual inductance in this way was suggested by Sharp,* his circuit being shown in Fig. 230; this is the mutual inductance analogue of the self inductance compensation in Fig. 227 (a). Balance is attained by adjustment of r and M. Little useful purpose would be served by working out the somewhat complex balance conditions since the application of mutual inductance in the secondary circuit can be made in a much simpler way suggested by the same investigator.

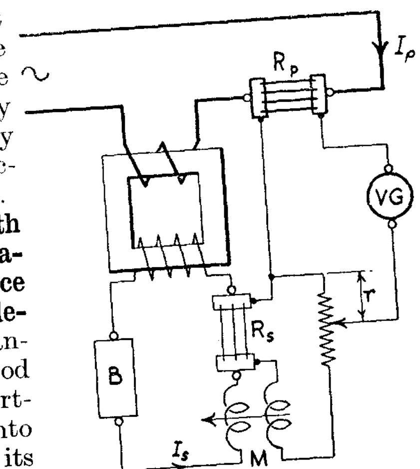


FIG. 230. SHARP'S BRIDGE METHOD WITH MUTUAL INDUCTANCE COMPENSATION FOR β

* C. H. Sharp, loc. cit. on p. 457 (1910).

Sharp's improved method as used at the Bureau of Standards is shown in Fig. 231 (*a*), R_p and R_s being four-terminal resistors of approximately equal volt-drop—about 0·25 to 1 volt—R_s consisting of a fixed resistor in series with a low-resistance slide-wire or of a fixed resistor shunted by a plug box. Balance is secured by adjustment of R_s and M until the galvanometer is undeflected; then, as the vector diagram of Fig. 231 (*b*)

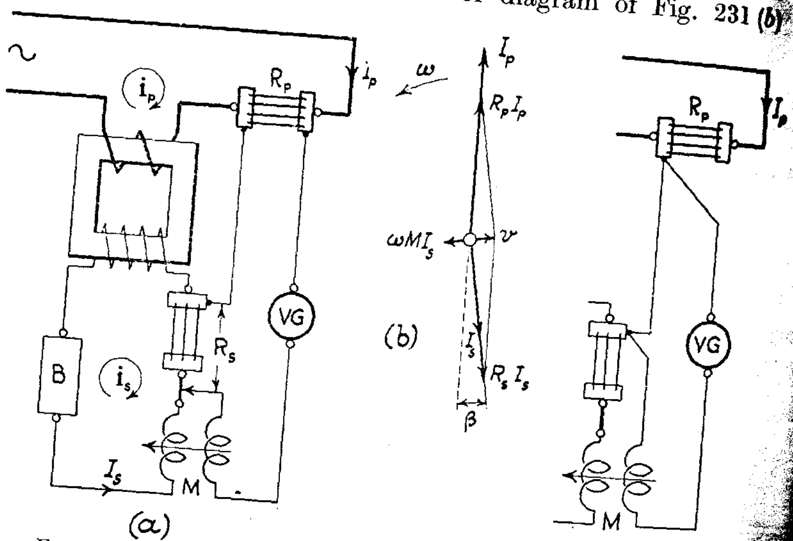


FIG. 231. BUREAU OF STANDARDS BRIDGE METHOD WITH MUTUAL INDUCTANCE COMPENSATION FOR β (SHARP)

shows, the resultant voltage v is in quadrature with I_s and is cancelled by ωMI_s. The diagram gives

$$R_pI_p \cos\beta = R_sI_s,$$

$$R_pI_p \sin\beta = \omega MI_s;$$

so that

$$K_c = I_p/I_s = R_s/R_p \cos\beta \doteqdot R_s/R_p$$

and also

$$\tan\beta = \omega M/R_s.$$

These conditions are readily established from the vector equations. Suppose the positive direction of the current $\boldsymbol{i}_g$ flowing in the detector mesh of impedance z when the bridge is unbalanced to be clockwise;

en with this convention M may be given a positive sign* for two rallel, similarly wound coils. Then

$$z\boldsymbol{i}_g - R_p\boldsymbol{i}_p - R_s\boldsymbol{i}_s + j\omega M\boldsymbol{i}_s = 0,$$

om which when $\boldsymbol{i}_g$ vanishes,

$$\boldsymbol{i}_p/\boldsymbol{i}_s = -(R_s - j\omega M)/R_p = -K_c\epsilon^{-j\beta}$$

he angle between $\boldsymbol{i}_p$ and $-\boldsymbol{i}_s$ is therefore

$$\beta = \arctan(\omega M/R_s), \text{ so that } M \text{ is positive when } \beta \text{ is positive;}$$

he ratio of the current amplitudes is

$$K_c = \sqrt{(R_s^2 + \omega^2M^2)}/R_p = R_s/R_p \cos\beta \doteqdot R_s/R_p$$

The simplicity, speed and high precision of this method make it one of the greatest practical value. Sharp and Crawford† claimed a precision of 1 part in 25 000 when the voltage across R_s was 0·125 volt with the rated current of 5 amperes, the detector used by them being a d.c. galvanometer in conjunction with a synchronous rectifying key. The method has been adopted for routine precision testing of current transformers at the Bureau of Standards,‡ using a vibration galvanometer; in this form the method is widely used in laboratories and test-rooms, where its simplicity, flexibility and almost unlimited range render it very valuable. The burden introduced by R_s and the primary of M is slight, being of the order of 0·1 ohm or 0·5 VA. Since current transformers are usually provided with standard 5 ampere secondaries, a single secondary resistor and mutual inductor serve for all tests. Drysdale§ points out that R_s may conveniently have a value of about 0·04 ohm and be shunted by a plug resistance box; with this value of R_s an inductor giving about 5 microhenries for M suffices to measure phase-angles up to 2° at 50 cycles per sec. A suitable series of primary resistances must be provided to meet the various values of nominal ratio found in practice.

* M is regarded as positive when positive current in one circuit causes positive flux to link the second circuit, i.e. flux in the same sense through the second circuit as would be caused by positive current in that circuit. The choice of positive current directions, with their accompanying positive fluxes is quite arbitrary and may be chosen to suit convenience. For a discussion of this topic see B. Hague, *A.C. Bridge Methods*, 3rd edition, pp. 55–57, 1932.

† C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, vol. 29, pp. 1517–1541 (1911).

‡ P. G. Agnew and F. B. Silsbee, "The testing of instrument transformers," *Trans. Amer. I.E.E.*, vol. 31, pp. 1635–1638 (1912). See also, R. S. J. Spilsbury, loc. cit. on p. 424 (1920); F. B. Silsbee, loc. cit. on p. 424 (1924).

§ C. V. Drysdale, "The testing of current transformers," *Journal Sci. Insts.*, vol. 3, pp. 57–58 (1925).

The method is subject to certain sources of error which must be taken into consideration in precise work. These are (i) effect of residual reactance in R_p and R_s; (ii) effect of impurity in M; (iii) influence of inductive interference due to stray fields; and (iv) capacitance and leakance effects.

(i) *Residual reactance effects.* It has been supposed that the resistors R_p and R_s are non-reactive, whereas actually each will have a small reactance arising from their respective residuals: Let l_p and l_s be the residual inductances of R_p and R_s respectively; note that either or both of these residuals may be negative quantities if capacitive residual effects should be preponderant. The condition for balance* will be

$$(R_p + j\omega l_p)\boldsymbol{i}_p + (R_s + j\omega l_s)\boldsymbol{i}_s - j\omega M\boldsymbol{i}_s = 0,$$

from which

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = -\frac{[R_s - j\omega(M - l_s)]}{R_p + j\omega l_p}.$$

Rationalizing,

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = -\left[\frac{[R_sR_p - \omega^2 l_p(M - l_s)] - j\omega[R_p(M - l_s) + R_s l_p]}{R_p^2 + \omega^2 l_p^2}\right],$$

so that the angle of lag of the primary current relative to the reversed secondary is given by the expression

$$\tan\beta = \frac{\omega[R_p(M - l_s) + R_s l_p]}{R_sR_p - \omega^2 l_p(M - l_s)}.$$

Now l_s, l_p and M are all small quantities, so that very closely their products may be neglected; to a high order of approximation,

$$\tan\beta = (\omega M/R_s) - (\omega l_s/R_s) + (\omega l_p/R_p).$$

There are two conditions that will make no error in the determination of β. The first occurs when $l_s = l_p = 0$, i.e. if the resistors are non-reactive. The second will be when both resistors are inductive or both capacitive and such that their time-constants are equal, i.e. $l_s/R_s = l_p/R_p$. In any other case, if l_s and l_p are known the error due to them is readily allowed for and may become important when testing precision transformers having small values of β.

Turning now to the ratio, the above operator gives

$$K_c = \frac{R_sR_p - \omega^2 l_p(M - l_s)}{(R_p^2 + \omega^2 l_p^2)\cos\beta}.$$

Again neglecting the products of small quantities involving the residuals,

$$K_c \doteqdot R_s/R_p \cos\beta$$

Hence the residuals have a negligible effect on the ratio determination.

(ii) *Impurity effects.* In an ideal mutual inductor the primary current and the secondary voltage are exactly in quadrature, such an

* See for further detailed analysis F. A. Laws, *Electrical Measurements*, pp. 581–583 (1917).

inductor being termed *pure.* In consequence of the effects of earth- and inter-capacities of the windings, of imperfect insulation and of eddy currents the exact quadrature relation may be somewhat disturbed; the mutual inductance is then said to be *impure.* Impurity* depends very considerably on frequency and is much more important at high frequencies than at low. Experience shows that a carefully constructed mutual inductor, with its windings well insulated and not too closely coupled, and in which unnecessary metal parts are avoided, is practically pure at ordinary power frequencies. Consequently the possibility of impurity causing any error in current transformer testing may usually be entirely disregarded.

(iii) *Inductive interference.* Inductive interference, particularly between the primary circuit of the transformer and the windings of the mutual inductor, may be a serious source of error. This can be avoided by the use of a properly designed astatic inductor, such as has been described on p. 374, combined with judicious arrangement of the circuit. The presence of inductive interference is easily detected† by arranging the connections as shown in Fig. 231 (*c*), disturbing them as little as possible from their original positions, setting M to zero, and then passing full primary current. If there is any stray field linking the detector loop this will be rendered evident by a deflection of the galvanometer; this deflection can then be reduced to zero by appropriate alterations in the relative positions of the primary and secondary leads, after which the detector leads may be restored to the potential points and the test of the transformer begun. Stray field troubles will be very slight if the primary, secondary and detector leads are all carefully twinned, so that loops of small area are enclosed by them.

(iv) *Capacitance and leakance effects.* Trouble is sometimes experienced from capacity and leakage currents entering the detector from the primary side of the test circuit. The current I_p is usually supplied from the secondary of a transformer, the primary of which may be at a fairly high potential relative to the test circuit. In order that undesired stray currents shall not find their way into the detector via leakage and capacitive paths it is essential that the power transformer have a high inter-winding leakage resistance and that the test circuit be set up remote from the power supply. To ensure stability of potential, the mid-point of the power transformer primary should be earthed; one side of the detector circuit, e.g. the junction of R_p and R_s, should be earthed also.

The Bureau of Standards uses this method for routine testing of current transformers up to primary currents of 12 000 amperes, with an accuracy of 0·01 per cent in ratio and 0·3 minute in phase-angle. Up to 2 500 amperes primary standard resistors are used. Above 2 500 amperes a standard ring-type multi-range current transformer with a secondary resistor replaces the primary resistor; the transformer is itself calibrated in terms of the resistance standards by grouping its

* See B. Hague, *A.C. Bridge Methods*, 3rd edition, pp. 113–117 (1932).

† I. M. Stein, *Trans. Amer. I.E.E.*, vol. 43, pp. 294–297 (1924).

windings for a current less than 2 500 amperes. A special room has been laid out for current transformer testing* containing all the necessary apparatus. Current is supplied to the room from motor-alternator sets in a distant part of the building; suitable supply transformers in the room deliver the current at the test-benches through leads consisting of a stranded conductor coaxially within a return conductor formed of a copper tube 1·5 in. diameter and $\frac{1}{16}$ in. thick. The rest of the heavy-current circuit is composed of copper slabs, 20 in. wide and $\frac{1}{2}$ in. thick, placed as close together as possible to reduce the stray field. Since the circuit must be opened at one place to receive the transformer under test, an ingenious device is used to reduce the stray field and the impedance of the circuit. The test transformer is enclosed within a "cage," a central vertical bar leading current through the transformer. The return is provided by four parallel vertical bars spaced symmetrically round the transformer in the form of a cage, equidistant from the central conductor.

The primary resistors for currents under 1 000 amperes are designed for air-blast cooling; those for the larger currents up to 2 500 amperes are of tubular oil-immersed construction; both types have been described in Chapter IX, see pp. 342 and 353. All operate with a volt-drop of 0·5 volt at rated current.

The standard transformer, used in conjunction with a secondary resistor R for 5 amperes, is used for currents above 2 500 amperes as shown in Fig. 232. Let K_{cs} be the ratio and β_s the phase-angle of the standard transformer, then (p. xxiv.)

$$\boldsymbol{i} = -(1/K_{cs})(\cos\beta_s + j\sin\beta_s)\boldsymbol{i}_p \doteqdot -(1/K_{cs})(1 + j\beta_s)\boldsymbol{i}_p.$$

Balance occurs when

$$(R + j\omega l)\boldsymbol{i} + (R_s + j\omega l_s)\boldsymbol{i}_s - j\omega M\boldsymbol{i}_s = 0.$$

Substituting for $\boldsymbol{i}$ and neglecting the products of small quantities,

$$-\frac{R}{K_{cs}}\left[1 + j\left(\frac{\omega l}{R} + \beta_s\right)\right]\boldsymbol{i}_p + R_s\left[1 - j\omega\frac{(M - l_s)}{R_s}\right]\boldsymbol{i}_s = 0.$$

In the simple resistance method of Fig. 231 (a) the balance condition is given on p. 464 and may be written as

$$R_p\left(1 + j\frac{\omega l_p}{R_p}\right)\boldsymbol{i}_p + R_s\left[1 - j\omega\frac{(M - l_s)}{R_s}\right]\boldsymbol{i}_s = 0,$$

* F. B. Silsbee, R. L. Smith, N. L. Forman and J. H. Park, "Equipment for testing current transformers," *Bur. Stds., Journal of Res.*, vol. 11, pp. 93–122 (1933).

so that the use of the standard transformer is equivalent to replacing R_p by R/K_{cs}, $\omega l_p/R_p$ by $(\omega l/R) + \beta_s$, and inverting the phase of $\boldsymbol{i}_p$. Making these changes

$$K_c \doteqdot \frac{K_{cs}R_s}{R\cos\beta}$$

$$\text{and } \tan\beta \doteqdot \frac{\omega M}{R_s} - \frac{\omega l_s}{R_s} + \frac{\omega l}{R} + \beta_s \equiv \frac{\omega M}{R_s} - \theta_s + \theta + \beta_s$$

where θ_s, θ are the phase-angles of the two resistors. The

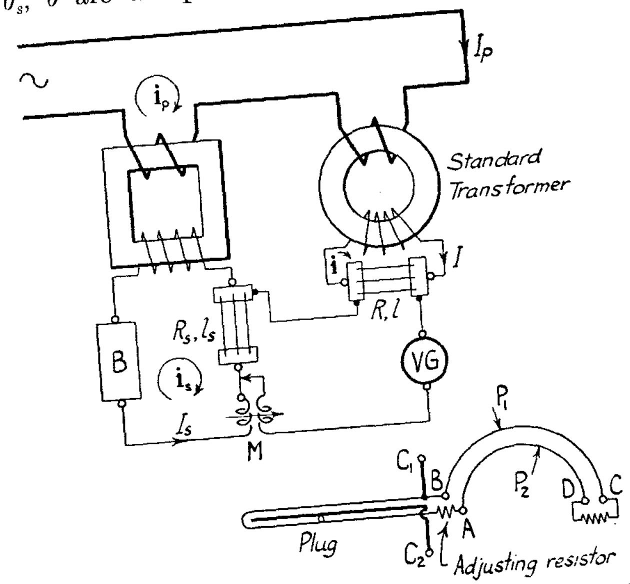
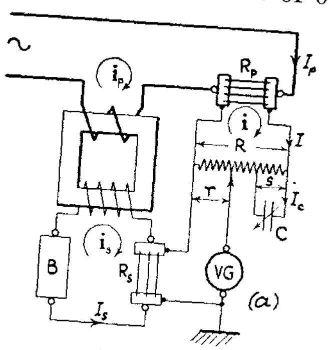


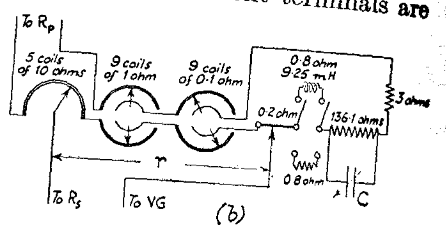
FIG. 232. DETAILS OF BUREAU OF STANDARDS EQUIPMENT

transformer has a silicon-iron ring core 20·7 cm. diameter outside, 19·8 cm. diameter inside, built up to 5·4 cm. thickness from 0·36 mm. stampings. The toroidal secondary is wound in sections to secure perfect insulation and has 2 400 turns capable of numerous uniform distributions, from 480 turns upward by series-parallel groupings. The primary consists of twenty-four fixed copper loops which can be arranged in various ways by the manipulation of a number of bolts and set-screws.

The mutual inductor M is of a modified Brooks and Weaver type with a maximum inductance of 56 μH, comprised of seven fixed sections of 6·6 μH each, seven fixed sections of 1·2 μH each, and a variable inductor giving $-1\ \mu$H to $+1{\cdot}3\ \mu$H. The winding carrying I_s has a resistance of only 0·007 ohm and an inductance of 2 μH.

The resistor R_s is of an interesting type, shown diagrammatically in Fig. 232. AB is a folded strip of resistance material having a total resistance of about 0·12 ohm; BC is a slide wire of 0·1 ohm; CD is a fixed coil of 9·4 ohms; and DA consists of twelve bifilar coils of 0·1 ohm each. Current terminals are





R_p ohms	R_s ohms 0·02	0·024	0·03	0·04	0·05	0·06	0·08
1·0				1/5			2/5
0·2	2·5/5	3/5		5/5		7·5/5	10/5
0·05			15/5	20/5	25/5	30/5	40/5
0·01	50/5	60/5	75/5	100/5	120/5	150/5	200/5
0·002	250/5	300/5		500/5	600/5	750/5	1000/5
0·0005		1200/5	1500/5	2000/5			

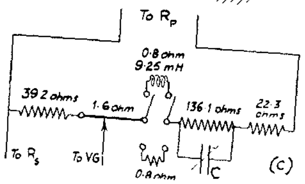


FIG. 233. SCHERING AND ALBERTI'S METHOD

applied to the strip at C_1 and C_2; the latter consists of a copper strip between the two layers of resistance material, and by means of contact blocks and plugs the position of C_2 may be changed in ten steps of 0·01 ohm. Potential contacts are provided at P_1, P_2. If R_c be the resistance plugged on the strip between C_1 and C_2, R_p the resistance tapped between P_1 and P_2 while S is the total resistance of the closed loop, it is not difficult to show that the four-terminal resistance of the device, i.e. the ratio of the p.d. between P_1 and P_2 to the current entering C_1, is R_cR_p/S. Marking C as zero and the middle of DA as 1, the resistance between these points is made 10 ohms, so that each step of DA changes R_cR_p/S by 1 per cent. An adjusting resistance enables S to be made 1000 times the resistance between adjacent plug tappings. The four-terminal resistance of the resistor is then 0·01 ohm per plug step. In

the plug is inserted to make the resistance equal to the product of the primary resistor and the nominal ratio of the transformer under test. Then the settings of P_1, P_2 give the true ratio in terms of the nominal ratio.

8. **Bridge methods with two resistances. Compensation by capacitance.** Standard condensers have the advantage over inductometers that they are not influenced by stray magnetic fields; consequently, their use in a testing network greatly reduces stray field troubles. Schering and Alberti* have introduced an excellent method, shown in Fig. 233 (*a*), which has been in use at the Reichsanstalt for a number of years as the routine method for current transformer testing. It has recently been adopted in a slightly modified form at the National Physical Laboratory for high precision testing, and it probably excels all other methods in simplicity, freedom from troublesome errors, and ease of operation.

Referring to the diagram, R_pI_p is chosen 4 to 8 times greater than R_sI_s and the resistor R_p is shunted by a resistance R of about 200 ohms. The phase of the shunt current I is adjusted by means of a condenser C shunted across a portion s of R; C and the fraction r/R are altered until the drop of voltage rI balances R_sI_s.

The theory of the method is as follows. Let $\boldsymbol{i}_p$, $\boldsymbol{i}_s$, $\boldsymbol{i}$, and $\boldsymbol{i}_c$ be the harmonic vectors representing the various currents; then applying Kirchhoff's rules,

$$r\boldsymbol{i} + R_s\boldsymbol{i}_s = 0,$$

$$(R + R_p)\boldsymbol{i} - R_p\boldsymbol{i}_p - s\boldsymbol{i}_c = 0,$$

and

$$[s + (1/j\omega C)]\boldsymbol{i}_c - s\boldsymbol{i} = 0.$$

Eliminating $\boldsymbol{i}$ and $\boldsymbol{i}_c$

$$\frac{\boldsymbol{i}_p}{\boldsymbol{i}_s} = -\left\{(R + R_p)\frac{R_s}{R_pr} - \frac{R_s}{R_p}\cdot\frac{s}{r}\cdot\frac{\omega^2C^2s^2}{(1+\omega^2C^2s^2)} - j\frac{R_s}{R_pr}\cdot\frac{\omega Cs^2}{(1+\omega^2C^2s^2)}\right\}.$$

$$= -(a - jb) = -K_c\epsilon^{-j\beta}$$

Hence the ratio is

$$K_c = \sqrt{(a^2 + b^2)},$$

and the angle of lead of $\boldsymbol{i}_s$ reversed on $\boldsymbol{i}_p$ is given by

$$\tan\beta = b/a.$$

* H. Schering and E. Alberti, "Eine einfache Methode zur Prüfung von Stromwandlern," *Arch. f. Elekt.*, vol. 2, pp. 263–275 (1914). Also Barbagelata, loc. cit. on p. 433 (1921) and F. Ahrberg *Elekt. Zeits.* vol. 46, p. 500 (1925).

The resulting expressions are rather unwieldy in their general form and will be reverted to later. In the apparatus used by Schering and Alberti it is permissible for most ordinary purposes to neglect $\omega^2C^2s^2$ in comparison with unity; moreover, in the worst possible case the omission of the frequency term only introduces an error of 8 parts in 10 000 in the value of a, and the error is usually much less. We may therefore write, with a high degree of approximation,

$$\frac{i_p}{i_s} \doteqdot - \left\{(R + R_p)\frac{R_s}{R_p r} - j\frac{R_s}{R_p r} \cdot \omega C s^2\right\}.$$

Again, $\omega C s^2$ is small in comparison with $R + R_p$—usually in the proportion of about 6 parts or less in 2 000—and has little effect upon the ratio. Hence, finally,

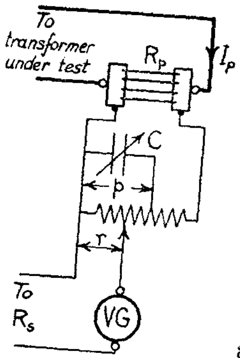


FIG. 234. MODIFICATION OF SCHERING AND ALBERTI'S METHOD FOR NEGATIVE VALUES OF β

$$K_c \doteqdot (R_s/R_p r)\,(R + R_p)$$
$$\tan\beta \doteqdot \omega C s^2/(R + R_p).$$

In most instances R_p is small in comparison with R, so that the simple approximations

$$K_c \doteqdot (R_s/R_p)\,(R/r)$$
$$\tan\beta \doteqdot \omega C s^2/R$$

are frequently sufficiently accurate.

It should be noted that the circuit of Fig. 233 (a) only serves for the measurement of positive or leading values of β. If β lags, Schering and Alberti make the necessary compensation by removing a small part of R between the sections r and s, and inserting a small inductance L having a resistance equal to that of the removed non-inductive portion. Then

$$\tan\beta \doteqdot (\omega C s^2/R) - (\omega L/R).$$

The use of this coil, however, may introduce stray field trouble. Arnold* has recently shown that its use is not necessary, proposing the circuit of Fig. 234. From this it will be seen that the apparatus is the same as in Fig. 233 (a), except that the condenser C has been transferred to the opposite end of p. Working out the theory in a similar manner gives

$$\frac{i_p}{i_s} = - \left\{(R + R_p)\frac{R_s}{R_p r} + j\frac{R_s}{R_p r} \cdot \omega C p(R + R_p - p)\right\}.$$

* A. H. M. Arnold, "Precision testing of current transformers," *Journal I.E.E.*, vol. 68, pp. 898–905 (1930).

reactive term in this operator is small in comparison with resistance term so that

$$K_c \doteqdot \frac{R_s}{R_p r}\,(R + R_p) \doteqdot \frac{R_s}{R_p} \cdot \frac{R}{r}$$

before. The angle of lag of i_s reversed on i_p is given by

$$\beta = -\omega C p + \frac{\omega C p^2}{R + R_p} \doteqdot -\omega C p + \frac{\omega C p^2}{R} \doteqdot -\omega C p\left(1 - \frac{p}{R}\right)$$

is method of dealing with negative phase angles has obvious vantages over the modification suggested for this purpose Schering and Alberti, and it is much to be preferred.

In either case the arrangement can be made to read the angle rectly in minutes. Suppose R to be fixed at 200 ohms. Then s in Fig. 233 (a) is set at $\sqrt{(2 \times 10^8/10\,800)}$ or 136·1 ohms, condenser of 1 μF will balance a leading angle of 100 minutes 50 cycles per sec. At the same frequency an auxiliary inductance of 9·25 mH will account for a lagging angle of 50 minutes. hus with a 1 μF condenser and an auxiliary inductance of 25 mH a range of β from -50 to $+100$ minutes can be overed at 50 cycles per sec. If in Arnold's method for lagging ngles, Fig. 234, p is fixed at 72·8 ohms a condenser of 1 μF ill balance 50 minutes of lag at the same frequency.

The process of attaining balance is very simple. With the nnections of Fig. 233 (a) the vibration galvanometer is unted and C set to zero, r being adjusted until a minimum flection is obtained. If this cannot be secured by rough djustments of r, the voltage drops over the four-terminal sistors are not in opposition; it will be necessary, therefore, reverse the potential connections to R_p and to repeat the ugh adjustment of r, gradually increasing the galvanometer nsitivity. Balance may now be completed by means of the ndenser shunting s and the fine adjustment of r; this will ffice if β is a positive or leading angle. If β is negative it will necessary in Schering and Alberti's method to insert the xiliary inductance and balance by adjustment of C; but is preferable to use Arnold's arrangement of Fig. 234 in hich the inductance is omitted, it being necessary merely to ansfer C to shunt the part p and to balance exactly as for sitive values of β.

The self-contained apparatus designed by Schering and Alberti is own in Fig. 233 (b). The resistance of R is made up of three dials of sistance coils, the first containing five coils of 10 ohms, the second nine

coils of 1 ohm, and the third nine coils of 0·1 ohm; the last two are com-
pensated on the well-known Feussner principle. In series with these
are a 0·2 ohm slide-wire, a fixed resistance of 136·1 ohms across which
the condenser C is shunted, a resistance of 0·8 ohms, and a final resist-
ance of 3 ohms to make up the total value of R to 200 ohms. The
entire set of resistances is contained in a box provided with the neces-
sary switches and terminals. The condenser C is a mica standard of
the decade pattern adjusted to a maximum of 1 μF by steps of 0·001 μF,
the value of 1μF at 50 cycles per second balacing a leading or positive
value of β equal to 100 minutes. To enable negative values of β to be
measured the 0·8 ohm resistance is replaced by a small astatically
wound coil of 0·8 ohm resistance and 9·25 mH inductance, this balancing
a lagging or negative angle of 50 minutes at 50 cycles. Resistance or
inductance can be inserted at will by the use of a change-over switch.
The entire detector circuit should be at some distance, not less than a
yard, from the main current circuits, especially the primary. The cur-
rent leads to R_s must be long enough to allow this condition to be ful-
filled; they should be twisted together and preferably adjusted to
0·1 ohm, representing a non-inductive burden of 2·5 volt-amperes, or
less if desirable. The resistance of the potential leads C, R_s and the
vibration galvanometer is of little importance. The leads from the
potential terminals of R_p to R must be sufficiently long to enable the
detector circuit to be kept far enough from the primary current; since
these leads properly form part of R their resistance must be kept low,
say not more than 0·04 ohm if error is to be negligible when the nominal
value of 200 ohms is assumed. Alternatively, the resistance of these
leads can be allowed for by slight corresponding reduction of the resist-
ances used in the construction of R.*

While a self-contained arrangement, such as has been described, is
of considerable utility it is, unfortunately, expensive to construct. A
much simplified apparatus can, however, be designed on the following
principle. Using the approximate formula for the ratio,

$$K_c \doteqdot \frac{R_s}{R_p} \cdot \frac{R}{r} \doteqdot K_{nc}\left[\frac{R_s}{R_p} \cdot \frac{R}{K_{nc}}\right]\frac{1}{r} = K_{nc}\frac{K}{r},$$

where K_{nc} is the nominal current ratio of the transformer. For each
value of the nominal ratio and each chosen value of R_p it is possibl
to choose R_s so that with $R = 200$ ohms, K has some constant valu
Schering and Alberti choose $K = 40$, so that $R_s = 0{\cdot}2\, K_{nc}R_p$. More
over, in practice the actual ratio seldom differs by more than $\pm$ 2 pe
cent from the nominal value; thus since K is 40 it is only necessar
for r to be variable between 39·2 and 40·8 ohms. These investigato
show that the simple circuit shown in Fig. 233 (*c*) is adequate for mo
practical purposes, the slide-wire being calibrated to read the rati

* Apparatus on this principle is made by Messrs. Hartmann & Braun
See, "Stromwandler-Prüfeinrichtung nach Angaben der P.T.R.," *Arch. f*
tech. Mess., Z224–2 (1932). For simplified arrangements see W. Geyge
"Vereinfachter Messzweig für Strom- und Spannungswandler-Prüfeinricht
ungen. Beitrag zur Bestimmung der Fehlergrössen von Strom- und Spann
ungswandlern mit den Kompensations-Messanordnungen von Schering und
Alberti," *Arch. f. Elekt.*, vol. 27, pp. 567–576 (1933).

directly while C gives a direct measure of the angle. Appropriate values of R_p and R_s are given in the table in Fig. 233 for various nominal ratios frequently occurring in practice; these four-terminal resistors are of bifilar construction for lower currents and of tubular water-cooled design for the higher values, for details see p. 345.

Arnold has described a slightly modified form of Schering and Alberti's simplified apparatus. The resistance R consists of two fixed sections of 48 ohms and 148 ohms respectively, with a 4 ohm slide-wire between them; this allows for a ratio error of $\pm$ 4 per cent and the slide-wire can be calibrated to give the ratio directly. The secondary four-terminal resistor, R_s, is fixed at 0·1 ohm dropping 0·5 volt, and is connected by long twisted leads of 0·1 ohm resistance to the transformer secondary; R_s with its current leads constitutes a non-inductive burden of 5 volt-amperes. The resistance R is provided with two tappings, one at $s = 136{\cdot}1$ ohms from the right-hand end, the other at $p = 72{\cdot}8$ ohms from the left-hand end. With the condenser across s the angle is read directly by the condenser setting, 0·01 μF corresponding with 1 minute of lead; while with the condenser across p the calibration is 0·02 μF per minute of lag, both at 50 cycles per second. The resistance R_p is chosen to suit the value of primary current and has a potential drop of 2 volts, i.e. four times that across R_s; a transformer entirely without ratio error would be balanced therefore at $r = 50$ ohms, the mid-point of the slide-wire. For large primary currents R_p is replaced by a nickel-iron cored transformer, the secondary circuit of which contains a standard four-terminal resistor; details will be discussed a little later. The apparatus is made by H. Tinsley & Co.

For primary currents up to a few hundred amperes air-cooled or oil-immersed resistors may be used for R_p. When currents of 1 000 amperes or more are to be dealt with it is usually necessary to provide the resistors with auxiliary water cooling, as described on p. 342, in order to carry off the considerable quantity of heat developed. Such heavy current resistors have many disadvantages and they are frequently replaced by a suitable high-ratio standard current transformer, the primary of which is connected in series with that of the transformer under test, while its secondary is closed through a four-terminal resistor carrying 5 amperes, in the manner shown in Fig. 235 (*b*). The first published statement of this principle was due to Ahrberg* in 1925, who pointed out that with the silicon-iron cored standard transformers in use at that time it would be necessary to make allowance for their quite appreciable ratio error and phase-angle. To overcome this defect he suggests the use of a Brooks and Holtz two-stage standard transformer (see p. 81) as

* Deutsches Reichspatentamt, Patentschrift Nr. 422 920, 15th Dec., 1925, "Verfahren und Einrichtung zur Prufung von Stromwandlern beispielweise nach der Methode Schering und Alberti." For an abstract of the patent see *E.u.M.*, vol. 45, pp. 373–374 (1927). F. Ahrberg, "Stromwandler-Prüfeinrichtung mit Normalwandler," *Elekt. Zeits.*, vol. 48, pp. 1411–1412 (1927).