voltage transformers, resulting in displacement of the neutral point and in certain peculiar conditions of oscillation and instability. Stigant and Lacey* have described an interesting case of voltage rise due to neutral inversion in two transformers stepping down from 33 000/110 V. The primary windings were connected in "vee" on the primary side and joined to the three-phase supply through three uncoupled single-pole isolating

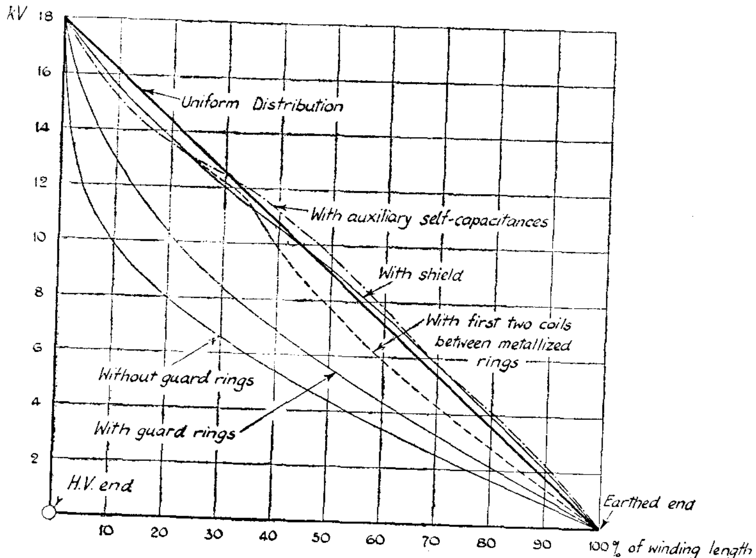


FIG. 152. INITIAL SURGE VOLTAGE DISTRIBUTION WITH VARIOUS METHODS OF PROTECTION

switches; the secondary windings were independent and unearthed. It was found that with one switch closed and the other two open one of the secondaries flashed over to earth and the insulators of the open switches showed an abnormal brush discharge, indicating a considerable rise of voltage. The effect was traced to the establishment of neutral point displacement and the setting up of resonant conditions between the earth-capacitances of the system and the inductance of the transformer windings. The radical cure was found to be the use of coupled isolating links and the connection of the

* S. A. Stigant and H. M. Lacey, "Abnormal voltage rises in e.h.t. potential transformers," *Elec. Times*, vol. 77, pp. 15–18 (1930). A. Boyajian and W. J. Rudge, "Inversion of system resulting from switching operations," *Gen. Elec. Rev.*, vol. 34, pp. 436–439 (1931).

secondary windings in "vee" with their common point solidly earthed. Other observers* have described similar phenomena in three star-star connected voltage transformers with earthed neutral point operating on a system which is otherwise unearthed.

The fundamental principle of these phenomena can be explained by reference to Fig. 153 (*a*) which shows a single-phase, two-wire circuit with equal earth-capacitances C_{ea}, C_{eb} joining the lines A, B to earth. A voltage transformer, represented by an inductance L, is connected between line A and earth; the whole circuit is equivalent to Fig. 153 (*b*); see Appendix VIII. If the inductance draws a negligible current and losses are neglected, the earthed neutral point N lies midway between the potentials of A and B, as shown by Fig. 153 (*c*), and the capacitances take equal currents in leading quadrature with the respective voltages across them. Suppose now the coil to take a certain magnetizing current; then the resultant current of C_{ea} and L in parallel will still be leading but reduced in amount, since the branch AN in Fig. 153 (*b*) has an apparently higher capacitive reactance than before. Consequently AN takes up more than half the line voltage and NB less, so that N is nearer to B than to A as in Fig. 153 (*d*). When the magnetizing current of L and the charging current of C_{ea} are equal, the branch AN behaves like an infinite impedance; the circuit takes zero current and the points B and N in Fig. 153 (*e*) then coincide. If the magnetizing current still further increases, the branch AN assumes an inductive character, the current lags and B moves down below N; the neutral point is thus outside the line voltage diagram, a condition described as *neutral point displacement* or *inversion*,† and illustrated by Fig. 153 (*f*). When the current taken by L is twice that in C_{ea} the resultant inductive impedance of the branch AN will be equal to the capacitive impedance of NB; i.e. the current in the circuit is infinite and lagging. The circuit

* C. T. Weller, "Saturation phenomena in potential transformers," *Elec. Eng.*, vol. 50, pp. 106–109 (1931). A. Boyajian and O. P. McCarty, "Physical nature of neutral instability," *ibid.*, pp. 110–113 (1931). C. W. La Pierre, "Theory of abnormal voltages," *ibid.*, pp. 114–116 (1931). J. R. Mortlock, "Earthed potential transformers on insulated systems," *Elec. Times*, vol. 87, pp. 69–71 (1935).

† It will be understood that the displacement is relative. Since the neutral point is earthed its potential is fixed and the displacement is actually that of the line potentials relative to N. The term *inversion* does not seem very apt; it is liable to confusion with the geometrical process on the one hand and with the idea that something has been turned upside-down on the other.

is then in resonance, A and B in Fig. 153 (f) moving off to infinity below. Thereafter, further increase in the magnetizing current brings these points back from infinity above, since the current in the circuit AB changes from lag to lead as resonance is passed through, until with an infinitely large magnetizing

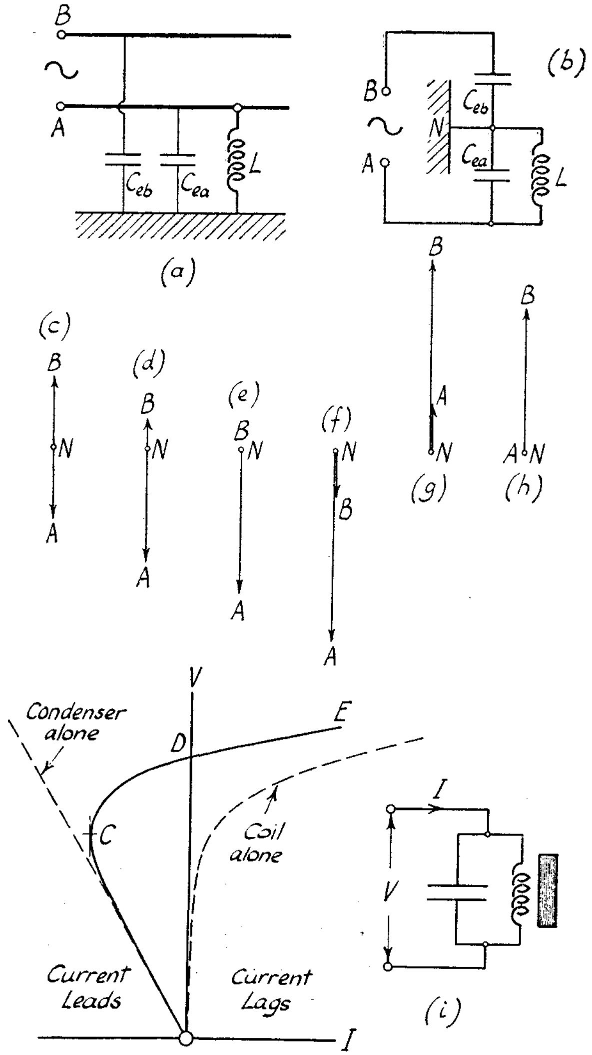


FIG. 153. ILLUSTRATING NEUTRAL-POINT DISPLACEMENT IN A SINGLE-PHASE CIRCUIT

current the line voltage is borne across NB, see Figs. 153 (g) and (h). It follows that inversion will occur for all values of magnetizing current in excess of the charging current of C_{ea} and the voltage to earth of one line at least will rise to not less than the line voltage. High voltages can thus be set up without the necessity for exact resonance, irrespective of whether L is iron-cored or not.

Conditions such as have been described may be established in a single-phase circuit if L is half the primary winding of a voltage transformer joined across the lines and earthed at its mid-point. If the connection to the lines is *via* two uncoupled switches the insertion of one switch only will suffice to reproduce Fig. 153 (a) exactly. If the normal magnetizing current of the transformer is less than the charging current of the line, B moves toward N (Fig. 153 (c)) resulting in over-excitation of the transformer and still further displacement of the diagram. The magnetizing current may attain many times the charging current so that the arrangement passes through resonance and returns to the state of Fig. 153 (h). The circuit is inherently unstable and may be set into a state of persistent and dangerous oscillation. Losses and saturation tend to reduce the amplitude of the resonance voltages and to change their phase, so that the locus of N relative to A and B in the vector diagrams is not the line AB but a closed curve; considerable losses may, indeed, result in a stable non-oscillatory state.

The phenomenon is somewhat modified by saturation, as indicated by Fig. 153 (i). The volt-ampere characteristic of the condenser is the dotted straight line drawn to the left; that of the coil is the saturation curve drawn to the right. The resultant characteristic is the full-line curve and has three regions: OC where the current leads in quadrature on the voltage and increases nearly in linear proportion to it, the circuit being capacitive and in a stable state; CD where the leading current decreases with rising voltage, so that the circuit has an instability somewhat of the same nature as that of an electric arc; DE where the current lags in quadrature on the voltage and increases very rapidly with rise of voltage, the circuit being again in stable equilibrium. At the point D the condenser current and the coil current are equal and opposite and the resultant current is zero; the system is in parallel resonance. It is well-known that the exciting current taken by a transformer on switching-in may be a large multiple of its normal value, according to the point on the voltage wave at which the switch is closed. Consequently, the transformer-earth-capacitance combination may easily be put on the unstable part *CD* or the part *DE* of its characteristic, resulting in inversion and considerable voltage rise. Roughly speaking, the effect of saturation is to reduce the maximum voltages but to increase the range of circuit parameters which may cause inversion, i.e. the probability of inversion occurring is greater with saturation than without.

Turning now to three-phase circuits, Fig. 153 (*a*) may be regarded as representing line-to-neutral conditions for one phase of a star-connected system with insulated star-point. In Fig. 154 (*a*) C_{e1}, C_{e2}, C_{e3} are the equal earth-capacitances of the lines, and L_1, L_2, L_3 are the inductances of the primary windings of three similar voltage transformers joined in star with the neutral point earthed. With all the switches closed and normal conditions, the neutral point is at the centre of figure of the line-voltage triangle, Fig. 154 (*b*); each line is, therefore, at a voltage above earth equal to the line voltage divided by $\sqrt{3}$ and represented by the heavy vectors in the diagram. Suppose now only one switch to be closed, giving the equivalent circuit of Fig. 514 (*c*). Then it is easy to explain, in the same way as for a single-phase circuit, how the voltage triangle will be displaced relative to the neutral point as the magnetizing current of L_1 increases from zero, Fig. 154 (*b*), to an appreciable value less than the charging current of C_{e1}, as in Fig. 154 (*d*). When the currents in L_1 and C_{e1} are equal, Fig. 154 (*e*), the combination carries zero resultant current and behaves as an open circuit; *N* then falls on the line voltage II, III. Further increase in the magnetizing current puts *N* outside the voltage triangle, Fig. 154 (*f*); this is the neutral inversion and the effective impedance of phase I is now inductive. In the absence of losses, when the lagging impedance of the L_1 C_{e1} combination is equal to the leading impedance of C_{e2} and C_{e3} in parallel, true resonance occurs; the total current is infinite and the voltage triangle passes off to infinity above. In this circumstance the current in L_1 is three times the current in C_{e1}. With still larger exciting currents the triangle returns from infinity below, as in Fig. 154 (*g*), and finally for infinite magnetizing currents *N* coincides with the vertex I, Fig. 154 (*h*); in this case the voltage to earth of lines II and III is the line voltage while I is at zero potential. When two switches are closed the argument is similar, only the circuit constants being different.

In an actual case the locus of *N* relative to the voltage

triangle is not the straight line perpendicular to II, III; losses and saturation make the locus a closed curve. A typical case

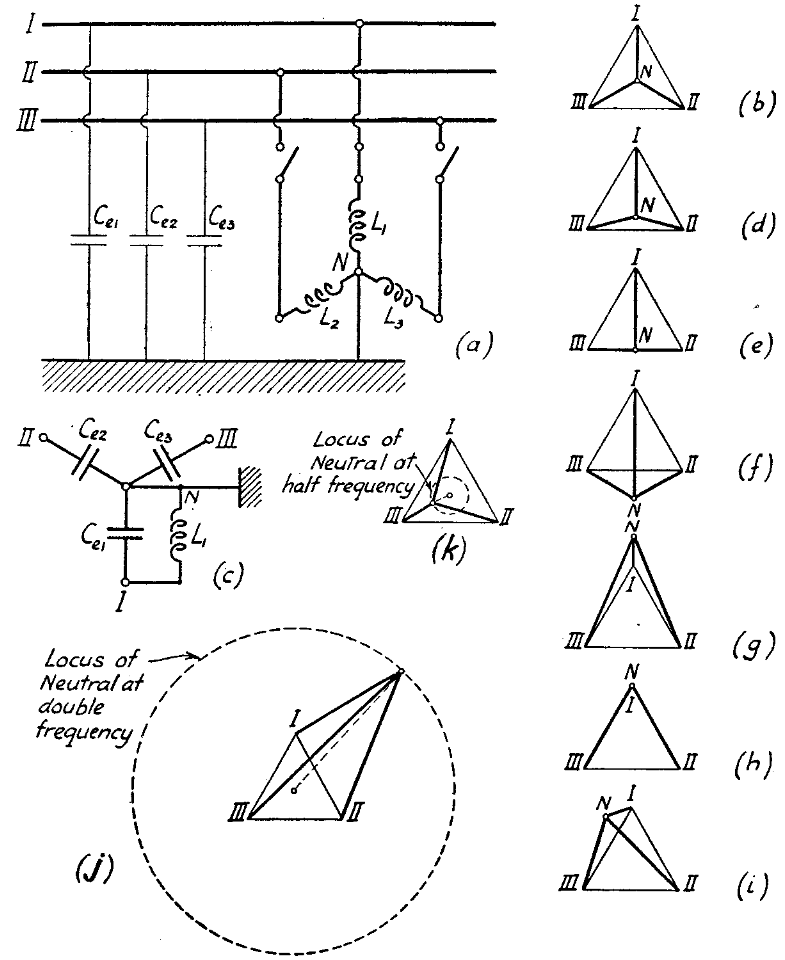


FIG. 154. ILLUSTRATING NEUTRAL POINT DISPLACEMENT IN A THREE-PHASE CIRCUIT

of neutral displacement is shown by Fig. 154 (*i*) where two transformers are subjected to approximately the line voltage while the third transformer has only a small voltage, and that of a very distorted wave-form, applied to it.

As in the case of single-phase circuits not only may there be neutral displacement in the three-phase circuit but the whole range of inversion may be gone through with a continued oscillation of the neutral point with respect to the voltage triangle. Weller (loc. cit. *ante*) has given oscillograms showing two interesting oscillatory conditions. In the first, Fig. 154 (*j*), the neutral point is displaced outside the voltage triangle and oscillates about the mid-point of its figure at double frequency. Approximately equal voltages of distorted wave-shape and about 2·7 times normal value appear on the three transformers, and these voltages are accompanied by equal, sharply-peaked currents of about ten times the rated full-load current in magnitude, indicating a high degree of saturation. Much audible vibration occurs and there is excessive corona. In the second, Fig. 154 (*k*), the neutral point oscillates within the triangle about the mid-point of its figure at half the normal frequency. Equal voltages of about 1·2 times normal value and of impure wave-shape appear on the three transformers, which take equal, peaked exciting currents of about fifteen times the full-load current, indicating a degree of saturation even greater than in the preceding case.*

Overvoltages due to neutral displacement can be limited in a number of ways: (i) By using coupled isolating links and ensuring that all three phases are simultaneously connected to the transformer. (ii) By working the transformers with a low saturation, so that the magnetizing current is small enough for the potentials of the lines to be established relative to earth by the earth-capacitances of the system, i.e. to ensure that the transformers work on the stable part of the volt-ampere characteristic, namely *OC* in Fig. 153 (*i*). (iii) By adding sufficient star-connected resistance burden on the secondary side of the transformers. This has the effect of lifting up the characteristic in Fig. 153 (*i*), so that a given resultant current, i.e. a given degree of inversion, requires a higher voltage for its production. The action is somewhat analogous to the use of swamping resistances in stabilizing an electric arc.

* It will be appreciated that the diagrams of Fig. 154 (*j*) and (*k*) are merely a convenient way of illustrating the phenomena; i.e. the triangle revolves about the centre of figure with velocity ω in order to project the line voltages and the point *N* moves round the locus with velocity 2ω or $\omega/2$ as the case may be. In actuality *N* is at rest, since it is the zero potential point. The triangle must revolve about its centre of figure with velocity 2ω or $\omega/2$ as the case may be, and the line joining *N* to the centre of figure must rotate about *N* with velocity ω.

L06

CHAPTER VI

INSTRUMENT TRANSFORMERS IN POWER OR ENERGY MEASUREMENTS

1. **Introductory.** When instrument transformers are used merely to operate ammeters, voltmeters or similar apparatus, it is only of importance that the ratio error should be sufficiently small, the phase-angle being of no significance. If transformers are used in conjunction with power- or energy-measuring instruments it is essential that the secondary quantities should bear a closely constant ratio to the corresponding primary quantities and at the same time maintain accurate phase relationships; in such cases, therefore, account must be taken of the ratio-error and phase-angle of the transformers. Wattmeters, power relays and watt-hour meters are the most important instruments of this class; the principles involved are the same throughout, though the instruments are necessarily different in construction, theory and application. It will be sufficient to work out the effect upon the ordinary dynamometer wattmeter when connected in circuit *via* instrument transformers; the conclusions will apply with even greater force to the more important watt-hour meter since, on account of its integrative principle, any errors introduced into its operation will be cumulative in their effect upon the readings of the meter. We shall first briefly review the theory of phase-error in a wattmeter unprovided with transformers and then show how the error is affected by the addition of current and voltage transformers, both in single-phase and in three-phase circuits.

2. **Theory of wattmeter without transformers.** It is well-known that the indications of a dynamometer wattmeter may be in error on account of four principal causes—

(i) The voltage-circuit of the instrument is not truly non-reactive.

(ii) The reading of the instrument includes with the power that it is desired to measure either the loss of power in the voltage coil or that in the current coil, according to the method of connection in circuit.

(iii) There is mutual inductance between the current and voltage coils, varying in amount according to the deflection of the moving system relative to the fixed coils.

(iv) There are eddy currents in the current coils, their leads and terminals, etc.

Of these it is our purpose to investigate (i) and (ii); (iii) is usually a small effect and may even be beneficial; (iv) is made as unimportant as possible by careful disposition of the metal parts, stranding of the conductors, etc.

Consider first the arrangement shown in Fig. 155 (*a*), and let the voltage across the terminals of the load, *BC*, be

$$v = v_1 \sin \omega t = (\sqrt{2}) V \sin \omega t,$$

represented by the harmonic vector $\mathbf{v}$ of length $v_1 = (\sqrt{2})V$ projected on the vertical. Let the impedance operator of the load be $z = R + jX$, so that the vector of current in the load is

$$\mathbf{i} = \mathbf{i}_c = \mathbf{v}/z,$$

having an instantaneous value

$$i = i_c = (v_1/Z) \sin (\omega t - \phi) = i_1 \sin (\omega t - \phi)$$
$$= (\sqrt{2}) I \sin (\omega t - \phi),$$

where $Z^2 = R^2 + X^2$, $\phi = \arctan (X/R)$ and I is the r.m.s. current in the load. If $z_c = R_c + j\omega L_c$ is the impedance operator for the current coil and $z_v = R_v + j\omega L_v$ that of the voltage circuit joined to AC, the current in the voltage circuit is the projection of the vector

$$\mathbf{i}_v = \frac{\mathbf{v} + (\mathbf{v}/z)z_c}{z_v} = \frac{z + z_c}{zz_v}\mathbf{v} = \frac{z_{AC}}{zz_v}\mathbf{v},$$

where

$$z_{AC} = (R + R_c) + j(X + \omega L_c) = z + z_c.$$

The instantaneous value of the voltage-circuit current is

$$i_v = (Z_{AC}/ZZ_v) v_1 \sin (\omega t + \phi_{AC} - \phi - \alpha),$$

where $Z_{AC}^2 = (R + R_c)^2 + (X + \omega L_c)^2$, $Z_v^2 = R_v^2 + \omega^2 L_v^2$,

ϕ_{AC} = phase-angle of load plus current coil $= \arctan (X + \omega L_c)/(R + R_c)$,

ϕ = phase-angle of load $= \arctan (X/R)$,

α = phase-angle of voltage circuit $= \arctan (\omega L_v/R_v)$.

The average torque on the moving system over a period of alternation, T seconds, is

$$\frac{a}{T}\int_0^T i_v\, i_c\, dt$$

where a is a factor depending on the design of the coil system. If the torque is resisted by a uniform spring control, the angular twist on which is θ radians and c is the torque per radian,

$$c\,\theta = \frac{a}{T}\int_0^T i_v\, i_c\, dt.$$

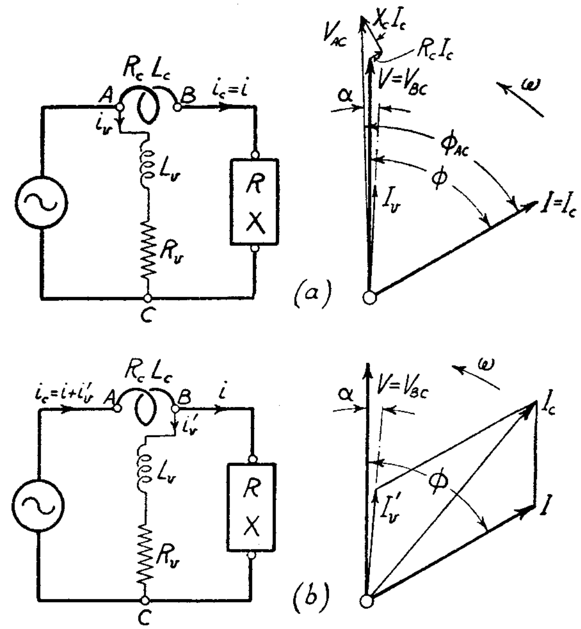


FIG. 155. ILLUSTRATING THEORY OF WATTMETER

Substituting,

$$c\theta = \frac{a}{T}\int_0^T \frac{Z_{AC}}{ZZ_v} v_1 i_1 \sin (\omega t + \phi_{AC} - \phi - \alpha) \,.\, \sin (\omega t - \phi) dt$$

$$= a \,.\, \frac{v_1 i_1}{2} \,.\, \frac{Z_{AC}}{ZZ_v} \cos (\phi_{AC} - \alpha) = a \frac{VI}{R_v} \,.\, \frac{Z_{AC}}{Z} \cos \alpha \,.\, \cos (\phi_{AC} - \alpha),$$

if V, I are the r.m.s. values of the voltage across and the current through the load. The reading in watts is

$$W_{AC} = \frac{c}{a} R_v \theta = \frac{VI}{Z} R_{AC} \frac{Z_{AC}}{R_{AC}} \cos \alpha \,.\, \cos (\phi_{AC} - \alpha)$$

$$= I^2 (R + R_c) \frac{\cos \alpha \,.\, \cos (\phi_{AC} - \alpha).}{\cos \phi_{AC}}$$

Transposing,

$$\frac{\cos \phi_{AC}}{\cos \alpha \, . \cos (\phi_{AC} - \alpha)} W_{AC} = I^2R + I^2R_c,$$

or

$$I^2R = K_{AC}\, W_{AC} - I^2R_c,$$

or *Power in load* $= K_{AC}$ (*wattmeter reading*) – *power lost in current coil*, where

$$K_{AC} = \frac{\cos \phi_{AC}}{\cos \alpha \, . \cos (\phi_{AC} - \alpha)}.$$

Consider now the connections shown in Fig. 155 (*b*), the voltage across the load at the points *BC* being again

$$v = v_1 \sin \omega t = (\sqrt{2}) V \sin \omega t;$$

and the current in the load

$$i = (v_1/Z) \sin (\omega t - \phi) = i_1 \sin (\omega t - \phi)$$
$$= (\sqrt{2}) I \sin (\omega t - \phi)$$

The current in the voltage coil is now

$$i_v' = (v_1/Z_v) \sin (\omega t - \alpha),$$

so that the current in the current coil of the wattmeter is

$$i_c = i + i_v' = i_1 \sin (\omega t - \phi) + (v_1/Z_v) \sin (\omega t - \alpha)$$

The expression for the average value of the resulting torque taken over a period T is

$$c\theta = \frac{a}{T}\int_0^T i_v' i_c dt$$
$$= \frac{a}{T}\int_0^T \frac{v_1}{Z_v}\sin (\omega t - \alpha) \left[i_1 \sin (\omega t - \phi) + \frac{v_1}{Z_v} \sin (\omega t - \alpha)\right] dt$$
$$= a\left[\frac{v_1 i_1}{2Z_v}\cos (\phi - \alpha) + \frac{v_1^2}{2Z_v^2}\right] = a\left[VI \frac{\cos \alpha}{R_v} \cos (\phi - \alpha) + \frac{V^2}{Z_v^2}\right]$$

The reading in watts will now be

$$W_{BC} = (c/a)\, R_v \theta = VI \cos \alpha \, . \cos (\phi - \alpha) + I_v'^2 R_v,$$

which becomes

$$\frac{\cos \phi}{\cos \alpha \, . \cos (\phi - \alpha)} (W_{BC} - I_v'^2 R_v) = K_{BC}\, (W_{BC} - I_v'^2 R_v)$$
$$= VI \cos \phi$$

or *Power in load* $= K_{BC}$ (*wattmeter reading – power lost in voltage circuit*), where

$$K_{BC} = \frac{\cos \phi}{\cos \alpha \, . \cos (\phi - \alpha)} \equiv \frac{\cos \phi_{BC}}{\cos \alpha \, . \cos (\phi_{BC} - \alpha)}$$

Comparing the two methods of connection we see that in the former the correction for the reactance of the voltage-circuit is made before allowing for the instrument loss, while in the second method the processes of correction are reversed.* It will also be observed that the correction factors K_{AC}, K_{BC} are of the same form though the one involves the phase-displacement in the load and current coil in series, ϕ_{AC}, while the other depends on the phase angle of the load alone, $\phi_{BC} \equiv \phi$. The difference is, however, usually very small since both R_c and ωL_c are negligible in most cases by comparison with R and X; the distinction may be important, however, when R and X are also small. Of the two methods the second is commonly preferred to the first, since the instrument loss correction is constant when the voltage across the load is also constant; either it may be allowed for in calibrating the meter scale, or one of the well-known methods of compensation for the loss may be applied, for details of which the reader is referred to Messrs. Drysdale and Jolley's classic treatise on measuring instruments. In what follows it will be assumed that the wattmeter is connected as in the second method, Fig. 155 (*b*), and that the voltage-circuit loss is either corrected for or compensated; then we may write

Power in load $= K_{BC} \times$ *wattmeter reading*

with

$$K_{BC} = \frac{\cos \phi}{\cos \alpha \, . \cos (\phi - \alpha)},$$

which is the correction factor originally introduced by Ayrton† in 1888, though in somewhat different form.

Transforming the trigonometrical terms,

$$K_{BC} = \frac{\cos \phi}{\cos \alpha \, . \cos (\phi - \alpha)} = \frac{1 + \tan^2 \alpha}{1 + \tan \alpha \, . \tan \phi} = \frac{1 + \omega^2 T_v^2}{1 + \omega T_v \tan \phi},$$

where $T_v = L_v/R_v$ is the time-constant of the voltage-circuit. The wattmeter will be correct only under two conditions, (i) $\alpha = 0$, i.e. when $L_v = 0$, thus assuming the voltage-circuit to be non-reactive; and (ii) $\alpha = \phi$, i.e. when the time-constants of the voltage-circuit and of the load are equal. In both these circumstances K_{BC} becomes unity; for all other values of α, K_{BC} is not unity and the amount of its deviation therefrom is a

* B. Hague, "The dynamometer wattmeter. Some notes on its theory. The application of corrections," *Elecn.*, vol. 92, pp. 96–97 (1924).

† W. E. Ayrton, *Journal I.E.E.*, vol. 17, pp. 172–175 (1888).

measure of the error introduced into the power measurement by the voltage circuit reactance. The variation of K_{BC} with ϕ is shown in Fig. 156 for the value $\tan\alpha = 0{\cdot}04$ or $\alpha = 2{\cdot}3°$ or 4·01 centiradians; this is much greater than the displacement usual in practice but has been specially chosen to exaggerate the peculiarities of the curve. For inductive loads with ϕ lying between α and $\pi/2$, K_{BC} is < 1, i.e. the wattmeter reads too high; for inductive loads with $\phi > 0 < \alpha$ and for capacitive loads with $\phi > 0 < -(\pi/2) + \alpha$, $K_{BC} > 1$, i.e. the instrument then reads too low. When $\phi = \alpha$, $K_{BC} = 1$ and the reading is correct; when $\phi = -(\pi/2) + \alpha$ the correction is indeterminate. Provided ϕ lies between limits of about $\pm\pi/3$, or $\cos\phi > 0{\cdot}5$, the factorial correction of the instrument reading is useful; in the neighbourhood of $\phi = \pm\pi/2$ the factor varies rapidly and for condensive loads shows an indeterminacy that renders it of little use in theory or practice.

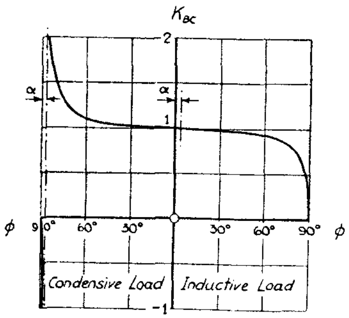


FIG. 156. CORRECTION-FACTOR FOR WATTMETER

The matter may be more usefully regarded from the following point of view. We may write

$$\textit{Wattmeter reading} - \textit{true power} = \textit{error},$$

and the *Fractional error* in terms of the true power is

$$\varepsilon_p = \frac{\textit{Wattmeter reading} - \textit{true power}}{\textit{true power}} = \frac{1}{K_{BC}} - 1$$

$$= \frac{\cos\alpha\,.\,\cos(\phi-\alpha)}{\cos\phi} - 1 = \cos^2\alpha + \cos\alpha\,.\,\sin\alpha\,.\,\tan\phi - 1.$$

Now α is small enough to take $\sin\alpha \doteqdot \tan\alpha \doteqdot \alpha$ and $\cos\alpha$ equal to 1, so that

$$\varepsilon_p \doteqdot \alpha\tan\phi \doteqdot (\omega L_v/R_v)\tan\phi \doteqdot \omega T_v\,.\,\tan\phi.$$

The error is

$$\varepsilon_p \times \textit{True power} \doteqdot \omega T_v \tan\phi\,.\,VI\cos\phi \doteqdot \omega T_v VI\sin\phi$$

and is subtracted from the wattmeter reading to obtain the true power. The correction in this form is due to Drysdale* and is valid without reservations for all values of ϕ. When the power-factor is low, which is the case where the factorial method of correction is invalid, $\sin\phi \doteqdot 1$ and the error becomes simply $\pm\,\omega T_v VI$, the upper sign relating to inductive and the lower sign to capacitive loads.

* C. V. Drysdale, "The theory and use of the alternate current wattmeter," *Elecn.*, vol. 46, pp. 774–778 (1901); "The theory of the dynamometer wattmeter," *Journal I.E.E.*, vol. 44, pp. 255–268 (1910).

3. **Theory of wattmeter with transformers.** Let the transformer be connected in circuit through voltage and current transformers* as shown in Fig. 157. It will be seen that the primary AB of the current transformer carries the load current I_p and the primary current of the voltage transformer; since V_p is constant the current taken by VT is also constant and it will be assumed that its effect is either corrected for or regarded as negligibly small. Turning to the vector diagram in Fig. 158, the true power in the load is

$$V_p I_p \cos\phi.$$

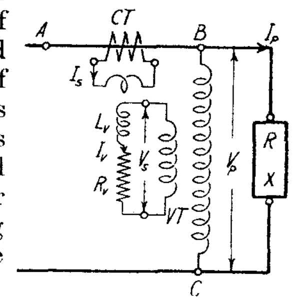


FIG. 157. WATTMETER WITH CURRENT AND VOLTAGE TRANSFORMERS

The voltage transformer applies a voltage $V_s = V_p/K_v$ to the voltage-circuit of the wattmeter, V_s being displaced from $-V_p$ by the angle γ. Similarly, the current transformer circulates a current $I_s = I_p/K_c$ through the current coil, displaced from $-I_p$ by the angle β. It will be remembered that β and γ are positive when the secondary quantity leads on the reversed primary quantity, as shown. The voltage V_s sends a current I_v through the voltage-circuit, I_v lagging on V_s by an angle $\alpha = \arctan(\omega L_v/R_v)$.

* The theory is given in the following: L. W. Wild, "Series transformers for wattmeters," *Elecn.*, vol. 56, pp. 705–706 (1906); E. S. Harrar, "The series transformer," *Elec. World*, vol. 51, pp. 1044–1046 (1908); W. Genkin, "Sur transformateurs d'intensité," *Lum. Élect.*, vol. 8, pp. 67–71 (1909); K. A. Sterzel, "Stromwandler für Wechselstrom-Leistungsmessungen," *Elekt. Zeits.*, vol. 30, pp. 489–491 (1909); K. Edgcumbe, "Some notes on the use of instrument transformers," *Elec. Rev.*, vol. 67, pp. 163–165 (1910); L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–1039 (1910); M. Rosenbaum, *Elecn.*, vol. 74, pp. 626–630 (1915); A. G. L. MacNaughton, *Journal I.E.E.*, vol. 53, pp. 269–271 (1915); R. H. Chadwick, "Errors due to the use of instrument transformers," *Elec. World*, vol. 66, pp. 1308–1310 (1915); A. Iliovici, *Lum. Élect.*, vol. 33, pp. 276–277 (1916); K. Takatsu, *Res. Elect. Lab. Tokyo*, vol. 95, pp. 1–74 (1921); E. Lienhard, "Errors in high tension power measurements when instrument transformers are employed," *B.B. Rev.*, vol. 12, pp. 168–173 (1925); R. van Cauwenberghe, "Note sur l'emploi des transformateurs de mesures," *Bull. Soc. Belge des Élecns.*, vol. 42, pp. 87–88 (1928); Bergtold, "Einfluss der Wandlerfehler auf die Messgenauigkeit von Elektrizitätszählern," *Elekt. Betrieb*, vol. 28, pp. 53–56 (1930); G. Hauffe, *Elekt. Zeits.*, vol. 52, pp. 900–901 (1931).

The torque on the moving system of the wattmeter is proportional to $I_v I_s \cos\phi'$, where ϕ' is the angle of phase-displacement between I_v and I_s. The reading in secondary watts will be

$$R_v I_v I_s \cos\phi' = V_s \cos\alpha \,.\, I_s \cos\phi'$$
$$= V_s I_s \cos\alpha \,.\, \cos(\phi + \gamma - \beta - \alpha)$$
$$= (V_p I_p / K_v K_c) \cos\alpha \,.\, \cos(\phi - \alpha - \beta + \gamma).$$

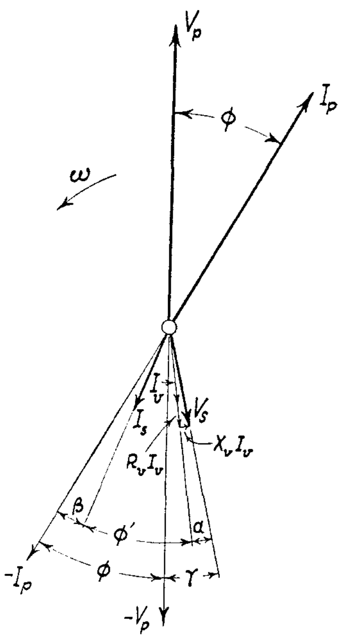


FIG. 158. VECTOR DIAGRAM FOR WATTMETER WITH INSTRUMENT TRANSFORMERS

The wattmeter is, however, usually scaled in terms of the nominal primary magnitudes, so that the reading of the instrument in primary units may be written

$$\frac{K_{nv}}{K_v} \,.\, \frac{K_{nc}}{K_c} \,.\, V_p I_p \cos\alpha \times \cos(\phi - \alpha - \beta + \gamma)$$
$$= \frac{1}{F_v \,.\, F_c} \,.\, V_p I_p \cos\phi \times \frac{\cos\alpha \,.\, \cos(\phi - \alpha - \beta + \gamma)}{\cos\phi}:$$

hence

True Power =

$$F_v F_c \frac{\cos\phi}{\cos\alpha \,.\, \cos(\phi - \alpha - \beta + \gamma)} \times \textit{Wattmeter reading.}$$

The factor $F_v F_c$ corrects for the ratio error of the two transformers. The second factor, correcting for the phase-angles of both the transformers and of the wattmeter voltage-coil can be written,

$$K = \frac{\cos\phi}{\cos\alpha \,.\, \cos(\phi - \delta)} = \frac{\cos\phi}{\cos\alpha \cos\phi'}$$

where $\delta = \alpha + \beta - \gamma$.

In this form it may readily be compared with the expression for K_{BC}, namely

$$K_{BC} = \frac{\cos\phi}{\cos\alpha \,.\, \cos(\phi - \alpha)}$$

Since α is small and $\cos\alpha$ is very nearly unity the presence of the transformers is equivalent to an alteration of α to δ in the second term of the denominator. It will be observed that a positive value of β is equivalent to an increased, and a positive value of γ to a decreased, value of α.* For this reason the French and American standard specifications for instrument transformers (see pp. 27 and 30) define a leading β and a lagging γ as positive, since each will then have a similar effect upon the phase-error in a power measurement. This adoption of two conventions for positive angles is unfortunate and confusing; it is, moreover, in contravention of the internationally adopted convention that a positive angle be measured counterclockwise, and is an unnecessary complication in mathematical work.

To express the effect of the transformers as a fractional error write as before,

$$\varepsilon_p = \frac{\textit{Wattmeter reading} - \textit{true power}}{\textit{true power}} = \frac{\cos\alpha}{F_v F_c} \,.\, \frac{\cos(\phi - \delta)}{\cos\phi} - 1.$$

By the definitions on pp. 11 and 8 the fractional errors in the ratios of the transformers are

$$\varepsilon_v = (1/F_v) - 1 \text{ and } \varepsilon_c = (1/F_c) - 1,$$
$$\text{or } 1/F_v = 1 + \varepsilon_v \text{ and } 1/F_c = 1 + \varepsilon_c.$$

Also

$$\cos(\phi - \delta)/\cos\phi = \cos\delta + \sin\delta \tan\phi \doteqdot 1 + \delta \tan\phi$$

when δ is small. Substituting in the expression for ε_p, taking $\cos\alpha \doteqdot 1$ and neglecting the products of small quantities as infinitesimals of the second order,

$$\varepsilon_p \doteqdot (\varepsilon_v + \varepsilon_c) + \delta \tan\phi = \varepsilon_v + \varepsilon_c + \varepsilon_\delta.$$

If ε_v and ε_c are in percentages and δ in centiradians, ε_p is the overall percentage error. The error, or amount to be subtracted from the wattmeter reading to obtain the true power, is

$$\varepsilon_p \times \textit{true power} = \varepsilon_p V_p I_p \cos\phi.$$

This is the general form of Drysdale's correction.

In practice α is almost always positive, i.e. I_v lags on V_s. With an inductive load ϕ is also positive, so that $\tan\phi$ is then

* This fact was recognized by A. C. Heap, *British Patent*, No. 47 (1903), who balanced β against γ by adjusting the leakage reactance of the current transformer by winding the primary and secondary on opposite limbs of the core. An even earlier attempt to eliminate the transformer error in a power measurement is due to Siemens Bros., *British Patent*, No. 17 263 (1899); β and γ are reduced to zero by applying suitable shunts to the secondary windings or to auxiliary tertiaries.

positive; the sign of the phase-error ε_δ is, therefore, determined by that of δ. Whatever signs are determined in this way will be reversed by a change to a capacitive load. If $\alpha + \beta$ exceeds γ, the following table illustrates the effect of the phase-error ε_δ on the wattmeter reading, both for inductive and for capacitive loads.

Load	Sign of			Sign of ε_δ	Wattmeter Reads too
	$\alpha + \beta$	γ	δ		
Inductive ϕ and $\tan\phi$ are +	+	+	+	+	high
	−	−	−	−	low
	+	−	+	+	high
	−	+	−	−	low
Capacitive ϕ and $\tan\phi$ are −	+	+	+	−	low
	−	−	−	+	high
	+	−	+	−	low
	−	+	−	+	high

If $\alpha + \beta$ is less than γ it is necessary to reverse the signs of δ and ε_δ, and to substitute "high" for "low" and conversely. The magnitude of the phase-error ε_δ is plotted for various constant values of δ as a function of $\cos\phi$ in Fig. 159, questions of sign being disregarded. For higher power-factors and larger values of δ the multiplication of such graphs is not convenient and an alternative process is required, as will be indicated below.

Nomographic representation has certain advantages. A chart with two sets of rectangular axes and a sloping index line has been published by Doyle.* A much simpler alignment chart with three parallel scales for ε_δ, δ and $\cos\phi$ respectively is given by W. Skirl, *Wechselstrom-Leistungsmessungen*, 3rd edition (1930).

The total power-error can be computed in the following way. Read the wattmeter and correct it for instrumental errors; determine also the volt-amperes from the indications of a voltmeter and an ammeter joined to the secondary sides of the transformers. The ratio of the watts to the volt-amperes gives the apparent power-factor $\cos\phi' = \cos(\phi - \delta)$. From the calibration curves of the transformers obtain ε_v, ε_c, γ and β; and from the particulars of the wattmeter find α. Thence calculate

* E. D. Doyle, "Correcting wattmeter readings for phase-angle errors," *Elec. World*, vol. 77, pp. 314 (1921).

δ, find ϕ' from the known value of $\cos\phi'$ and thus obtain ϕ and $\cos\phi$. Using Fig. 159 or, if the figures fall outside the values therein plotted, calculating directly from the expression

$$\varepsilon_\delta = \delta \tan\phi$$

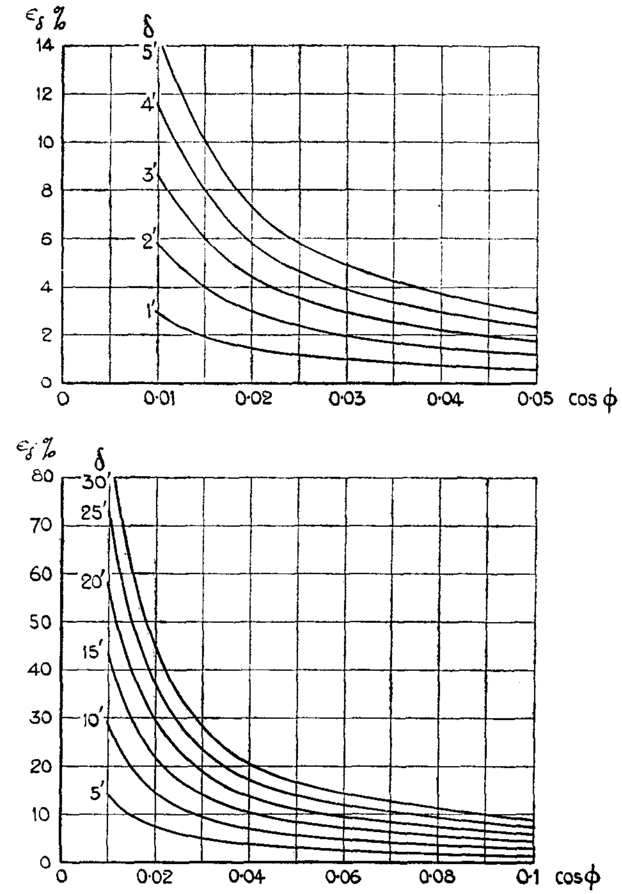


FIG. 159. ERROR IN POWER MEASUREMENT DUE TO PHASE-ANGLE, ε_δ AS FUNCTION OF δ AND COS ϕ

find ε_δ and thus

$$\varepsilon_p = \varepsilon_v + \varepsilon_c + \varepsilon_\delta$$

is finally obtained. Note that α is constant, as also is γ for a given voltage and burden; β varies with the primary load current and is the only variable in usual practical cases.

This round-about process, though not very troublesome when only a few calculations are to be made, is rather tedious when a large number of readings require correction. A factorial procedure introduced by Robinson* is often more convenient. As before, find $\cos(\phi - \delta)$ from the measured watts and volt-amperes, and compute $\delta = \alpha + \beta - \gamma$ from the transformer characteristics. Then from p. 300,

$$\textit{True power} = \frac{F_v F_c}{\cos\alpha} \cdot \frac{\cos\phi}{\cos\phi'} \times \textit{wattmeter reading}.$$

The factor

$$F_v F_c = 1/(1 + \varepsilon_v)(1 + \varepsilon_c) \doteqdot (1 - \varepsilon_v)(1 - \varepsilon_c) \doteqdot 1 - \varepsilon_v - \varepsilon_c$$

corrects for the ratio error of the transformers; $\cos\alpha$ may be taken as unity. The phase-angle error is correctedby the factor $\cos\phi/\cos\phi'$; to obtain this tables are constructed giving $\cos\phi/\cos\phi'$ as a function of δ for various constant values of $\cos\phi'$. Such tables were calculated by Robinson and later extended with improvements by Weller† and by Werres.‡ They are reproduced in Appendix II together with illustrative examples of their use.

The question of wattmeter errors has recently been examined from a novel standpoint by Vahl§ and by Nierenberger.‖ From p. 301

$$\textit{Wattmeter reading} - \textit{True power} = \left(\frac{\cos\alpha}{F_v F_c} \cdot \frac{\cos(\phi - \delta)}{\cos\phi} - 1\right) \times V_p I_p \cos\phi.$$

In most national standard rules for the accuracy of measuring instruments it is usual to express the error not in terms of the true value of the measured quantity corresponding with the position of the pointer on the scale but as a fraction of the maximum scale-reading. On this basis,

$$\textit{Error} = \frac{V_p I_p \cos\phi}{\textit{max. reading}} \cdot \left(\frac{\cos\alpha}{F_v F_c} \cdot \frac{\cos(\phi - \delta)}{\cos\phi} - 1\right)$$

$$= \frac{V_p I_p}{\textit{max. reading}} \cdot \frac{1}{F_v F_c}[\cos\alpha \,.\, \cos(\phi - \delta) - F_v F_c \cos\phi]$$

Substituting for F_v and F_c their approximate values in terms of ε_v and

* L. T. Robinson, *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–1039 (1910).

† C. T. Weller, "Revised table of phase-angle correction factors for use in power measurements," *Gen. Elec. Rev.*, vol. 28, pp. 202–206 (1925).

‡ C. O. Werres, "A simplified method of applying instrument transformer correction factors to wattmeter readings," *Gen. Elec. Rev.*, vol. 36, pp. 462–463 (1933).

§ H. Vahl, "Der Einfluss der Wandlerfehler auf die Genauigkeit bei Leistungsmessungen," *Elekt. Zeits.*, vol. 54, pp. 29–30 (1933).

‖ R. Nierenberger, "La correction des erreurs dans les mesures wattmétriques indirectes," *Rev. Gén. de l'Él.*, vol. 33, pp. 177–181 (1933).

ε_c, taking δ and α as small enough to regard the angles equal to their sines

$$\textit{Error} \doteqdot \frac{V_p I_p}{\textit{max. reading}} \,.\, (1 + \varepsilon_v + \varepsilon_c)\,[(\varepsilon_v + \varepsilon_c)\cos\phi + \delta\sin\phi]$$

The first term is the ratio of the volt-amperes to the maximum reading; the second term is the usual factorial ratio error. The third term contains the effect of phase-error and power-factor and is worthy of further study. Suppose that the maximum reading is obtained with rated voltage and current at unity power-factor; then the reading will vary

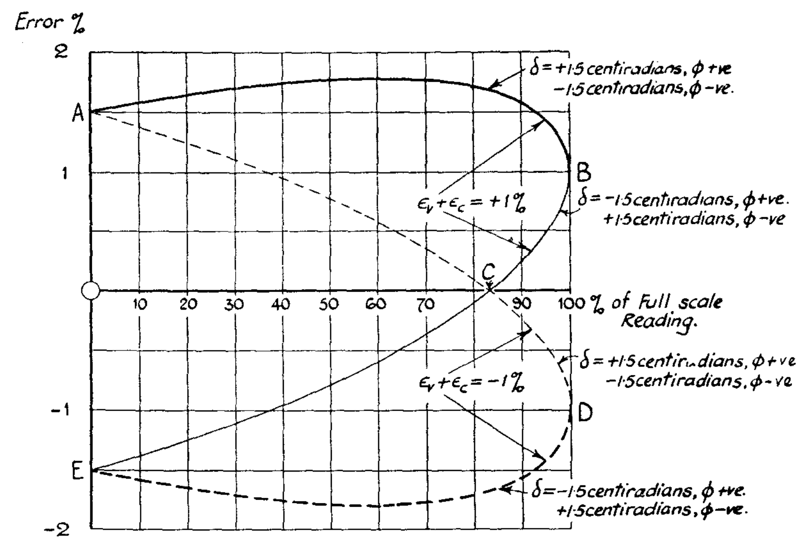


FIG. 160. MAXIMUM POSSIBLE POWER ERROR AS FUNCTION OF SCALE READING; VOLT-AMPERES CONSTANT AND TRANSFORMERS SATISFYING CLASS 0·5 LIMITS OF ACCURACY

with $\cos\phi$ for constant rated volt-amperes. Let the transformers satisfy certain standard limits of error; for example, with the I.E.C. Class 0·5 limits, the voltage transformer has $\varepsilon_v = \pm 0{\cdot}5$ per cent and $\gamma = \pm 20'$, and the current transformer $\varepsilon_c = \pm 0{\cdot}5$ per cent and $\beta = \pm 30'$ as the rated limits of error. Assuming the transformers to be combined in such a way as to make the greatest power error $\varepsilon_v + \varepsilon_c = \pm 1{\cdot}0$ per cent and $\beta - \gamma = \pm 50' \doteqdot \pm 1{\cdot}5$ centiradian; neglecting α this can be taken as the value of δ. There are now four possibilities, $+ 1{\cdot}0$ with $+ 1{\cdot}5$, $+ 1{\cdot}0$ with $- 1{\cdot}5$, $- 1{\cdot}0$ with $- 1{\cdot}5$ and $- 1{\cdot}0$ with $+ 1{\cdot}5$. Using these in the expression within square brackets enables the maximum possible error expressed as a function of the maximum scale reading to be calculated for different positions on the scale, the volt-amperes being constant at the rated value. The result is plotted in Fig. 160 for positive and negative values of ϕ. Since these graphs bound the maximum error, the actual error must fall within the area enclosed within $ABCDEA$; the error represents the fraction of the maximum reading that must be subtracted from the instrument indication to give the true power. This method of correction has obvious practical advantages.

4. **Power measurement in three-phase circuits.** Power is often measured in three-phase, three-wire circuits by the use of two wattmeters or of an equivalent single instrument with two active elements, connected as shown in the left-hand diagram of Fig. 161; this is the well-known Aron connection or two-wattmeter method. Assuming an equivalent star-connected load, the vectorial relationships between the currents and voltages associated with the two meters will be as shown in the

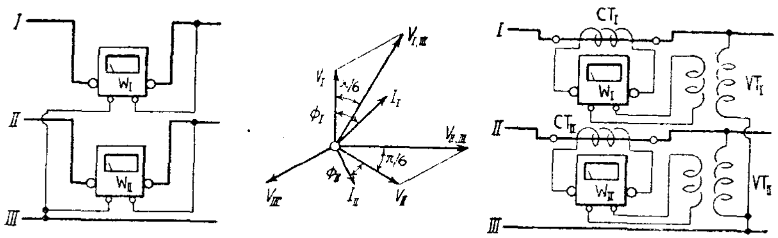


FIG. 161. TWO-WATTMETER METHOD FOR POWER MEASUREMENT IN THREE-PHASE CIRCUIT

centre diagram. Neglecting the phase-angle of the voltage circuits, the wattmeters will together indicate the total power in the system, namely,

$$V_{\mathrm{I,III}}\, I_{\mathrm{I}} \cos\, [\phi_{\mathrm{I}} - (\pi/6)] + V_{\mathrm{II,III}}\, I_{\mathrm{II}} \cos\, [\phi_{\mathrm{II}} + (\pi/6)],$$

assuming the star-voltages V_{I}. V_{II}, V_{III} to form a symmetrical three-phase system and the load to be unbalanced. In the particular case where the load is balanced $I_{\mathrm{I}} = I_{\mathrm{II}} = I_p$ say, and $\phi_{\mathrm{I}} = \phi_{\mathrm{II}} = \phi$; writing $V_{\mathrm{I,III}} = V_{\mathrm{II,III}} = V_p$ gives

$$V_pI_p \cos\, [\phi - (\pi/6)] + V_pI_p \cos\, [\phi + (\pi/6)] = (\sqrt{3}) V_pI_p \cos \phi.$$

In high voltage practice the wattmeters are connected to the network through voltage and current transformers, the most obvious way* being shown in the right-hand diagram of Fig. 161.

* The question of the various methods of connecting wattmeters in circuit via transformers falls outside the scope of the present volume and is a special branch of meter technology; we are only concerned here with the general principle and the contribution of correctly connected transformers to the overall error. The reader may consult *inter alia*, J. Auchincloss, "Various methods of connecting indicating wattmeters," *Gen. Elec. Rev.*, vol. 31, pp. 257–263, 376–381 (1928). J. Möllinger, "Eine eigenartige Stromwandler-Fehlschaltung bein Anschluss von Drehstromzählern," *Siemens Zeits.*, vol. 7, pp. 161–169 (1927), discusses the error due to a wrong connection of current transformers. W. Beetz, "Beachtenwertes beim Anschluss von Messgeräten

This shows two voltage transformers, designed for normal operation at the line voltage, joined on the primary side in "vee"-connection. The transformers will have ratio and phase errors, and the wattmeter voltage circuits will be reactive. Let $F_{v\mathrm{I}}$, $F_{c\mathrm{I}}$, α_{I}, β_{I}, γ_{I} refer to the first meter and $F_{v\mathrm{II}}$, $F_{c\mathrm{II}}$, α_{II}, β_{II}, γ_{II} to the second. Then the total watts read on the meter scales in primary units will be $(V_pI_p/F_{\mathrm{I}v}F_{c\mathrm{I}}) \cos \alpha_{\mathrm{I}}$. $\cos [\phi - (\pi/6) - \delta_{\mathrm{I}}] + (V_pI_p/F_{v\mathrm{II}}\, F_{c\mathrm{II}}) \cos \alpha_{\mathrm{II}} \cos [\phi + (\pi/6) - \delta_{\mathrm{II}}]$, assuming a balanced load and writing $\delta_{\mathrm{I}} = \alpha_{\mathrm{I}} + \beta_{\mathrm{I}} - \gamma_{\mathrm{I}}$ and $\delta_{\mathrm{II}} = \alpha_{\mathrm{II}} + \beta_{\mathrm{II}} - \gamma_{\mathrm{II}}$, while the true power is $(\sqrt{3})\, V_pI_p \cos \phi$. The fractional error is

$$\varepsilon_p = \frac{\textit{wattmeter reading} - \textit{true power}}{\textit{true power}}$$

$$\doteqdot \frac{(1 + \varepsilon_{v\mathrm{I}})(1 + \varepsilon_{c\mathrm{I}}) \cos [\phi - (\pi/6) - \delta_{\mathrm{I}}] + (1 + \varepsilon_{v\mathrm{II}})(1+\varepsilon_{c\mathrm{II}}) \cos [\phi + (\pi/6) - \delta_{\mathrm{II}}]}{(\sqrt{3}) \cos \phi} - 1,$$

on substituting the fractional ratio errors in place of the ratio factors and writing $\cos \alpha_{\mathrm{I}} \doteqdot \cos \alpha_{\mathrm{II}} \doteqdot 1$. Simplifying the trigonometrical terms by writing $\cos \delta_{\mathrm{I}} \doteqdot \cos \delta_{\mathrm{II}} \doteqdot 1$, also $\sin \delta_{\mathrm{I}} \doteqdot \delta_{\mathrm{I}}$ and $\sin \delta_{\mathrm{II}} \doteqdot \delta_{\mathrm{II}}$ gives, on neglecting second order products,

$$\begin{aligned}\varepsilon_p \doteqdot\, & (1 + \varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}})\, [\tfrac{1}{2}(1 + \delta_{\mathrm{I}} \tan \phi) + (1/2\sqrt{3})\, (\tan \phi - \delta_{\mathrm{I}})] \\ & + (1 + \varepsilon_{v\mathrm{II}} + \varepsilon_{c\mathrm{II}})\, [\tfrac{1}{2}(1 + \delta_{\mathrm{II}} \tan \phi) \\ & \qquad - (1/2\sqrt{3})\, (\tan \phi - \delta_{\mathrm{II}})] - 1\end{aligned}$$

Again expanding and omitting products of small quantities gives finally*

an Stromwandler," *Elekt. Zeits.*, vol. 54, pp. 1092–1095 (1933), works out the error in power measurement due to transformers of various classes connected in different ways. G. W. Stubbings, "Current transformers. Methods of interconnection in three-phase circuits," *Elec. Rev.*, vol. 113, p. 278 (1933), discusses the use of Z-connected current transformers with protective gear. See also some of the books on meter technology referred to in the Bibliography.

* J. Goldstein, "Die Zusammensetzung der Einzelfehler der Messwandler zum resultierenden Fehler des Messaggregates in Drehstromnetzen und die daraus resultierende zweckmässige Anordnung der Wandler," *Bull. Schw. Elekt. Verein*, vol. 11, pp. 304–311 (1920). G. Hauffe, "Drehstromleistungsmessung in Aronschaltung mit Messwandlern," *Arch. f. Elekt.*, vol. 19, pp. 10–14 (1927). H. Nützelberger and R. Resch, "Diagramm zur Ermittlung des durch Messwandler entstehanden Fehlers bei Leistungsmessungen in Drehstrom-Dreileiteranlagen," *Arch. f. Elekt.*, vol. 24, pp. 29–36 (1930). The more complicated case of unbalanced load is treated by the method of symmetrical co-ordinates by A. Iliovici, "Erreur introduite par les transformateurs dans la mésure d'une puissance (et d'une energie)," *Bull. Soc. Franç. des Élecns.*, vol. 7, pp. 1311–1324 (1927).

$$\varepsilon_p \doteqdot \tfrac{1}{2}(\varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}})\ [1 + (1/\sqrt{3})\tan\phi] + \tfrac{1}{2}(\varepsilon_{v\mathrm{II}} + \varepsilon_{c\mathrm{II}}).$$
$$[1 - (1/\sqrt{3})\tan\phi] + \tfrac{1}{2}(\delta_{\mathrm{I}} + \delta_{\mathrm{II}})\tan\phi - (1/2\sqrt{3})\ (\delta_{\mathrm{I}} - \delta_{\mathrm{II}})$$
$$\doteqdot \tfrac{1}{2}(\varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}} + \varepsilon_{v\mathrm{II}} + \varepsilon_{c\mathrm{II}}) \mp (1/2\sqrt{3})\ (\varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}} - \varepsilon_{v\mathrm{II}} - \varepsilon_{c\mathrm{II}}).$$
$$\tan\phi + \tfrac{1}{2}(\delta_{\mathrm{I}} + \delta_{\mathrm{II}})\tan\phi - (1/2\sqrt{3})\ (\delta_{\mathrm{I}} - \delta_{\mathrm{II}}),$$

in which the first two terms express the fractional error introduced by the transformer ratios, and the second pair that due to their phase-angles and to the wattmeter voltage circuit reactances. When the wattmeters and their transformers are identical $\delta_{\mathrm{I}} = \delta_{\mathrm{II}} = \delta$, $\varepsilon_{v\mathrm{I}} = \varepsilon_{v\mathrm{II}} = \varepsilon_v$ and $\varepsilon_{c\mathrm{I}} = \varepsilon_{c\mathrm{II}} = \varepsilon_c$; then

$$\varepsilon_p \doteqdot \varepsilon_v + \varepsilon_c + \delta \tan\phi$$

exactly as in single-phase power measurement.

The above expression for ε_p has some very interesting properties. Considering first the contribution of the ratio errors, in general,

$$(1 - F_vF_c) = 1 - [1/(1 + \varepsilon_v)(1 + \varepsilon_c)] \doteqdot 1 - (1 - \varepsilon_v - \varepsilon_c)$$
$$\doteqdot \varepsilon_v + \varepsilon_c.$$

Hence the portion of ε_p due to the ratio errors will vanish entirely if

$$F_{v\mathrm{I}}F_{c\mathrm{I}} = F_{v\mathrm{II}}F_{c\mathrm{II}} = 1.*$$

The second term of this part of the total error will vanish if $F_{v\mathrm{I}}F_{c\mathrm{I}} = F_{v\mathrm{II}}F_{c\mathrm{II}}$. Hence the transformers should be selected so that the products of the ratio factors are as nearly equal as possible and as close to unity as can be obtained. It is to be noted that with inductive loads and power-factors between about 0·8 and 0·2 most of the power is measured by the reading of W_{II}; hence, the transformers used with it should be chosen to make $F_{v\mathrm{I}}F_{c\mathrm{I}}$ as near to unity as possible.

Usually the ratio error is very small, and greater importance is attached to the phase-angle error expressed by

$$\tfrac{1}{2}(\delta_{\mathrm{I}} + \delta_{\mathrm{II}})\tan\phi - (1/2\sqrt{3})\ (\delta_{\mathrm{I}} - \delta_{\mathrm{II}}).$$

The four transformers should be grouped in such a way as will make the gross error the least. To illustrate the point consider the following numerical example: $\alpha_{\mathrm{I}} = \alpha_{\mathrm{II}} = 0$, $\beta_{\mathrm{I}} = 20' =$ 0·582 centiradian, $\beta_{\mathrm{II}} = 50' =$ 1·454 centiradian, $\gamma_{\mathrm{I}} = -20' =$ − 0·582 centiradian, $\gamma_{\mathrm{II}} = -40' =$ − 1·164 centiradian, $\cos\phi$ = 0·5, $\tan\phi$ = 1·732. Then there are four possibilities—

* That is if $\varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}}$ and $\varepsilon_{v\mathrm{II}} + \varepsilon_{c\mathrm{II}}$ are zero individually. There is also the possibility $(\varepsilon_{v\mathrm{I}} + \varepsilon_{c\mathrm{I}}) = -(\varepsilon_{v\mathrm{II}} + \varepsilon_{c\mathrm{II}})$ as well as $(\varepsilon_{v\mathrm{I}} + \varepsilon_{v\mathrm{II}}) = -(\varepsilon_{c\mathrm{I}} + \varepsilon_{c\mathrm{II}})$ to be noted.

(i) *Using* β_{I} *and* γ_{I} *with* W_{I}, β_{II} *and* γ_{II} *with* W_{II}. Then the percentage error is 0·866 (1·164 + 2·618) − 0·289 (1·164 − 2·618) = 3·70 per cent.

(ii) *Using* β_{I} *and* γ_{II} *with* W_{I}, β_{II} *and* γ_{I} *with* W_{II}. Then the percentage error is 0·866 (1·746 + 2·036) − 0·289 (1·746 − 2·036) = 3·36 per cent.

(iii) *Using* β_{II} *and* γ_{I} *with* W_{I}, β_{I} *and* γ_{II} *with* W_{II}. Then the percentage error is 0·866 (2·036 + 1·746) − 0·289 (2·036 − 1·746) = 3·20 per cent.

(iv) *Using* β_{II} *and* γ_{II} *with* W_{I}, β_{I} *and* γ_{I} *with* W_{II}. Then the percentage error is 0·866 (2·618 + 1·164) − 0·289 (2·618 − 1·164) = 2·86 per cent.

In this case the fourth arrangement shows a marked superiority over the others and justifiesthe choice of this particular sequence of transformers.

As regards the actual calculation of errors in a given instance, each wattmeter may be treated individually as though it were making a single-phase measurement and corrected from a knowledge of its reading, the volt-amperes, and apparent power-factor, exactly as described in Section 3, making use of the same expressions, graphs or tables of correction factors. By this principle the method may be extended to apply to three-wattmeter measurements in four-wire circuits, to the common case of three wattmeters operated by star-connected voltage transformers, and to more complex cases.

5. **The use of instrument transformers with energy meters.** The theory given in Sections 3 and 4 is important since it shows the nature of the error introduced into a power measurement by the use of instrument transformers and the possible methods of correction. The corrections to be applied are necessary in all accurate power measurements, especially in those made in laboratory work at low power-factors. The effects are of even greater importance in commercial energy measurements made by watt-hour meters, since the errors are integrated and accumulate in their effect on the registration of the meter. Consequently, commercial energy metering necessitates the use of high-grade instrument transformers, particularly chosen in regard to smallness of phase-angle.

The great majority of a.c. energy meters operate on the induction principle, a discussion of which is outside the scope of this book. At unity power-factor the fluxes set up by the current and voltage elements of the meter should be in quadrature; this condition can be satisfied for a particular value of the meter current but will be somewhat upset at other values of current. Consequently, the meter will have an error dependent upon the current strength, and this error will be further changed by alteration of the power-factor. If the meter is connected into circuit via instrument transformers, the ratio errors and phase-angles of these will influence the accuracy of the measurement of energy in a way quite analogous to that described above for power measurement.*

Since the primary voltage is constant, the ratio of the voltage transformer is also constant and may be allowed for in the calibration of the meter. The matter is not quite so simple, however, in the case of the current transformer. With a normal transformer the ratio increases for lower values of primary current, i.e. the secondary current diminishes, and this would tend to make a perfectly adjusted meter run too slowly at low loads. To counteract this effect the light-load adjustment of the meter may be altered to cause the meter to be intrinsically a little too fast. Then if the error curve of the meter is the same shape as the ratio error curve of the transformer, i.e. falling with increasing load, the compensation for ratio error will be almost automatic.

The phase-angle errors of the transformers cannot be compensated completely by adjustments of the meter in any such simple way as will deal satisfactorily with the ratio error. As the phase errors are much more important than the ratio errors, especially at low power-factors, attention must be directed to keeping the phase-angles of the transformers as small and as constant as possible.† In the case of current transformers, which present the greater difficulties in this respect, systems of compounding, two-stage transformation, nickel-iron cores and numerous other devices have enabled very small values of β to be attained with but slight variation as the primary current changes from no-load to full-load. These topics have been fully discussed in Chapter III.

* A. Maxwell, "Measurement of energy with instrument transformers," *Trans. Amer. I.E.E.*, vol. 31, pp. 1545–1550 (1912). A. Mieg, "La question des mesures d'énergie aux très hautes tensions," *Bull. Soc. Franç. des Élecns.*, vol. 2, pp. 182–189, 190–194 (1932).

† For a discussion of the desirable accuracy of measurement see E. C. Wentz, "Desirable accuracies in instrument transformers," *Elec. Eng.*, vol. 50, pp. 657–659 (1931). Using probability analysis it is shown that in a network with numerous meters no practical advantages accrue, either to the producer or to the consumer of energy, by specifying overall accuracy of an individual meter and its transformers better than 0·5%. This justifies the claim made by many engineers that constancy of errors, i.e. small changes with load, is of more practical importance than their smallness.

6. Limits of error. National rules. Practically all the National Rules for Instrument Transformers specify classes of transformers suitable for use in power and energy metering, and lay down appropriate limits of error. In general, transformers for accurate power or energy metering have an accuracy similar to Class 0·5 of the I.E.C. Publication 44; transformers for house-service meters or other rather less accurate purposes have limits of error like Class 1. Most of the Standard Rules obtain the desired accuracy of power measurement by imposing certain limits on the ratio error and phase-angle of the transformers, e.g. Great Britain,* Germany, Italy, Switzerland, etc., deal with the question in this way. The reader may, in this connection, consult the summaries of the national rules given in Chapter I. Certain countries, however, lay down limits to the error that shall be made in a power measurement by the inclusion of a transformer, and it is to these that attention will now be given.

In the *United States* (see p. 30), A.I.E.E. Standard No. 14, March, 1925, defines the ratio and phase-angle correction factors given in the expression deduced on p. 304, namely,

$$\textit{True power} = \frac{F_v F_c}{\cos\alpha} \cdot \frac{\cos\phi}{\cos\phi'} \cdot \textit{wattmeter reading}.$$

The N.E.M.A. Rules lay down limits for F_v, F_c, γ and β that have already been discussed on p. 33. The A.E.S.C. Standard No. C 12—1928 gives a Code for Electricity Meters which reaffirms the above definitions and establishes Weller's Table of $\cos\phi/\cos\phi'$ (see Appendix II) for regular use; limits of ratio and phase-angle for suitable metering transformers are given. It has already been pointed out that the American convention for positive sign of γ is the reverse of that internationally adopted, but that a promised revision of the rules will remove this discrepancy.

In *France* (see p. 27) Publication No. 92 of the Union des Syndicats de l'Électricité gives limits of error for ε_v, ε_c, γ and β for transformers suitable for various purposes. In addition, limits are imposed upon the permissible error that may be

* G. L. E. Metz, "Electricity meter accuracy. The ratio and phase-angle effects on the accuracy of watt-hour meters brought about by the introduction of current transformers, in the light of the requirements of the B.E.S.A. specifications," *Elec. Rev.*, vol. 102, pp. 277–278 (1928), mentions that a meter conforming to B.S.S. Spec. 37 and a transformer to B.S.S. Spec. 81 when combined, may not, in the extreme case, conform to the limit of error of 2·5% imposed upon transformer-operated meters.

introduced by transformers of the two higher classes when used in power or energy measurements. Writing the power error ε_p from p. 301 in the form

$$\varepsilon_p = \varepsilon_v + \varepsilon_c + \delta \tan\phi = (\varepsilon_v - \gamma \tan\phi) + (\varepsilon_c + \beta \tan\phi) + \alpha \tan\phi$$

$$\equiv \varepsilon_{pv} + \varepsilon_{pc} + \alpha \tan\phi,$$

ε_{pv} is the error introduced by the voltage transformer and ε_{pc} that by the current transformer. In the French Rules the minus sign in ε_{pv} is changed to plus, since their convention of a positive value of γ is when V_s lags on $-V_p$, i.e. the reverse of the usual convention now universally adopted by all other lands.

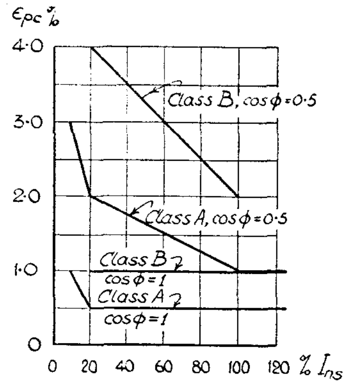


FIG. 162. FRENCH STANDARD SPECIFICATION FOR ERROR ε_{pc} INTRODUCED INTO A POWER MEASUREMENT BY A CURRENT TRANSFORMER

With *Voltage transformers*, if the secondary burden does not exceed the rated value and its power-factor lies between 0·5 and 1, the following limits for ε_{pv} are not to be exceeded for all values of secondary voltage between 0·8 and 1·2 times rated value: *Class A transformers*; $\pm$ 0·5% with $\cos\phi = 1$ and $\pm$ 1·0% with $\cos\phi = 0{\cdot}5$; *Class B transformers*; $\pm$ 1·0% with $\cos\phi = 1$; $\pm$ 1·5% with $\cos\phi = 0{\cdot}5$ and a non-inductive secondary circuit; $\pm$ 3·0% with $\cos\phi = 0{\cdot}5$ and a secondary circuit with a power factor of 0·5. With *Current transformers* the prescribed limits for ε_{pc} with the rated secondary burden are not to exceed the following—

Class A transformers; $\pm$ 0·5% from 20% to 125% of I_{ns} and $\pm$ 1% at 10% of I_{ns} when $\cos\phi = 1$, with linear variations between these points; $\pm$ 1% from 100% to 125% of I_{ns}, $\pm$ 2% at 20% of I_{ns} and $\pm$ 3% at 10% of I_{ns} when $\cos\phi = 0{\cdot}5$, with linear variation.

Class B transformers; $\pm$ 1% from 20% to 125% of I_{ns} when $\cos\phi = 1$; $\pm$ 2% at 100% of I_{ns} and $\pm$ 4% at 20% of I_{ns} with $\cos\phi = 0{\cdot}5$; in both cases with linear graphs.

The limits of error for ε_{pc} are plotted in Fig. 162. Although Class A current and voltage transformers are those intended for accurate power metering, it is specified that Class B transformers may be used for metering provided that the meter is the sole burden on the transformers.

The *Swedish Rules* (p. 29) specify two classes of instrument transformers suitable for power and energy metering, distinguished respectively as the "W" Class and the "kWh" Class. The "W" Class is intended for ordinary switchboard

service, while the more accurate "kWh" Class is used mainly for measurements upon which consumers are charged. The Rules are unique in that the accuracy of both classes is fixed by imposing limits on ε_p and not at all by statements of ratio error and phase-angle, these last being settled by a "control diagram" which forms an essential part of the specification.

For *Voltage transformers* the rated secondary burden is to be at least 10 VA with a power-factor of 0·95 for Class "W" and 0·8 for Class "kWh." With all values of burden down to 1 VA and for secondary voltages between 85 per cent and 105 per cent of the rated value the following limits are imposed on $\varepsilon_{pv} = \varepsilon_v - \gamma \tan\phi$. *For Class W* the limits are $\pm$ 1·25 per cent when $\cos\phi$ lies between 1 and 0·4 lagging. With these values of $\cos\phi$, $\tan\phi$ lies between 0 and 2·29, so that there is a linear relation between ε_v and γ, namely, $\pm 1{\cdot}25 = \varepsilon_v - 2{\cdot}29\gamma$. With the upper sign and either $\tan\phi$ or γ equal to zero, $\varepsilon_v = +1{\cdot}25$; with $\gamma \neq 0$ and $\tan\phi = 2{\cdot}29$ the relation between γ and ε_v is the sloping line in the lower half of the upper diagram in Fig. 163, its slope being 1/2·29. The area within the triangle bounded by this line and the part of the axis of ε_v between $+1{\cdot}25$ and $-1{\cdot}25$ must contain all values of ε_v and γ which will make $\varepsilon_{pv} + \leqslant 1{\cdot}25$ per cent. Similarly the upper triangle contains all the values which make $\varepsilon_{pv} \leqslant -1{\cdot}25$ per cent. *For Class "kWh"* the limits are $\pm$ 0·5 per cent for inductive loads with $\cos\phi$ between 1 and 0·4 ($\tan\phi$ between 0 and 2·29) and for capacitive loads with $\cos\phi$ between 1 and 0·7 ($\tan\phi$ between 0 and $-$ 1·02). The lines defining the limits of ε_v and γ for these limits of ε_{pv} are plotted in the lower diagram of Fig. 163, the shaded area referring to capacitive loads and the unshaded areas to inductive loads. For *Current transformers*, somewhat similar control diagrams set limits within which ε_c and β must be confined for specified limits in $\varepsilon_{pc} = \varepsilon_c + \beta \tan\phi$. The power factor of the burden in both cases is to be 0·8; the least rated burden for a rated secondary current is 5 VA and the limits of error are to be satisfied for all values of the burden from 25 per cent to 100 per cent of its rated value. *For Class "W"* the limits for ε_{pc} are $\pm$ 1·25 per cent at a secondary current of

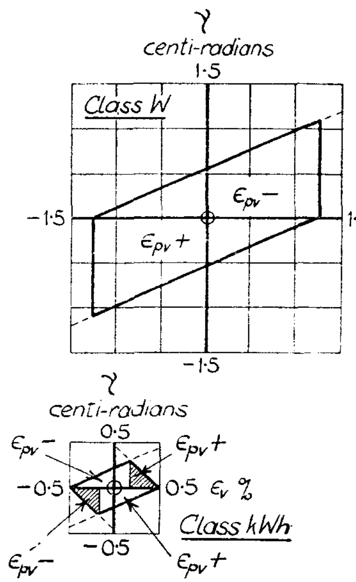


FIG. 163. SWEDISH "CONTROL DIAGRAM" FOR ERROR ε_{pv} INTRODUCED INTO A POWER MEASUREMENT BY A VOLTAGE TRANSFORMER

5 amperes and $\pm$ 2 per cent at 1 ampere, the value of $\cos\phi$ being between 1 and 0·6 lagging ($\tan\phi$ between 0 and 4/3). The limit lines for these values of secondary current are plotted in the control diagrams in the upper portion of Fig. 164, the method of construction being similar to that outlined for voltage transformers; all values of ε_c and β which satisfy the given limits of ε_{pc} must lie within the appropriate areas of the control diagram. *For Class "kWh"* the limits for ε_{pc} at a secondary current of 5 amperes are $\pm$ 0·5 per cent for inductive loads with $\cos\phi$ between 1 and 0·6 ($\tan\phi$ between 0 and 4/3) and $\pm$ 1·0 per cent for capacitive loads with $\cos\phi$ between 1 and 0·7 ($\tan\phi$ between 0 and $-$ 1·02). At a secondary current of 1 ampere the limits for ε_{pc} are $\pm$ 1·25 per cent for inductive loads with $\cos\phi$ between 1 and 0·4 ($\tan\phi$ between 0 and 2·29) and $\pm$ 2·0 per cent for capacitive loads with $\cos\phi$ between 1 and 0·7 ($\tan\phi$ between 0 and $-$ 1·02). The control diagrams are plotted in the lower diagram of Fig. 164 with the capacitive areas shaded.

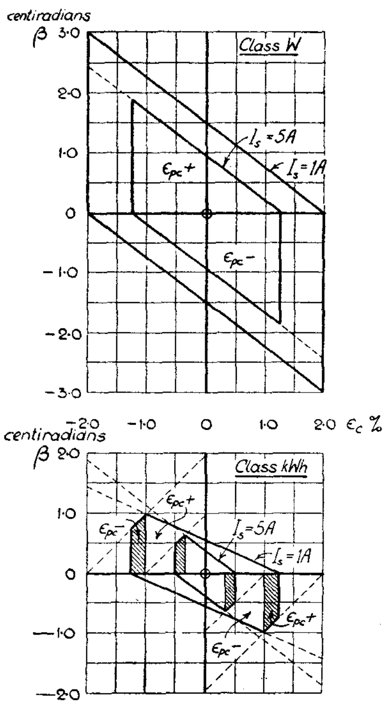


Fig. 164. Swedish "Control Diagram" for Error ε_{pc} Introduced into a Power Measurement by a Current Transformer

7. Reactive volt-ampere measurements in three-phase circuits. A further measurement in which the influence of instrument transformer errors is important is the metering of reactive volt-amperes or reactive volt-ampere-hours in a three-phase circuit by the use of two instruments or of a single instrument with two elements.* The scheme of connections is shown in Fig. 165, which should be compared with the corresponding diagram in Fig. 161. It will be seen that the wattmeter $W_{\rm I}$ has the voltage $V_{\rm II,\,III}$ applied to its voltage circuit, while $W_{\rm II}$ has the voltage $-$ $V_{\rm I,\,III}$. Referring to the vector diagram in Fig. 161 it is easy to see that the sum of the instrument readings is $V_{\rm II,\,III}\, I_{\rm I} \cos\,[(\pi/2) - \phi_{\rm I}] - V_{\rm I,\,III}\, I_{\rm II} \cos\,[(\pi/2) + \phi_{\rm II}]$,

* M. Peltier, "Comparaison des erreurs dues aux transformateurs de mesure dans l'enregistrement de l'énergie active et dans celui de l'énergie réactive," *Rev. Gén. de l'Él.*, vol. 21, pp. 295–298 (1927); see also A. Iliovici, *loc. cit.*

that is, $V_{\rm II,\,III}\, I_{\rm I} \sin\phi_{\rm I} + V_{\rm I,\,III}\, I_{\rm II} \sin\phi_{\rm II}$, neglecting the phase-angle of the voltage circuits. In the particular case of balanced load this sum becomes

$$2\, V_p I_p \sin\phi.$$

But the total reactive volt-amperes in the system is $(\sqrt{3}) V_p I_p$. $\sin\phi$; hence

Reactive volt-amperes
$= (\sqrt{3}/2)$ *sum of wattmeter readings.*

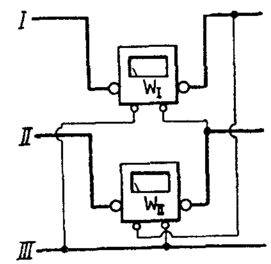


Fig. 165. Two-wattmeter Method for Reactive Volt-ampere Measurement in Three-phase Circuit

It will be observed that in this case the instruments each read one-half the total; thus when the load is balanced and the meters are perfect one instrument is really adequate.

In practice, however, the meters each have some reactance in their voltage circuits and they are connected to the network through voltage and current transformers. Using the same notation as on p. 307, the sum of the readings is actually

$$(1/F_{{\rm I}v}F_{{\rm I}c}) V_p I_p \cos\alpha_{\rm I} \sin(\phi - \delta_{\rm I}) + (1/F_{v{\rm II}}F_{c{\rm II}}) V_p I_p \cos\alpha_{\rm II} \sin(\phi - \delta_{\rm II})$$

Making the same approximations as before, the fractional error in the measurement is

$$\varepsilon_r = \frac{\textit{wattmeter readings}}{\textit{true volt-amperes}} - 1$$

$$\doteqdot \frac{(1 + \varepsilon_{v{\rm I}})(1 + \varepsilon_{c{\rm I}}) \sin(\phi - \delta_1) + (1 + \varepsilon_{v{\rm II}})(1 + \varepsilon_{c{\rm II}}) \sin(\phi - \delta_{\rm II})}{2 \sin\phi} - 1$$

$$\doteqdot (1 + \varepsilon_{v{\rm I}} + \varepsilon_{c{\rm I}})\, [\tfrac{1}{2} - \tfrac{1}{2}\,\delta_{\rm I} \cot\phi] + (1 + \varepsilon_{v{\rm II}} + \varepsilon_{c{\rm II}})\, [\tfrac{1}{2} - \tfrac{1}{2}\,\delta_{\rm II} \cot\phi] - 1$$

$$\doteqdot \tfrac{1}{2}(\varepsilon_{v{\rm I}} + \varepsilon_{c{\rm I}} + \varepsilon_{v{\rm II}}) - \tfrac{1}{2}(\delta_{\rm I} + \delta_{\rm II}) \cot\phi$$

The first term is the error due to the ratios and the second that due to the phase-angles. When the transformers are identical,

$$\varepsilon_r \doteqdot \varepsilon_v + \varepsilon_c - \delta \cot\phi = \varepsilon_v + \varepsilon_c + \{ - \delta \tan\,[(\pi/2) - \phi]\}$$

Comparing this with

$$\varepsilon_p \doteqdot \varepsilon_v + \varepsilon_c + \delta \tan\phi$$

we see that the error ε_r is the same as in an active power-measurement with a phase-angle $(\pi/2) - \phi$ in the load and a phase-error angle of $-\delta$. Just as before, the same methods of computing the corrections may be employed, taking into account these differences in the formulae for ε_r and ε_p.

CHAPTER VII

PRELIMINARY DISCUSSION OF INSTRUMENT TRANSFORMER TESTING

1. **Introductory.** The preceding chapters have given the reader a general view of the most important characteristics of instrument transformers; the purpose of the present chapter is to sketch in a general way the methods that are used in practice to measure the various characteristic magnitudes. A fully detailed consideration of the subject of instrument transformer testing is given in Parts 3, 4, and 5, the immediate object being to deal with broad principles.*

By far the most important test to be made on an instrument transformer is to find how correctly the secondary current or voltage, as the case may be, is an exact reduced copy of the corresponding primary quantity, i.e. to measure the ratio error and phase-angle and to determine how these are affected by the conditions under which the transformer is to operate. For this purpose numerous experimental processes have been introduced, classified as *Indirect* or *Direct* according to the particular principle involved.

In addition to the errors of transformation there are other matters requiring test. One of the most important of these auxiliary tests is to verify the polarity of the transformer's terminals. Other tests of practical interest relate to the safety of instrument transformers under normal and abnormal

* The following papers contain critical reviews of the subject: H. Gewecke, "Strom- und Spannungswandler und die Verfahren ihrer Untersuchung," *Elekt. Kraft. u. Bahn*, vol. 12, pp. 141–148 (1914); R. S. J. Spilsbury, "Instrument Transformers," *Beama J.*, vol. 6, pp. 505–513 (1920); F. A. Kartak, "Testing of instrument transformers," *Elec. World*, vol. 75, pp. 1368–1370 (1920); A. Barbagelata, "Sulla prova di trasformatori di misura," *L'Elettro*, vol. 8, pp. 165–175 (1921); T. Rosskopf, "Over de eigenschappen van Meettransformatoren in het bijzonder van Stroomtransformatoren," *Ver. v. Direct. v. Elect. in Nederland*, pp. 1-26, (7th Dec. 1923); F. B. Silsbee, "Methods for testing current transformers," *Trans. Amer. I.E.E.*, vol. 43, pp. 282–294 (1924); W. Janvier, "Le contrôle de la consommation d'énergie électrique dans les réseaux de distribution," *Rev. Gén. de l'Él.*, vol. 16, pp. 195–203, 237–248 (1924); G. W. Stubbings, "Current transformers. A critical study of various methods of determining errors," *Elec. Rev.*, vol. 108, pp. 462–463 (1931); G. Keinath, "Fehlergrössen des Stromwandlers. Experimentelle Bestimmung," *Arch. f. tech. Mess.*, Z224–1 (1932).

conditions of operation. For example, the mechanical strength under the influence of the large forces set up during short-circuit conditions, particularly in the case of current transformers; the ability to withstand thermal damage due to short-circuit current heating effects; and the possibility of damage being done to the insulation by voltage rises and surges, are all questions involving special experimental investigation. For the present, attention will be directed to the methods for testing ratio error and phase-angle to indicate their fundamental principles and the apparatus usually employed in them; the other classes of tests will be dealt with in later chapters of Part 5.

2. **Indirect method for measuring ratio and phase-angle.** One of the most obvious ways of finding the ratio error and phase-angle of an instrument transformer is to use some modification of the methods used to measure the regulation of a power transformer. The theory of instrument transformers being first investigated, as has been done in Chapters II and IV, to deduce expressions for the ratio and phase-angle and show how they depend upon the magnetization characteristics of the core, the reactance and resistance of the windings and of the secondary burden, etc., measurements are made upon the transformer to find the various quantities which enter into the equations. These measurements correspond with the well-known open-circuit, short-circuit and resistance tests made on power transformers, but with some considerable refinement in experimental detail because of the much smaller magnitude of the quantities involved. With the aid of these results the desired errors are easily calculated.

The method has been used by a number of workers for tests on current transformers, particularly for those having very large primary currents; a full discussion will be found in Chapter XVIII. It is now, however, superseded for this purpose by more convenient direct methods. The indirect process has been but little applied in voltage transformer testing since direct methods of greater precision and convenience are available, see Chapter XXV.

3. **Direct methods for measuring ratio and phase-angle.** In contradistinction to the indirect method are the *direct* methods, in which the ratio and phase-angle are directly measured by observing the ratio of the primary and secondary magnitudes and the phase-displacement between them when the instrument transformer is operating under its working conditions. Direct

methods are of two main classes: *absolute* or *independent*, wherein the ratio and phase-angle are found in terms of laboratory standards by measurements made directly upon the transformer; and *relative* or *comparative*, wherein the characteristics of a given transformer are compared with those of a standard transformer, the latter having been calibrated by an absolute method.

Both absolute and relative methods are further divisible into two classes, designated *deflectional* methods and *null* methods respectively. In the deflectional methods the required characteristics are computed from the readings of appropriate instruments suitably applied to the primary and secondary circuits. In the null methods it is usual to arrange some form of compensating circuit containing an alternating current detector, suitable resistance and reactance adjustments being made to reduce the deflection of the detector to zero; the ratio and phase imperfections of the transformer are thus balanced out by the auxiliary circuit, the balance setting of which enables the desired quantities to be calculated. Null methods resemble in many points of principle and technique the well-known a.c. bridge methods so widely used for impedance measurements.

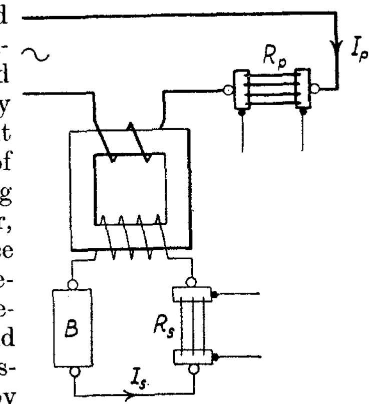


FIG. 166. ILLUSTRATING THE PRINCIPLE OF DIRECT METHODS FOR TESTING CURRENT TRANSFORMERS

4. **Direct methods for current transformers.** The general principle of the majority of direct absolute methods for testing current transformers is illustrated in Fig. 166. A suitable four-terminal resistor R_p is put into the primary circuit and a similar resistor R_s in the secondary circuit; these resistors are chosen to give approximately equal drops of voltage and must be as non-reactive as possible, so that the potential differences at their potential terminals are exactly in phase with and proportional to the respective currents entering their current terminals. Moreover, since R_s imposes a certain burden upon the secondary of the transformer, the load burden B should be a combination of resistances and reactances such that together with R_s the total burden with which the transformer

is to work in practice will be imitated. Alternatively, B may be some instrument (ammeter, watt-hour meter, etc.) provided R_s is kept small enough to have a negligible effect on the errors of transformation. The principle is then to compare the magnitudes of the voltage drops, R_pI_p and R_sI_s, and to find the phase-angle between them; this can be done in a variety of ways. In the deflectional absolute methods the drops are compared in magnitude and phase by one or more separately-excited dynamometers, or by an electrometer; alternatively, the resultant vector difference of R_pI_p and R_sI_s can be similarly measured. In the null absolute methods this resultant voltage is compensated by an auxiliary voltage of adjustable magnitude and phase; adjustments are made until a suitable detector, such as a vibration galvanometer, shows that balance has been obtained. Other different procedures may be adopted; for example, R_p and R_s may be replaced by suitable mutual inductors, the secondary voltages of which can be compared or their resultant balanced in a similar way to that just described. These and numerous other devices receive detailed explanation in Part 3.

Direct relative methods for testing current transformers fall into two groups. In the first, the unknown and the standard transformers have their primary windings excited in series; the behaviour of certain instruments when operated first from the secondary of the one transformer and next from the secondary of the other is observed, the relative values of the ratio and phase-angle errors of the two transformers being computed from the observations. The instruments may be ammeters, dynamometers or watt-hour meters, and the methods are essentially deflectional. In methods of the second group the primaries are again excited in series; the secondary currents are arranged to circulate their vectorial difference through a branch of the test network common to both secondary windings, measurement being made of the resultant secondary magnitude. This may be done in a deflectional method by the use of a separately-excited dynamometer; alternatively a null method may be devised by provision of some auxiliary device for compensating this resultant combined with a suitable balance detector. These methods are also considered fully in Part 3.

5. **Direct methods for voltage transformers.** Direct absolute methods for testing voltage transformers are based on the principle illustrated by Fig. 167. A high resistance voltage-

divider* R is put in parallel with the primary winding; the drop of voltage down a fraction r of this resistance is approximately equal to and is compared with the secondary voltage V_s. The comparison may be made deflectionally, using a separately-excited dynamometer to measure rV_p/R, V_s, or their vector difference, in magnitude and phase. Alternatively, the two voltages may be opposed through a suitable auxiliary circuit provided with means for compensating the small phase-defect between them, a null indication of an appropriate detector indicating when balance is obtained. Such methods are treated in Part 4.

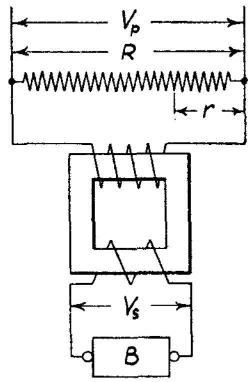


FIG. 167. ILLUSTRATING THE PRINCIPLE OF DIRECT METHODS FOR TESTING VOLTAGE TRANSFORMERS

Direct relative methods for voltage transformers consist, as a rule, in supplying the primary windings of the standard and unknown transformers in parallel and comparing their secondary voltages in relative magnitude and phase. The comparison may be made either by a deflectional process; or by suitable compensating arrangements a null technique can be employed. These methods are described in Part 4.

6. **Apparatus.** It will be appreciated from what has been said in Sections 4 and 5 that an essential preliminary to the detailed discussion of methods of testing instrument transformers for ratio and phase-angle will be an examination of the apparatus required for the purpose, some of which is of a special character; this is the object of Part 2. The general plan of this Part is to discuss the principles and properties of the standards of resistance, self- and mutual inductance and capacitance specially required in this branch of measurement, followed by a description of alternating current potentiometers and magnetic potentiometers, and of the standard current and voltage transformers required for use in relative methods. Sources of supply, phase-shifting devices, and regulating arrangements then engage brief attention, followed by the treatment of the requisite types of indicating instruments and of balance detectors. The Part concludes with a discussion of the choice of natural and artificial burdens with which the secondary circuits are loaded during the tests.

* At the highest voltages, condenser potential dividers are to be preferred as they are easier to construct free from impurity.

Some of the apparatus is of a kind familiar in general a.c. testing practice and this will receive but brief attention. Other apparatus, e.g. standard resistances of very low and very high values, standard transformers, etc., has been specially developed for instrument transformer testing and has not received adequate treatment in readily accessible form. Consequently, the space devoted in this book to apparatus is given chiefly to those details which the reader will be unable to find in the usual textbooks; for full particulars of the more familiar things the student is referred to the standard sources.

PART 2
APPARATUS USED IN TESTING INSTRUMENT TRANSFORMERS FOR RATIO AND PHASE-ANGLE

CHAPTER VIII
INTRODUCTORY

THE general purpose of Part 2 has been briefly indicated in Section 6 of the preceding chapter; it is now our object to discuss this purpose in somewhat closer detail.

The apparatus used in testing instrument transformers for ratio and phase-angle may be conveniently divided into two groups, viz. (*a*) Standards and (*b*) General Equipment and Instruments. In the first group are the standards of resistance, self- and mutual inductance, and capacitance used in the arrangement of the measuring circuit, and in terms of which the ratio and phase-angle of the transformer under investigation are expressed. Related closely to these standards are the a.c. potentiometers and the magnetic potentiometers. In this group the standard transformers, used in the relative methods of testing frequently employed in works practice, are also included. Each class of standard possesses particular properties which it is our present purpose to describe; stress is laid solely on questions of fundamental principle, so that the reader may assess for himself the merits of any piece of standard apparatus he may encounter in practice or be called upon to design. No attempt is made to describe all commercially available standard apparatus, but rather to discuss typical examples wherein the theoretical and practical requirements are satisfactorily fulfilled.

In dealing with the second group it is our object to indicate merely the special features to be possessed by general equipment and instruments if the testing of instrument transformers is to be carried out with the greatest facility and adequate accuracy. The chapters in which these subjects are discussed are by no means to be regarded as a complete treatment of the equipment and operation of a standardizing laboratory or test-room.*

In addition to the apparatus required for accuracy tests the most important is that necessary for high voltage testing of the dielectric strength of the transformers and for investigation of their behaviour under surge conditions. The discussion of h.v. testing transformers and surge generators lies outside the scope of this work; such apparatus in no way differs from that used in ordinary h.v. laboratory practice and the reader is expected to refer to the well-known extensive literature on this subject for details of design and operation.

* For a description of the National Physical Laboratory arrangements see R. S. J. Spilsbury, *Journal I.E.E.*, vol. 72, pp. 30–36 (1933).

CHAPTER IX

RESISTANCE STANDARDS

1. **Introductory.** The standard resistances required in the circuits used for Instrument Transformer Testing fall into three classes—

Low resistances, below 1 ohm in value;

Medium resistances, lying between the approximate limits of 1 ohm and 10 000 ohms; and

High resistances, having values from about 10 000 ohms upwards.

A resistor in any of these classes should conform as closely as possible to the ideal of purity, i.e. the alternating current flowing into the resistor and the potential difference across its terminals should be in phase. In addition to this non-reactive property, the magnitude of the resistance must be accurately known and be independent of various possible disturbing factors such as the arrangement of the resistor in the circuit, skin effect, temperature rise, and so on. The practical fulfilment of these ideals in each class involves quite distinct problems and the use of radically different methods of construction; it is our purpose in this chapter to examine in some detail the essential fundamental principles underlying each class of resistor.

The case of *Medium Resistances* can be briefly dismissed first, leaving the other classes for fuller discussion in subsequent sections. The resistance material is usually manganin, or some similar high-resistivity alloy, suitably insulated and wound into a compact form upon a bobbin or other support; a resistance unit of high accuracy, permanent value and very small temperature coefficient is readily constructed. The method of winding adopted varies according to the value of the resistance, having for its object the reduction of the reactance of the unit to a minimum; it is not possible, in general, to make the reactance accurately zero, a small residual reactance being usually present. In the lower resistances the residual reactance is inductive; in the higher resistances, the much greater bulk of the unit results in self- and earth-capacitances of such amount that the residual is predominantly capacitive; resistances of about 100 ohms are most perfectly non-reactive. It is not difficult to design resistors between about 100 and 1 000 ohms in value in which the time-constant is less than a few hundredths of a microsecond, which at 50 cycles per sec. corresponds with a phase-difference of the order of 3×10^{-6} radian or 0·01 minute between the current and voltage; this small phase-angle is usually negligible in practice. Phase-angles in excess of this figure occur in resistors outside these limits, owing to the greater difficulty in sufficiently reducing the residual reactance. A number of resistance units are commonly grouped to form a resistance box, most conveniently arranged in decade form with dial switches enabling any desired value of resistance to be obtained by the addition of units in series. As a rule the minimum resistance value obtained from a resistance box is large in comparison with the contact resistances at the terminals; consequently the two-terminal construction is sufficient to ensure definiteness of value when such a resistance box is joined in circuit with other apparatus. The current for which these medium-valued resistors are designed is usually small, not greater than about 0·5 ampere, so that the cross-section of the resistance material is small; consequently skin-effect is inappreciable, especially at the low frequencies used in transformer testing. The properties of medium-valued resistance standards will not be further discussed here, since the subject has been fully examined in the author's *A.C. Bridge Methods*, pp. 65–96 (1932), to which the reader is referred for greater detail.*

Resistances much outside the approximate lower and upper limits of 1 and 10 000 ohms present specially difficult problems. The low-valued resistors, usually less than 0·1 ohm, carry considerable currents and it is not possible to ensure definiteness of value by the ordinary two-terminal construction; it is necessary, therefore, to arrange them on the four-terminal plan. Since the resistance is low, the resistor will be reasonably free from phase-defect only if the residual reactance is reduced to an exceedingly small figure; this involves very special arrangements of the resistance material and the connecting leads joined to the current and potential terminals. In the case of high resistances, frequently having a value of several megohms, trouble is encountered principally on account of the importance of distributed self- and earth-capacitances of the bulky resistor units. This is especially the case in the design of potential dividers used in the testing of voltage

* Also see A. Campbell and E. C. Childs, *The Measurement of Inductance, Capacitance, and Frequency* (1935).

transformers with very high primary voltages. It is the purpose of the following sections to examine in some detail the theory, principles of design and construction of low and high resistances.

Low Resistances

2. **General considerations.** It has been pointed out on page 319 that the primary and secondary currents of a current transformer are usually compared by passing each through a suitable low-value resistor and comparing the drops of potential therein both in magnitude and phase. The resistor included in the secondary circuit forms a part of the secondary test burden, the major part being the instruments or other apparatus forming the normal secondary load under operating conditions; consequently the volt-amperes expended in the resistor should be kept small. While the secondary resistor seldom carries a current greater than 5 amperes, the resistor included in the primary circuit may carry thousands of amperes; in order that the power wasted in it shall not be excessive its resistance must be low, often less than a thousandth of an ohm. In both cases, therefore, to keep down the power dissipation the resistances are of low value; the drop of potential across them never exceeds 10 volts and is more usually 2 volts or less. Even so the power wasted in heat in the resistors used with very large currents may be quite considerable and renders special cooling arrangements necessary to ensure a low temperature-rise of the resistance material.

Since resistance is defined as the ratio of potential difference to current it becomes important in these low resistors to specify exactly where the p.d. is to be measured; this is ensured by the provision of *potential terminals*. Moreover, if the resistance is to have a definite value, the p.d. between the potential terminals must be proportional to the current and independent of the manner in which the current enters and leaves the resistor; this necessitates the use of *current terminals*. The standard low resistor for alternating current is, therefore, of the four-terminal construction already commonly in use for d.c. standards of low value and for ammeter shunts; this design is adopted primarily to ensure definiteness of resistance independently of the arrangement of the connecting leads.

The construction and theory of four-terminal resistors used in d.c. circuits is well known; the problem is, however, much more complex in the case of resistors for a.c. work on account of the influence of self-inductance in the resistance element and mutual inductance between the current and potential circuits. Since the voltages are low, at the most about 10 volts, capacitance effects are entirely negligible. If an alternating current is passed into the current terminals of a resistor a potential difference is set up at the potential terminals; if the resistor were entirely free from inductive effects this p.d. would be proportional to the current and in phase with it. In general a resistor will possess some residual inductance, in consequence of which the current and p.d. will be out of phase by a small phase-angle determined by the ratio of the inductance of the resistor to its resistance, i.e. by its *time-constant*. Since the resistance is low it follows that the impurity of the resistor will only be small if the effective inductance is also reduced to an exceedingly small absolute value; this is by no means easy to attain, especially in resistors used with very large currents, where it is necessary to obtain all possible cooling surface to carry away the considerable generated heat.* In these it is not possible, therefore, to keep current-carrying parts with currents flowing in opposite directions *very* close together and the phase-defect in them is greater than in higher resistances.

Again, when a resistor is used with alternating current, especially if the current terminals and resistance material have considerable bulk the current distribution, and with it the resistance, may be a function of frequency, in consequence of the influence of "skin effect." Let $\boldsymbol{i}$ be the current passed through the resistor, $\boldsymbol{v}$ the p.d. across the potential terminals, both assumed to be harmonic vectors with sinusoidal time-variation; then

$$z = R' + j\omega L = \boldsymbol{v}/\boldsymbol{i}$$

is the impedance operator of the resistor, R' its effective resistance and L its effective inductance. The various methods by which L is arranged to be a small and definite quantity are discussed in later sections, as also are the ways for making R' closely equal to the d.c. resistance R, definite in value and independent of moderate temperature changes.

3. **Desirable qualities in low resistances.** A low resistance standard for alternating current should possess the following essential qualities.

* A resistor of 0·002 ohms for 1 000 amperes, for example, absorbs 2 kilowatts.

(*a*) Permanence of value.
(*b*) Low resistance-temperature coefficient.
(*c*) Facilities for satisfactory cooling.
(*d*) Easy exact adjustment of resistance value.
(*e*) Resistance to be definite independently of changes in the position of the leads connected to the current and voltage terminals.
(*f*) Lowest possible residual inductance and time-constant.
(*g*) Small external magnetic field.
(*h*) Negligible "skin effect."

Of these (*a*) to (*e*) and (*g*) are important also in standards used with d.c. for which many designs have been made satisfying these essential conditions; (*f*) and (*h*), on the other hand, present problems which are peculiar to a.c. work and necessitate modifications in the design and construction of the resistors to enable them to be regarded as satisfactory in these respects.

Experience has shown that a resistor having great permanence and stability of value can be obtained by the use of carefully selected, well-annealed manganin wire, tube or sheet.* This material is an alloy of copper (about 85 per cent), manganese (about 12 per cent), nickel (about 5 per cent), and iron (a few tenths per cent); the resistivity is about 35 to 55 microhm-centimetres, according to the composition of the alloy. The annealing must be carefully done; the manganese in the surface layer suffers selective oxidation, leaving a skin relatively richer in copper and increasing the resistance-temperature coefficient. This can be avoided by annealing the material in an atmosphere of carbon dioxide. After annealing, the outer surface is removed by means of emery paper or by pickling in acid, the manganin being then silver-soldered to the copper current-terminals; the resistance material should be supported so that deformation is avoided, and be protected from oxidation by a coating of enamel or varnish. A resistor tends to rise in value about 0·1 per cent in the first year; it should be kept, therefore, for at least a year before finally adjusting and standardizing. Thereafter the rate of ageing is very small. The resistance-temperature coefficient of manganin is exceedingly low, frequently not more than a few parts in a million per degree centigrade; by appropriate proportioning of the

* For a full discussion of the properties of manganin see *Dictionary of Applied Physics*, vol. 2, "Resistance, Standards and Measurements of," pp. 705–710 (1922).

constituents of the alloy it is possible to bring the coefficient to zero or even to make it negative. In building a resistor with multiple resistance elements it is often possible, by selecting material with positive and negative coefficients, to construct the resistor with practically no resistance-temperature variation. Since the resistance value is so slightly influenced by rise of temperature it follows that quite considerable temperature rise may be tolerated; even so, a resistor for large currents may become very bulky in order that sufficient cooling surface may be secured so that the large amount of power generated in the resistor may be dissipated without excessive heating. Numerous cooling devices have been used in practice, some of which are noticed in later sections.

Constantan is sometimes used in preference to manganin on account of its greater freedom from corrosion troubles and the ease with which it can be soldered. This material is an alloy of copper (66 per cent) and nickel (34 per cent) and has the disadvantage of a high thermal e.m.f. with respect to copper. While this may be troublesome in standards used with direct currents it is entirely unimportant in a.c. standards. This material has been adopted for the a.c. standard resistors at the National Physical Laboratory.

These preliminary remarks sufficiently dispose of the desirable qualities (*a*), (*b*), and (*c*). The remaining conditions are somewhat closely linked together and need not be separately discussed; the essential principles governing the design of resistors intended to satisfy these conditions can only be appreciated by first considering the theory of the four-terminal conductor, which we now proceed briefly to summarize.

4. **Definitions.** Consider a mass of conducting material upon the surface of which four limited areas A, B, C, D are taken to act as terminals for connection of the mass to other conductors. If a direct current enters one area and leaves at another there is, in general, a difference of potential between the other pair; the ratio of this p.d. to the current is one of the resistances of the conductor. For example, in Fig. 168 let unit current enter at A and leave at B, the p.d. between C and D being observed; then the resistance is defined by the symbols $[CD/AB]$, where the first pair of letters denotes the places between which the p.d. is measured and the second pair the areas at which unit current enters and leaves respectively. Using all four terminals it would appear that the conductor has twenty-four such resistances, but a simple consideration

will show that in actuality there are not more than six. For example, sending unit current from B to A instead of from A to B reverses the p.d. between C and D; also with the current in a given direction the p.d. between C and D is equal and opposite to that between D and C. Thus we have,

$$[CD/AB] = -[CD/BA] = -[DC/AB] = [DC/BA].$$

There will be five additional, similar relations when the current terminals are AC, AD, BC, BD, and CD respectively. These six relations reduce the original twenty-four four-terminal resistances of the conductor to six, and it will be shown that of these only two are really independent.

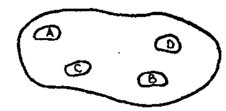


FIG. 168. DEFINING THE RESISTANCES OF A FOUR-TERMINAL CONDUCTOR

For any one of the resistances so defined to be definite in value, the ratio of the p.d. at the potential terminals to the current entering and leaving at the current terminals must be independent of the parts of each of the areas A, B, C, D used to make contact with the leads joining the conductor to an external circuit. That is, the p.d. between all parts of the potential terminals to which the potential leads are attached must be the same, and this p.d. must be unaffected by the distribution of current over the areas serving as current terminals. Methods of attaining this ideal in practice will be discussed later.

When such a conducting mass is supplied with alternating current the question is more complex. A sinusoidal a.c. entering at A and leaving at B will set up a p.d. between the areas C and D; this p.d. will not, however, be in phase with the current. It may be resolved into two components, in phase and in quadrature with the current respectively. The ratio of the in-phase component of p.d. to the current is the effective resistance; in consequence of skin effect this will not, in general, be equal to the resistance value obtained with d.c., and one of the problems of resistor design is to make the effective a.c. and the d.c. resistances as nearly equal as possible. The ratio of the quadrature component of p.d. to the current is the reactance of the conductor with the specified arrangement of current and voltage terminals. In other words, the ratio of the e.m.f. induced in a circuit composed of the conductor and one pair of leads to the rate of change of current in a circuit composed of the conductor and the second pair of leads is the inductance for the given arrangement. There are clearly as many reactances or inductances as there are resistances.

If any one of the inductances is to be definite in value, the terminals must be brought out so that the relative positions of the current and potential leads are invariable and definite. Moreover, since the conductor must behave as nearly like a pure resistor as possible, the inductance should be vanishingly small.

Further consideration of the four-terminal conductor is greatly simplified by the use of the reciprocal theorem, which enables the relationships between the various resistances or inductances to be more usefully developed.

5. **The reciprocal theorem.** The reciprocal theorem may be stated as follows: *In any four-terminal conductor the p.d. between A and B due to unit current entering C and leaving D is equal to the p.d. between C and D if unit current enters A and leaves B.*

This theorem was first stated and proved for linear conductors, i.e. circuits composed of thin wires, by Kirchhoff* in 1847, two years after the publication of the well-known "Kirchhoff's rules."

Helmholtz† in 1853 developed the theorem to apply to non-linear conductors composed of a solid, homogeneous, isotropic material.‡ The generalization for non-homogeneous, anisotropic conductors was made by Rosen in 1887. All these earlier workers assumed direct current, and we shall first establish the theorem for this condition, using a form of proof suggested by Heaviside and published by G. F. C. Searle§ in an important paper which should be consulted by the reader interested in the complete theory of the four-terminal conductor.

Consider a conducting mass, as in Fig. 169, into which a direct current i enters at a *point* A and leaves at a second point B. At any point P in the mass a tube of current flow of small cross-section $d\alpha$ carries a current $di = u \, . \, d\alpha$, where u is the current density at P. Now let i be stopped and a second current i' enter at a point C and leave at D. Potential differences will be set up between other pairs of points in the mass; let the electric force e' at P due to i' have a direction θ relative

* G. Kirchhoff, "Ueber die Auflösung der Gleichungen, auf welche man bei der Untersuchungen der linearen Vertheilung galvanischer Ströme geführt wird," *Ann. der Phys. u. Chem.*, vol. 72, pp. 497–508 (1847).

† H. von Helmholtz, "Ueber einige Gesetze der Vertheilung elektrischer Ströme in körperlichen Leitern mit Anwendung auf die thierisch-elektrischen Versuche," *Ann. der Phys. u. Chem.*, vol. 89, pp. 211–233, 353–377 (1853).

‡ A body is *homogeneous* if it consists of the same kind of material throughout. A body is *isotropic* if it has identical physical properties in all directions. For example, a piece of rolled steel plate is homogeneous but not magnetically isotropic, having a different permeability along the direction of rolling from that across. In the discussion given in the text we are concerned with electrical isotropy or its absence. A body which is not isotropic is referred to as *anisotropic* or *allotropic*.

§ G. F. C. Searle, "On resistances with current and potential terminals," *Elecn.*, vol. 66, pp. 999–1002, 1029–1033 (1910); vol. 67, pp. 12–14, 54–58 (1911).

to the tube of flow APB due to the original current. We shall calculate the value of

$$Q = \int ue' \cos\theta \, . \, dv,$$

throughout the whole mass, where dv is an element of volume. Take an element of the tube APB, ds in length; the elementary volume $dv = d\alpha \, . \, ds$, so that the contribution of the element to the integral is $u \, . \, d\alpha \, . \, e' \cos\theta \, . \, ds$. The contribution of the whole tube is thus,

$$di \int_A^B e' \cos\theta \, . \, ds,$$

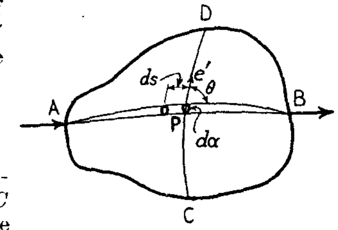


FIG. 169. THE RECIPROCAL THEOREM

the integral being the excess of potential of A over B when i' flows from C to D; this is clearly independent of the particular tube considered and is proportional to i'. Let $[AB/CD]$ denote the resistance measured by the p.d. between A and B when unit current passes from C to D. Then,

$$\int_A^B e' \cos\theta \, . \, ds = i'[AB/CD], \text{ and } Q = i'[AB/CD]\int di = ii'[AB/CD].$$

In a similar manner we can show that

$$Q' = \int u'e \cos\theta' \, . \, dv = ii'[CD/AB]$$

The reciprocal theorem will be established if we can show that $Q = Q'$, which implies $ue' \cos\theta = u'e \cos\theta'$. So far no mention has been made of the nature of the body which, in general, is both non-homogeneous and anisotropic. In an anisotropic body the electrical resistivity is different in different directions about a point; resolving the current-density at a point into three components parallel to suitable cartesian axes, u_x, u_y, u_z say, the corresponding components of electric force are $\rho_x u_x$, $\rho_x u_y$, $\rho_y u_z$ where ρ_x, ρ_y, ρ_z are the resistivities in the axial directions at the point. Consequently, in an anisotropic mass the resultant current-density and the resultant electric force at a point cannot have the same direction.

Consider now the simpler case of an isotropic conductor of resistivity ρ; then u coincides in direction with e and u' with e'. Moreover, $e = \rho u$, $e' = \rho u'$ and $\theta = \theta'$; hence

$$ue' \cos\theta = \rho uu' \cos\theta,$$

$$u'e \cos\theta' = \rho uu' \cos\theta,$$

and hence

$$[AB/CD] = [CD/AB]$$

which establishes the theorem for the particular case of isotropy. Notice that the mass need not be homogeneous; the theorem applies, therefore, to a body composed of a number of different isotropic pieces joined by isotropic solder.

Searle [loc. cit.] has generalized the proof of the theorem for the case of an anisotropic mass carrying direct current. The most general proof of all, however, has been supplied by Wenner,* who has shown that the theorem applies to a non-homogeneous, anisotropic conductor carrying alternating current, the current and potential contacts being made at finite areas upon the surface of the mass, and not at points. Hence the reciprocal theorem may be regarded as established for all practical conditions.

6. **Theory of four-terminal conductor.** The twenty-four four-terminal resistances (or impedances) of a conductor reduce by the condition of symmetry given on p. 331 to the following six—

$$[AB/CD],\ [AC/BD],\ [BC/AD],\ [CD/AB],\ [BD/AC],\ [AD/BC].$$

Applying the reciprocal theorem,

$$[AB/CD]=[CD/AB];\ [AC/BD]=[BD/AC];\ [BC/AD]=[AD/BC],$$

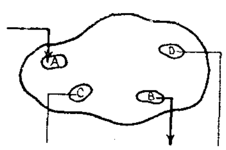


FIG. 170
THE FOUR-TERMINAL CONDUCTOR

so that only three of the four-terminal resistances are different. We shall now show that there is a relationship between them.

Referring to Fig. 170 let A, B be chosen as current terminals and C, D as potential terminals; then with unit current entering A the four-terminal resistance is $[CD/AB]$. Associated with it is the two-terminal resistance $[AB/AB]$ and the three-terminal resistances $[CB/AB]$, $[DB/AB]$, $[AC/AB]$, and $[AD/AB]$. Now the p.d. between C and D is equal to that between C and B minus that between D and B; hence

$$[CD/AB] = [CB/AB] - [DB/AB].$$

Similarly, if unit current had been passed between C and B we should have had

$$[AD/CB] = [AB/CB] - [DB/CB] \equiv [CB/AB] - [DB/CB]$$

using the reciprocal theorem. Again if unit current had passed from D to B we should have obtained

$$[CA/DB] = [CB/DB] - [AB/DB] \equiv [DB/CB] - [DB/AB].$$

Adding these relations,

$$[AD/CB] + [CA/DB] = [CB/AB] - [DB/AB] \equiv [CD/AB].$$

Referring now to Fig. 171 (*a*), which shows a four-terminal resistor diagrammatically, the term $[CD/AB]$ may be called the *direct resistance*; $[AD/CB]$ is the resistance that would be

* F. Wenner, "The four-terminal conductor and the Thomson bridge," *Bull. Bur. Stds.*, vol. 8, pp. 559–609 (1913); also see *Phys. Rev.*, vol. 32, pp. 614–616 (1911).

found by sending unit current from one current terminal to the opposite potential terminal and measuring the p.d. between the other pair, it is conveniently called the *diagonal resistance*; $[CA/DB]$ is the ratio of the p.d. between a current terminal and its adjacent potential terminal when unit current is passed between the other pair, it is called the *cross resistance*. Then

$$[CD/AB] = [AD/CB] + [CA/DB],$$

or *Direct resistance* = *diagonal resistance* + *cross resistance.*

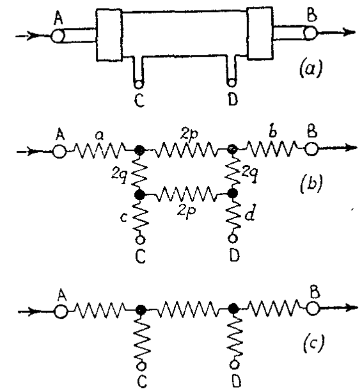


FIG. 171. EQUIVALENT CIRCUITS FOR FOUR-TERMINAL CONDUCTOR

Dr. Searle has shown that since a four-terminal conductor requires for its specification six related resistances, it is equivalent to the network of wires shown in Fig. 171 (*b*), this also having six degrees of freedom; it should be noted that some of the branches of the network may have negative values, so that in general it is not possible to construct a model of real wires having the same electrical properties as the actual resistor. If, however, the cross resistance $[CA/DB] \to 0$, so that the direct and the diagonal resistances are equal, the simple five-wire network of Fig. 171 (*c*) is accurately equivalent to the resistor, which is then termed *linear*. This is usually the case in practice where the current terminals are attached to substantial copper blocks by lugs of sufficient length. For example, in the well-known Reichsanstalt pattern 0·001 ohm resistor the ratio $\frac{[CD/DB]}{[CD/AB]}$ is less than 10^{-6}; in a similar 0·0001 ohm resistor less than 3×10^{-5}; in a 0·1 ohm at the National Physical Laboratory less than 5×10^{-8}; and in the standard mercury ohm about 10^{-100}. Hence with proper design a practical four-terminal resistor is very closely linear and is usually assumed to be so.

It will be realized that this argument is immediately applicable to a.c. resistors provided that the various relationships are taken between the impedance operators instead of the d.c. resistances.

7. **Current and potential terminals.** The question of proper location of the current and potential terminals is of great importance in determining the definiteness of value of a given resistor; the subject has been fully considered by Dr. Searle in the paper cited, to which the reader is referred for complete details.

Finite currents must necessarily enter a resistor through an area and not at a point. If current enters an area at A and leaves a second area at B the p.d. between C and D will be definite if the distribution of current across the areas is definite, i.e. if the lines of flow of the current in that part of the resistor to which C and D are attached are independent of the way in which the current leads make contact with the areas at A and B. This can only be secured in practice by designing the resistor so that A and B are sufficiently far from C and D to ensure that the current flow in the resistance material does not much depend on accurate placing of the current leads upon their terminals. Dr. Searle has illustrated the magnitude of the effect to be expected by solving the two-dimensional flow problem shown in Fig. 172 (a), where the conductor consists of a semi-infinite, thin, conducting strip of width a with an electrode A attached to one corner of the edge AY.

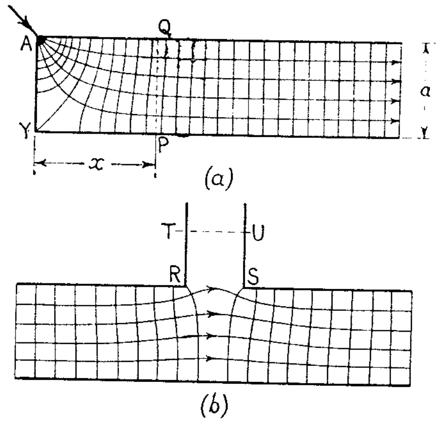


FIG. 172. ILLUSTRATING THE THEORY OF POTENTIAL TERMINALS

At a very short distance from the end of the strip the equipotentials are practically straight lines perpendicular to its sides. Consequently, the p.d. between two points Q and P equidistant from the end on opposite sides of the strip will be very small. Dr. Searle shows that if x/a is about 3, the p.d. between Q and P is only about 1/10 000 of the fall of p.d. over a length of strip equal to a where the current has become uniformly distributed. Hence it would appear that if a point potential electrode were to be applied anywhere on the line QP the potential assumed by it would be practically independent of the situation of the current electrode along AY, provided $x/a > 3$. This suggests that the current terminals should be situated at the end of lugs, the length of each of which is at least three times its greatest width, Fig. 171 (a); the accurate placing of leads in the current terminals will then have a negligible effect on definiteness of p.d. between the potential points.

In practice, however, potential contacts must also be made at areas and not at points; the potential terminals must be so designed, therefore, that on each there is an area over which the potential is sensibly invariable and dependent only upon the magnitude of the current. Dr. Searle has shown that this can be attained by attaching the potential terminals at the ends of rods or tongues soldered to the resistance

material, as at C and D in Fig. 171 (a). In the two-dimensional problem of an infinitely long tongue, Fig. 172 (b), the p.d. between opposite points T and U is less than 1/100 000 of that between R and S, where the tongue joins the resistor, if US is about four times RS; the potential terminal could, therefore, be attached anywhere on the line TU without seriously affecting its potential. Here again, in practice, with tongues of finite length, great definiteness is obtained if the potential terminals are at the ends of rods or tongues of which the length is not less than three times their width.

The system of potential tongues has the practical inconvenience of making adjustment of the value of the resistor somewhat difficult, and permits of adjustment only in one direction. It is necessary to attach the tongues so that the resistance is a little low, bringing it up to the correct value by making sawcuts in the edge of the manganin strip, a method not applicable to tubular types of resistor. This form of potential electrode, moreover, does not lend itself readily to schemes for the reduction of the effective inductance in a.c. working.

The method of attachment shown in Fig. 173 (a) is frequently preferred, and is equally applicable to strip and to tubular resistors. The terminals C, D are fixed at the ends of tongues formed by making U-shaped sawcuts in the manganin strip or tube; lengthening the saw cut at C decreases and at D increases the resistance, thus providing adjustment in both directions. Definiteness is secured if the terminal is at the end of a tongue having a length two or three times its width.

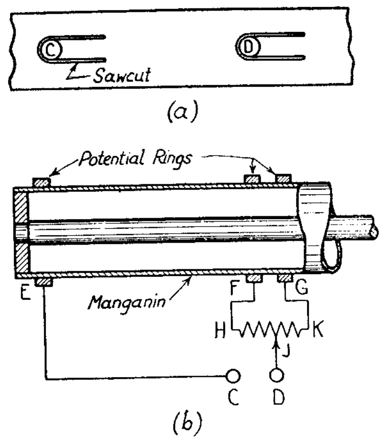


FIG. 173. TYPES OF POTENTIAL TERMINALS

Tubular resistors are frequently used on account of their freedom from magnetic leakage field and their very small residual inductance. The cutting of slots as in Fig. 173 (a) materially affects the uniformity of the current stream lines and results in a small local magnetic flux linked with the potential circuit, which adds to the residual. The arrangement shown in Fig. 173 (b) is free from this defect, and though shown in application to a tubular resistor is equally applicable to other forms. Three potential rings E, F and G are provided, the resistance between E and F being slightly lower and between E and G slightly higher than the nominal value. A fairly high resistance HK connects F and G, having a value several thousand times the resistance of the main conductor between F and G. Adjustment of the value is made by soldering the tapping J to an appropriate point in HK, the range of variation being approximately FG/EG.*

* This device may also be used to provide a variable four-terminal resistor by making HK a slide wire upon which J is the sliding contact. Such a device is frequently required in current transformer testing.

8. **"Open" and "closed" resistors.** We come now to the problem of reducing the time-constant of a resistor used with a.c. Neglecting skin-effect, if a sinusoidal current $\boldsymbol{i}$ flows in the resistor, the p.d. at its potential terminals would be $R\boldsymbol{i}$ if inductive effects were absent. Actually the material of the resistor has effective self inductance l, so that the p.d. becomes $(R + j\omega l)\boldsymbol{i}$. In addition, the proximity of the current and potential circuits results in the injection into the latter of an e.m.f. of mutual

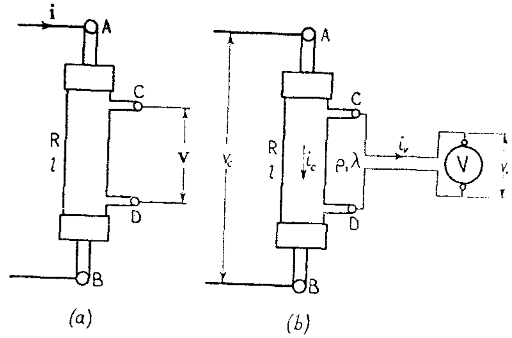


FIG. 174. ILLUSTRATING THE INDUCTION COEFFICIENTS OF A FOUR-TERMINAL RESISTOR

induction; let m be the effective mutual inductance between the two circuits, then the p.d. at the ends of the potential leads will become finally $[R + j\omega(l + m)]\boldsymbol{i} = \boldsymbol{v}$, as indicated in Fig. 174 (*a*); m may be a positive or a negative quantity.

Writing this as $(R + j\omega L)\boldsymbol{i} = \boldsymbol{v}$, the time-constant of the resistor is L/R seconds and its phase-angle is arctan $(\omega L/R) \doteq \omega L/R$ radians very nearly, since L/R is small. The smallness of the time-constant is a useful measure of the perfection of a resistor, since the phase-angle between the current and voltages is directly proportional to it and to the frequency; in the ideal resistor L/R should be zero, so that in practice we must aim at reducing L as nearly to zero as possible.

This simple theory enables us to classify resistors according to the means adopted to deal with the effective inductance $L = l + m$. In one type sufficient mutual inductance is provided to be equal to l but of opposite sign; then L becomes zero. The resistance material in this case forms an *open* inductive element; such arrangements are not necessarily astatic. In the second type the resistance material forms practically a *closed* circuit, so that m is zero, while l is made as small as possible by appropriate arrangement of the go and

return conductors in proximity. This classification into open and closed types is convenient for the purpose of analysing the different methods of construction adopted in practice.*

9. **Construction of open-type resistors.** The method of reducing the effective inductance of a resistor to zero by making $m = -l$ was first suggested by A. Campbell† utilizing a principle described by Lichtenstein.‡ As shown in Fig. 175 the potential leads consist of strips equal in width to the working resistance placed as close to it as possible, but insulated from it, and tied securely in position. By this arrangement the self-inductance of the resistance material and the mutual-inductance between it and the potential leads are made sensibly equal; exact equality could only be secured by making the two parts strictly coincident, but very accurate compensation may be secured in practice.§

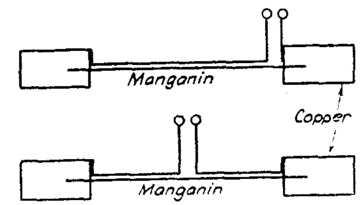


FIG. 175. CAMPBELL'S OPEN-TYPE RESISTORS

Referring to Fig. 174 (*b*), the resistor is shown as a shunt to some voltage measuring device V; the various currents and voltages vary with time in any desired way. If R, l are the resistance and inductance of the working material, and ρ, λ those of the potential leads up to the terminals of V, at which the p.d. is v_v; then

$$v_c = Ri_c + l\frac{di_c}{dt} + m\frac{di_v}{dt} \doteq \rho i_v + \lambda\frac{di_v}{dt} + m\frac{di_c}{dt} + v_v,$$

neglecting the influence of cross-resistances, m being the mutual inductance between R, l and ρ, λ. If the potential leads lie very close to the resistor and are symmetrically situated with respect thereto we can take $l = \lambda = m$ numerically, then

$$v_c \doteq Ri_c + m\frac{d}{dt}(i_c + i_v) \doteq \rho i_v + v_v + m\frac{d}{dt}(i_c + i_v);$$

so that, $$v_v \doteq Ri_c - \rho i_v,$$

* F. B. Silsbee, "A study of the inductance of four-terminal resistance standards," *Bull. Bur. Stds.*, vol. 13, pp. 375–421 (1916). See also Campbell and Childs, loc. cit.

† A. Campbell, "On compensation for self induction in shunt resistances," *Elecn.*, vol. 61, pp. 1000–1001 (1908).

‡ L. Lichtenstein, "Zur Theorie der Wechselstromkreise," *Dingler's Poly. J.*, vol. 321, pp. 38–41, 109–112, 118–123 (1906).

§ When the potential lead follows exactly the shape of the working material, l is always greater than m on account of the flux in the insulating layer between them; the like is also true when the potential lead consists of a single central wire or a number of wires in parallel arranged symmetrically about the width of the strip. In these cases therefore the effective inductance is always positive. This is not necessarily true when the potential leads do not ensheath the resistor or when they are not symmetrically placed.

exactly as if the inductances were absent. Note that this result is independent of the law of time-variation followed by the voltages and currents, and that the behaviour resembles that of a d.c. shunt whether the potential circuit carries current or not.

This principle was adopted by Paterson and Rayner* in the resistors designed by them for the National Physical Laboratory in 1909. The working resistance consists of a seamless manganin tube with thin walls, in which skin effect

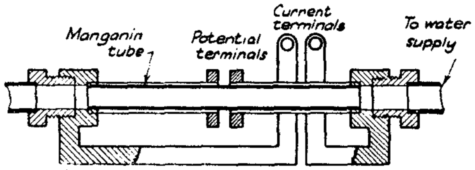


FIG. 176. PATERSON AND RAYNER'S TUBULAR RESISTOR

will be very slight; the cross-section is proportioned so that a length of about 40 cm. gives the desired resistance. After annealing at a red heat the tube is silver-soldered to copper rings which are screwed and sweated into the current terminal lugs, the whole being arranged to be almost astatic, as shown in Fig. 176; the surface of the tube is removed by immersion in acid (see p. 329), after which the inner surface is stove-enamelled to protect it from corrosion. The potential leads consist of thin copper tubes, composed of foil 0·04 mm. thick, coaxial with the resistance material and insulated from it by varnished cloth 0·2 mm. thick. The resistors are designed for a drop of 2 volts with normal current, but if a temperature change of resistance of 2 parts in 10 000 is permissible much higher currents can be carried. The tubes are mounted with their axes in the vertical plane, a vigorous flow of cold water being maintained through them at a rate of about 15 litres per minute; the cooling water enters at the bottom and is discharged at the top, thus ensuring that the tubes are always full and cannot be damaged if switched into circuit before the water supply is turned on. This system provides excellent cooling, permitting a current-density of 25 amperes per sq. mm. (16 000 amperes per sq. in.) in the manganin to be safely used. The table on p. 341 gives the principal characteristics of the resistors; "normal current" gives 2 volts drop, while

* C. C. Paterson and E. H. Rayner, "Non-inductive, water-cooled standard resistances for precision alternating current measurements," *Journal I.E.E.*, vol. 42, pp. 455–470 (1909).

PRINCIPAL CHARACTERISTICS OF RESISTORS

Resistance in ohms	0·04	0·02	0·01	0·002	0·001
Normal current, amperes	50	100	200	1000	2000
Maximum current, amperes	115	260	450	1300	2500
Outside diameter of tube, mm.	6	10	15	30	40
Thickness of wall, mm.	0·25	0·30	0·40	1·00	1·50
Length of tube, cm.	35·5	40	39	48	42·5
kW at maximum current	0·53	1·35	2·00	3·40	6·25
Time-constant microseconds	0·16	0·27	0·34	1·85	3·00
Calculated inductance, cm.	6·5	5·4	3·4	3·7	3·0
Phase-angle at 50 cycles per sec. { radians	$50{\cdot}2 \times 10^{-6}$	$84{\cdot}8 \times 10^{-6}$	107×10^{-6}	581×10^{-6}	943×10^{-6}
Phase-angle at 50 cycles per sec. { minutes	0·173	0·292	0·368	2·00	3·24

"maximum current" can be safely carried without the resistance value changing more than 2 in 10 000.

Hartshorn* has recently given the results of inductance measurements made on some of the Paterson-Rayner resistors, the values being given in the table—

Resistance in ohms	0·02	0·01	0·002	0·001
Calculated inductance, cm.	5·4	3·4	3·7	3·0
Measured inductance, cm.	5·8	4·1	2·2	3·2

Agreement is as close as can be expected, considering that the calculation takes no account of the gap at the middle of the potential sheath and the fact that the insulation has been renewed since the calculations were originally made. Experience with these resistors† during the past twenty-six years shows that the enamel film is apt to crack, the cooling water then causing corrosion of the resistance material, necessitating fairly frequent adjustment and measurement of the resistance. The residual inductance is neither as small nor as constant as is desirable, especially in the heavy-current resistors; this has caused them to be gradually superseded at the National Physical Laboratory by an improved design described on p. 349.

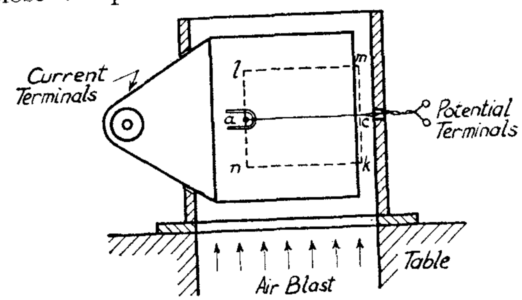
A well-designed series of resistors using the compensation principle has recently been described by Silsbee,‡ presenting many interesting features, a typical example being shown diagrammatically in Fig. 177. They range in value from 0·05 to 0·001 ohm and give a volt-drop of 0·5 volt. The resistance material is folded into a series of loops, the sides of which are about 9 mm. apart, attached to copper current-terminals; both sides of the manganin strip are exposed to the cooling action of an air blast supplied by a motor-driven blower through a trunk in the top of the table. The air velocity is about 1 600 cm. per sec. (36 m.p.h.) and dissipates 0·017 watt per sq. cm. per ° C. rise; the 500 ampere resistor has a temperature rise of about 5° C. and the others considerably less. Air cooling has the advantage that the resistance material is not subjected to any corrosive action, as may occur with water or with acid-contaminated oil; it eliminates, moreover, the leakage and mess usually associated with oil-cooling.

* L. Hartshorn, "Standards of phase-angle," *World Power*, vol. 8, pp. 171–180, 234–240 (1927); "The measurement of the inductances of four-terminal resistance standards," *Proc. Phys. Soc.*, vol. 39, pp. 377–387 (1927).

† Resistors of the Paterson-Rayner type for 200, 1 000 and 2 400 amperes have also been designed at the P.T.R.; see Schering and Alberti, *Arch. f. Elekt.*, vol. 2, pp. 263–275 (1914).

‡ F. B. Silsbee, "Notes on the design of four-terminal resistance standards for alternating current," *Bur. Stds. Journal of Res.*, vol. 4, pp. 73–107 (1930).

The potential leads are attached to U-shaped lugs at *a* and *b*, as in Fig. 173 (*a*); they consist of flattened No. 30 wire* insulated with silk and are securely tied to the manganin strip as indicated. In the 10, 25, 50, and 100 ampere resistors, where the strip is relatively narrow, the potential leads are placed along the centre of width of the strip as at *ac*, this giving a very close compensation for effective inductance and skin



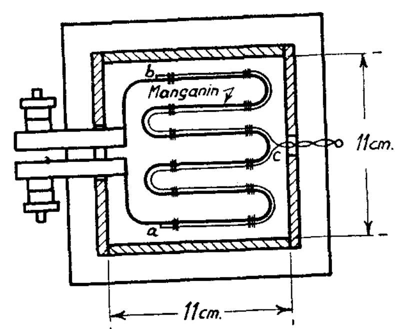


FIG. 177. SILSBEE'S AIR-COOLED RESISTOR

effect as well as rendering the phase-angle of the resistor practically immune from the influence of stray fields. In the 200 ampere resistor with wider manganin strip the leads follow the line *almc*, the part *lm* being situated at a distance of 2/10 of the width of the strip from the edge; this is shown by theory and experiment to give a more accurate compensation than

* The use of thin wire instead of a strip equal in width to the manganin interferes as little as possible with cooling from both surfaces of the manganin strip and has practically the same compensating effect provided the manganin strip is not too wide.

the central lead. In the 500 ampere resistor, where the strip is still wider, two leads in parallel are used, following the lines *almc, ankc*; this restores the immunity to stray fields sacrificed by the use of a single asymmetrically situated lead in the preceding resistor, as well as giving very good compensation. The following table gives the principal particulars of the resistors.

Resistance, ohms	0·05	0·02	0·01	0·005	0·0025	0·001
Current, amperes	10	25	50	100	200	500
Length, cm.	75	75	75	65	76	72
Width, cm.	1·6	4·0	8·0	7·0	13·3	20·0
Thickness, cm.	0·056	0·056	0·056	0·081	0·10	0·14
Watts per sq. cm.	0·021	0·021	0·021	0·055	0·050	0·083
Phase-angle at 60 cycles per sec. measured, min.	+ 0·4	+ 0·3	+ 0·1	− 0·3	− 0·4	− 2·0
Time-constant, microseconds	+ 0·31	+ 0·23	+ 0·08	− 0·23	− 0·31	− 1·52

10. **Construction of closed-type resistors.** In the closed-type of four-terminal resistor the resistance element is constructed in such a way as to have a very small residual magnetic field (sometimes termed the *residual*), i.e. so that l is the least possible, while m is made as nearly zero as possible by the use of short potential leads lying outside the influence of the residual field. To fulfil these conditions the resistance material may take the form either of a flat strip folded back upon itself at the middle of its length, or a concentric tubular construction may be used.

The folded strip construction appears to be due to Ayrton and Mather* who, in 1892, described a resistor consisting of twelve elements, each composed of a platinoid strip 4 cm. × 0·25 mm. × 6 m. in length doubled upon itself, with varnished silk between the halves. The resistance of each element was about ¼ ohm and they could be grouped in a variety of ways according to the current to be dealt with.

Orlich† in 1909 described a series of resistors, shown diagrammatically in Fig. 178, composed of folded manganin strip with mica insulation 0·1 to 0·3 mm. in thickness between the halves. The strip is very thin, so that skin-effect is negligible at supply frequencies. In the higher-valued resistors, the construction shown on the left is used, while for the lower values that on the right is preferred; the space occupied by insulation

* W. E. Ayrton and T. Mather, "The construction of non-inductive resistances," *Proc. Phys. Soc.*, vol. 11, pp. 269–275 (1892).

† E. Orlich, "Ueber Starkstromwiderstände mit kleiner Selbstinduktion," *Zeits. f. Inst.*, vol. 29, pp. 241–256 (1909).

is much exaggerated, the two halves of the element and the copper current terminals actually coming as close together as possible. The resistors are immersed in oil which is vigorously stirred; about 1 watt per sq. cm. of surface in contact with the oil can be dissipated effectively without undue temperature rise. The table gives essential particulars of the resistors.

Resistance, ohms	0·03	0·01	0·003	0·001
Current, amperes	40	100	333	1000
Volt drop, volts	1·2	1·0	1·0	1·0
Length of strip, cm.	51	42	49	69
Width ,, ,,	1·42	3·6	6·85	14·5
Thickness ,, mm.	0·5	0·5	1·0	2·0
kW	0·048	0·100	0·333	1·00
Time-constant, microseconds	0·57	0·58	1·7	5·3
Inductance, cm.	17	5·8	5·0	5·3

Potential terminals are situated in a plane perpendicular to that containing the current terminals and are joined by very short leads to the magnanin strip.

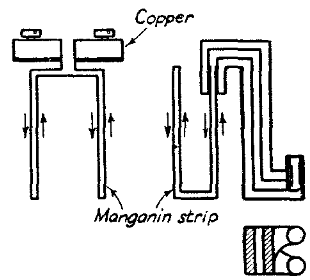


FIG. 178. ORLICH'S CLOSED-TYPE RESISTORS

A further group of bifilar resistors was designed by Schering and Alberti (loc. cit. *ante*) in 1914 for use in the method for testing current transformers developed at the Reichsanstalt and described on p. 469. Particulars are given in the table, those for currents up to 10 amperes having natural air-cooling, while those above 10 amperes are cooled by immersion in oil.

Resistance, ohms	0·08	0·06	0·05	0·04	0·03	0·024	0·02	1·005	0·2002	0·05
Current, amperes	5	5	5	5	5	5	5	2	7·5	40
Length of strip, cm.	2 × 48	2 × 36	2 × 30	2 × 24	2 × 18	2 × 29	2 × 24	2 × 60	2 × 90	2 × 120
Width of strip mm.	10	10	10	10	10	20	20	5	7·5	40
Thickness of strip, mm.	0·5	0·5	0·5	0·5	0·5	0·5	0·5	0·1	0·5	0·5

Silsbee in 1930 has shown that by appropriate arrangement of the potential leads a bifilar loop may be made to have exactly zero effective inductance. In Fig. 179 the resistance material of width w and thickness t is folded into the form AEB with current terminals attached at A and B. The potential leads run from points C and D, parallel to the length of the strip at the middle of its width, for a distance l_1, after which they are taken out at right angles to the axis and are kept close together up to the potential terminals. By calculating the inductive effect of the two portions of the current element, one of length l_1 and the other of length l_2, upon the potential leads, Silsbee finds that the effective inductance of the resistor may be written as

$$L = (4\pi/w)\,[l_2(g - \tfrac{1}{3}t) - \tfrac{1}{3}l_1 t]\ \text{cm.}$$

Hence if we choose l_1 and l_2 so that

$$l_1/l_2 = (g - \tfrac{1}{3}t)/\tfrac{1}{3}t$$

then L will be zero. It is assumed that the thickness and spacings are small compared with w.

It will be seen from the figures that have been previously given that resistors for large currents have a much greater residual than those for smaller currents. The smaller resistors are easily constructed in bifilar form, either with twisted wire or closely folded strip; the larger ones, in order to cool satisfactorily, must have considerably greater surface and be more widely spaced, with consequent increase in the residual. Many attempts have been made to construct heavy current resistors with small residuals by putting in parallel a number of bifilar units, since the time-constant of a number of similar resistors in parallel is the same as that of the individual elements, provided mutual inductance between them is negligible. In illustration of this principle the following measured figures given by Hartshorn (loc. cit. *ante.*) are very instructive. The 1 ohm unit consists of a single bifilar loop of wire; the lower resistances are composed of 10 and 25 such units in parallel; all are air-cooled.

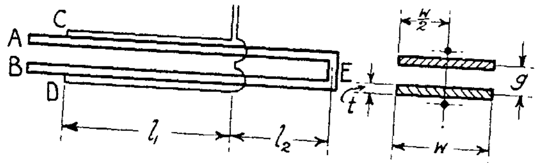


FIG. 179. THEORY OF POTENTIAL LEADS ON BIFILAR RESISTOR

Resistance, ohms	1	0·1	0·04
Current, amperes	2	20	50
Time-constant, microseconds	0·23	0·23	0·2
Effective inductance, cm.	230	23	8
Phase-angle at 50 cycles per sec., radian	0·00007	0·00007	0·00006

The first use of this paralleling principle appears to have been due to Dr. Drysdale* in 1910, his arrangement being shown

* C. V. Drysdale, "A new form of non-inductive low resistance standard or shunt," *Elecn.*, vol. 66, pp. 340–343 (1910). For currents up to 20 000 amperes or more, a cylindrical construction is suggested, the current blocks fitting together like the two halves of a claw coupling; oil cooling is then essential. Resistors of 0·01, 0·001 and 0·00024 ohm for 200, 1 000 and 5 000 amperes based on Drysdale's design are described by C. H. Sharp and W. W. Crawford, *Trans. Amer. I.E.E.*, vol. 29, p. 1537 (1911), but a faulty design of current terminal resulted in phase-angles as large as 0·007 radian (20 min.) at 60 cycles per sec.

in Fig. 180 (*a*). The current terminals are forked as shown, the bifilar loops being silver-soldered to the interlacing prongs; the potential leads are applied to the loops according to Campbell's method, p. 339. An air-cooled resistor of 0·0005 ohms for 1 000 amperes had a calculated phase-angle at 50 cycles per sec. of about 0·0003 radian, i.e. a time-constant of about 1 microsecond.

It is somewhat difficult to make a satisfactory piece of work with straight or bent strips, which are liable to buckle during silver-soldering. Dr. Drysdale* has therefore developed improved designs on the bifilar principle, shown in Figs. 180 (*b*), (*c*), and (*d*), using a tubular construction for the resistance elements; these resistors are very widely used in test-roms and are made by Messrs. H. Tinsley & Co. For resistances of 0·005 ohm and higher the arrangement shown in Fig. 180 (*b*), which is identical in principle with the construction used by Orlich in Fig. 178, is adopted; the diagram illustrates a resistor of 0·01 ohm for 150 amperes. For lower resistances, usually 0·001 ohm and 0·0005 ohm to carry 700 and 1 000 amperes respectively, the arrangement is that given in Fig. 180 (*c*). The current terminals consist of two large horizontal plates separated by a thin layer of insulation; from each of these eight lugs project, those from the upper plate passing through clearing holes in the lower. The resistance element consists of two coaxial manganin tubes 6 in. and 6¼ in. diameter, 6 in. long and 0·01 to 0·02 in. in thickness according to the resistance. Each tube is silver-soldered to two tinned copper rings which are screwed to the lugs at the upper end; the lower rings are screwed together with eight copper distance pieces between them and the whole sweated up. For still lower resistances used with very large currents of the order of 10 000 amperes, the necessary number of parallel sections is obtained by the ingenious construction shown in Fig. 180 (*d*), which is self-explanatory.

All the resistors are immersed in oil, provided with mechanical stirrers and in the heavy current types with water-cooling pipes also.

The inductance of these resistors can be calculated approximately from their dimensions without much difficulty. Hartshorn (loc. cit.

* C. V. Drysdale, "New low resistance standards," *Elecn.*, vol. 77, pp. 629–633 (1916).

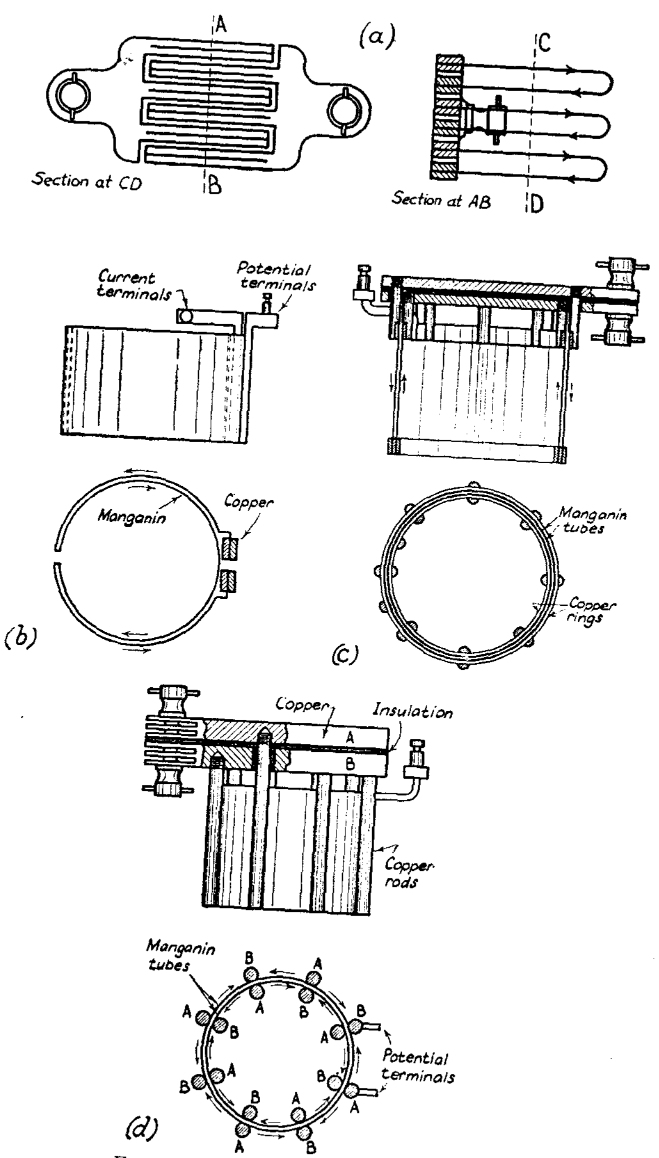


FIG. 180. DRYSDALE'S RESISTORS

ante) has given the following measured values on a series of Drysdale-Tinsley resistors, showing that their residuals are quite small.

Resistance, ohms	1	0·1	0·005	0·001	0·0005
Current, amperes	3	22	200	700	1000
Time-constant, microseconds	0·75	1·34	0·22	1·4	2·5
Inductance, cm.	750	134	1·1	1·4	1·3

Spilsbury and Arnold* have designed a precision resistor, for use at the National Physical Laboratory, consisting of bifilar, air-cooled elements in parallel; the construction is shown in Fig. 181. A range of resistors for 1, 2, 5, 10, 20, 50, 100, and 200 amperes has been prepared, each giving a drop of 2 volts at normal current; the diagram illustrates the largest size. The resistance units, H, are 2 ohms each and carry a current of 1 ampere; the 200 ampere resistor is composed of 200 such units in parallel. Each unit consists of two double-silk insulated constantan wires laid flat and parallel, and then single silk-covered over all; a length of about 40 in. of this bifilar conductor constitutes the unit. The wires are soldered together at one end to a bus-bar D, which forms the mid-point of the resistor and is needed when used in conjunction with an electrostatic wattmeter. The free ends are joined to the lower bus-bars C which act as current terminals for the units. The time-constant of the unit is about 0·2 microsecond, and the inductance of the bus-bars adds little to this figure in the low-current resistors. In the high-current resistors, where the bus-bars are both long and of considerable section, the inductance of the bus-bars becomes the predominating factor in determining the time-constant of the complete resistor, and necessitates careful attention to the placing of the potential terminals. The bus-bars may be regarded as a transmission line with resistance and inductance, upon which there is an inductive leak represented by the resistance units. Spilsbury and Arnold show that if the voltage points are taken on the bus-bars at the opposite end to the current terminals E the inductance of the resistor will be equal to that of the units in parallel minus one-sixth of the inductance of the bus-bars; if the inductance of the bus-bars exceeds six times the inductance of the units the resistor will have a leading phase-angle. If, however, the voltage points are situated at the same end as the current

* R. S. J. Spilsbury and A. H. M. Arnold, "Some accessory apparatus for precise measurements of alternating current," *Journal I.E.E.*, vol. 68, pp. 889–897 (1930).

terminals the inductance of the resistor is equal to the sum of the inductance of the units and one-third of that of the bus-bars. Hence, if the inductance of the bus-bars is more than six times that of the units it is possible to find voltage

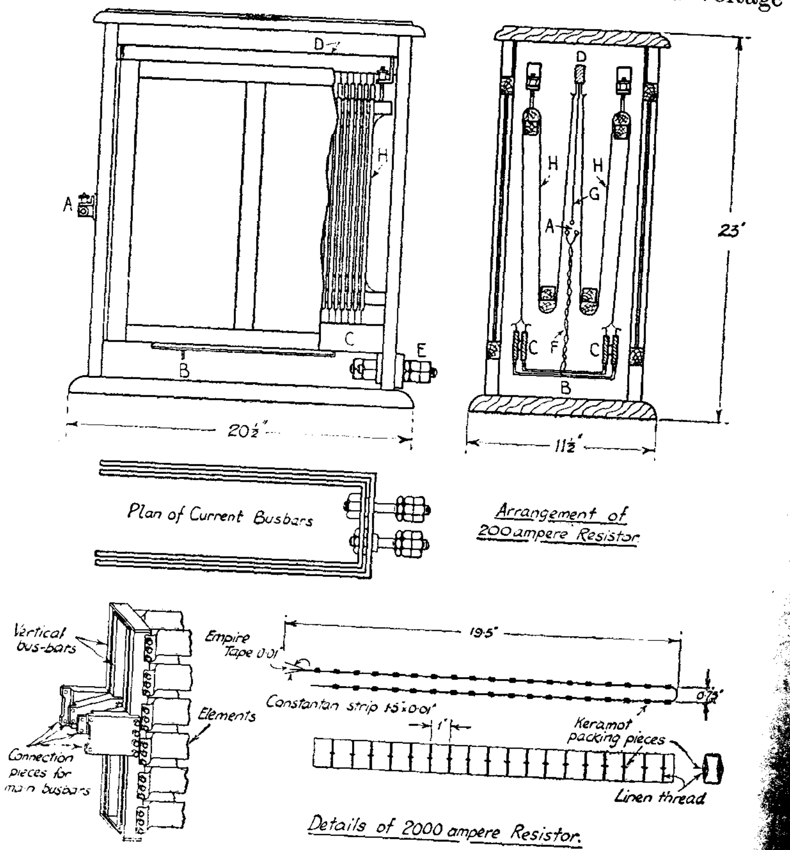


FIG. 181. NATIONAL PHYSICAL LABORATORY BIFILAR RESISTORS FOR 200 AND 2 000 AMPERES

points upon them that will make the resistor non-inductive; the exact position of the points is found by trial. To do this a copper wire is run under each bus-bar, soldered thereto near each end, the voltage connectors being attached to the wires as at *B*; a twinned conductor *F* runs from the voltage points *B* and a lead *G* from the mid-point bus-bar *D* to a special three-point voltage terminal *A* situated at the opposite end

of the wood case containing the resistor to the current terminals *E*. In order to render the bus-bars astatic with respect to external magnetic fields they are arranged in duplicate, resistance units being attached symmetrically to each side. The time-constant of the completed 200 ampere resistor is less than 0·05 microsecond, corresponding with a phase-angle of about 0·05 minute at 50 cycles per sec.; these figures represent the highest degree of perfection so far attained in the design of four-terminal resistors. The temperature rise is considerable, 40 to 50 centigrade degrees at full load, necessitating the use of resistance material with very low resistance-temperature coefficient if changes in the resistance value with temperature are to be kept small. This has been attained by the judicious selection of constantan units with positive and negative temperature coefficients in such a way that the resistance does not alter more than 1 part in 10 000 between no-load and full-load.

Arnold* has recently described an extension of this principle applied to a resistor of 0·001 ohm for a current of 2 000 amperes. A unit of the resistor is shown in Fig. 181 and consists of a strip of constantan 1½ in. wide and 0·01 in. thick bent back on itself at its mid-point, the halves being separated by two strips of empire tape each 0·005 in. thick. The strips are bound at intervals of an inch with strong thread, a small piece of keramot ensuring that the strips are maintained close together. Each unit has a resistance of 0·096 ohm and requires 77 in. of strip; to save space it is bent over as shown to give an overall length of 19·5 in. When in use the element is mounted horizontally with the flat faces of the strips vertical. The complete resistor consists of ninety-six units joined in parallel. Six units are arranged one above the other in a vertical row clamped at intervals of 3½ in. to wooden bars fixed between the top and bottom framework of the resistor. There are sixteen such rows and astaticism is secured by arranging the currents in successive rows to flow in opposite directions. The ends of the units in each vertical row are screwed and soldered to vertical brass bus-bars of 1 in. × 0·25 in. section. The sixteen pairs of vertical bars are paralleled by screwing and soldering connecting pieces from the middle of each bar to horizontal copper bars running the whole length of the resistor. These horizontal bars consist of a central bar between upper and lower bars;

* A. H. M. Arnold, "A non-inductive natural-air-cooled four-terminal resistance standard for alternating currents up to 2 000 amperes," *Journal I.E.E.*, vol. 76, pp. 95–100 (1935).

the latter are joined in parallel to preserve astaticism. The voltage points are located on these bars by trial to give the lowest time-constant, which is + 0·1 microsecond, corresponding with a phase-angle of about 0·1 minute at 50 cycles per sec.

It is well known that a conductor in which the outward and return leads are coaxial circular cylinders, one within the

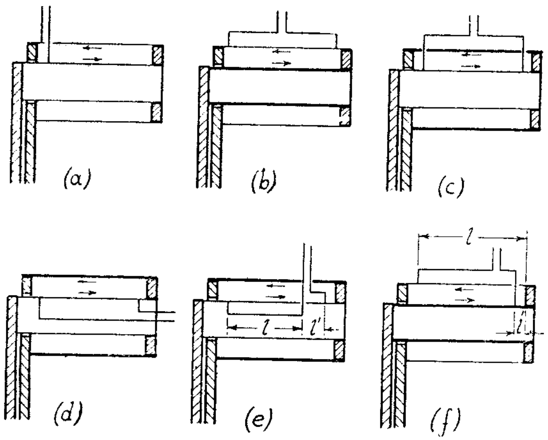


FIG. 182. THEORY OF POTENTIAL LEADS ON COAXIAL TUBULAR RESISTORS

other like a concentric cable, produces no magnetic field at external points, and has the further advantage that the resistance, skin-effect and inductance may all be readily computed* from the measured dimensions of the cylinders. It would appear, therefore, that this form would be a particularly advantageous one for standard resistors. Silsbee† in 1916 gave a very complete theoretical and experimental treatment of the coaxial cylindrical resistor, distinguishing the six cases shown in Figs. 182 (*a*) to (*f*) which are of practical interest.

Fig. 182 (*a*) shows two concentric tubes of resistance material and is equivalent to Drysdale's arrangement shown in Fig. 180 (*c*); it has the disadvantage that there are two soldered joints between the potential terminals. In the other arrangements there is one copper tube (thick) and one of manganin (thin). The arrangement shown in Fig. 182 (*b*)

* This is true also of parallel wire resistors, for examples of which the reader is referred to *A.C. Bridge Methods*, pp. 90–94, and particularly pp. 281–282 (1932). Parallel wires are useful, however, only for small currents, while tubular resistors may be constructed for the largest currents encountered in testing practice.

† F. B. Silsbee, *Bull. Bur. Stds.*, vol. 13, pp. 375–421 (1916).

has been used by Moore,* in a resistor for 0·0001 ohm having a manganin tube 13·4 cm. inside diameter, 0·1 cm. thick and 10 cm. long, for which a time-constant of 1·5 microsecond is claimed. Silsbee points out that in cases (*b*) and (*d*) the effective inductance is actually negative but smaller than in (*a*) and (*c*). The cases illustrated by Figs. 182 (*e*) and (*f*) are interesting since by proper choice of l and l' the inductance can be made accurately zero. In the former case this occurs if

$$l/l' = 2 + 3(s/t) + 6(u/t)$$

while in the latter

$$l/l' = 3 + 2(t/s) + 6(u/s)$$

where $s = (a_4 - a_3)/a_4$, $t = (a_2 - a_1)/a_2$ and $u = (a_3 - a_2)/a_3$, a_1 and a_3 being the inner radii while a_2 and a_4 are the outer radii of the two tubes.

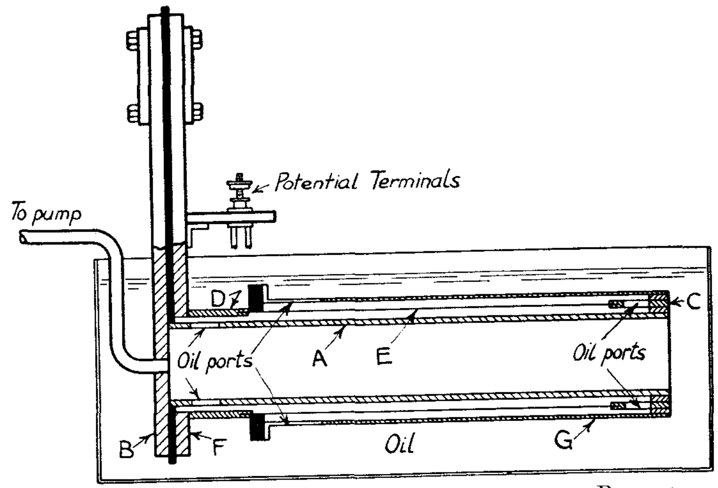


FIG. 183. BUREAU OF STANDARDS COAXIAL TUBULAR RESISTOR

The tubular construction has recently been employed at the Bureau of Standards† in two resistors for 1 000 and 2 500 amperes respectively, the former having a resistance of 0·0005 ohm while the latter has two sets of potential terminals giving 0·00025 and 0·0002 ohm. The general arrangement is shown in Fig. 183. A copper tube *A* is connected to the terminal slab *B* at one end and to the copper ring *C* at the other end. The manganin tube *E* is attached to *C* and to a similar ring *D*, which in turn is fixed to the terminal slab *F*. Manganin tube of the required size was not commercially obtainable, so that the tubes were built up by bending sheet material to form

* A. E. Moore, "A new concentric standard dynamometer wattmeter for heavy currents, and concentric non-inductive standards of low resistance," *Journal I.E.E.*, vol. 55, pp. 380–402 (1917).

† F. B. Silsbee, *Bur. Stds. Journal of Res.*, vol. 4, pp. 73–107 (1930).

portions of a circular cylinder. The smaller tube is made of two pieces subtending an angle of 180°; the larger tube is constructed of three pieces covering 120°. All are hard-soldered to the copper ring and after all work upon them is complete are annealed in a nitrogen atmosphere at 450° C. Over the manganin tube is the brass tube G. Cooling is effected by a stream of oil circulated by a pump through ports at the left-hand end of A, along the inner surface of E, through ports in C, over the outer surface of E, being discharged through ports at the left-hand end of G; the temperature rise of E is about 11° C. above the oil. The tube is provided with four potential leads in parallel for each potential point, spaced at 90° round the tube. The potential leads at the left-hand end consist of four thin brass pipes passing through the ports in G to the manganin. Adjustment of the resistance value is provided by the shunting method of Fig. 173 (*b*), potential points at the right-hand end being situated at about 2·5 per cent of the resistance apart; leads from these points consist of flattened insulated manganin wires tied down against the tube running back to the left-hand end where they are drawn through the brass pipes, which serve not only as potential leads but also as conduits, thus avoiding closed loops. The final potential connection is provided by concentric terminals. Particulars of the resistors are given in the subjoined table.

Resistance, ohms	0·0005	0·00025	0·00020
Current, amperes	1 000	2 500	
Outside diam. of tube, cm.	8·57	15·2	15·2
Thickness of wall, cm.	0·1	0·2	0·2
Length between potential points, cm.	35	64	51
Watts per sq. cm.	0·27	0·26	
Phase-angle at 60 cycles per sec., minutes	− 0·7	− 2·9	
Time-constant, microseconds	− 0·54	− 2·24	

11. **Skin-effect in resistors.** In normal laboratory practice resistance standards are intercompared and maintained by bridge measurements using direct current, even when such standards will ultimately be used with alternating current. The tacit assumption in such a case is that the effective resistance with a.c. does not appreciably differ from the d.c. value, which is by no means necessarily true, especially in resistances for large currents. Consequently, numerous methods* for

* See L. H. Hartshorn, *Proc. Phys. Soc.*, vol. 39, p. 377 (1927); N. F. Astbury, "A simple method for measurements of residual inductance on potentiometers and four-terminal resistance coils," *Journal Sci. Insts.*, vol. 8, pp. 221–223 (1931); A. H. M. Arnold, "The calibration of four-terminal resistance standards with alternating current at power frequencies," *Journal I.E.E.*, vol. 69, pp. 1013–1018 (1931). Also consult *A.C. Bridge Methods*, p. 279 (1932).

measuring the effective resistance and the time-constant of four-terminal resistors with alternating current have been devised and are recommended for adoption wherever possible; these arrangements usually take the form of an a.c. Kelvin double bridge and the technique of their operation is given in the papers cited.

In cases where only a d.c. calibration has been made it is desirable to be able to estimate the extent to which skin-effect will modify the resistance value; this problem has been investigated by Silsbee (loc. cit. *ante*, 1930), who has given some useful formulae.

For the case of two thin, wide strips close together

$$\frac{R'}{R} = 1 + \frac{m^4t^4}{45} - \frac{m^8t^8}{4725},$$

where R' is the effective resistance, R is the d.c. resistance, t is the thickness of the strips and $m^2 = 8\pi^2 f/\rho$, f being the frequency in cycles per sec. and ρ the resistivity in c.g.s. units (10^9 ohm-cm.). If the strips are far apart, so that they exert no appreciable effect on one another, current tends to crowd toward the edges of the strips; then

$$R'/R = 1 + 0{\cdot}0087432p^4 - 0{\cdot}000384p^8 + 0{\cdot}0000189p^{12},$$

where $p^2 = m^2tw/\pi$, w being the width of the strips. The intermediate case of strips at a moderate distance apart can only be treated empirically with the aid of test results and the principle of similarity.

For tubular resistors consisting of a thin-walled tube of thickness t with the return conductor at a distance,

$$R'/R = 1 + (m^4t^4/45)$$

When the resistor consists of two concentric tubes with walls that are thin compared with the diameters of the tubes

$$R' = \frac{\rho[1 + (m^4t^4/45)]}{\pi(c^2 - b^2)} + \frac{\rho[1 + (m^4t_1^4/45)]}{2\pi[at_1 - (t_1^2/2)]}$$

where c is the outer and b the inner radius of the external tube, $t = c - b$ is its thickness and t_1 the thickness of the internal conductor of which the external radius is a.

12. **Use of a resistor with a current transformer for large currents.** As the preceding sections have shown, it is possible to construct satisfactory resistors for currents up to 2 000 amperes or more, but these will be costly, bulky, and will waste very considerable amounts of energy in heat. Moreover, if a wide range of currents is to be measured several resistors will be necessary, thus further adding to the cost of the equipment. All these defects can be completely removed by the method adopted at the National Physical Laboratory. It is now possible to construct ring-type current transformers with nickel-iron cores in which the ratio error and phase-angle are very small and practically independent of the primary current; for details, see p. 119 and p. 136. By connecting a resistor of 0·4 ohm in the secondary circuit of such a transformer, e.g. with a ratio of 5 000/5 amperes, the combination is equivalent

to a 5 000 ampere resistor with a drop of 2 volts, wasting only a few watts instead of 10 kW. Only one resistor is required for any number of transformers, and this is of a type which can be very simply and accurately constructed. Currents lower than the maximum value corresponding with one primary turn can be measured by looping a suitable number of turns of the primary cable through the transformer. There is no doubt that the nickel-iron transformer with a secondary resistor renders the construction of high-current resistors quite unnecessary for most practical purposes, since the total error of the combination need not exceed a few parts in 10 000 for resistance or 2 minutes for phase-angle. Arnold has pointed out, however, that a properly designed primary resistor has some advantages over the transformer-resistor combination that may be important in certain precise measurements. They are: (*a*) The time-constant can be made very small, nearly independent of frequency and independent of the current. (*b*) If the resistor is not exposed to strong magnetic fields the time-constant is more definite than that of the transformer. (*c*) The resistance is independent of frequency. (*d*) The resistor is available for use with direct or alternating currents.

HIGH RESISTANCES

13. **General considerations.** In measuring the ratio and phase-angle of a voltage transformer, a standard method is to join a high resistance across the primary terminals and to compare the drop of voltage down a fraction of this resistance with the secondary voltage of the transformer. Since such a resistor must withstand the rated primary voltage and should take only a small current (usually about $\frac{1}{20}$ ampere) it will contain a considerable amount of resistance material and will, in consequence, be bulky. It will, therefore, have appreciable self-capacitance and considerable earth-capacitance with respect to surrounding objects.

If the resistance units were wound in the form of a simple coil their inductance would be considerable. The inductance is easily reduced to a very small amount by winding the wire upon mica cards, so that the turns enclose only a small area; the direction of winding may be reversed after every few turns, or the wire may be wound on in a series of bifilar loops. The self-capacitance can be reduced to as much as desired by subdividing the resistance unit into a number of sections in series,

the self-capacitance effect being reduced approximately in proportion to $1/n^2$, n being the number of sections. By these means it is not difficult to prepare high resistance units with very small time-constants.*

When a number of such resistance units are combined in series to form a voltage-divider, the result is by no means satisfactory. The units present a considerable surface and there will be large distributed capacitances between them and from them to earth. The current varies from point to point along the resistor, both in magnitude and phase; consequently the fall of potential over any portion of the resistor is determined not only by the resistance of the portion but also by its earth capacitance, exactly as in a transmission line. Hence the fall of potential down various fractions of the resistor will not be in proportion to the resistances of those fractions. To make the capacitance effects quite definite and to reduce their influence on the division of potential, the various resistance units must be shielded, the potentials of the shields being maintained at appropriate values; various methods of accomplishing this will be considered in Section 16.

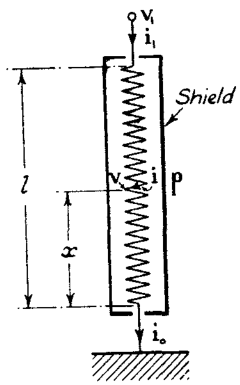


FIG. 184. THEORY OF A SHIELDED RESISTOR UNIT

14. **Theory of high resistance earthed at one end.** High voltage voltage-dividers for transformer testing are almost always used with one end earthed. As a preliminary, consider a resistor surrounded by a shield, as in Fig. 184. Let ρ be the resistance and κ the distributed capacitance to the shield, each per unit length; assume the inductance and leakance to be negligible. Then with the origin at the earthed end, let $\boldsymbol{v}$ and $\boldsymbol{i}$ be the potential and current at a point x and $\boldsymbol{p}$ be the potential of the shield. With all quantities varying sinusoidally with a pulsatance $\omega = 2\pi f$, the equations for resistance drop through and capacitance current from an element of length dx will be

$$\rho \, . \, \boldsymbol{i} dx = d\boldsymbol{v},$$

$$j\omega\kappa(\boldsymbol{v} - \boldsymbol{p})dx = d\boldsymbol{i};$$

or

$$\partial \boldsymbol{v}/\partial x = \rho \boldsymbol{i} \text{ and } \partial \boldsymbol{i}/\partial x = j\omega\kappa(\boldsymbol{v} - \boldsymbol{p}),$$

* For a detailed discussion *see A.C. Bridge Methods*, p. 65, *et seq.* (1932); and also Campbell and Childs, loc. cit. *ante*.

whence

$$\partial^2 \boldsymbol{v}/\partial x^2 = j\omega\kappa\rho(\boldsymbol{v} - \boldsymbol{p}) = \alpha^2(\boldsymbol{v} - \boldsymbol{p}),$$

where $\alpha^2 = j\omega\kappa\rho.$

The solution of this differential equation is

$$\boldsymbol{v} = \boldsymbol{A} \cosh \alpha x + \boldsymbol{B} \sinh \alpha x + \boldsymbol{p},$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are harmonic vectors to be found from the conditions

$$\boldsymbol{v} = 0 \text{ when } x = 0 \text{ and } \boldsymbol{v} = \boldsymbol{v}_1 \text{ when } x = l.$$

The final solution is

$$\boldsymbol{v} = \frac{\boldsymbol{v}_1 - \boldsymbol{p}(1 - \cosh \alpha l)}{\sinh \alpha l} \sinh \alpha x - \boldsymbol{p} \cosh \alpha x + \boldsymbol{p}$$

and consequently

$$\boldsymbol{i} = \frac{\alpha}{\rho}\left[\frac{\boldsymbol{v}_1 - \boldsymbol{p}(1 - \cosh \alpha l)}{\sinh \alpha l} \cosh \alpha x - \boldsymbol{p} \sinh \alpha x.\right]$$

Let $\boldsymbol{i}_0$ be the current at the earthed end $x = 0$, then

$$\boldsymbol{i}_0 = \frac{\alpha}{\rho \sinh \alpha l}[(\boldsymbol{v}_1 - \boldsymbol{p}) + \boldsymbol{p} \cosh \alpha l]$$

Similarly, if $\boldsymbol{i}_1$ is the current at the high-voltage end $x = l$,

$$\boldsymbol{i}_1 = \frac{\alpha}{\rho \sinh \alpha l}[(\boldsymbol{v}_1 - \boldsymbol{p}) \cosh \alpha l + \boldsymbol{p}]$$

Now write $\boldsymbol{p} = k\boldsymbol{v}_1$ where k is a real quantity, then

$$\boldsymbol{i}_0 = \frac{\alpha}{\rho \sinh \alpha l}[(1 - k) + k \cosh \alpha l]\, \boldsymbol{v}_1,$$

and

$$\boldsymbol{i}_1 = \frac{\alpha}{\rho \sinh \alpha l}[(1 - k) \cosh \alpha l + k]\boldsymbol{v}_1.$$

At power frequencies αl is a small quantity; expanding in series and taking two terms as a sufficient approximation,

$$\frac{\sinh \alpha l}{\alpha} = \frac{1}{\alpha}\left[\alpha l + \frac{\alpha^3 l^3}{3!} + \ldots\right] \doteqdot l + \frac{\alpha^2 l^3}{3!} \doteqdot l\left[1 + \frac{\alpha^2 l^2}{6}\right],$$

$$\cosh \alpha l = 1 + \frac{\alpha^2 l^2}{2!} + \ldots \doteqdot 1 + \frac{\alpha^2 l^2}{2},$$

and

$$\frac{\alpha \cosh \alpha l}{\sinh \alpha l} \doteqdot \frac{1}{l}\left[1 + \frac{\alpha^2 l^2}{3}\right].$$

Substituting these approximate values gives

$$\boldsymbol{i}_0 \doteqdot \frac{1}{l\rho}\left[1 + \frac{\alpha^2 l^2}{6}(3k - 1)\right]\boldsymbol{v}_1 \doteqdot \frac{1}{l\rho}\left[1 + j\frac{\omega\kappa\rho l^2}{6}(3k - 1)\right]\boldsymbol{v}_1,$$

$$\boldsymbol{i}_1 \doteqdot \frac{1}{l\rho}\left[1 - \frac{\alpha^2 l^2}{6}(3k - 2)\right]\boldsymbol{v}_1 \doteqdot \frac{1}{l\rho}\left[1 - j\frac{\omega\kappa\rho l^2}{6}(3k - 2)\right]\boldsymbol{v}_1.$$

But $l\rho = R$, the total resistance, while $l\kappa = C$ the total capacitance to the shield; hence, finally,

$$\boldsymbol{i}_0 \doteqdot \frac{1}{R}\left[1 + j\omega CR\left(\frac{k}{2} - \frac{1}{6}\right)\right]\boldsymbol{v}_1 \doteqdot \sqrt{2}\,\frac{V_1}{R}\left[1 + j\omega CR\left(\frac{k}{2} - \frac{1}{6}\right)\right]\varepsilon^{j\omega t}\boldsymbol{1},$$

and

$$\boldsymbol{i}_1 \doteqdot \frac{1}{R}\left[1 - j\omega CR\left(\frac{k}{2} - \frac{1}{3}\right)\right]\boldsymbol{v}_1 \doteqdot \sqrt{2}\,\frac{V_1}{R}\left[1 - j\omega CR\left(\frac{k}{2} - \frac{1}{3}\right)\right]\varepsilon^{j\omega t}\boldsymbol{1},$$

where V_1 is the r.m.s. value of the voltage and $\boldsymbol{1}$ is a unit vector along the axis $t = 0$. Several interesting cases occur, as will now be shown.

(i) *Earthed shield, $k = 0$.* The operator in the expression for $\boldsymbol{i}_0$ becomes

$$1 - \frac{j\omega CR}{6} \doteqdot \left(1 + \frac{\omega^2 C^2 R^2}{72}\right)\angle -\omega CR/6.$$

Hence the current at the earthed end of the resistor is greater than V_1/R in the ratio of $1 + (\omega^2 C^2 R^2/72)$ to 1 and *lags* on the voltage applied to the resistor by an angle $\omega CR/6$. This would also express the magnitude and phase of the drop of voltage down a small portion of the resistor at the earthed end relative to the total voltage, as would occur in voltage transformer testing; the amplitude factor is usually a very small correction, but the phase-angle is of the utmost importance.

Similarly the operator in the expression for $\boldsymbol{i}_1$ becomes

$$1 + j\frac{\omega CR}{3} \doteqdot \left(1 + \frac{\omega^2 C^2 R^2}{18}\right)\angle +\omega CR/3$$

The magnitude correction is thus four times as great as at the earthed end, while the current at the high-voltage end *leads* on the voltage applied to the resistor by an angle $\omega CR/3$.

(ii) *Shield at mid-potential, $k = \frac{1}{2}$.* When the shield is maintained at the mid-potential of the resistor both operators are the same, namely,

$$1 + j\frac{\omega CR}{12} \doteqdot \left(1 + \frac{\omega^2 C^2 R^2}{288}\right)\angle +\omega CR/12.$$

The amplitude correction is usually negligible. In this instance the currents at both ends of the resistor *lead* the applied voltage by an angle $\omega CR/12$.

(iii) *Shield at one-third potential*, $k = \frac{1}{3}$. When the shield is at a potential equal to $\frac{1}{3}$ of the applied voltage, the operator for the current at the earthed end is unity. In this case the

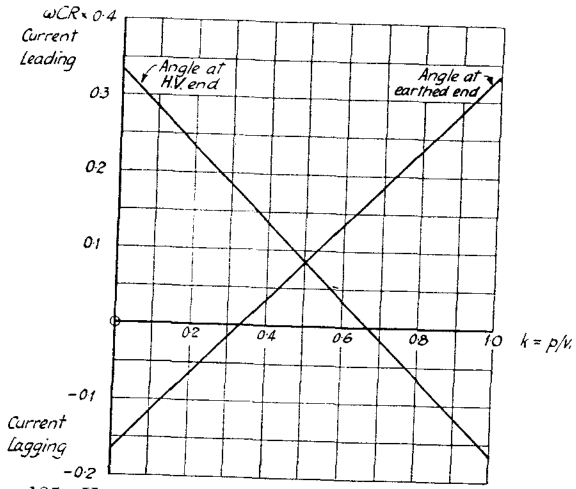


FIG. 185. VARIATION OF PHASE-ANGLE OF CURRENTS AT ENDS OF A RESISTOR WITH POTENTIAL OF SHIELD

current at the earthed end is equal to V_1/R and is in phase with the applied voltage.

The variation of the phase-difference between the currents at both ends of the resistor and the applied voltage is given in Fig. 185 as a function of k.

To the above phase-displacement between the current and voltage there must be added the additional displacement caused by the residual inductance and the self-capacitance of the resistor. If the inductance be L and if the self-capacitance be represented by a condenser C_1 across the terminals of the unit, then the current taken by the combination will be obtained by operating on the vector $\boldsymbol{v}_1$ with the admittance operator

$$\frac{R + j\omega\,[C_1R^2 - L(1 - \omega^2C_1L)]}{R^2 + \omega^2L^2} \doteqdot \frac{1}{R}\left[1 + j\omega\left(C_1R - \frac{L}{R}\right)\right],$$

neglecting ω^2CL in comparison with 1 and ω^2L^2 in relation to

R^2. The residuals thus account for an angle of approximately $\omega[C_1R - (L/R)]$ between the current and voltage. Assuming mid-potential shielding, the total phase angle is

$$\omega\left[\frac{CR}{12} + \left(C_1R - \frac{L}{R}\right)\right];$$

since C_1 is predominant and L is small, the total angle is a lead. By artificially increasing L by the addition of an inductance in series with the resistor, the total phase-angle will be zero if the value of the inductance is

$$R^2[C_1 + (C/12)].$$

15. **Theory of resistance units in series.** It is clear that earth-capacitance effects will have no influence upon the distribution of potential along a resistor if some form of continuous shielding could be provided, such that the potential of the shield at each point was equal to that of the resistor at the same point. By this means the capacitance current to the shield would vanish, the current at all parts of the resistor would be the same and the fall of potential down it would be uniform. In practice it is not easy to provide such infinitely-finely graded shielding (see, however, p. 369), so one must be content with an approximation; to this end high resistances are composed of a limited number of sections joined in series, each section being enclosed within a shield maintained at an appropriate potential. We shall now examine the theory of such a resistor, consisting of m similar shielded units in series, as shown in Fig. 186.

Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \ldots \boldsymbol{v}_n, \ldots \boldsymbol{v}_m$ be the harmonic vectors of p.d. over the various sections, and suppose the shields to be maintained at fractions $k_1, k_2, k_3 \ldots k_n, \ldots k_m$ of the p.d.'s across the sections they enclose. Adapting the equations on p. 359, the current at the lower end of the nth unit can be written

$$\boldsymbol{i}_{n-1} = \frac{1}{R}\left[1 + jwCR\left(\frac{k_n}{2} - \frac{1}{6}\right)\right]\boldsymbol{v}_n,$$

and the current at the upper end of the same unit is

$$\boldsymbol{i}_n = \frac{1}{R}\left[1 - j\omega CR\left(\frac{k_n}{2} - \frac{1}{3}\right)\right]\boldsymbol{v}_n.$$

But $\boldsymbol{i}_n$ is also the current at the lower end of unit $n + 1$, i.e.

$$\boldsymbol{i}_n = \frac{1}{R}\left[1 + j\omega CR\left(\frac{k_{n+1}}{2} - \frac{1}{6}\right)\right]\boldsymbol{v}_{n+1},$$

where R is the resistance of any unit and C the total capacitance of any unit to its shield. Equating these expressions and remembering that the phase-displacements are all small we obtain

$$v_{n+1} = \left[1 - j\omega CR\left(\frac{k_{n+1} + k_n}{2} - \frac{1}{2}\right)\right]v_n = [1 - j\psi_n]v_n,$$

where $\psi_n = (\omega CR)(k_{n+1} + k_n - 1)/2$. The vector diagram of

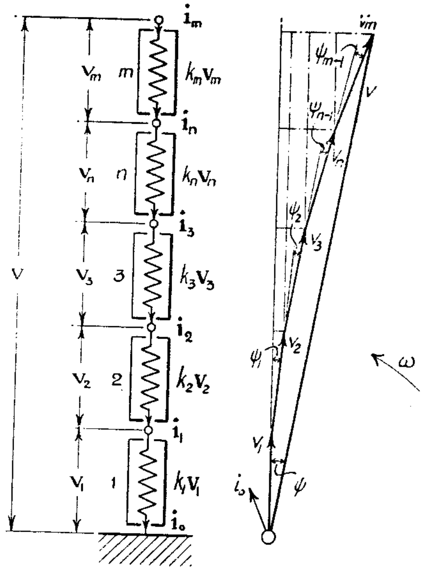


FIG. 186. THEORY OF SHIELDED RESISTORS IN SERIES

the voltages is also shown in Fig. 186, from which it will be seen that the components of v along and perpendicular to v_1 are

$$v_1 + v_2 \cos\psi_1 + v_3 \cos(\psi_1 + \psi_2) + \ldots v_n \cos(\psi_1 + \psi_2 + \ldots \psi_{n-1}) + \ldots v_m \cos(\psi_1 + \psi_2 + \ldots \psi_{m-1}),$$

and

$$v_2 \sin\psi_1 + v_3 \sin(\psi_1 + \psi_2) + \ldots . v_n \sin(\psi_1 + \psi_2 + \ldots . \psi_{n+1}) + \ldots . v_m \sin(\psi_1 + \psi_2 + \ldots . \psi_{m-1}).$$

In a well-designed resistor all the phase-angles will be small and the voltages across the several sections will be sensibly equal. To a close approximation, therefore, the components of v_1 can

be taken as mv_1 and $v_1\ [\psi_1(m-1) + \psi_2(m-2) + \psi_3(m-3) + \ldots \psi_n(m-n) + \ldots \psi_{m-1}]$, where v_1 is the amplitude of v_1. With the same degree of approximation we may write

$$v \doteq m(1 - j\psi)v_1,$$

where

$$\psi \doteq [\psi_1(m-1) + \psi_2(m-2) + \psi_3(m-3) + \ldots . \psi_n(m-n) + \ldots \psi_{m-1}]/m.$$

The current at the earthed end of the first section is

$$i_0 = \frac{1}{R}\left[1 + j\omega CR\left(\frac{k_1}{2} - \frac{1}{6}\right)\right]v_1$$

$$= \frac{1}{mR}\left[1 + j\omega CR\left(\frac{k_1}{2} - \frac{1}{6}\right)\right] \cdot \frac{1}{(1 - j\psi)}v$$

whence,

$$i_0 \doteq \frac{1}{mR}\left\{1 + j\left[\omega CR\left(\frac{k_1}{2} - \frac{1}{6}\right) + \psi\right]\right\}v.$$

Substituting for ψ and summing the resulting series gives

$$i_0 \doteq \frac{1}{mR}\left\{1 + j\frac{\omega CR}{2}\left[k_1\left(2 - \frac{1}{m}\right) + k_2\left(2 - \frac{3}{m}\right) + k_3\left(2 - \frac{5}{m}\right) + \ldots \frac{3}{m}k_{m-1} + \frac{1}{m}k_m + \frac{1}{6} - \frac{m}{2}\right]\right\}v.$$

Making the simplification $k_1 = k_m$ and $k_2 = k_3 = \ldots k_{m-1}$

$$i_0 \doteq \frac{1}{mR}\left\{1 + j\frac{\omega CR}{2}\left[2k_1 + k_2(m-2) + \frac{1}{6} - \frac{m}{2}\right]\right\}v.$$

Two interesting cases arise. In the first case the shield potentials are adjusted to make the phase-displacement between i_0 and v equal to zero, so that the whole arrangement behaves as a pure resistance mR. To secure this,

$$2k_1 + k_2(m-2) + (1/6) - (m/2) = 0,$$

or

$$k_1 = m(1/4 - k_2/2) - (1/12) + k_2.$$

For this to be independent of the number of units necessitates

$$k_2 = 1/2 \text{ and hence } k_1 = 5/12$$

The phase-angle will be zero, therefore, if $k_1 = k_m = 5/12$ and $k_2 = k_3$, etc. $= 1/2$.

In the second case all the shields are maintained at the mid-potentials of the units, i.e. $k_1 = k_2 = k_3$, etc $= 1/2$ and the phase-angle is a lead of $\omega CR/12$, exactly as for a single unit earthed at one end.

It is necessary that the shields should be maintained accurately at the desired potentials. For example, if the units are all similarly shielded the phase-angle is

$$(\omega CR/12)\ [1 + 6m(k_1 - \tfrac{1}{2})]$$

which will be $\omega CR/12$ if all the shields are correctly at mid-potential. If they should deviate by ε per cent from the true value, $k_1 = \frac{1}{2}(1 + (\varepsilon/100))$ and the angle becomes

$$(\omega CR/12)\ [1 + 3m(\varepsilon/100)]$$

so that there is an error proportional to the number of units and to three times the deviation.

To establish the correct potentials it is usual to connect a second m-unit resistor in parallel with the first, suitable points on the auxiliary resistor being connected to the shields. The auxiliary is usually unshielded. With very high resistances, however, the earth-capacitance of the auxiliary resistor results in a non-uniform fall of potential along it and may be an important factor in fixing the potentials of the shields on the main resistor. To avoid trouble due to this cause it is necessary to shield the auxiliary and to maintain the potentials of its shields by the use of a third resistor. The use of auxiliary resistors may be entirely avoided by connecting the shields of the main resistor to tappings on a transformer. Both methods have been used in practice and actual examples will be found in Section 16.

16. **Types of shielded resistors.** In this Section we shall examine some examples of shielded resistors designed upon the principles worked out in Sections 14 and 15. Four methods of construction have been used, and we shall consider these in turn.

(i) *Shielded to give zero phase-angle.* It has been shown on p. 363 that the phase-angle at the earthed end of a number of shielded resistors joined in series will be zero if the shields of the two end units are maintained at 5/12 of the p.d. across the resistors contained within them while the remaining shields are at mid-potential. Resistors on this principle have been constructed by Orlich and Schultze* at the Reichsanstalt and by Kouwenhoven† at Karlsruhe.

The main resistor in Orlich and Schultze's design consists

* E. Orlich and H. Schultze, "Über einen Spannungsteiler für Hochspannungsmessungen," *Arch. f. Elekt.*, vol. 1, pp. 1–15, 88–94, 232 (1913).

† W. B. Kouwenhoven, "Über Hochspannungsmessungen," *Arbeiten Elekt. Inst. Karlsruhe*, vol. 3, pp. 1–47 (1913).

of eight shielded units, each of about 200 000 ohms, in series between the points C and B, Fig. 187. Each unit consists of a thick-walled porcelain tube, 100 cm. long and 6 cm. diameter, wound with 5 000 to 6 000 turns of silk-covered manganin wire 0·05 mm. diameter; the unit can withstand about 3 000 volts, i.e. 15 milliamperes, without overheating. The units are mounted vertically within a rectangular wooden frame, from which they are insulated by porcelain insulators. A tubular

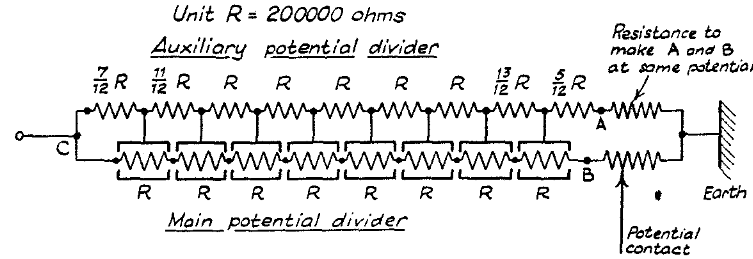


Fig. 187. Orlich and Schultze's Shielded Resistor for 25 000 Volts

metal shield 10 cm. diameter, slit along its entire length to avoid eddy currents, surrounds each unit coaxially and is insulated from the frame by ebonite rods. Between B and earth is a resistance of a few thousand ohms with a potential contact. The auxiliary resistor consists of nine unshielded units, similar to those of the main resistor, connected between C and A; their values are respectively 5/12, 13/12, five sections 12/12, 11/12 and 7/12 of 200 000 ohms, thus ensuring the appropriate potentials for the shields of the main resistor. The auxiliary units are accommodated in the same frame as the main units. To maintain A and B at the same potential, a resistor of the necessary amount is joined between A and earth. The phase-angle between $\boldsymbol{i}_0$ and $\boldsymbol{v}$ at 50 cycles per sec. is 1 minute and is entirely attributed to the inductance of the unifilar spiral-wound coils; each unit has an inductance of 0·18 henry.

Kouwenhoven's resistor is shielded in a similar way but the units consist of Jena glass tubes containing a solution of 121 grammes of mannite, 41 grammes of boric acid and 0·06 gramme of potassium chloride per litre of water; the inductance is quite negligible and the phase-angle very small.

(ii) *Shielded at mid-potential using auxiliary resistor.* The practical disadvantage of the preceding method of shielding is that if less than the total number of the main resistor units are in use for lower voltages it is necessary to rearrange the connections to the auxiliary resistor. This is not the case when mid-potential shielding is used; the shielding circuit is made up permanently and it is only necessary to use any desired number of main units along with the corresponding number of auxiliary units.

Mid-potential shielded resistors have been used in connection with wattmeter measurements at very high voltages* and as potential dividers in research on X-ray apparatus†. One of the latter consists of "Multohm" resistances, which are glass tubes 43·3 cm. long and 2·8 cm. diameter, coated inside with a thin carbon film. The tubes have a resistance of 4 to 5 megohms each and are suitable for 20 to 30 kV; the inductance is negligible. Ten such tubes are used in series, each surrounded by a tubular shield 10 cm. in diameter, the shields being maintained at mid-potentials by connecting them to a second resistor also composed of "Multohm" units. For precision work, however, wire-wound resistors are preferred on account of their greater permanence and stability.

Fig. 188 shows the diagrammatic arrangement of a resistor with mid-potential shielding, examples of this type being in use at the Bureau of Standards and at the National Physical Laboratory. As pointed out on p. 361 the capacitances of the units to their shields cause the currents both at the high voltage and at the earthed ends of the resistor to lead on the applied voltage by an angle $\omega CR/12$, where C is the shield capacitance and R is the resistance of a unit, independently of the number of units in use. In addition, self-capacitance C_1 and inductance L may make an additional displacement of $\omega[C_1R - (L/R)]$.

The Bureau of Standards resistor‡ is for use up to 30 000 volts and consists of twenty-seven units. Two of these have a resistance of 10 000 ohms and the remainder are 20 000 ohms each; the whole totals 520 000 ohms. Manganin wire is wound upon mica cards 14 cm. by 5·6 cm., each card having a resistance of 1 250 ohms. A 20 000 ohm unit contains sixteen cards hung vertically in the shield on two horizontal glass rods passing through holes in the upper corners of the cards and resting in notches in the side of the shield. These consist of brass boxes 20 cm. by 20 cm. by 13 cm. filled with oil. The

* R. Hiecke, "Über Wechselstrommessungen," *E.u.M.*, vol. 33, pp. 505–509 (1915).

† J. E. Lilienfeld and W. Hoffman, "Konstante hochohmige Mess- und Belastungswiderstände," *Elekt. Zeits.* vol. 41, pp. 870–873 (1920). A. Karolus, "Untersuchungen über das kontinuierliche Röntgenspektrum bei verschiedenen Entladungsfrequenzen," *Ann. der Phys.*, vol. 72, pp. 595–616 (1923).

‡ F. B. Silsbee, "A shielded resistor for voltage transformer testing," *Bur. Stds. Sci. Papers*, vol. 20, pp. 489–514 (1926).

auxiliary resistor is constructed from similar cards which are not, however, shielded; they are assembled in racks, each of which corresponds with two adjacent shielded boxes. The whole equipment is mounted on shelves composed of wooden bars spaced by porcelain supports; four boxes of the main resistor and two racks of the auxiliary are accommodated on each shelf. It is estimated that the resistance of the resistor does not differ more than 0·01 per cent from its d.c. value and that the phase-angle does not exceed 0·3 minute.

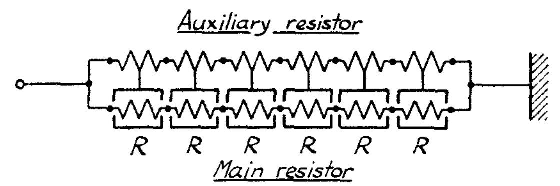


FIG. 188. DIAGRAM OF MID-POTENTIAL SHIELDED RESISTOR

The National Physical Laboratory resistor* consists of six units, each of 100 000 ohms resistance; the unit at the earthed end is 2 000 ohms short to allow of the insertion of a variable resistor in series with it. The resistance wire is 0·0076 in. diameter, single-silk covered constantan wound upon paxolin cards 29 cm. by 8·5 cm. by 0·3 cm. Each card has a resistance of 2 500 ohms and 40 cards are used in each unit arranged in two tiers in a wooden framework supported by porcelain pieces of special design. By judicious selection of the resistance material it has been found possible to reduce the temperature coefficient practically to zero. The main and auxiliary resistors are similarly constructed; in the latter the spacing of the cards is uniform, but in the former a wider space is left at the middle of both tiers to accommodate a total inductance of 0·118 henry per unit to compensate for the time constant $CR/12 + (C_1R - L/R)$ and thus to render the unit non-reactive. To facilitate cooling, the shielded units of the main resistor are contained in perforated zinc boxes 106 cm. by 47 cm. by 47 cm.;

* R. Davis, "The errors associated with high resistances in alternating current measurements," *Journal Sci. Insts.*, vol. 5, pp. 305–312, 354–361 (1928). "The design and construction of a shielded resistor for high voltages," *Journal I.E.E.*, vol. 69, pp. 1028–1034 (1931). L. Hartshorn and R. M. Wilmotte, "Note on shielded non-inductive resistances," *Journal Sci. Insts.*, vol. 4, pp. 33–37 (1926). L. Hartshorn, "Standards of phase angle," *World Power*, vol. 8, pp. 171–180, 234–240 (1927). R. M. Wilmotte, "A general theorem on screened impedances," *Phil. Mag.*, vol. 6, pp. 788–795 (1928).

the auxiliary resistor is open to the air and unshielded. Separate wooden frames carry the main and auxiliary resistors, as shown in Fig. 189. Tests on the completed resistor show that at voltages up to 40 kV the magnitude error and phase-angle are respectively 5×10^{-4} and 2×10^{-4} radian (0·7 minute) at 50 cycles per sec. for all conditions of use.

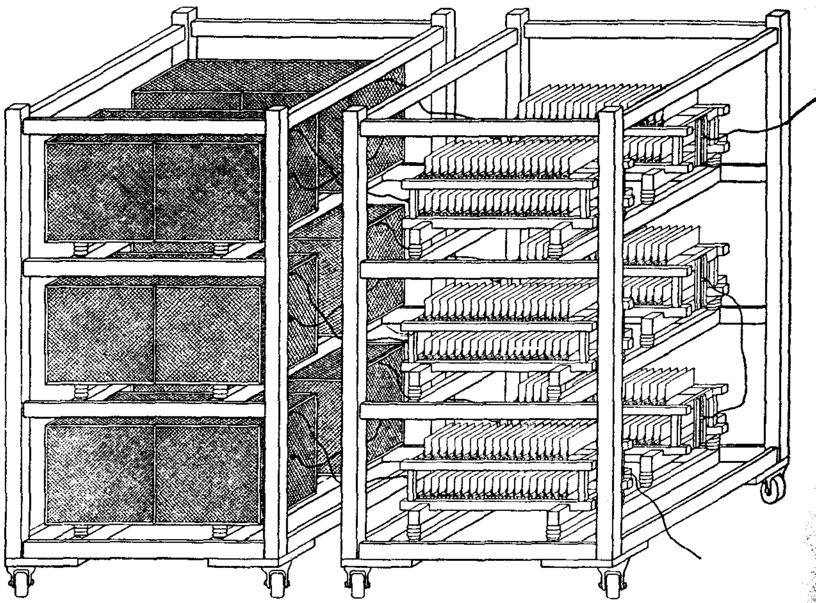


Fig. 189. General View of N.P.L. Shielded Resistor for 40 kV

(iii) *Shielded at mid-potential using transformer.* Silsbee has pointed out that at very high voltages the earth capacitances of the shields and of the auxiliary resistor sections may cause a serious disturbance of the shield potentials and greatly increase the phase-angle of the main resistor; the effect varies approximately with the square of the voltage. To overcome the trouble it is necessary either to shield the auxiliary resistor and provide a tertiary resistor to maintain its shields at the correct potentials, or to abandon the use of resistors entirely as a means of fixing the shield potentials on the main resistor, a transformer being used instead. This method has been adopted in the 132 kV resistor designed in the testing laboratory of the General Electric Co. at Schenectady.* The

* C. T. Weller, "132 kV shielded potentiometer for determining the accuracy of potential transformers," *Journal Amer. I.E.E.*, vol. 48, pp. 312–316 (1929).

main resistor-unit is 100 000 ohms for use at 11 kV; each consists of ten 10 000 ohm sections, which are in turn composed of twelve vertically mounted resistance tubes of $833\frac{1}{3}$ ohms each. The tubes are 12 in. long and 1·25 in. diameter, made of sheet brass insulated with paper and wound with two layers of 0·011 in. diameter silk-insulated manganin wire; the layers are wound in opposite directions to reduce the inductive residual to a minimum. The tubes are mounted three deep and four wide in an insulating frame about 36 in. by 8 in., the ten frames being contained in a sheet-steel tank 5 ft. by 3·5 ft. by 5·5 ft. filled with 530 gallons of oil. Four such units are grouped about an autotransformer, as shown diagrammatically in Fig. 190, provided with suitable tappings to fix the potentials of the resistor tanks at the correct values. Three such groups are provided for series connection to attain 132 kV. The autotransformers have coupling windings to ensure the uniform division of the total voltage between them when they are joined in series. Each resistor tank and each transformer stands upon insulating cylinders. The resistors weigh about 2·8 tons each, the transformers about 10 tons each; the complete equipment weighs about 65 tons, occupies a floor space 42 ft. by 9 ft. and is 12 ft. high. The phase-angle at 60 cycles per sec. is about 7 minutes for one 44 kV group, 6 minutes with two groups in series for 88 kV, and 4 minutes at 132 kV with three groups in series. It is intended to extend the arrangement to 220 kV.

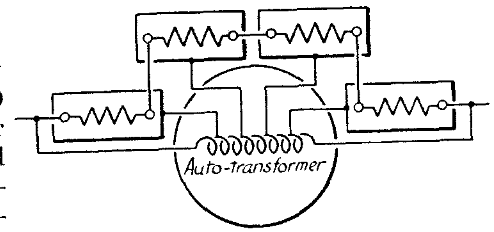


Fig. 190. Diagram of G.E.C. Shielded Resistor for 44 kV

(iv) *Approximate continuous shielding.* It has been stated on p. 361 that if a resistor could be shielded in such a way that the potential of the shield at each point is equal to the potential of the resistor at that point, then earth-capacity currents could not flow and the phase-angle of the resistor would be due entirely to its inductance and self-capacitance. The Laboratoire Central d'Électricité in Paris* have designed resistors in

* P. de la Gorce, "La réalisation d'une résistance pour les mesures en très haute tension," *Comptes Rendus*, vol. 191, pp. 1297–1299 (1930); *Rev. Gén. de l'Él.*, vol. 29, pp. 427–428 (1931).

which an attempt has been made to provide a close approximation to such graded or continuous shielding, Fig. 191 illustrating an example for 150 kV. The main resistor consists of an even number of flat coils wound successively in opposite directions to reduce the inductance; 0·1 mm. diameter constantan wire is used and the total resistance is 3 megohms. The shield circuit consists of a similar winding with a resistance of 6 megohms, the main and shield resistors being arranged coaxially and joined in parallel as shown. Both are contained

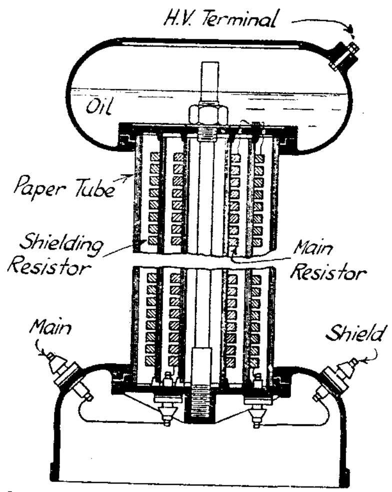


FIG. 191. CONTINUOUSLY SHIELDED RESISTOR FOR 150 kV

in an insulating container with metal end-caps, oil filling being provided for cooling and insulating the coils. The phase-angle at 50 cycles per sec. is 2 minutes leading. Other designs have been prepared on this principle for use at 40 kV (portable) and 350 kV.

The same principle has been used with a different method of construction in the 33 kV resistor designed by Jimbo and Sakimura.* This consists of three resistors, each of 660 000 ohms, joined in parallel; one is the main resistor, the other two acting as the shielding circuit in a manner to be described. Each resistor is composed of thirty-four units, two of these having resistances of 10 000 ohms and the remainder 20 000 ohms each. A unit contains three groups of twenty bakelite cards, one each for the shield resistors and for the main resistor,

* S. Jimbo and H. Sakimura, "A new method of testing potential transformers," *Res. Elect. Lab. Tokyo*, No. 363, pp. 1–42 (1934). In English.

the cards measuring 15 cm. by 5·4 cm. by 0·15 cm.; they are wound with 0·16 mm. diameter constantan wire. In a 20 000 ohm unit each card is wound with ten bifilar sections of 100 ohms. The cards are mounted in two tiers, the space between adjacent cards being 5 mm.; every card of the main resistor is arranged to lie between two corresponding cards of the shielding resistor. The phase-angle is about 11 minutes at 50 cycles per sec.

CHAPTER X

INDUCTANCE AND CAPACITANCE STANDARDS

1. **Introductory.** Standards of resistance, such as have been described in Chapter IX, may be otherwise regarded as standards of zero phase-angle, since in a perfect resistor the current and voltage are exactly in phase. All practical resistors to some extent fall short of this ideal and we have seen that quite elaborate precautions are necessary to ensure that the impurity or defect from zero phase-displacement in low resistances and in high resistances does not become excessive.

Standards of inductance and capacitance, on the other hand, may be regarded as standards of quadrature, since the current and voltage in perfect inductors and condensers are exactly a quarter-period apart, lagging in the one case and leading in the other. This degree of perfection is not quite attained in practical apparatus, for a variety of reasons, but the impurity or deviation from exact quadrature is usually very slight at the usual industrial frequencies of 25 to 60 cycles per sec.; and in any case the imperfection is usually much less than in resistors. In the following sections we shall examine the types of inductance and capacitance standards that find application in instrument transformer testing, but for a full treatment of the subject the reader is referred to the author's *A.C. Bridge Methods*, where complete details of theory, design, and construction are given.*

2. **Inductance standards.** Self inductances are not much used in instrument transformer test circuits, though they find application in the preparation of burdens. Many methods of testing, however, employ mutual inductors to couple a detector or measuring circuit with either the secondary or the primary circuit of the transformer which is being tested. Such mutual inductors may be fixed or variable, usually the latter, and should satisfy the following conditions—

(i) The secondary voltage and the primary current should be exactly in quadrature.

(ii) The inductor should be unaffected by stray magnetic fields of the test frequency.

* See also Campbell and Childs, loc. cit. *ante*.

(iii) In variable inductors the scale should be reasonably uniform to admit of easy interpolation.

Deviation from quadrature is due to two principal causes, (*a*) self-capacitance of the windings and inter-capacitance between them and (*b*) eddy currents in the coils and their fittings. In general, the deviation only becomes appreciable at frequencies above 500 cycles per second, and in the low industrial frequency range of 25–60 cycles per sec. is quite negligible, provided all the usual precautions are observed in construction, such as careful insulation of the coils, the use of stranded conductors, and the removal of the terminals to a situation of weak magnetic field.

Complete freedom from interference by external fields can only be obtained by the use of toroidal ring windings; this method of construction is exceedingly troublesome to undertake but may be necessary in certain extreme circumstances. As a rule sufficient astaticism can be obtained by dividing the windings into two groups connected up in such a way that the e.m.f.'s induced in the groups by an external field are in opposition.

A suitable scale characteristic in variable inductors can be secured by appropriate shaping and proportioning of the coils. Although a uniform scale has some advantage and is favoured by many, there is much to be said for a logarithmic scale in which the percentage accuracy of reading is the same at all parts.

In voltage transformer testing, whether the inductor has one winding in the primary or in the secondary circuit of the transformer, the current to be carried is seldom more than a few tenths of an ampere. In such cases any of the well-known types of mutual inductor used in a.c. bridge methods are quite suitable; pre-eminent among these are the Campbell inductor and the Butterworth-Tinsley inductor described in the places cited. They are not, however, astatic; though this is not likely to cause much trouble since the stray fields in voltage transformer test circuits are not very important.

In current transformers testing the lack of astaticism and the low current-carrying capacity may be serious drawbacks to the use of these excellent inductors. When one winding of the inductor is connected in the secondary circuit and will, therefore, carry up to 5 amperes, the Bureau of Standards*

* H. B. Brooks and F. C. Weaver, "A variable self and mutual inductor," *Bull. Bur. Stds.*, vol. 13, pp. 569–580 (1917).

astatic disc-type inductor with link-shaped coils is specially suitable. As shown in Fig. 192 it consists of three ebonite discs in which the coils are embedded. The central disc may be rotated between the other two, varying the mutual inductance between the fixed and moving systems from − 272 μH to + 278 μH. The proportions shown give a high time-constant with all the coils in series as a self inductor (3·4 milliseconds) and an almost uniform scale; the dimension r is 36·8 mm. and the diameter of the discs is 35·5 cm. Each fixed coil is wound with nine layers of two turns each, the wire consisting of seven insulated copper strands each 0·8 mm. diameter, the four fixed coils are connected astatically in series. The moving coils have nine layers of four turns each and are similarly astatic.

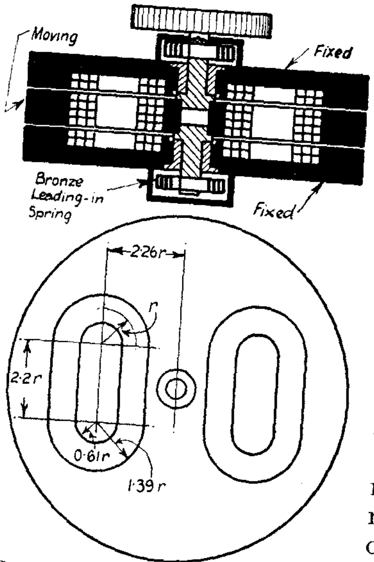


FIG. 192. BROOKS AND WEAVER MUTUAL INDUCTOR

When one winding of a mutual inductor is to be connected in the primary circuit of a current transformer it is necessary to design it to carry large currents and to make it perfectly astatic. Fortescue has met these requirements by winding toroidal inductors upon marble rings of circular cross-section; his apparatus is described in the appropriate place on p. 483. Modern practice would avoid entirely the use of heavy-current inductors by inserting in the primary circuit of the test transformer a nickel-iron cored standard current transformer, in the 5 ampere secondary of which an inductor of the Brooks and Weaver or other suitable type is connected.

3. **Capacitance standards for high voltages.** Resistance voltage-dividers of the shielded pattern, such as have been described in Chapter IX, are frequently used in the testing of voltage transformers. At high voltages, however, the resistors are far from perfect and require elaborate systems of shielding to keep the phase-error down to a reasonably small value; the highest voltage for which a resistance has been constructed

is 132 kV. and the apparatus is extremely bulky and costly. A considerable amount of testing must now be done at 220 kV and there is no doubt that higher voltages are likely in the future; experience with resistors leads one to suppose that they are unlikely to be satisfactory standards at such high voltages. The great success of high-voltage condensers with gas dielectric, when used in a.c. bridges for dielectric loss measurements, has led to numerous methods of testing voltage transformers in which condenser voltage-dividers are employed. These consist of a high-voltage condenser of small capacitance in series with a much larger mica condenser; the former carries the greater proportion of the total voltage and can be readily built to withstand the largest voltages encountered in practice. Its purity is much better than that of a resistor and is attained without elaborate constructional features. Two principal types are of interest for the present purpose. In the first the dielectric is air at atmospheric pressure; in the second the dielectric is air or other gas under high pressure.

The electrodes in air condensers may be either parallel flat plates or coaxial circular cylinders. The dimensions must be such that the dielectric stress is well below breakdown, and losses due to brush discharge and corona must be avoided by adequate spacing of the electrodes and careful rounding of their edges. The low-voltage electrode must be provided with a guard electrode to protect it against external electric fields and also to define the area of the low-voltage electrode in such a way that the capacitance of the condenser can be calculated with high precision from its dimensions. The greater perfection of shielding and ease of construction render cylindrical condensers superior to flat plate condensers, and they are now very widely used; several examples of both types are described in the author's book previously cited.

As a typical instance of the carefully designed cylindrical condenser the design illustrated in Fig. 193 may be mentioned, this being due to Churcher and Dannatt* of the Metropolitan-Vickers Co. The condenser is for 150 kV and has electrodes of machined cast iron protected against rust by a coating of cellulose enamel. The h.v. electrode is outside; the inner l.v. electrode is in three parts, of which the middle part is the working section while the end parts are guard electrodes. The capacitance is 100 μμF. At 300 kV a modified design is

* B. G. Churcher and C. Dannatt, "The use of air condensers as high voltage standards," *Journal I.E.E.*, vol. 69, pp. 1019–1027 (1931).

preferred; the l.v. electrode stands on the ground and is on the outside; being at earth potential much less clearance is needed round the condenser. A special tripod support holds the h.v. electrode coaxially within the outer cylinder by means of a micarta tube. It has been shown that it is possible to determine the capacitance of such condensers within 1 part

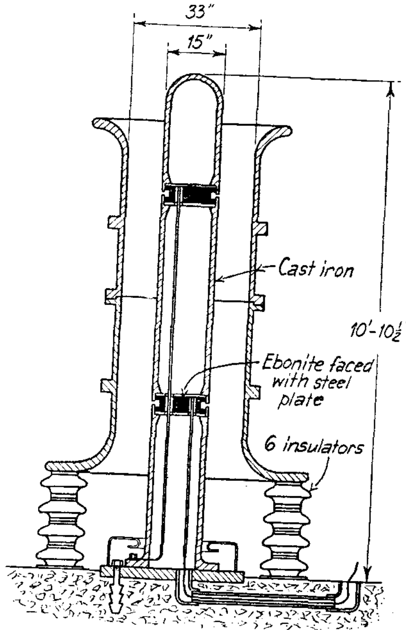


FIG. 193. CHURCHER AND DANNATT'S AIR CONDENSERS FOR 150 KV

in 10^4 and the phase-angle is certainly less than 10^{-5} radian in defect of exact quadrature.

The principal defect of high-voltage air condensers is their considerable bulk, but this can be greatly reduced by using gas under pressure as the dielectric. The dielectric strength of a gas increases with its pressure; for example air at 10 atmospheres has a dielectric strength about equal to that of oil; the permittivity, on the other hand, changes very little with the gas pressure. Compressed-gas condensers are compact, light and portable; they are constant in capacitance, entirely free from losses, and completely shielded. The casing must be gas-tight; but any slight leakage of gas can be readily made up with the aid of a cylinder of compressed or liquefied gas in order to maintain the dielectric strength.

Fig. 194 shows a compressed-gas condenser designed at the Reichsanstalt by Schering and Vieweg.* The body of the condenser consists of a paper tube with substantial metal flanges and lids, by means of which it can be closed and made gas-tight. The high-voltage electrode is a tube attached to the upper lid; the low-voltage electrode is a coaxial tube supported

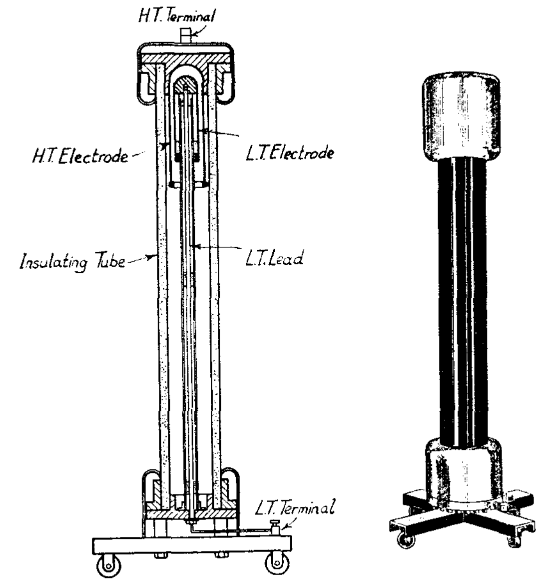


FIG. 194. COMPRESSED GAS CONDENSER FOR 230 KV

on a rod passing through the lower lid and insulated therefrom. This rod is shielded by a concentric metal tube attached to the lower earthed cover. Nitrogen or carbon dioxide at 12–14 atmospheres is used, the gas pressure falling about 1 atmosphere per month; a manometer mounted on the base enables a check on the pressure to be made at any time. The working voltage is 230 kV, the capacitance 54 $\mu\mu$F; the overall height is 225 cm., the floor space 92 cm. by 92 cm. and the weight 300 kg. (660 lb.).

Another condenser, due to Palm,† is shown in Fig. 195; it is suitable for 140 kV and has a capacitance of 100 $\mu\mu$F. The

* H. Schering and R. Vieweg, "Ein Messkondensator für Höchstspannungen," *Zeits. f. tech. Phys.*, vol. 9, pp. 442–445 (1928).

† A. Palm, "Über neuere Hochspannungsmessgeräte und ihre Anwendung," *Elekt. Zeits.*, vol. 47, pp. 873–875, 904–907 (1926).