150

INSTRUMENT TRANSFORMERS

ALSO BY B. HAGUE

ALTERNATING CURRENT BRIDGE METHODS

For the Measurement of Inductance, Capacitance, and Effective Resistance at Low and Telephonic Frequencies

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INSTRUMENT TRANSFORMERS

THEIR THEORY CHARACTERISTICS AND TESTING

A THEORETICAL AND PRACTICAL HANDBOOK
FOR TEST-ROOMS AND RESEARCH
LABORATORIES

BY

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TO

EMERITUS-PROFESSOR T. MATHER, F.R.S.

THIS VOLUME IS INSCRIBED AS

AN AFFECTIONATE TRIBUTE

FROM

ONE OF HIS GRATEFUL STUDENTS

PREFACE

THIS book had its origin in a systematic study, begun nearly nine years ago, of the numerous methods that have been devised for the testing of Instrument Transformers. It was originally intended that this study should be published as a sequel to a book on alternating current bridge measurements,* to which the subject is closely related, but pressure of other duties prevented this intention being carried into effect. In revising the work from time to time it became clear that a wider scope would be desirable, since it is not easy to discuss the testing of instrument transformers without at the same time examining the theory, construction, and characteristic features of the transformers in some detail.

The resolve to prepare a book dealing with this wider aspect of the subject was strengthened by observing that there is no complete analysis published in the English language of the very extensive literature dealing with instrument transformers, though there are several excellent reviews of portions of the field in some recent books. On the Continent there are a few fairly complete treatises, in French, German, and Russian, though even these do not deal with the most recent developments and are, moreover, beyond the reach of many English-speaking engineers. Since most of the useful material concerning instrument transformers is to be found in periodical literature, it was felt that it would be useful for one person to read and summarize the large number of papers that has been published, in order to give the busy engineer, research worker and advanced student a general view of the whole subject. With what success the author has attempted to perform this task the reader alone can judge. Being unconnected either with the manufacturing or with the operating side of the electrical industry, the author may fairly claim to be reasonably free from bias in favour of this or that type of transformer or method of use; he has endeavoured, as a teacher, to concentrate chiefly on an exposition of fundamental principles, illustrated by a description of apparatus in which those principles

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* *Alternating Current Bridge Methods for the Measurement of Inductance, Capacitance and Effective Resistance at Low and Telephonic Frequencies,* third edition, 1932 (Sir Isaac Pitman and Sons, Ltd.). Referred to throughout this volume as *A.C. Bridge Methods.*

seem most satisfactorily to be embodied. If some parts of the subject are not so completely treated as the reader, and indeed the author, would have wished, consolation may be found in the fact that full references to the literature are given, so that anyone who is interested to do so may easily find further information.

In writing the book certain limitations were imposed at the outset. First, the book deals primarily with the transformers used in electrical measurements; transformers used in connection with protective systems are only given secondary consideration, insofar as they conform to the common general principles. Much of the interest in protective transformers lies not in the transformers themselves but in the particular circuits to which they are connected, and it was felt that this subject lay outside the real purpose of this book; moreover, the use of protective transformers has recently been ably and fully discussed by Mr. G. W. Stubbings (see p. 521) in a very useful and practical way. Second, attention is confined to transformers used at supply frequencies; those used in radio-frequency circuits are entirely omitted as they are adequately treated elsewhere in more appropriate fashion. Third, problems of design, in the ordinary practical meaning of the term, are not considered; each individual works has its own way of simultaneously satisfying the requirements of electromagnetic principles and the stern laws governing the economic production of a saleable article. Design in this sense is not a suitable topic for academic discussion. The reader is again reminded that the foremost thought in the author's mind has been to set out fundamental principles; with these clearly stated and understood all else follows without much difficulty. While it is perhaps true that the present volume will be primarily of interest to the specialist, it is the author's hope that the correlation of the various subjects dealt with will interest students and others who may be concerned with the theory and practice of A.C. measurements, while the bibliographical study of British and foreign technical literature which it provides—much of which is not readily accessible to the general reader—will be useful to the research worker in many branches of electrotechnical and physical investigations.

A few words regarding the plan of the book may not be out of place. Part 1 is devoted to the theory of instrument transformers and to an examination of such constructional or other practical features as may exert an important influence upon their operating characteristics. On the constructional side this Part is to be regarded neither as a complete catalogue of types nor as a critical comparison of the products of different manufacturers; again the illustration of principles has been the paramount idea. An attempt has been made in this Part to discuss the various national and international rules that have been proposed as the standard for satisfactory performance, a subject that seems likely to undergo drastic revision in the near future. Part 2 approaches the subject of apparatus used in testing transformers. Certain topics, such as the design of low- and high-value resistors, are very fully treated since little is to be found on such subjects in other books, while other things are treated with brevity where fuller information can easily be found elsewhere. Parts 3 and 4 deal respectively with the testing of current transformers and voltage transformers for ratio and phase-angle errors, while Part 5 examines other tests, such as those for temperature rise, dielectric strength, polarity, etc.

The reader of Parts 3 and 4 will be impressed by the considerable number of different methods that have been proposed, by the small proportion of these that have been generally adopted in practice, and by the great waste of effort, due to lack of co-ordination, that has resulted in the repeated re-introduction of an old method as new. With regard to the number of methods, it is the author's experience in research that a method not generally adopted for its original purpose may prove to be useful in quite other fields, and for this reason is worthy of being put on record. As regards the waste of effort, to which reinvention bears witness, it is the author's hope that the co-ordination provided by the bibliographical study given in Parts 3 and 4 may do something to prevent in the future much unnecessary duplication of work. At the same time an attempt has been made at a critical appraisal of those methods which have been found best for transformer testing.

The diagrams have all been specially prepared for this book. The bulk of the illustrations are line drawings, but in cases where it has been necessary to convey some idea of external appearance shaded drawings have been used in preference to photo-blocks, since they give a much more striking impression of the features it is desired to bring to the reader's attention. Thanks are offered to the firms mentioned below for their generous provision of drawings or photographs which have been used in making many of the illustrations. Particular acknowledgment will here be made to the Institution of Electrical Engineers for permission to use material in the *Journal* from which Figs. 55, 181, 189, and 193 have been prepared.

In preparing a book of this kind, where so much is in the nature of a compilation of existing data, the author is necessarily deeply in debt to those whose experimental results, theoretical skill and practical experience have been so freely placed at his disposal; numerous individual acknowledgments of this debt will be found in the text. This opportunity is taken to thank collectively all those who have given help, but who cannot be separately mentioned on account of limitations of space. In response to a questionnaire circulated in 1928 and to later requests, the following manufacturers generously supplied a wealth of useful information, only a fraction of which could be used in the space available in this book; to these firms and to their engineers sincere thanks are tendered—Allgemeine Elektrizitäts Gesellschaft; Brown-Boveri & Cie.; Elliott Bros.; Everett, Edgcumbe & Co.; Cie. pour la fabrication des Compteurs et Matériel d'Usines à Gaz, Eau et Électricité; Ferranti Ltd.; General Electric Co. (of Great Britain) and Chamberlain & Hookham Ltd.; General Electric Co. (of Schenectady) and British Thomson-Houston Co.; Koch & Sterzel A.G.; Landis & Gyr Ltd.; Metropolitan-Vickers Electrical Co.; Nalder Bros. & Thomson Ltd.; Sachsenwerk Licht und Kraft A.G.; Sangamo Electric Co.; Siemens & Halske A.G.; Westinghouse Electric and Manufacturing Co. of Pittsburg; Zenith Electrical Co. It is appropriate here to acknowledge the valuable assistance given by the British Standards Institution and by the similar organizations responsible for the preparation of standards in various foreign countries.

To Mr. A. J. Small, B.Sc., the author's thanks are due for the great assistance he has given in the task of proof-reading; to his painstaking care and vigilance the volume owes much in accuracy and clarity of statement. Finally, cordial thanks are offered to Professor G. W. O. Howe, whose kindly criticism and friendly counsel were ever at the author's disposal; without the stimulus his continuous encouragement has afforded it is doubtful whether this long, interesting, but often wearying task could have reached a satisfactory conclusion.

B. HAGUE.

GLASGOW.
April, 1936.

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INSETS

ABBREVIATIONS

A.C.E.C. Journal.	Journal des Ateliers de Constructions Électriques de Charleroi.
A.E.G. Mitt.	A.E.G. Mitteilungen.
Ann. der Phys.	Annalen der Physik.
Ann. der Phys. u. Chem.	Annalen der Physik und Chemie.
Arbeiten Elekt. Inst. Karlsruhe	Arbeiten aus dem elektrotechnisches Institut in Karlsruhe.
Arch. f. Elekt.	Archiv für Elektrotechnik.
Arch. f. tech. Mess.	Archiv für technisches Messen.
A.S.E.A. Journal.	Allmänna Svenska Elektriska Aktiebolaget Journal.
Atti dell'Assoc. Elett. Ital.	Atti dell'Associazione Elettrotecnica Italiana.
B.B. Rev.	Brown-Boveri Review.
Beama J.	Beama Journal (British Electrical and Allied Manufacturers Association), continued as *World Power*.
Bell Syst. Tech. J.	Bell System Technical Journal.
B.T.H. Activities	B.T.H. Activities (British Thomson-Houston Co.).
Bull. Bur. Stds.	Bulletin of the Bureau of Standards; continued as *Bur. Stds., Journal of Res.*
Bull. Schw. Elekt. Verein	Bulletin des schweitzerische elektrotechnische Verein.
Bull. Soc. Belge des Elecns.	Bulletin de la Société Belge des Électriciens.
Bull. Soc. Franç. des Elecns.	Bulletin de la Société Française des Électriciens.
Bull. Soc. Int. des Elecns.	Bulletin de la Société Internationale des Électriciens.
Bur. Stds., Journal of Res.	Bureau of Standards, Journal of Research.
Comptes Rendus	Comptes Rendus hebdomadaires des séances de l'Academie des Sciences.
Dingler's Poly. J.	Dingler's polytechnisches Journal.
Écl. et Force Mot.	Éclairage et Force Motrice.
Elec. Eng.	Electrical Engineering.

Elecn.	The Electrician.
Elec. Rev.	Electrical Review.
Elec. Times.	Electrical Times.
Elec. World.	Electrical World.
Elect. Club J.	Electric Club Journal; continued as *Elect. J.*
Elect. Comm.	Electrical Communication
Elect. J.	Electric Journal.
Elekt. Betrieb	Elektrische Betrieb.
Elekt. Kraft u. Bahn.	Elektrische Kraft und Bahnen.
Elekt. Wirts.	Elektrische Wirtschaft.
Elekt. Zeits.	Elektrotechnische Zeitschrift.
E.N.T.	Elektrische Nachrichten-technik.
E.u.M.	Elektrotechnik und Maschinenbau.
Gen. Elec. Rev.	General Electric Review.
Helios	Helios.
Int. Conf. H.T.E.S.	International Conference on Large High-Tension Electrical Systems.
Journal Amer. I.E.E.	Journal of the American Institute of Electrical Engineers; continued as *Elec. Eng.*
Journal Frank. Inst.	Journal of the Franklin Institute.
Journal I.E.E.	Journal of the Institution of Electrical Engineers.
Journal I.E.E. Japan	Journal of the Institution of Electrical Engineers of Japan.
Journal Sci. Insts.	Journal of Scientific Instruments.
K.u.S. Mitt.	Koch und Sterzel Mitteilungen.
L'Elettro.	L'Elettrotecnica.
Lum. Élect.	La Lumière Électrique.
Messtechnik.	Messtechnik.
M.V. Gaz.	Metropolitan-Vickers Gazette.
Phil. Mag.	Philosophical Magazine.
Phys. Rev.	Physical Review.
Phys. Zeits.	Physikalische Zeitschrift.
Power.	Power.
Proc. Amer.I.E.E.	Proceedings of the American Institute of Electrical Engineers.
Proc. N.E.L.A.	Proceedings of the National Electric Light Association.
Proc. Phys. Soc.	Proceedings of the Physical Society.
Proc. Roy. Soc. A.	Proceedings of the Royal Society of London, Section A.
Res. Elect. Lab. Tokyo	Researches of the Electrotechnical Laboratory, Ministry of Communications, Tokyo.
Rev. Gén. de l'Él.	Revue Général de l'Électricité.
Schw. Elekt. Zeits.	Schweitzerische elektrotechnische Zeitschrift.
Siemens Jahrbuch	Siemens Jahrbuch.
Siemens Nach.	Siemens Nachrichten.
Siemens Zeits.	Siemens Zeitschrift.
Stahl u. Eisen	Stahl und Eisen.
Trans. Amer.I.E.E.	Transactions of the American Institute of Electrical Engineers.
Trans. S. Afr.I.E.E.	Transactions of the South African Institution of Electrical Engineers.
Univ. of Illinois, Bull.	Bulletin of the University of Illinois.
Univ. of Toronto Eng. Res. Bull.	University of Toronto Engineering Research Bulletin.
V.D.E. Fachberichte	Fachberichte des Verbandes Deutsche-Elektrotechniker.
Ver. v. Direct. v. Elect. Nederland	Verhandlung van Directoren van Electricitat in Nederland.
Wiss. Veröff. Siemens Konz.	Wissenschaftliche Veröffentlichungen aus dem Siemens Konzerns.
World Power	World Power.
Zeits. des V.D.I.	Zeitschrift des Vereines Deutscher Ingenieure.
Zeits. f. Electrochemie und ang. phys. Chem.	Zeitschrift für Electrochemie und angewandte physikalische Chemie.
Zeits. f. Inst.	Zeitschrift für Instrumentenkunde.
Zeits. f. tech. Phys.	Zeitschrift für technische Physik.

ERRATA

Page xvii, line 13. *For* (3) *read* (c).

Page 23, line 3. *For* "between $\frac{1}{4}$ and $\frac{1}{4}$" *read* "between $\frac{1}{4}$ and $\frac{1}{2}$."

Page 68, line 15. "$B_{max}/at_s = B_{max}/at_0$" *read* "$B_{max}/at_s = \partial B_{max}/\partial at_0$."

Page 221, line 8. *For* "$\doteqdot I\gamma_0 \, (R_{wp} \sin \zeta - X_{wp} \cos \zeta)/V_p$" *read* $\gamma_0 \doteqdot I_0 \, (R_{wp} \sin \zeta - X_{wp} \cos \zeta)/V_p$."

Page 333, line 28. *For* "$\rho_x u_x, \rho_x u_y, \rho_y u_z$" *read* "$\rho_x u_x, \rho_y u_y, \rho_z u_z$."

Page 555, line 5. *For* "$\{\omega L - [\omega C s^2/(1 + \omega^2 C^2 s^2)]\}^2$" *read* $\{\omega L - [\omega C s^2/(1 + \omega^2 C^2 s^2)]\}$."

Page 561, line 9. *Amend to read* "$K_v \doteqdot M/C_1 rS$."

Page 635, line 14 (left). *For* "623" read "621."

Page 645, line 20. *Amend to read* "Great Britain, 591, 633."

Page 645, line 28. *Amend to read* "National Specifications, 595–6, 633."

Page 650, line 56. *For* "621" *read* "622."

Page 651, line 2. *Add after* "209," "621."

Page 654, line 55. *For* "277" *read* "275."

(T.5722)

SUMMARY OF DEFINITIONS

CURRENT TRANSFORMERS

I_{np} = Nominal or rated primary current.
I_{ns} = Nominal or rated secondary current.
K_{nc} = Nominal or rated current ratio = I_{np}/I_{ns}.
I_p = Primary current.
I_s = Secondary current.
K_c = Current ratio = I_p/I_s.
ε_c = Current ratio error in per cent = $[(K_{nc}/K_c) - 1]100$ = $[(1/F_c) - 1]100$.
F_c = Current ratio factor = $K_c/K_{nc} = 1/[1 + (\varepsilon_c/100)]$.
K_T = Turns ratio = T_s/T_p.
ε_T = Turns ratio error in per cent = $[(K_{nc}/K_T) - 1]100$.
β = Phase-angle = angle between I_p and I_s reversed; positive when $-I_s$ leads on I_p; usually in centiradians.

VOLTAGE TRANSFORMERS

V_{np} = Nominal or rated primary voltage.
V_{ns} = Nominal or rated secondary voltage.
K_{nv} = Nominal or rated voltage ratio = V_{np}/V_{ns}.
V_p = Primary voltage.
V_s = Secondary voltage.
K_v = Voltage ratio = V_p/V_s.
ε_v = Voltage ratio error in per cent = $[(K_{nv}/K_v) - 1]100$ = $[(1/F_v) - 1]100$.
F_v = Voltage ratio factor = $K_v/K_{nv} = 1/[1 + (\varepsilon_v/100)]$
K'_T = Turns ratio = T_p/T_s.
ε'_T = Turns ratio error in per cent = $[(K_{nv}/K_T') - 1]100$.
γ = Phase-angle = angle between V_p and V_s reversed; positive when $-V_s$ leads on V_p; usually in centiradians.

POWER MEASUREMENT

α = Angle of lag of current in voltage circuit relative to voltage applied to it = arctan $(\omega L_v/R_v)$ usually in centiradians.
δ = $\alpha + \beta - \gamma$ usually centiradians.

ϕ = Phase displacement of current in main circuit relative to voltage; positive when voltage leads on current.

$\phi' = \phi - \delta$.

ε_p = Power error in per cent $= \varepsilon_v + \varepsilon_c + \varepsilon_\delta$
$= \varepsilon_{pv} + \varepsilon_{pc} + \alpha \tan \phi$.

$\varepsilon_\delta = \delta \tan \phi$.

$\varepsilon_{pv} = \varepsilon_v - \gamma \tan \phi$ = overall power error due to voltage transformer.

$\varepsilon_{pc} = \varepsilon_c + \beta \tan \phi$ = overall power error due to current transformer.

FRACTIONAL ERRORS

It is sometimes convenient to use ε_c, ε_v, ε_p, ε_δ, ε_{pv} and ε_{pc} as fractions rather than percentages. Their definitions follow from those given by omitting the factor 100. Likewise it may be more convenient to express the angles β, γ, α, and δ in radians. In all cases it will be clear from the context which definition is intended.

VECTOR NOTATION

The notation for harmonic vectors explained in *A.C. Bridge Methods* will be used. For a *current transformer*, if ε_c is the fractional ratio error and β is in radians, the vector relation between the primary current $\boldsymbol{i}_p$ and the secondary current $\boldsymbol{i}_s$ is

$$\begin{aligned}\boldsymbol{i}_s &= \frac{1}{K_c}\varepsilon^{-j(\pi-\beta)}\,\boldsymbol{i}_p = -\frac{1}{K_c}\varepsilon^{j\beta}\,\boldsymbol{i}_p = -\frac{1}{K_{nc}}\cdot\frac{1}{F_c}\varepsilon^{j\beta}\,\boldsymbol{i}_p\\ &= -\frac{1}{K_{nc}}(1+\varepsilon_c)(\cos\beta + j\sin\beta)\,\boldsymbol{i}_p \doteqdot -\frac{1}{K_{nc}}(1+\varepsilon_c)(1+j\beta)\,\boldsymbol{i}_p\\ &\doteqdot -\frac{1}{K_{nc}}[(1+\varepsilon_c)+j\beta]\,\boldsymbol{i}_p.\end{aligned}$$

Similarly for a *voltage transformer*,

$$\begin{aligned}\boldsymbol{v}_s &= \frac{1}{K_v}\varepsilon^{-j(\pi-\gamma)}\,\boldsymbol{v}_p = -\frac{1}{K_v}\varepsilon^{j\gamma}\,\boldsymbol{v}_p = -\frac{1}{K_{nv}}\cdot\frac{1}{F_v}\varepsilon^{j\gamma}\,\boldsymbol{v}_p\\ &= -\frac{1}{K_{nv}}(1+\varepsilon_v)(\cos\gamma + j\sin\gamma)\,\boldsymbol{v}_p \doteqdot -\frac{1}{K_{nv}}(1+\varepsilon_v)(1+j\gamma)\boldsymbol{v}_p\\ &\doteqdot -\frac{1}{K_{nv}}[(1+\varepsilon_v)+j\gamma]\boldsymbol{v}_p.\end{aligned}$$

INSTRUMENT TRANSFORMERS

PART 1

THEORY AND CHARACTERISTICS OF INSTRUMENT TRANSFORMERS

CHAPTER I

GENERAL INTRODUCTION

1. **Preliminary and historical.** In modern electrical engineering practice electrical energy is generated in steam or hydraulic power stations, and transmitted by polyphase alternating currents to the distributing network, whence it is utilized by the consumers to whom the energy is sold as a commercial commodity. The annual financial value of the electrical energy used throughout the world reaches a tremendous figure; it is not surprising, therefore, that considerable attention has been devoted to its accurate measurement. A highly developed art of electrical measurement has been gradually evolved, including within its scope not only the measurements required by the engineer in practice but also those used in the laboratory for purposes of standardization and research. In all of these branches *Measuring* or *Instrument Transformers* are an essential feature in modern measurement technique. Ammeters, voltmeters, wattmeters and watt-hour meters, of switchboard or laboratory types, as well as relays and protective devices are, for convenience, accuracy and safety, invariably operated in the secondary circuits of suitable instrument transformers. These are of two main kinds, *Current Transformers* and *Voltage Transformers*, according as their purpose is to operate current-measuring or voltage-measuring apparatus.

The use of instrument transformers appears to have become general toward the close of the nineteenth century, when the development of alternating current distribution on a large scale became possible after the introduction of the three-phase system of transmission in 1890 and the improvement made in the earlier types of power transformer during the last decade of the century. Currents and voltages increased rapidly in magnitude with the growth in power generation and transmission, and it soon became apparent that for convenience and safety the measuring instruments should be connected to the network through suitable transformers. The idea seems to have occurred simultaneously and independently to several engineers, so that it is hardly possible to assign the invention of instrument transformers to a single individual.

There are, however, certain historical facts* that are of considerable interest in the early development of instrument transformers. The earliest account of a current transformer appears in a patent specification filed by Ferranti and Thompson† in 1882, in which a description is given of an a.c. electrolytic quantity meter with a self-contained air-cored transformer. The main current is passed through the primary, which is a thick copper spiral, while the secondary terminals are joined to the electrodes. In 1887 Dick and Kennedy‡ first suggested the use of a step-down transformer in conjunction with a Cardew hot-wire voltmeter, their object being to eliminate the use of the delicate electrostatic instruments which were the only means known at that time for the measurement of high voltages; their invention constituted the earliest application of the voltage transformer.

During the next five years numerous patents were taken out for instruments of various types operated by transformers, but none were of remarkable interest until in 1893 Ayrton and Mather § patented the iron ring or toroidal type of instrument transformer threaded over one of the a.c. mains. Their original purpose was to compensate, by the voltage induced in the secondary windings, the reading of an electrostatic voltmeter for drop in the feeder, but their patent also provides for the measurement of current by joining the secondary directly to an electrostatic instrument; both arrangements are shown in Fig. 1 at (*a*) and (*b*). It will be appreciated that the secondary is virtually on open-circuit and to obtain secondary voltages proportional to the current the iron core is provided with an air-gap. Ayrton and Mather also suggested the totalizing of the currents in several feeders by passing them through a common toroid, and they further described the use of a split or hinged core to clip over cables or bus-bars, thus anticipating many recent inventions.

The first description of a current transformer with a closed secondary

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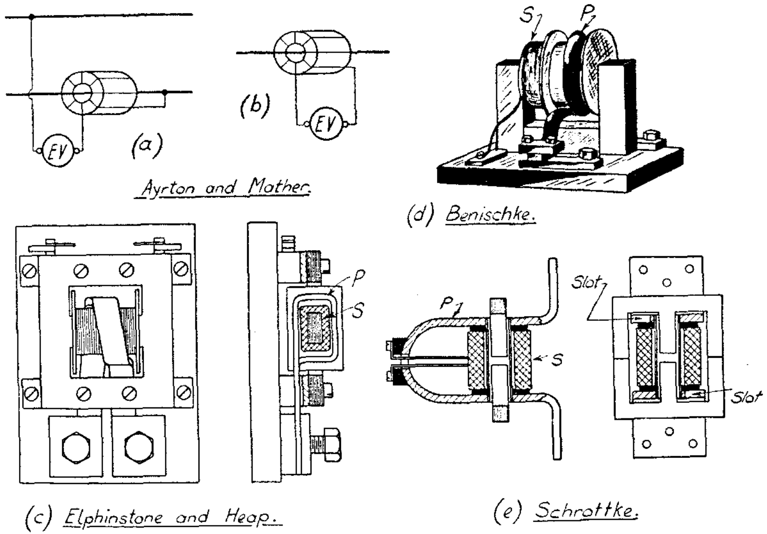
* The author is much indebted to Professor T. Mather, F.R.S., and to Professor Dr. George Keinath for many valuable suggestions.

† S. Z. de Ferranti and A. Thompson, *Brit. Pat.* No. 4 596, Sept. 27th (1882). See also Konrad Norden, "Elektrolytische Zähler," p. 145 (1908), for a brief description. Also correspondence in *E.u.M.*, vol. 51, pp. 319–320 (1933), between Professor Benischke and the author.

‡ R. Dick and Rankin Kennedy, *Brit. Pat.* No. 4 027, Mar. 17th (1887).

§ W. E. Ayrton and T. Mather, *Brit. Pat.* No. 24 217, Dec. 16th (1893).

circuit is due to Albert Campbell* in 1896. Using an air-cored transformer with a Kelvin balance in the secondary circuit, Campbell showed that the primary and secondary currents are proportional to one another and independent of frequency if the secondary circuit is sufficiently inductive, and he further showed this to be true when the transformer had an iron core. Though Campbell's work referred only to laboratory measurements there is little doubt that it had a considerable influence on the adoption of current transformers in practice and gave engineers faith in their performance. By 1898 Elphinstone and Heap† had produced a shell-type current transformer with a closed iron core of quite modern design, as Fig. 1 (*c*) shows.



(*a*) (*b*) Ayrton and Mather. (*d*) Benischke. (*c*) Elphinstone and Heap. (*e*) Schrottke.

FIG. 1. VARIOUS EARLY TYPES OF CURRENT TRANSFORMERS

Considerable progress was made in Germany about this time. Benischke‡ in 1898 described various types of current and voltage transformers made by the A.E.G., one of the former being shown in Fig. 1 (*d*); while the products of Siemens & Halske were described by Schrottke § in 1901, though there is no doubt that this firm had also used instrument transformers some four or five years earlier than this date. It is interesting to note that some of the early German current

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* A. Campbell, "On the measurement of very large and very small alternating currents," *Phil. Mag.*, 5th series, vol. 42, pp. 271–277 (1896).

† G. K. B. Elphinstone and A. C. Heap, *Brit. Pat.* No. 18 272, Aug. 25th (1898).

‡ G. Benischke, "Neue Wechselstrom-Messinstrumente und Bogenlampen der Allgemeinen Elektricitäts Gesellschaft," *Elekt. Zeits.*, vol. 20, pp. 82–89 (1899).

§ F. Schrottke, "Ueber Drehfeldmessgeräte," *Elekt. Zeits.*, vol. 22, pp. 657–667 (1901).

transformers were provided with an air-gap in the core, a typical example being shown in Fig. 1 (*e*).

From about 1900 instrument transformers became increasingly important in practice* and an improvement in their accuracy was necessary. In the case of voltage transformers no serious problems were presented, but it was not until 1904 that the theory of the current transformer and its rational design were understood. The fundamental principles were explained in papers by Wilder and by Curtis in America, by Görner in Switzerland and Germany and by Drysdale in England in the interval 1904–08.† In 1909 Keinath‡ published a dissertation which greatly stimulated development in the Continent, as Drysdale's work had done in Great Britain. Thereafter progress was continuous and a discussion of it is the subject-proper of this book.

2. The advantages and properties of instrument transformers. Instrument transformers are designed to fulfil one of two purposes, namely, the operation of measuring instruments or of protective devices. They are of two distinct types, according as they are intended to deal with the main current or with the supply voltage. The *current* or *series transformer*, the circuit of which is shown diagrammatically in Fig. 2 (*a*), serves to transform a given primary current I_p into a relatively smaller current I_s which operates an ammeter or other current-measuring device A. The *voltage, potential*, or *shunt transformer*, Fig. 2 (*b*) is used to convert the voltage V_p between the mains into a relatively lower voltage V_s which is applied to a voltmeter or other voltage-measuring device V. The use of current and voltage transformers with a wattmeter or a watt-hour meter is shown in Fig. 2 (*c*).

Instrument transformers are used in all cases where heavy currents or high voltages are concerned, in order to reduce them to values more convenient for measurement. By the use of transformers the instruments and protective devices carry only a small current or are worked at a low voltage; hence it is unnecessary to bring heavy cables or bus-bars to the switchboard and high-voltage conductors are eliminated therefrom, with a consequent cheapening of the wiring and enhanced safety for the operator. As a further safeguard it is usual to earth the core and one secondary terminal of each transformer. The insulation between the primary and secondary windings of the transformers serves to isolate all the measuring instruments from the high voltage of the primary circuit, and must be designed with great care. Since a current transformer carries the main current in its primary winding it will necessarily be subjected to the short-circuit currents which may flow in the primary network under abnormal conditions. Hence it must be designed so that its windings will not be displaced by the severe mechanical forces acting on them when abnormal currents flow, and the transformer must not be damaged by the excessive heat generated by these currents. It is clear, therefore, that instrument transformers must be reliable links between the main and the measuring circuits.

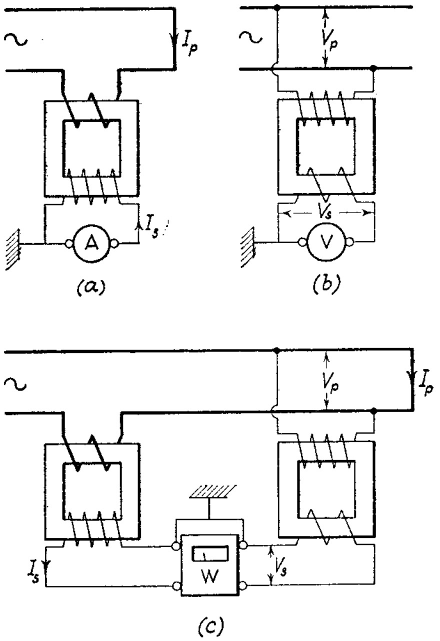

FIG. 2. FUNDAMENTAL CIRCUITS OF CURRENT AND VOLTAGE TRANSFORMERS

Instrument transformers are also advantageous in another

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* See F. Punga, "Measuring instrument transformers," *Elecn.*, vol. 51, pp. 1008–1001 (1903). K. Edgcumbe and F. Punga, "Direct reading instruments for switchboard use," *Journal I.E.E.*, vol. 33, pp. 620–669, 671–693 (1904).

† E. L. Wilder, "Operation of the series transformer," *Elect. Club J.*, vol. 1, pp. 451–455 (1904). J. Görner, "Stromwandler," *Schw. Elekt. Zeits.*, vol. 3, pp. 434–435, 444–445, 455–457, 474–475, 489–490, 504–506 (1906); "Vortrag über Stromwandler für Messgeräte," *Elekt. Zeits.*, vol. 27, 208–209 (1906). K. L. Curtis, "The current transformer," *Trans. Amer. I.E.E.*, vol. 25, pp. 715–734 (1907). C. V. Drysdale, "The use of shunts and transformers with alternate current measuring instruments," *Phil. Mag.*, 6th series, vol. 16, pp. 136–153 (1908).

‡ G. Keinath, "Untersuchung an Messtransformatoren," Dissertation. Technische Hochschule in München (1909).

direction, since their use makes it possible to standardize the measuring instruments by the adoption of fixed secondary ratings. It is modern practice to use a rated full-load secondary current of 5 amperes in current transformers, and from 100 to 150 volts at the secondary terminals of voltage transformers. Such a measure of standardization greatly cheapens the cost of manufacture of the instruments and of the transformers themselves.

3. **The imperfections of instrument transformers.** In an ideal instrument transformer the secondary quantity would be a copy of the primary quantity to a reduced scale and exactly opposite to it in phase, independently of all changes in the operating conditions. In such a transformer the primary and secondary ampere-turns would be equal and opposite in their effect on the core [see Section 8], the ratio of primary current/secondary current equal to the ratio of secondary turns/primary turns, while the value of primary voltage/secondary voltage would equal that of primary turns/secondary turns.

Practical transformers do not fulfil these ideal conditions for a number of reasons, prominent among which are the following—

(i) Some portion of the primary ampere-turns is utilized to set up in the core of the transformer the magnetic flux requisite to induce the secondary voltage:

(ii) The magnetization characteristic of the core material is not linear:

(iii) The primary current contains a component accounting for the loss of energy by hysteresis and eddy currents in the iron core:

(iv) Since there is inevitable magnetic leakage the magnetic flux in the core does not link both the primary and secondary windings equally.

In consequence of these and other reasons the secondary current in a current transformer is not equal to primary turns/secondary turns times the primary current; similarly, in a voltage transformer the secondary voltage is not equal to secondary turns/primary turns times the primary voltage. Not only are the magnitudes of the secondary quantities somewhat in error but they are not exactly in opposition of phase to the primary quantities; moreover, on account of the characteristics of the iron core, these errors in ratio and phase are not constant, but vary with the operating conditions of the transformer. It is the object of the designer to reduce these

imperfections to the minimum and to approach the ideal condition as closely as possible; this may be done by skilful design and the use of various devices discussed in Chapters III and V.

4. **Definitions. Current transformers.** A current transformer is intended to operate normally with the rated current of the network into which its primary winding is inserted and, with this current flowing, to deliver in its secondary circuit a specified rated current. Let I_{np} be this rated primary current and I_{ns} the rated secondary current; then the *nominal* or *rated current ratio* for which the transformer is designed is

$$K_{nc} = I_{np}/I_{ns},$$

sometimes also called the *marked ratio*, since it is the value inscribed upon the rating plate of the transformer. The value of I_{ns} is universally standardized at 5 amperes, except in special instances to which reference is made on p. 142. K_{nc} is expressed as an unreduced proper fraction showing the rated values of I_{np} and I_{ns}, e.g. 200/5.

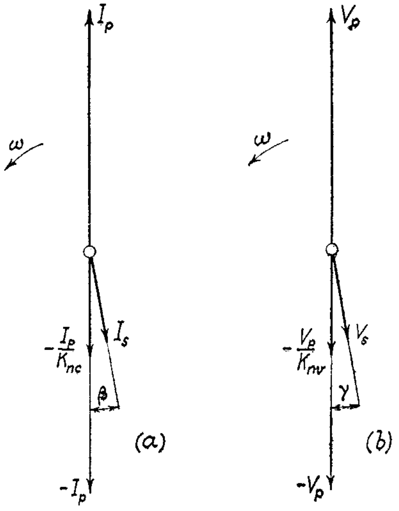

FIG. 3. RATIO AND PHASE-ANGLE DEFINITIONS

In practice, as has been indicated in Section 3, the actual ratio of a current transformer does not remain constant as the primary current is varied, and its value may be appreciably different from K_{nc}.* Let I_p be any value of primary current and I_s the corresponding value of the secondary current; then the true or actual *current ratio* will be

$$K_c = I_p/I_s:$$

the current I_s will differ from the ideal value I_p/K_{nc} in magnitude, and from exact opposition of phase to I_p by a small angle, β, called the *phase-angle*, or *phase-error* or *phase displacement* of the transformer. These fundamental relationships are illustrated in Fig. 3 (*a*), assuming sinusoidal time-variation

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* See M. G. Lloyd, "What is the ratio of a transformer?" *Elec. World*, vol. 52, pp. 77–78 (1908).

of the various currents. The factor by which the nominal ratio must be multiplied to obtain the actual ratio is called the *current ratio factor*, defined by

$$F_c = K_c/K_{nc}.$$

The purpose of a current transformer is to measure the primary current in terms of the secondary current; consequently, the instrument in the secondary circuit is scaled in nominal primary values. Let I_s be the secondary current, then the value of primary current read upon the instrument scale will be $K_{nc}I_s$; but, in consequence of the transformer's imperfections its true ratio is K_c and the actual value of the primary current is $I_p = K_cI_s$. The current read upon the scale is thus in error by the amount

$$\textit{Read current} - \textit{true current} = \textit{error} = K_{nc}I_s - K_cI_s$$

In terms of the true current the *percentage current error* is

$$\varepsilon_c = 100\,\frac{K_{nc}I_s - K_cI_s}{K_cI_s} \equiv 100\,\frac{K_{nc}I_s - I_p}{I_p} \equiv 100\,\frac{I_s - (I_p/K_{nc})}{(I_p/K_{nc})}$$
$$\equiv 100\,\frac{K_{nc} - K_c}{K_c} \equiv 100\left(\frac{K_{nc}}{K_c} - 1\right) \equiv 100\left(\frac{1}{F_c} - 1\right).$$

The *error in a current measurement is positive* when, with a given secondary current, the primary current indicated by the instrument exceeds the actual value, i.e. when the instrument reads too high; or with a given primary current the actual secondary current is in excess of its ideal value I_p/K_{nc}; or again, when the nominal ratio exceeds the actual ratio. This error, which is due to incorrectness and variation of ratio is frequently known as the *percentage ratio error* or more concisely as the *current error* or *ratio-error*; and as defined above is recommended by the International Electrotechnical Commission for general adoption.[*] Remembering that ε_c is usually small, the *current ratio factor* can be written.

$$F_c = \frac{1}{1 + (\varepsilon_c/100)} \doteqdot 1 - \frac{\varepsilon_c}{100}.$$

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[*] Definitions equivalent to this are contained in the Standard Specifications of France, Germany, Great Britain, Italy, Japan, and Switzerland. The Specification of the United States makes exclusive use of the Ratio Factor. It may be added that there is frequent lack of clarity in the definitions.

Then since

$$\frac{\varepsilon_c}{100} = \frac{\textit{read current} - \textit{true current}}{\textit{true current}},$$

$$\textit{True current} = \frac{\textit{read current}}{1 + (\varepsilon_c/100)} = F_c \times \textit{read current}.$$

Rewriting the expression on p. 8,

$$\textit{True current} = \textit{read current} - \textit{error} \equiv \textit{read current} + \textit{correction},$$

so that,

$$\textit{Correction} = -\,\textit{error}.$$

Expressing the correction as a percentage of the reading,

$$c_c = 100(K_c - K_{nc})/K_{nc} \doteqdot -\,\varepsilon_c,$$

since K_{nc} is usually not very different from K_c. The quantity c_c, described as the *percentage current correction* or the *percentage error in the ratio of transformation*, is adopted in the Swedish Standards and is defined also in those of France. The relative advantages of the use of ε_c or of c_c are clearly argued by Hauffe,[*] who concludes in favour of the former.

The *phase-angle* β is usually very small, seldom greater than 3 degrees and usually much less. It has become a regular practice to express the angle in minutes of arc and, in deference to this common usage, this unit is frequently employed in this book. The practice has, however, nothing to commend it and the author would take this opportunity to enter an emphatic protest against its international adoption. It will be shown in Chapter VI and elsewhere that it is almost always the trigonometrical functions of β that are required; since β is a small angle these functions are obtainable at once without the use of tables from the approximate relations

$$\sin\beta \doteqdot \tan\beta \doteqdot \beta \text{ and } \cos\beta \doteqdot 1,$$

when β is expressed in radians. Hence it would be much more rational to express β in radians, or even better in centiradians, than in minutes.[†] Moreover, as will be seen in Part 3, all the methods used for measuring β give the result directly in radian

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[*] G. Hauffe, "Uebersetzungsfehler und Messfehler bei Messungen mit Wandlern," *Elekt. Zeits.*, vol. 52, pp. 900–901 (1931).

[†] See A. Campbell, "Angular unit for small phase-differences," *Journal Sci. Insts.*, vol. 3, pp. 93–94 (1925). The Standard Specifications of France, Germany, Great Britain, Switzerland and the United States contain a definite recommendation to measure β in minutes; Italy prescribes the centiradian; while Sweden specifies either unit as acceptable, as also does the I.E.C.

measure. In order to make easy conversion from one system of units to the other we note that

1 radian $= 57{\cdot}3 \times 60 = 3\,438$ minutes,
so that 1 centiradian $= 10^{-2}$ radian $= 34{\cdot}38$ minutes,
and 1 minute $= 0{\cdot}02908$ centiradian $= 0{\cdot}0002908$ radian;
also 1 degree $= 1{\cdot}745$ centiradians.

The *phase-angle is positive* when the reversed secondary current vector is *ahead* of the primary current vector, i.e. when the actual secondary current I_s leads on the ideal secondary current I_p/K_{nc} in Fig. 3 (*a*). This convention is universally adopted in all countries.

A current transformer is designed to operate with a definite impedance joined to the terminals of its secondary winding, this impedance consisting of some piece of apparatus, such as an ammeter or a watt-hour meter current coil, together with its connecting leads. This external load connected to the secondary terminals is called the *burden* of the transformer. The *rated burden* is the burden with which the transformer can be loaded, so that with the rated secondary current flowing (usually 5 amperes) the values of ε_c and β do not exceed the limits laid down as permissible by the national standard specification. The rated burden may be expressed either in *ohms*—in which case it is sometimes called the *rated secondary impedance*—or in *volt-amperes*—sometimes called the *rated secondary output*; in either case the power-factor of the rated burden must also be stated. The phase-angle of the burden is conventionally regarded as positive when the voltage at its terminals leads on the current through it, i.e. when the burden has an inductive impedance.

5. **Definitions. Voltage transformers.** The fundamental definitions for a voltage transformer are analogous to those just given for a current transformer and may be very briefly stated. A voltage transformer is intended to operate normally with the rated voltage of the network to which its primary winding is connected, and to provide at its secondary terminals a specified rated voltage. It will be realised, however, that the conditions under which a voltage transformer operates are quite different from those to which a current transformer is subjected, and from the point of view of accuracy are much less severe. The current transformer is required to maintain accuracy of ratio and smallness of phase-angle when the primary current fluctuates widely from the rated value in consequence of load variations in the primary network, i.e. when the voltage applied to the primary winding is far from constant. The voltage transformer, on the other hand, works at the line voltage which in practice is nearly constant and is, in consequence, much less subject to sources of error in ratio and phase-angle. Let V_{np} be the rated primary voltage and V_{ns} the rated secondary voltage; then the *nominal* or *rated voltage ratio* for which the transformer is designed is

$$K_{nv} = V_{np}/V_{ns},$$

sometimes also called the *marked ratio.* The value of V_{ns} is not the same in all countries but lies between the limits of 100 and 150 volts; it is unfortunate that a definite figure, such as 100 volts, has not been agreed upon for international adoption.* K_{nv} is expressed as an unreduced proper fraction showing the values of V_{np} and V_{ns}, e.g. 15 000/100.

In practice the actual ratio of a voltage transformer varies with changes in the primary voltage, the secondary load and other factors, and may be appreciably different from K_{nv}. Let V_p be any value of primary voltage and V_s the corresponding value of the secondary voltage; then the true or actual *voltage ratio* will be

$$K_v = V_p/V_s.$$

The voltage V_s will differ from the ideal value V_p/K_{nv} in magnitude and from exact phase-opposition to V_p by a small angle, γ, called the *phase-angle, phase-error* or *phase displacement* of the transformer. These relationships are shown for sinusoidal quantities in Fig. 3 (*b*). The *voltage ratio factor* is

$$F_v = K_v/K_{nv};$$

and the percentage error in a voltage measurement, the *percentage voltage error* or *ratio error* is

$$\varepsilon_v = 100\,\frac{K_{nv}V_s - K_vV_s}{K_vV_s} \equiv 100\,\frac{K_{nv}V_s - V_p}{V_p} \equiv 100\,\frac{V_s - (V_p/K_{nv})}{(V_p/K_{nv})}$$

$$\equiv 100\,\frac{K_{nv} - K_v}{K_v} \equiv 100\left(\frac{K_{nv}}{K_v} - 1\right) \equiv 100\left(\frac{1}{F_v} - 1\right)$$

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* The I.E.C. Recommendation is 100 to 125. Germany and Switzerland adopt 100; Great Britain and Sweden, 110; the United States, 115; Japan, 100 to 115; Italy, 100 to 120; France, 100 to 150.

so that

$$F_v = \frac{1}{1 + (\varepsilon_v/100)} \doteq 1 - \frac{\varepsilon_v}{100}$$

The *error in a voltage measurement is positive* when, with a given secondary voltage, the primary voltage indicated by the instrument in the secondary circuit exceeds the actual value, i.e. when the instrument reads too high; it is understood that this instrument is scaled in nominal primary values. Otherwise, error is positive when, with a given primary voltage, the actual secondary voltage is in excess of its ideal value V_p/K_{nv}; or again, when the nominal ratio exceeds the actual ratio. Further

$$\textit{True voltage} = \frac{\textit{read voltage}}{1 + (\varepsilon_v/100)} = F_v \times \textit{read voltage}.$$

The *phase-angle* γ is generally very small and, though customarily expressed in minutes of arc, would be more rationally stated in centiradians. The *phase-angle is positive* when the reversed secondary voltage vector is *ahead* of the primary voltage vector, i.e. when the actual secondary voltage V_s leads on the ideal secondary voltage V_p/K_{nv} in Fig. 3 (*b*). This convention is, unfortunately, not adhered to by all countries,[*] in spite of its recommendation by the I.E.C. for international adoption; the question is discussed in Chapter VI, p. 311.

The external load, consisting of a voltmeter or other instrument with its connecting leads, connected to the secondary terminals of a voltage transformer is its *burden* and, as before, may be specified either in *ohms* or in *volt-amperes* with a stated power-factor. The *rated burden* is that with which the transformer can be loaded at the rated secondary voltage without the values of ε_v and γ exceeding the limits laid down in the national standard specification.

6. I.E.C. recommendations for instrument transformers.

The importance of instrument transformers as accurate measuring units has long been recognized in many countries by the issue of national standard specifications to which the accuracy and performance of such transformers should conform. The progress effected in the design and construction of transformers during recent years has rendered the provisions of many of these specifications out of date and some attempt has been made in preparing their revision to correlate the various definitions and standards upon an international basis. The grounds for this action will be evident when it is remembered that (i) electrical energy is now a commodity sold across national frontiers and that a considerable measure of standardization of electrical machinery and apparatus has already been made by international action; and (ii) instrument transformers are manufactured in all countries not only for national use but also for export. For these and other reasons it is very desirable that the general principles, if not the details, of standardization should be the same throughout the world. As a step toward the unification of standard specifications the International Electrotechnical Commission has compared the various national rules and at a plenary meeting held in Oslo in July, 1930, approved a specification entitled "I.E.C. Recommendations for Instrument Transformers," Publication 44 (pp. 1–9), issued by the I.E.C. at 28 Victoria Street, Westminster, S.W.1.[*]

For CURRENT TRANSFORMERS the recommendations are made under the following headings—

1. *Phase-displacement*, 2. *Ratio error*, 3. *Correction factor*, these being equivalent to the definitions given in Section 4 for β, ε_c and $1 - \frac{\varepsilon_c}{100}$ respectively.

4. *Rated primary currents* to be recognized as standard are, in amperes,

5	—	—
10	100	1 000
15	150	1 500
20	200	2 000
30	300	3 000
40	400	4 000
50	500	5 000
75	750	—

5. *Rated secondary current* shall be 5 amperes, unless a lower power consumption in the connecting leads is desirable, when 1 ampere may be used.

6. *Rated secondary burden* is not to be less than 5 volt-amperes; the burden may have any desired larger value and may be expressed either in volt-amperes, at the rated secondary current, or in ohms.

7. *Information given on the nameplate* shall include (*a*) the class of

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[*] The notable exceptions are the United States and France. The U.S.A., however, have their Standard Specification under revision and propose to alter their convention to agree with that given here; see *Journal Amer. I.E.E.*, vol. 49, pp. 749–750 (1930). France defines γ as *negative* when the reversed secondary voltage vector *leads* on the primary voltage vector and adheres to this in the new specification of 1930.

[*] The following abstract of this publication is made by kind permission of the International Electrotechnical Commission.

the transformer (see No. 11); (*b*) The rated burden; (*c*) Optionally the value of β.

8. *Temperature rise* shall conform to the I.E.C. Rules for Power Transformers.

9. *High voltage tests* consists of (*a*) The test voltage of the primary circuit, to be applied between primary and secondary, core, frame and/or case, connected together, shall correspond with that laid down by the I.E.C. Rules. (*b*) The test voltage of the secondary circuit applied for one minute between secondary and the primary, core and frame joined together to be 2 000 volts. (*c*) In transformers with divided secondaries or primaries, the sections shall withstand 2 000 volts applied between them for one minute.

10. A current transformer shall withstand without permanent injury the opening of the secondary circuit during a period of one minute when rated current flows in the primary.

11. *Two classes are recognized*, corresponding with the permissible percentage ratio error at rated current and burden, designated Classes 0·5 and 1 respectively. *Limits of error* when tested with an inductive burden having a power-factor of 0·8 and any value lying between 25 per cent and 100 per cent of rated burden shall not exceed the following limits—

Class	Percentage of Rated Primary Current I_{np}	ε_c ± %	β	
			In Minutes	In Centi-radians
0·5	10 20 100 120	1·0 0·75 0·50 0·50	60 40 30 30	1·745 1·163 0·872 0·872
1	10 20 100 120	2·0 1·5 1·0 1·0	120 80 60 60	3·490 2·326 1·745 1·745

If the transformer has a protective resistance it shall be in place when the tests are made. These limits are plotted in Fig. 4 and will be referred to later in discussing the corresponding classification of current transformers under the various national standard rules.

For VOLTAGE TRANSFORMERS the recommendations are made under the following headings—

12. *Phase-displacement*, 13. *Ratio error*, 14. *Correction factor*, these being equivalent to the definitions given in Section 5 for γ, ε_v and $1-(\varepsilon_v/100)$ respectively.

15. *Rated secondary voltage* shall have a value between 100 and 125 volts.

16. *Rated secondary load* is not to be less than 10 volt-amperes but may have any higher value. It shall be expressed in volt-amperes at rated secondary voltage.

17. *Information to be given on the nameplate* shall include (*a*) The class of the transformer (see No. 20); (*b*) The rated load; (*c*) Optionally the value of γ.

18. *Temperature rise* shall conform to the I.E.C. Rules for Power Transformers.

19. *High voltage tests* are the same as in Item 9 above.

20. *Limits of error* when tested with a secondary load having a lagging power-factor of 0·8 and any value lying between 25 per cent and 100 per cent of the rated secondary load shall not exceed the following limits—

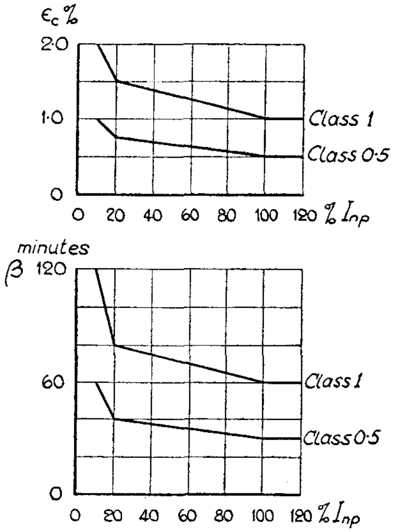

FIG. 4. I.E.C. LIMITS OF ERROR FOR CURRENT TRANSFORMERS

Class	Percentage of Rated Primary Voltage V_{np}	ε_v ± %	γ	
			In Minutes	In Centi-radians
0·5	80 to 120 inclusive	0·5	20	0·582
1	80 to 120 inclusive	1·0	40	1·163

The I.E.C. Recommendations apply to transformers suitable for use with house service meters and wattmeters for industrial purposes. It is proposed at a later date to issue a specification dealing with transformers of higher precision suitable for laboratory or other special purposes.

7. National specifications for instrument transformers. Specifications for the standardization of instrument transformers are in force in the following countries: Australia, Canada, Czechoslovakia, France, Germany, Great Britain, Italy, Japan, Poland, Sweden, Switzerland, and the United States. Austria, Belgium, Denmark, and Holland have not yet adopted any rules. In this Section we shall examine the general provisions of some of these specifications, particularly in regard to the classification of transformers and their permissible limits of error. Certain topics, such as dielectric tests, limits of temperature rise, overload limitations etc., are most usefully dealt with in detail in other parts of the book where these subjects receive full consideration and will not be given more than passing mention at this stage. The intercomparison of the various national rules is of great interest and brings out very clearly the differences of technical practice followed in different countries* and also the need for still further international unification; we shall employ the I.E.C. Recommendations as a standard of reference by means of which the national specifications may be correlated.

7a. **Great Britain, Australia, and Canada.** It is natural in a book addressed primarily to British readers to begin with a description of the standards at present in use in Great Britain. These are contained in *British Standard Specification for Instrument Transformers*, No. 81—1927, pp. 1–23, April, 1927, issued by the British Standards Institution.† An identical specification is in force in Australia under the title *Tentative Australian Standard Specification for Instrument Transformers*, No. C45—1928. T., pp. 1 29, June, 1928, published by the Australian Commonwealth Engineering Standards Association. The Canadian specification is quite different, more nearly resembling that of the U.S.A., and will be noticed at the conclusion of this section. B.S.S. No. 81 is now out of date and is by no means representative of the state of the art of instrument transformer design in the British Isles at the present time, particularly as regards limits of error. It is, however, at present under revision and its provisions will be considerably amended

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* For an earlier comparison of the principal rules, see G. Keinath, "Regeln für Messinstrumente und Messwandler in verschiedenen Ländern," *Bull. Schw. Elekt. Verein*, vol. 14, pp. 166–177, 597–598 (1923).

† The abstract given below is made by permission from *British Standard Specification No.* 81, *Instrument Transformers*, official copies of which can be obtained from the British Standards Institution, 28 Victoria Street, London, S.W.1., price 2s. 2d. post-free.

and extended to deal with the modern situation, but, at the time of going to Press, the revised specification has not yet made its appearance. (See Appendix IX.)

The Specification is comprised within twenty-five numbered paragraphs or clauses; added to these are seven appendices which are given for information only and are not mandatory. Clauses 1 to 10 inclusive are definitions substantially in agreement with certain parts of Sections 4 and 5. Clauses 11 to 15 inclusive consist of a general specification for instrument transformers and comprise the following: 11. The standard frequency shall be 50 cycles per sec. 12. Transformers for use at frequencies other than 50 shall be considered as conforming to the British Standard if they satisfy the requirements of the Specification at the frequency for which they are intended. 13. High voltage tests of the primary and secondary windings, details of which are reserved until p. 595. 14. Limits of temperature rise, dealt with on p. 591. 15. Tests at the maker's works acceptable as evidence of compliance with the requirements of the Specification shall be the high-voltage tests of Clause 13 and measurements of ratio error, in the case of current transformers at full and half the rated primary current and in voltage transformers at full and half the rated burden with full rated primary voltage. It is noteworthy that no test of temperature rise or of phase-angle is regarded as obligatory.

Current Transformers are given particular attention in Clauses 16 to 20 inclusive. The rated secondary current I_{ns} is to be 5 amperes and the following values of I_{np}, the rated primary current in amperes, are regarded as standard—

5	—	—
10	100	1 000
15	150	1 500
20	200	2 000
—	250	—
30	300	3 000
40	400	4 000
50	500	5 000
60	600	—
80	800	—

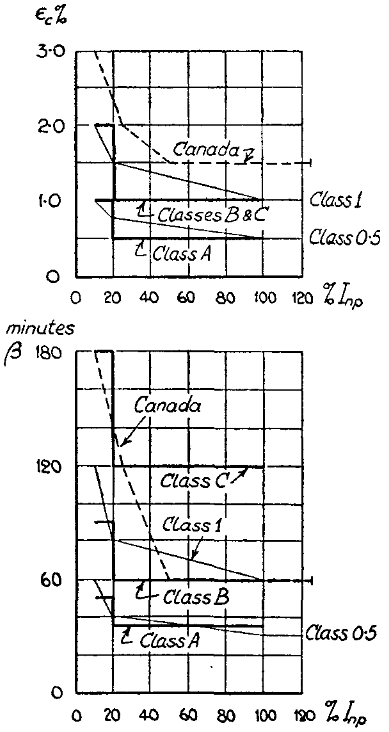

FIG. 5. LIMITS OF ERROR FOR CURRENT TRANSFORMERS: GREAT BRITAIN, AUSTRALIA, AND CANADA

These values are similar to those recommended by the I.E.C. with the substitution of 60 and 80 amperes for 75, of 600 and 800 for 750, and the addition of a 250-ampere range.

Four classes, distinguished by an arbitrary letter notation A, B, C, D, are specified, conforming to the particulars given in the following table—

Class	Type	Use	Rated Burden at 50 c.p.s. and Unity Power Factor	Percentage of Rated Primary Current	ε_c ± %	β Minutes
A	Portable Precision Type	For laboratory and precision work (Up to 11 000 volts only)	7½ VA	100 to 20 inclusive	0·5	35
				Below 20 to 10	1·0	50
B	Switchboard and Portable Industrial Type	For metering equipments where close accuracy is required (Up to 33 000 volts only)	5 and 15 VA	100 to 20 inclusive	1·0	60
				Below 20 to 10	2·0	90
C	Switchboard and Portable Industrial Type	For ordinary industrial requirements	5, 15 and 40 VA	100 to 20 inclusive	1·0	120
				Below 20 to 10	2·0	180
D	Switchboard and Portable Industrial Type	For ammeters, relays, trip-coils, etc., where phase-angle is unimportant and where a larger ratio error than in Class C is permissible	Not exceeding 40 VA	100 to 20 inclusive	5	—

These limits of error are not to be exceeded when the transformer is loaded with the tabulated rated burden and are plotted for Classes A, B and C in Fig. 5. It will be seen that Classes A and B correspond approximately with Classes I.E.C. 0·5 and 1 respectively and are, indeed, somewhat more stringent than the latter, especially at the lower values of primary current.

Every current transformer conforming to B.S.S. 81—1927 must be indelibly marked with the following particulars: (*a*) The manufacturer's name. (*b*) The manufacturer's serial number. (*c*) The value of K_{nc} expressed as an unreduced proper fraction I_{np}/I_{ns}, e.g. 1 000/5. (*d*) Rated frequency. (*e*) Rated burden in volt-amperes. (*f*) The class letter. It is noteworthy that no information is required either of the normal voltage of the circuit in which the transformer may be used or of the test voltage to which it has been subjected.

Voltage Transformers are dealt with in Clauses 21 to 25 inclusive. The rated secondary voltage V_{ns} is 110 and the following values of rated primary voltage V_{np} are standardized—

110, 440, 550, 660, 1 100, 2 200, 3 300, 5 500, 6 600, 11 000,
22 000, 33 000, 49 500, 66 000, 110 000, 132 000

Four classes, A, B, C and D, are specified corresponding in type and use with the classes of current transformers; their ratings and limits of error are tabulated below—

Class	Rated Burden at 50 Cycles per Sec. and Unity Power-factor		ε_v ± %	γ Minutes
	Single Phase	Three Phase		
	VA	VA per Phase		
A	10	—	0·5	20
B	15, 50, 100 and 200	25, 50 and 100	1·0	30
C	15, 50, 100 and 200	25, 50 and 100	1·0	60
D	No standard	No standard	5·0	—

The tabulated limits of error are not to be exceeded at rated primary voltage and with any secondary burden not greater than rated burden, at unity power-factor; they are plotted for Classes A, B and C in relation to the I.E.C. Recommendations in Fig. 6.

Every voltage transformer satisfying the requirements of B.S.S. 81—1927 must be indelibly marked with the following particulars: (*a*) The manufacturer's name. (*b*) The manufacturer's serial number. (*c*) The value of K_{nv} expressed as an unreduced proper fraction, e.g. 6 600/110. (*d*) The number of phases. (*e*) The rated burden in volt-amperes per phase. (*f*) Rated frequency. (*g*) The class letter.

The *Appendices* deal with the following topics: (i) Conditions affecting the construction of transformers for special services. (ii) Polarity and terminal marking (see p. 583). (iii) Selection of current transformers for use in large generating systems. (iv) Limitations of single-turn current transformers. (v) Cautions against opening the secondary circuit of a current transformer and using a voltage transformer at other than rated frequency. (vi) Rating of current transformers at other than 50 cycles per sec. (vii) Methods recommended for the measurement of temperature-rise (see p. 590).

The Canadian Engineering Standards Association in their publication *Standard Requirements for Alternating Current*

Watt-hour Meters, No. C17—1925, pp. 1–20, Jan., 1925, give a specification for a single class of instrument transformer, namely, one suitable for use with a.c. energy meters.

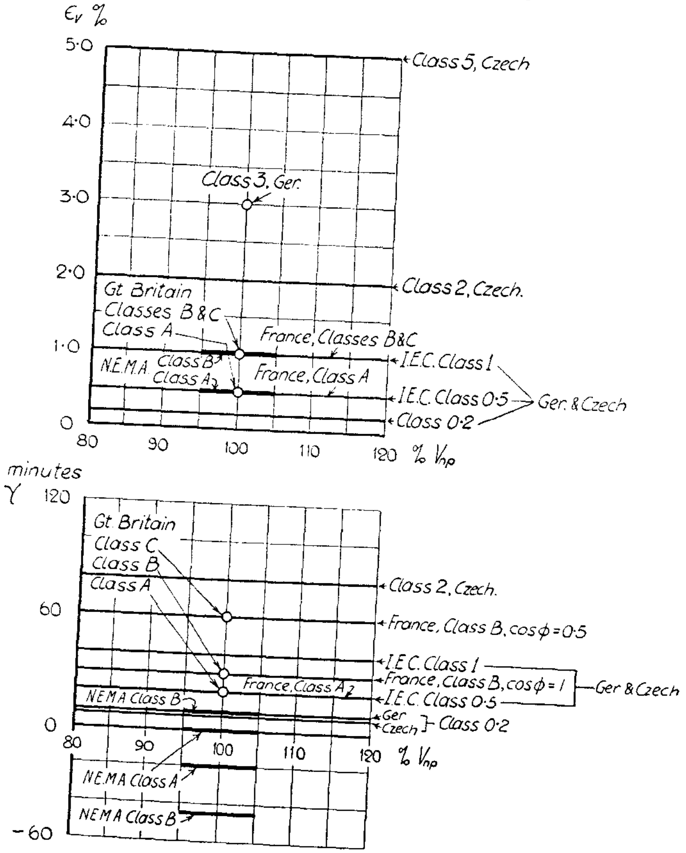

FIG. 6. LIMITS OF ERROR FOR VOLTAGE TRANSFORMERS: VARIOUS COUNTRIES

After specifying the nameplate requirements and the limit of temperature-rise for *Current Transformers* the following limits of accuracy are imposed—

Percentage of I_{np}	10	25	50	75	100	125
ϵ_c% ±	3	2	1·5	1.5	1·5	1·5
β minutes	180	120	60	60	60	60

The burden is to consist of a non-reactive resistance sufficient to consume at rated secondary current (5 amperes) the rated compensated VA, i.e. the output at which the ratio has been adjusted to be equal to the nominal value; before testing the transformer it is to be demagnetized. These figures are plotted in Fig. 5, showing the transformer to be inferior in accuracy of ratio to Classes B and C but intermediate between them for phase-angle.

For *Voltage Transformers* the rated voltages are 110 and multiples thereof. The standard rated compensated outputs, at which the ratio is equal to the nominal value, are 15, 25 and 40 VA, the burdens being non-reactive. With rated primary voltage the ratio shall not change by more than 1½ per cent when the secondary current changes from 200 per cent of the rated compensated value to zero; for any value of secondary current not exceeding the rated compensated value γ shall not be more than 60 minutes in 60 cycle transformers or 120 minutes in 25 cycle transformers. With rated compensated current in the secondary circuit, variation of the primary voltage from 90 per cent to 100 per cent of rated value shall not change the ratio by more than ± 0·5 per cent or γ by more than 60 minutes.

7*b*. **Germany.** The German standards, which are among the most complete of all national codes, are contained in *Regeln für Wandler*, No. VDE 0414. R.E.W./1932, pp. 1–26, 1st Jan., 1932, issued by the Verband Deutscher Elektrotechniker E.V. The Rules are based upon the I.E.C. Recommendations and are a logical amplification of them in several directions.

The Specification consists of five parts divided into thirty-five numbered sections or clauses, with an appendix of explanatory notes.* These parts are as follows: I. Validity (§§ 1–2). II. Definitions (§§ 3–8). III. Accuracy (§§ 9–10). IV. Standard values (§§ 11–14). V. Regulations. A. General (§§ 15–18). B. Heating (§§ 19–23). C. Insulation (§§ 24–27). D. Identification marks and rating plates (§§ 28–34). E. Tolerances (§ 35).

The Rules apply to transformers for a frequency range of 15–60 cycles per sec. Definitions of rated currents, voltages, burdens, etc., and of ϵ_c, ϵ_v, β and γ follow those given in Sections 4 and 5 of this chapter. Four additional definitions for current transformers are to be noted: *Auslösebürde*, i.e. Tripping burden of a transformer used in conjunction with tripping relays, is the value of the burden applied for a short time which with normal primary current produces an error ϵ_c of 10 per cent without regard to the value of β, the power factor being 0·6. It is chosen in preference to the term "Limiting burden," which for a current transformer has no definite meaning. *Überstromziffer*, i.e. overload factor, is the multiple of normal primary current which, with rated secondary burden, produces an error ϵ_c of 10 per cent

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* G. Keinath, "Die neuen V.D.E.-Regeln für Wandler," *Elekt. Zeits.*, vol. 52, pp. 657–659 (1931). "Ueber die Anforderungen an Stromwandler in Kraftwerken," *Elekt. Wirts.*, vol. 30, pp. 60–68 (1931). "V.D.E.-Regeln für Stromwandler," *Arch. f. tech. Mess.*, Z20-1, T80 (1931). "Stromwandler. Begriffserklärungen. Definitionen," *Arch. f. tech. Mess.*, Z20-2, T15 (1932).

(see p. 64). *Thermischer Grenzstrom*, i.e. the thermal current limit, is the primary current which can be carried for 1 second without damage to the transformer, based upon a final temperature of 200° C. *Dynamischer Grenzstrom*, i.e. dynamic current limit, is the first primary current amplitude which the transformer can carry without mechanical damage, the secondary being short-circuited. The first two definitions take account of the growing use of current transformers in protective networks; the second pair deal with the thermal and mechanical safety of transformers under abnormal conditions.

Current Transformers have a rated secondary current I_{ns} of 5 amperes as the usual value, though 1 ampere may be adopted in special cases. The rated primary currents I_{np} in amperes are

5	—	—
10	100	1 000
—	150	
20	200	
30	300	
—	400	
50	500	
—	600	
75	750	
—	800	

which correspond with the I.E.C. values with the omission of 15 and 40 amperes and the addition of 600 and 800 amperes, though the I.E.C. extend the range upwards to 5 000 amperes. Rated secondary burdens at 5 amperes are, inclusive of leads, to have the following impedances and volt-amperes—

0·2	0·6	1·2 ohm
5	15	30 VA

Five classes of current transformers are specified by figures,* Class 0·2, 0·5, 1·0, 3·0, and 10, of which Classes 0·5 and 1·0 correspond with the I.E.C. Recommendations; the figures specify the permissible ratio error at rated current. Their various purposes are as follows—

Class 0·2. For the most accurate laboratory measurements, especially of power with large phase-displacements.
Class 0·5. For laboratory measurements and also for accurate industrial measurements of power and energy.
Class 1·0. For industrial measurements of power and energy.
Class 3·0. Of high thermal and dynamical strength; suitable only for ammeters and relays.
Class 10. Bar primary type for small rated currents; suitable for overload relays and transformer-operated tripping devices.

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* This notation, in agreement with that of the I.E.C., replaces an earlier letter notation comprising Classes E, F, G, H, I. Of these, Classes E and F correspond approximately with Classes 0·5 and 1 respectively, but the others have no exact counterparts. The letters E, F, G are the initials of the German adjectives *edel*, *fein* and *gut* (literally, noble, fine, and good) as descriptions of quality.

The permissible limits of error are given in the following table and are to be satisfied in transformers of Class 0·2, 0·5 and 1 for burdens between $\frac{1}{4}$ and $\frac{1}{4}$ of the rated value and in Classes 3 and 10 from $\frac{1}{2}$ to $\frac{1}{1}$ of the rated value. The power factor of the burden is 0·8; the value of the burden with 5 ampere rated secondary current is not to be less than 0·15 ohm or with 1 ampere rated current 1·5 ohm.

Class	ε_c per Cent			β Minutes		
	1·0 I_{np}	0·2 I_{np}	0·1 I_{np}	1·0 I_{np}	0·2 I_{np}	0·1 I_{np}
0·2	0·2	0·35	0·5	10	15	20
0·5	0·5	0·75	1·0	30	40	60
1·0	1·0	1·5	2·0	60	80	120
3·0	3·0	—	—	—	—	—
10·0	10·0	—	—	—	—	—

The limits for 100 per cent of rated primary current hold good also in Class 0·2, 0·5 and 1 at 120 per cent and in Classes 3 and 10 at 50 per cent; they are plotted in Fig. 7 for the first four classes. All transformers are to be demagnetized before testing.

Voltage Transformers have a rated secondary voltage of 100 volts and rated primary voltages of

1, 3, **6**, 10, **15**, 20, **30**, 45, **60**, 80, **100**, 150, **200** and 300 kilovolts

at 50 cycles per sec.; the figures in heavy type are recommended for general use in three-phase circuits. This series corresponds with the nominal I.E.C. Standard Voltages given in Publication 38/1927.* The standard ratings in volt-amperes at normal voltages are 15, 30 and 60; in three-phase transformers the total output is the rated output. Four classes, denoted by Class 0·2, 0·5, 1·0 and 3·0, are distinguished, the first three having the same uses as the corresponding current transformers while Class 3·0 is for use with relays only. The limits of error given in the table and plotted in Fig. 6 apply to outputs between $\frac{1}{4}$ and $\frac{1}{1}$ of the rated value at normal primary voltage and with a secondary power factor of 0·8. During the tabulated voltage variation the secondary burden shall remain fixed at the impedance which absorbs rated output at rated voltage.

Class	Voltage	ε_v %	γ Minutes
0·2	0·8 to 1·2 V_{np}	0·2	10
0·5	,,	0·5	20
1·0	,,	1·0	40
3·0	V_{np}	3·0	—

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* The nominal voltage is that at the consumer's terminals. The maximum voltage is that at the generating station and is taken as 10 per cent greater than these figures.

As before, Classes 0·5 and 1 correspond with the I.E.C. Recommendations.

The German Rules for polarity and terminal marking (see p. 583), temperature rise (see p. 592), and high-voltage tests (see p. 596) will be deferred until later. The requirements for identification markings on the nameplate are more detailed than in Great Britain, a specially notable feature being the statement of both normal and test voltages. For current transformers the thermal and dynamical current limits,

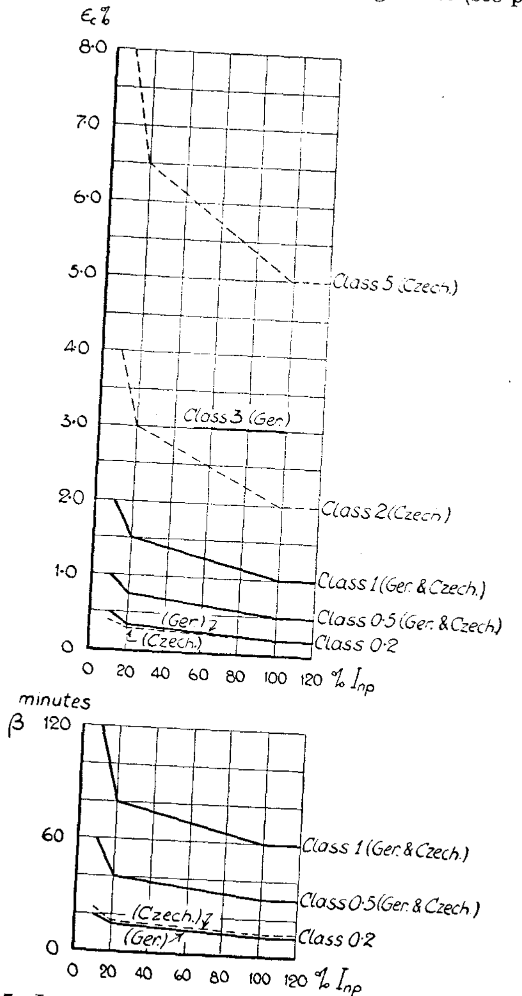

FIG. 7. LIMITS OF ERROR FOR CURRENT TRANSFORMERS: GERMANY AND CZECHOSLOVAKIA

the tripping burden and the overload figure are also required. A concise four-letter notation for transformer type designation is used, the first letter denoting the kind of insulation between the windings, the second the method of construction, the third the type of winding, and the fourth the method of mounting the finished transformer.

7c. Czechoslovakia and Poland. The Czechoslovakian standards,* which resemble the German Rules in many respects as regard thoroughness and direct development from the I.E.C. Recommendations, are contained in *Měřicí Transformátory*, No. ČSN-ESČ 64*a*, pp. 1–30, 1933 (Decimal Index 621.314.2: 621.317), published by the Elektrotechnický Svaz Československý. The Rules are in the Czech language, but are to be published later in German; they are in force both in Czechoslovakia and in Poland.

The Specification consists of eight parts, each divided into numbered clauses, and an appendix of explanatory notes. In their general provisions the Rules resemble the German, but with certain differences that will be noted here and elsewhere.

Current Transformers are to be insulated for the I.E.C. rated voltages of 1, 3, 6, 10, 15, 20, 30, 45, 60, 80, 100, 150, 200, 300 kV, except in the case of transformers for laboratory purposes. The rated primary currents follow the I.E.C. Recommendations, though 75 may be replaced by 60 and 80 amperes where convenient; the rated secondary current is 5 amperes, with 1 ampere for special cases. The rated secondary burdens are 15, 30, 60 and 90 volt-amperes, the power-factor being 0·8.

Five classes, specified as Class 0·2, 0·5, 0·1, 2 and 5 respectively are provided, having the following tabulated limits of error for burdens between 25–100 per cent of the rated value.

Class	$\varepsilon_c \pm$ per Cent			$\beta \pm$ Minutes		
	1 to 1·2I_{np}	0·2 I_{np}	0·1 I_{np}	1 to 1·2I_{np}	0·2 I_{np}	0·1 I_{np}
0·2	0·2	0·3	0·4	12	16	24
0·5	0·5	0·75	1·0	30	40	60
1·0	1·0	1·5	2·0	60	80	120
2·0	2·0	3·0	4·0	—	—	—
5·0	5·0	6·5	8·0	—	—	—

These are plotted with the German limits in Fig. 7. Classes 0·5 and 1 agree with the I.E.C. Class 0·2 has a slightly lower ratio-error and greater phase-angle than the corresponding German class. Classes 2 and 5 replace the German Classes 3 and 10, and are intended for similar purposes.

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* "Tschechoslowakische Normen für Messwandler," *Elekt. Zeits.*, vol. 47, p. 1398 (1926), gives a German abstract of an earlier edition of the Rules, now much modified.

The rated primary voltages for *Voltage Transformers* are the I.E.C. standard values given above; the rated secondary voltage is 100, but may if convenient be increased to 110. The rated burdens are 15, 30, 60, 120, 250, 500 and 1 000 VA, the power-factor being 0·8. As with current transformers, five classes are distinguished having the tabulated limits of error for all burdens between 25–100 per cent of the rated value.

Class	Voltage	ε_v ± %	γ ± Minutes
0·2	0·8 to 1·2 V_{np}	0·2	8
0·5	,,	0·5	20
1·0	,,	1·0	40
2·0	,,	2·0	80
5·0	,,	5·0	—

These limits are included in Fig. 6.

The remaining items in the Rules dealing with insulation, heating, polarity and the use of instrument transformers in power measurement will be referred to in their appropriate places later in the book.

7*d*. **Switzerland.** The requirements for instrument transformers in Switzerland are restricted to a single class, namely, transformers for use with supply meters; the necessary information is given in *Vollziehungsverordnung über die amtliche Prüfung von Elektrizitätsverbrauchsmessern*, pp. 1–23, 1st July, 1933, issued by the Eidgenössisches Amt fur Mass und Gewicht in Bern. The I.E.C. definitions of ε_c, β, ε_v and γ are adopted.

It is provided that the least rated burden of a *Current Transformer* shall be 10 VA, or in a bar-type transformer 5 VA. The limits of error are those of I.E.C. Class 0·5, a tolerance of 0·1 per cent in the ratio-error and 5 minutes in the phase-angle being permitted; protective shunts are to be in place during the test for accuracy and previous demagnetization with alternating current is essential. The transformer shall not suffer injury if the secondary circuit is opened for one minute with the full rated primary current flowing.

The *Voltage Transformer* for metering purposes has a rated burden not less than 30 VA, with limits of error corresponding with I.E.C. Class 0·5; tolerances equal to those allowed for current transformers are permitted. In three-phase transformers the total burden is the rated burden. In single-phase transformers for star-star connection, as also in five-limbed transformers, the star voltage is taken as the rated voltage.

7*e*. **Italy.** The Italian Rules[*] are contained in *Norme per i Trasformatori di Misura*, pp. 1–12, 1st Aug., 1932, compiled by a subcommittee of the Comitato Elettrotecnico Italiano and published by the Associazione Elettrotecnica Italiana. They are in general agreement with the I.E.C. Recommendations, but with certain small differences of detail.

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[*] “Schema di Norme per i Trasformatori di Misura,” *L'Elettro.*, vol. 18, pp. 884–886 (1931), gives the original draft of the Rules.

The Specification has five chapters, each containing decimally numbered sections, the main topics being: I. Definitions; II. General requirements; III. Requirements for current transformers; IV. Requirements for voltage transformers; V. Rules for voltage transformers with an earthed primary terminal. The definitions follow the regular course.

Three classes of *Current Transformers* are designated Classes P, Q and R respectively. Of these, Classes P and Q are identical with I.E.C. Classes 0·5 and 1, except that the limits of error are to be satisfied from 0 to 100 per cent of rated burden and for power-factors from 0·6 lagging to 1. Class R has the ratio error of Class Q but is unlimited as to phase-angle.

Two classes of *Voltage Transformers* are designated Classes P and Q corresponding with I.E.C. Classes 0·5 and 1, but with the range of burden and secondary power-factor adopted for current transformers.

7*f*. **France.** The French standards are contained in *Normalisation des Transformateurs de Mesure*, Publication No. 92, pp. 1–31, 4th June, 1930, issued by the Union des Syndicats de l'Electricité. These Rules were put into force, therefore, almost at the same time as the appearance of the I.E.C. Recommendations, from which they differ in a few noteworthy particulars.

The Specification contains six parts divided into thirty-eight numbered sections or clauses, the various topics being: I. Terminology and Classification (§§ 1–7); II. Normal working conditions of instrument transformers (§§ 8–13); III. Accuracy (§§ 14–16); IV. Heating (§§ 17–29); V. Dielectric strength (§§ 30–36); VI. Overload and voltage rises (§§ 37–38). We shall be concerned here with I, II and III, leaving the other portions of the Rules to be examined in later chapters.

The usual definitions of rated quantities, etc., follow the normal lines and are commendably clear. Definitions are given in addition of the percentage errors introduced by instrument transformers into a power measurement, on the basis of which the French rules adopt a convention for the positive sign of γ contrary to that used by all other countries, see Chapter VI, p. 301.

Transformers are classed according to their degree of accuracy into three groups, having the following properties—

Class A. For precise laboratory measurements.
Class B. For industrial power and energy measurements on switchboards.
Class C. For industrial current and voltage measurements.

In addition there is a group called “ordinary transformers” for which no limits of error are specified.

Current Transformers have a rated secondary current, I_{ns}, of 5 amperes. The rated primary currents, I_{np}, are certain members of a geometric progression* with a common ratio of $10^{\frac{1}{20}}$ and an initial term of 1, the values chosen being

5, 6·4, 8
10, 12·5, 16, 20, 25, 32, 40, 50, 64, 80
100, 125, 160, 200, 250, 320, 400, 500, 640, 800
1 000, 1 250, 1 600, 2 000, 2 500, 3 200, 4 000, 5 000, 6 400, 8 000, 10 000

These differ considerably from the I.E.C. Recommendations.

The standard rated burden is 15 volt-amperes, the secondary power-factor being unity. With the rated burden the limits of error must not exceed the values tabulated below and plotted in Fig. 8, from which it will be seen that Classes A and B are approximate equivalents of Classes 0·5 and 1 of the I.E.C., though rather more severely restricting the value of β.

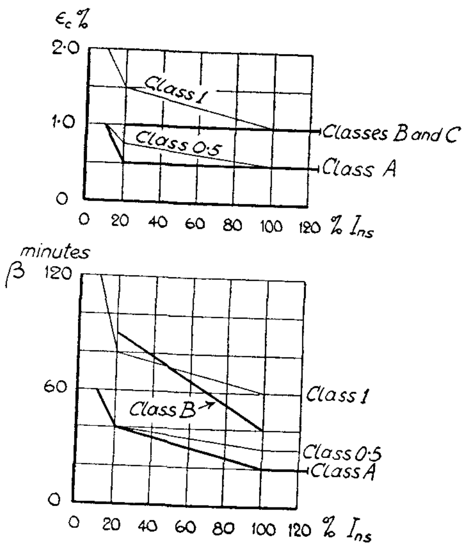

FIG. 8. LIMITS OF ERROR FOR CURRENT TRANSFORMERS: FRANCE

Class	ε_c per Cent ±				β Minutes ±			
	$1{\cdot}25\ I_{ns}$	$1{\cdot}0\ I_{ns}$	$0{\cdot}2\ I_{ns}$	$0{\cdot}1\ I_{ns}$	$1{\cdot}25\ I_{ns}$	$1{\cdot}0\ I_{ns}$	$0{\cdot}2\ I_{ns}$	$0{\cdot}1\ I_{ns}$
A	0·5	0·5	0·5	1·0	20	20	40	60
B	1·0	1·0	1·0	—	—	40	90	—
C	1·0	1·0	1·0	—	—	—	—	—

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* See Publication No. 185, *Normalisation des intensités de courant pour l'appareillage électrique*, 4th June, 1924.

The rated secondary voltage of *Voltage Transformers* may lie between the limits of 100 to 150 volts. The primary voltages are not standardized, but it is recommended that the numerical value of K_{nv} should be 10, 15, 25, 30, 40, 50, 60, 80, 100, or these multiplied by an integral power of 10. The rated secondary burden is 30 volt-amperes per phase. Limits of error are—

Class	Voltage	ε_v % ±	γ Minutes ±
A	$0{\cdot}8$ to $1{\cdot}2\ V_{ns}$	0·5	20
B	,,	1·0	30, 60
C	,,	1·0	—

For Class A and for the ratio tests in Classes B and C the limits apply for any secondary power-factor between 0·5 and 1. The lower limit for γ in Class B applies to a power-factor of 1 and the higher limit to one of 0·5. The figures are plotted in Fig. 6, showing that the limits of ε_v and γ for Class A are the same as Class 0·5. The limit of ε_v for Class B is that of Class 1 but the value of γ is different, being above or below the I.E.C. Recommendation according to the secondary power-factor.

In addition to the ratio and phase-angle limits of error just described, both current and voltage transformers must also satisfy other conditions limiting the error they introduce in power measurements; these will be discussed in Chapter VI, p. 312.

7*g*. **Sweden.** The Swedish Rules are given in *Normer för Mättransformatorer*, Svenska Elektrotekniska Normer SEN 9, pp. 1–33, 1932 published by the Svenska Elektrotekniska Kommittén. This consists of seven sections, divided into 48 sub-sections, as follows: A. General definitions and requirements; B. Current transformers; C. Voltage transformers; D. Tolerances; E. Ratings and name-plate particulars; F. Appendix explaining the method of defining the errors.

The definitions and general requirements are similar to those usually accepted, though there are certain radical differences* from those usual in other countries in the specification of dielectric strength and of temperature rise. It is also to be noted that the ratio correction, $c = -\,\varepsilon$, is defined instead of the ratio error; see p. 9.

Rated primary currents for *Current transformers* follow the I.E.C. values with the addition of 60, 250, 600, 6 000 and 7 500 amperes; rated secondary current is 5 amperes, or 1 ampere in special cases. Four classes are specified as follows—

Class kWh: For energy metering at consumer's terminals.

Class W: For current, power and energy measurement on switchboards.

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* For a discussion of these differences from the German standpoint, *see* G. Keinath, "Schwedische Regeln für Stromwandler," *Arch. f. tech. Mess.*, Z20–3 (Aug. 1932).

Class A3: For operation of overload relays in normal circumstances.

Class A25: For operation of overload relays under abnormal conditions.

The secondary burden in Classes kWh and W is to have a power-factor of 0·8 and in Classes A3 and A25 of 0·5; with a rated secondary current of 5 amperes the minimum inductive impedance is to be 0·2 ohm, i.e. 5 volt-amperes. The limits of error for Classes kWh and W are specified with respect to the total effect of ratio error and phase-angle upon power or energy measurements, and are displayed in an ingenious "control diagram" which will be explained in Chapter VI, p. 314. For Class A3 the ratio error is not to exceed ± 3 per cent for burdens between 25 and 100 per cent of rated burden and secondary currents from 80 to 200 per cent of I_{ns}. For Class A25 the error is ≯ ± 25 per cent with burdens from 25 to 100 per cent of rated value and currents from 80 to 160 per cent of I_{ns}. The ratio and phase-angle errors in Classes kWh and W are to be found by Schering and Alberti's method or some other method of comparable accuracy; the ratio errors in Classes A3 and A25 are satisfactorily found by an ammeter method. In all cases previous demagnetization is essential.

The standard rated voltages for single-phase or three-phase *Voltage transformers* are 0·55, 1·65, 3·3, 6·6, 11, 22, 33, 44, 55, 66, 77, 88, 110, 132, 154, 176 and 220 kV; for single-phase transformers intended for three-phase star-connection the rated voltages are these values divided by $\sqrt{3}$. Of these Nos. 3, 4, 5, 6, 7, 10, 12, 13, and 17 are the I.E.C. rated generator voltages (*see* German Rules on p. 23). Rated secondary voltage is 110, or $110/\sqrt{3}$ for single-phase transformers when connected in a star-star bank. Two classes, Classes kWh and W, are specified, exactly as for current transformers, and their total error in power measurement again set out in a "control diagram" which is given in Chapter VI on p. 313.

7h. **United States of America.** The standards for instrument transformers in the U.S.A. are contained in three publications, sponsored by different authorities, approved as American Standard by the American Engineering Standards Committee (A.E.S.C.). The first of these is *Instrument Transformers*, being No. 14 of the American Institute of Electrical Engineers (A.I.E.E.) Standards, pp. 1–8, March, 1925, reprinted Nov., 1926, and adopted by the A.E.S.C. on 15th Dec., 1925, by whom it is given the notation A.E.S.C. No. C22—1925. This publication confines itself to general definitions, statement of service conditions, heating, dielectric tests, short-circuit limitations, polarity and such matters, which form the basis for the preparation of contracts; questions of classification, rating and limits of error are not included. This pamphlet is under revision,* when its main deviation from standard usage—the converse of the usual positive sign convention for γ—will be corrected and the whole publication brought into line with modern practice.

The second publication is *Code for Electricity Meters*, pp. i–viii, 1–122, being A.E.S.C. Standard No. C12—1928. It is sponsored by the Association of Edison Illuminating Cos., the National Electric Light Association (N.E.L.A.), and the Bureau of Standards, the 3rd edition being approved by the A.E.S.C. on 20th Feb., 1928, and published by the N.E.L.A. On pp. 48–55 and 73–81 the Code deals with instrument transformers for use with consumers' watt-hour meters, adopting the general requirements of A.I.E.E. No. 14—1925.

After initial demagnetization *current transformers for use with meters* are tested, both at 60 cycles per sec. and at the lowest frequency of the range for which they are designed, under two conditions of secondary load: (*a*) with a non-inductive burden of 0·15 ohm, i.e. at 5 amperes a burden of 3·75 VA; (*b*) with a non-inductive burden absorbing rated volt-amperes at rated secondary current. The limits of error are not to exceed—

Percentage of I_{np} . .	10	20	40	60	100
ε_c % ±	2	1·5	1·5	1·5	1·5
β minutes . . .	150	90	60	45	45

These figures are plotted in Fig. 9 from which, and from Fig. 5, it will be seen that transformers conforming to this specification are inferior in accuracy to I.E.C. Class 1 or Great Britain Class B, both of which are intended for industrial energy metering. Supply meters not infrequently sustain overload and even short-circuit currents and it is essential that the accuracy of the metering equipment shall not be impaired thereby. In the latter case the current transformers may become permanently magnetized (see p. 126) and it is necessary to ensure that their ratio and phase-angle shall not be seriously altered. As a test of this, after the normal test given above, a direct current of full rated value is to be passed through the primary or secondary, according to convenience, and the values of ε_c and β again determined with burden (*a*). The maximum *deviations* from the previous values shall not exceed—

Percentage of I_{np}	10	100
Deviation in ε_c % ±	1·0	0·5
Deviation in β, minutes ±	60	30

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* See *Journal Amer. I.E.E*, vol. 49, pp. 749–750 (1930).

Voltage transformers for use with watt-hour meters are subjected to two tests at the above frequencies. The first is a load test at constant rated voltage, the limits of error being—

Percentage of rated burden	0	100	100
Secondary power-factor	—	1	0·1
ε_v % ±	1·0	2·0	2·0
γ minutes	30	60	90

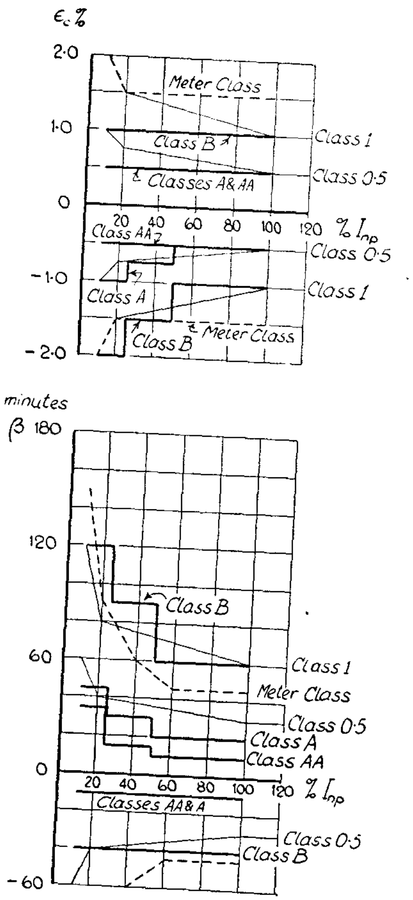

FIG. 9. LIMITS OF ERROR FOR CURRENT TRANSFORMERS: UNITED STATES

The second test is at no-load with variable primary voltage, when the maximum *deviations* from the previous no-load test at rated voltage shall not exceed—

Percentage of V_{np}	90	110
Deviation ε_v % ±	0·5	0·5
Deviation in γ, minutes	30	30

The third publication is the *National Electrical Manufacturers Association Handbook of Apparatus Standards*, May, 1928, on pp. 309–312 of which particulars are given of standards to be conformed to by transformers intended to operate the usual types of *indicating* voltmeters, ammeters, wattmeters, etc., and also tripping devices. Amendments are given in N.E.M.A. Standards Bulletin 126 of 18th June and 19th Aug., 1929, quotations from which are given by kind permission of the N.E.M.A.*

Four Classes of *Current Transformers* are specified, denoted as Classes AA, A, B. and C, with the following limits of error—

Class	ε_c per Cent			β Minutes		
	10–25% I_{np}	25–50% I_{np}	50–100% I_{np}	10–25% I_{np}	25–50% I_{np}	50–100% I_{np}
AA	− 0·5 to + 0·5	− 0·5 to + 0·5	− 0·5 to + 0·5	+ 35 to − 10	+ 15 to − 10	+ 10 to − 10
A	− 1·0 to + 0·5	− 0·75 to + 0·5	− 0·5 to + 0·5	+ 45 to − 10	+ 30 to − 10	+ 20 to − 10
B	− 2·0 to + 1·0	− 1·5 to + 1·0	− 1·0 to + 1·0	+ 120 to − 40	+ 90 to − 40	+ 60 to − 40

These are plotted in Fig. 9. The fourth grade, Class C, includes relay transformers with values of ε_c and β outside the limits of Class B. This is the only national specification in which the positive and negative limits of error are unequal. Tests of accuracy are to be made at 60 cycles per sec. with standard burdens, X, Y, or Z, having the following particulars—

Burden X: 2·5 VA, cos ϕ = 0·9; 0·09 ohm and 0·117 millihenry.
Burden Y: 15 VA, cos ϕ = 0·9; 0·54 ohm and 0·69 millihenry.
Burden Z: 50 VA, cos ϕ = 0·5; 1·0 ohm and 4·6 millihenrys.

Standard rated secondary current is 5 amperes; primary currents are not standardized. A transformer is graded as Class AY, for example, if it gives an accuracy within the limits of Class A when tested with burden Y, and so on.

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* J. Gibbs, "The Accuracy of Current Transformers," *Elect. J.*, vol. 27, pp. 204–206, 231 (1930).

Neither the primary nor the secondary voltage of a *voltage transformer* is specified, but it is recommended that the nominal voltage ratios should be 2, 4, 5, 20, 40, 60, 100, 120, 200, 300, 400, 600, 800, 1 000, 1 200, 1 400 and 2 000 to 1. The standard outputs are 25, 50* and 200* VA for transformers working at 6 900 volts and below, and 200* VA for those working above 6 900 volts; the figures marked with an asterisk are preferred ratings. The limits of error for the three classes are—

Class . . .	A	B	C
ε_v % . . .	− 0·5 to + 0·5	− 1·0 to + 1·0	Greater than B limits
γ, minutes . .	0 to − 20	+ 10 to − 45	

These limits apply from 95 to 105 per cent of V_{np}; they are plotted in Fig. 6. Tests are to be made with the following standard burdens—

Burden W . . .	13 VA	with power factor of		0·1
Burden X . . .	25 VA	,,	,,	0·7
Burden Y . . .	75 VA	,,	,,	0·85
Burden Z . . .	200 VA	,,	,,	0·85

8. **Some fundamental principles in transformer theory.** Before we can determine how closely the performance of instrument transformers can be expected to conform to standard specifications such as have been described in Section 7, it is necessary to investigate the theory upon which the design of the transformers is based; this is done in Chapter II for current transformers and in Chapter IV for voltage transformers. The general theory of the transformer is worked out in numerous textbooks and is assumed to be known to the reader. There are, however, certain conventions and fundamental principles that require definite statement at the outset in order to unify the discussions given later in the book; it is to these basic questions that attention will be given here.

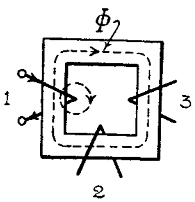
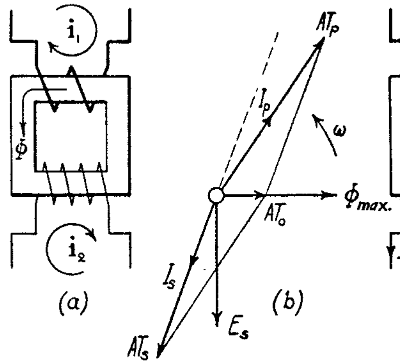


FIG. 10. ILLUSTRATING THE THEORY OF FLUX LINKAGE

Consider a number of loops, such as 1, 2, 3, wound upon an iron core, as in Fig. 10, and let loop 1 carry a varying current. A magnetic flux will be produced through the loop. A small portion of this flux will complete its path in air, but by far the greater proportion of the flux will be confined within the iron core and will link through the loops 2 and 3 in the same way as through loop 1. Consequently, if Φ is this varying core flux, equal electromotive forces of amount $-d\Phi/dt$, will be induced in all the loops. In a transformer, loop 1 would be a constituent of the primary winding, while 2, 3, etc., might be elements of a secondary winding, in the closed circuit of which the induced e.m.f. can circulate the secondary current. It is important for our purpose in this book to establish a convention by which the positive direction to be ascribed to the current in a loop may be specified. We shall take as the positive direction round any loop that sense in which current must flow to cause a flux in the core having the same direction as Φ; in other words, if the self-flux of a secondary loop and the mutual flux entering it from the primary loop are in the same direction round the core, the current producing the self-flux is regarded as flowing in the positive sense.



(a) (b) (c)

FIG. 11. POSITIVE AND INSTANTANEOUS CURRENT DIRECTIONS

In the transformer in Fig. 11 (*a*) if the positive direction of i_1 in the primary mesh is taken as clockwise, the positive direction of the core flux is anti-clockwise when the primary coil is a right-handed spiral. Then, if the secondary coil is also right-handed, the convention just laid down will make the positive direction of i_2 in the secondary mesh clockwise also. If the secondary coil is a left-handed winding, then the positive sense of i_2 will be anti-clockwise in the mesh in order that the current round the loops shall remain unchanged in direction.

The greatest care is necessary in transformer theory to avoid confusion between the *positive* directions of the primary and secondary currents and their *instantaneous* directions round their respective circuits. Fig. 11 (*b*) shows the essential portions of the transformer vector diagram, assuming sinusoidal time variations. A definite core flux of amplitude Φ_{max} requires for its production a definite number of ampere-turns AT_0. This flux will induce in the secondary winding an e.m.f. E_s, lagging by a quarter-period on the flux. This e.m.f. will circulate in the inductive secondary circuit a lagging current I_s which, flowing in the secondary turns, superposes on the core a certain number of ampere-turns AT_s. In order to retain AT_0, current must flow in the primary of a sufficient magnitude and in such a phase as to produce AT_p ampere-turns. It will be observed that AT_p and AT_s, with their corresponding currents I_p and I_s, are instantaneously approximately in opposition so that, as it were, one momentarily magnetizes while the other demagnetizes the core, leaving a resultant AT_0 sufficient to produce Φ_{max}. Consequently, except for a short interval just after the secondary current becomes zero, the primary and secondary currents cannot coexist in their positive directions since the one is very nearly opposite to the other.

In many circuit diagrams, such as those displaying the relative polarities of the primary and secondary terminals or those illustrating methods of transformer testing, it is often more useful to indicate the instantaneous directions of the currents than their positive directions. The example in Fig. 11 (*c*) is the "instantaneous diagram" corresponding with the "positive-sense diagram" of Fig. 11 (*a*). In general we shall adopt the convention of marking positive directions upon cyclic mesh currents, the instantaneous directions being marked on the conductors in which the particular currents are flowing.

The general theory of the transformer can be considered in several different ways, between which there are some very interesting relationships,* but only two of these are strictly applicable to the iron-cored transformer. (i) The first method expresses the theory in terms of the self- and mutual-induction coefficients of the windings, in the manner made classic by Maxwell. Provided that these coefficients remain constant the theory is completely satisfactory, but this condition is, unfortunately, only satisfied when the magnetic paths in the transformer have constant permeability. Its application is limited in practice, therefore, to the air-core transformer, since with an iron core the inductances are no longer constant but vary with the current strength. (ii) The second method may be conveniently called the "method of partial fluxes," represented by the classic theory of the transformer and the induction motor introduced by André Blondel. The primary ampere-turns acting alone are supposed to set up a certain flux, of which a certain fraction links the secondary coil. Similarly, the secondary ampere-turns independently give rise to a flux of which some portion links the primary coil. It is then assumed that the fluxes in the normal régime are obtained by superposition of the partial fluxes provided by the windings individually; this can only be true when the iron core is such that the fluxes in it are proportional to the ampere-turns impressed on it, i.e. when there is no saturation. (iii) The most widely used theory recognizes the physical fact that fluxes are the result of ampere-turns, so that we must superpose the ampere-turns upon a magnetic circuit containing iron in order to find the distribution of flux; the converse process of superposing partial fluxes is inapplicable. In the transformer, the resultant of the primary and secondary ampere-turns sets up in the core the so-called *main flux* which links commonly through both windings. In addition each winding is linked by a *leakage flux* set up in the leakage paths (mostly air spaces) by the ampere-turns of the winding in question. This theory is the one usually employed by engineers and is the only theory taking adequate account of the ferromagnetic properties of the core.

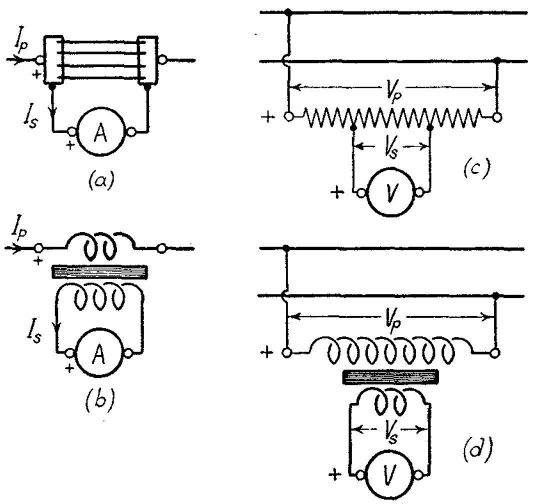


FIG. 12. EQUIVALENT CIRCUITS OF CURRENT AND VOLTAGE TRANSFORMERS

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* H. Barkhausen, "Zur Theorie des Transformators," *Elekt. Zeits.*, vol. 52, pp. 1463–1466 (1931).

(iv) Finally, there is the theory which replaces the transformer in effect by an "equivalent network" of impedances. Fig. 12 indicates the principle applied to current and voltage transformers. In either case the transformer can be represented with regard to the effects at its terminals by a four-terminal unit, which may take many forms. For the current transformer, this is shown as a shunt; for the voltage transformer as a voltage-divider. Suitable choice of the resistances and reactances of the networks can be made in such a way that the networks become electrically equivalent in magnitude and phase to the transformers. These ideas are also used in practice, often in conjunction with the principles of (iii), and will be familiar to the reader who has studied the usual theory of the power transformer; for a mathematical comparison of the various principles the reader is referred to the paper by Barkhausen cited above.

CHAPTER II

THEORY OF THE CURRENT TRANSFORMER

1. Vector diagram of the current transformer. The theory of the current transformer[*] is most easily developed from its vector diagram, assuming the currents, fluxes and electromotive forces to vary sinusoidally with time. The essential parts of the general transformer diagram are shown in Fig. 13, all unnecessary vectors being omitted.

The secondary winding is connected to a given external burden, consisting of the instrument or other device which is to be operated by the secondary current of the transformer; this burden together with the leakage impedance of the secondary

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[*] The theory of the current transformer is treated by numerous writers, notably the following: J. Görner, "Stromwandler," *Schw. Elekt. Zeits.*, vol. 3, pp. 434–435, 444–445, 455–457, 474–475, 489–490, 504–506 (1906); *Elekt. Zeits.*, vol. 27, pp. 208–209 (1906). K. L. Curtis, "The Current Transformer," *Trans. Amer. I.E.E.*, vol. 25, pp. 715–734 (1907). C. V. Drysdale, "The use of shunts and transformers with alternate current measuring instruments," *Phil. Mag.*, 6th series, vol. 16, pp. 136–153 (1908). E. S. Harrar, "The series transformer," *Elec. World*, vol. 51, pp 1044–1046 (1908). W. Genkin, "Sur transformateurs d'intensité," *Lum. Élect.*, vol. 8, pp. 67–71 (1909). A. P. Young, "The theory and design of current transformers," *Journal I.E.E.*, vol. 45, pp. 670–678 (1910). L. T. Robinson, "Electrical measurements on circuits requiring current and potential transformers," *Trans. Amer. I.E.E.*, vol. 28, pp. 1005–1039 (1910). E. Rosa and M. G. Lloyd, "The determination of the ratio of transformation and of the phase relations in transformers," *Bull. Bur. Stds.*, vol. 6, pp. 1–30 (1910). P. G. Agnew, "A study of the current transformer with particular reference to iron loss," *Bull. Bur. Stds.*, vol. 7, pp. 423–474 (1911). J. Görner, "Messtransformatoren und deren Verwendung," *Bull. Schw. Elekt. Verein.*, vol. 2, pp. 97–113 (1911). E. Wirz, "Ein Beitrag zur Theorie und Berechnung der Stromwandler," *Bull. Schw. Elekt. Verein*, vol. 4, pp. 365–392 (1913); *Elekt. Zeits.*, vol. 37, p. 147 (1916). P. G. Agnew and F. B. Silsbee, "Accuracy of the formulas for the ratio, regulation and phase-angle of transformers," *Bull. Bur. Stds.*, vol. 10, pp. 279–293 (1914). M. Rosenbaum, "The current transformer," *Elecn.*, vol. 74, pp. 626–630 (1915). A. G. L. MacNaughton, "The current transformer," *Journal I.E.E.*, vol. 53, pp. 269–271 (1915). R. S. J. Spilsbury, "Instrument transformers," *Beama J.*, vol. 6, pp. 505–513 (1920). A. Barbagelata, "Sulla prova dei trasformatori di misura," *L'Elettro.*, vol. 8, pp. 165–175 (1921). K. Takatsu, "On current transformers," *Res. Elect. Lab. Tokyo*, No. 95, pp. 1–74 (June, 1921). T. Rosskopf, "Over de eigenschappen van Meettransformatoren in het bijzonder van Stroomtransformatoren," *Ver. v. Direct. v. Elect. in Nederland*, pp. 1–26 (7th Dec., 1923). C. T. Melling, "The design of current transformers," *Journal I.E.E.*, vol. 65, pp. 283–284 (1927). J. T. Hattingh, "The current transformer, its theory, characteristics and applications," *Trans. S. Afr. I.E.E.*, vol. 19, pp. 27–44, 44–46, 103–105 (1928). A. Hobson, "Instrument Transformers," *Elec. Rev.*, vol. 110, p. 410 (1932).

winding constitutes a total secondary load of impedance Z_s and phase-angle ϕ_s. To circulate a current I_s in the secondary circuit requires the induction in the secondary winding of a voltage $E_s = Z_s I_s$, leading on I_s by an angle ϕ_s when the load is inductive,* as is usually the case. The voltage E_s is produced by the main flux Φ_{max} in the transformer core, Φ_{max} leading on E_s by $\pi/2$ radians.

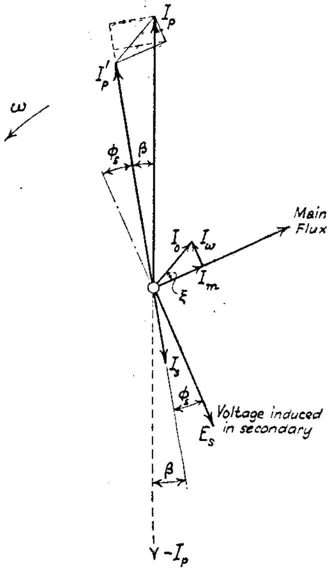

FIG. 13. VECTOR DIAGRAM OF THE CURRENT TRANSFORMER

Turning now to the primary side, the primary current must contain a component opposite in phase to I_s and of such magnitude that the secondary and primary load ampere-turns are equal. If I'_p is this component, T_p and T_s are the numbers of primary and secondary turns respectively, then

$$I'_p T_p = I_s T_s \text{ numerically,}$$

or $$I_p' = (T_s/T_p) I_s = K_T I_s,$$

where K_T is the ratio of secondary to primary turns. In addition, a certain component of primary current must be provided in order to give the ampere-turns required to produce the main-flux Φ_{max}; this is the magnetizing component I_m drawn in phase with the flux, and is necessarily wattless. The hysteresis and eddy-current waste of power in the iron core is represented by a wattful component I_w leading on I_m and Φ_{max} by $\pi/2$ radians. The total exciting current I_0 required for the magnetization of the core and the supply of its iron losses is the vector sum of I_m and I_w; it has a magnitude $I_0 = \sqrt{(I_m^2 + I_w^2)}$ and leads on the flux Φ_{max} by an angle $\xi = \arctan I_w/I_m$. Compounding I_0 with I_p' gives the total primary current I_p as shown. The ratio of the transformer is then $K_c = I_p/I_s$ and its phase-angle is β.

The primary current I_p is determined by the momentary conditions existing in the network to which the primary winding of the transformer is connected. Every variation in the primary current alters the magnitude of the p.d. impressed upon the terminals of the primary winding, necessitating a corresponding change in the primary induced e.m.f. and, with this, in the flux Φ_{max} and the two components of the exciting current, I_m and I_w.* The shape and size of the triangle composed of I_0, I_m and I_w will change, therefore, with the primary current, as will be seen later (see Fig. 17). Hence K_c and β will vary with the magnitude of the primary current, or with that of the corresponding secondary current, and cannot be constants throughout the operating range of the transformer, except in certain special circumstances.

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* ϕ_s is positive when the voltage leads on the current, i.e. with inductive impedances, and is negative when it lags, i.e. with capacitive impedances.

X

2. **Expressions for ratio and phase-angle.** Referring to Fig. 13, resolve I_p parallel and perpendicular to $I_p' = K_T I_s$, then

$$I_p \cos\beta = K_T I_s + I_m \sin\phi_s + I_w \cos\phi_s,$$
$$I_p \sin\beta = I_m \cos\phi_s - I_w \sin\phi_s.$$

Squaring and adding on both sides,

$$I^2_p = K_T^2 I_s^2 + 2\,K_T I_s (I_m \sin\phi_s + I_w \cos\phi_s) + (I_m^2 + I_w^2),$$

whence

$$K_c = \frac{I_p}{I_s} = K_T\left[1 + \frac{2}{K_T I_s}(I_m \sin\phi_s + I_w \cos\phi_s) + \frac{I_0^2}{K_T^2 I_s^2}\right]^{\frac{1}{2}} \qquad (1)$$

Now $K_T I_s = I_p' \doteqdot I_p$ since I_0 is usually a very small proportion of the primary current. Hence, since the two terms in the square bracket are small compared to unity, the square root may be taken approximately by the binomial theorem. Thus,

$$K_c \doteqdot K_T\left[1 + \frac{1}{K_T I_s}(I_m \sin\phi_s + I_w \cos\phi_s) + \tfrac{1}{2}\cdot\frac{I_0^2}{K_T^2 I_s^2}\right] \qquad (1a)$$

As a rule the third term is negligible in comparison with the second, so that a still simpler formula for the ratio is

$$K_c \doteqdot K_T + \frac{I_m \sin\phi_s + I_w \cos\phi_s}{I_s} = K_T + \frac{I_0}{I_s}\sin(\phi_s + \xi), \qquad (1b)$$

remembering that $I_m = I_0 \cos\xi$ and $I_w = I_0 \sin\xi$. It is a simple matter in any given numerical instance to determine which formula, (1), (1a) or (1b) will give adequate accuracy, as has been shown by Agnew and Silsbee in the paper cited on p. 39.

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* For a form of circle diagram illustrating all these variations, see H. G. Nolen, "Diagramme des Stromtransformators," *Elekt. Zeits.*, vol. 36, pp. 272–273 (1915).

To determine the phase-angle β, divide the second of the original equations by the first, giving

$$\tan\beta = \frac{I_m\cos\phi_s - I_w\sin\phi_s}{K_TI_s + I_m\sin\phi_s + I_w\cos\phi_s} = \frac{I_0\cos(\phi_s+\xi)}{K_TI_s + I_0\sin(\phi_s+\xi)} \quad (2)$$

It is frequently sufficiently accurate to neglect $I_0\sin(\phi_s+\xi)$ in comparison with K_TI_s, so that very nearly

$$\tan\beta \fallingdotseq \frac{I_m\cos\phi_s - I_w\sin\phi_s}{K_TI_s} \fallingdotseq \frac{I_0}{K_TI_s}\cos(\phi_s+\xi) \quad . (2a)$$

From Equation 1*b* it will be seen that if the total secondary load has a positive phase-displacement* all the terms are positive and the current-ratio K_c will be greater than the turns-ratio K_T; this is the usual condition in practice. When the phase-displacement of the load is negative† the term $I_m\sin\phi_s$ becomes negative; K_c will be greater than K_T until $I_m\sin\phi_s + I_w\cos\phi_s$ becomes zero, when K_c and K_T are equal. For greater negative angles K_c will be less than K_T. From Equations 2 or 2*a*, the angle β will be positive, i.e. the reversed secondary current will lead on the primary current, so long as $I_m\cos\phi_s$ remains greater than $I_w\sin\phi_s$; this will be the case for all except large positive values of ϕ_s.

Before examining the properties of the formulae in greater detail it is interesting to consider some simple cases. At a high power-factor in the secondary circuit we may write $\phi_s \fallingdotseq 0$; then

$$K_c \fallingdotseq K_T + (I_w/I_s) \text{ and } \tan\beta \fallingdotseq I_m/K_TI_s.$$

In this important practical case the ratio is affected principally by I_w and the phase-angle by I_m; this is illustrated graphically by the vector diagram Fig. 14 (*d*) for the case of $K_T = 1$, which shows clearly that $I_p > I_s$, i.e. $K_c > 1 > K_T$, and β leads. At a very low power-factor, as when the secondary supplies a trip-coil (positive ϕ_s) or a condenser (negative ϕ_s), we may write $\phi_s \fallingdotseq \pm\pi/2$; then

$$K_c \fallingdotseq K_T \pm (I_m/I_s) \text{ and } \tan\beta \fallingdotseq \mp I_w/K_TI_s$$

the upper sign referring to the inductive and the lower to the condensive load. In these cases the ratio is now affected principally by I_m and the phase-angle by I_w. This is illustrated

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* That is if E leads on I_s, the condition with inductive load.

† That is if E_s lags on I_s, the condition with capacitive load.

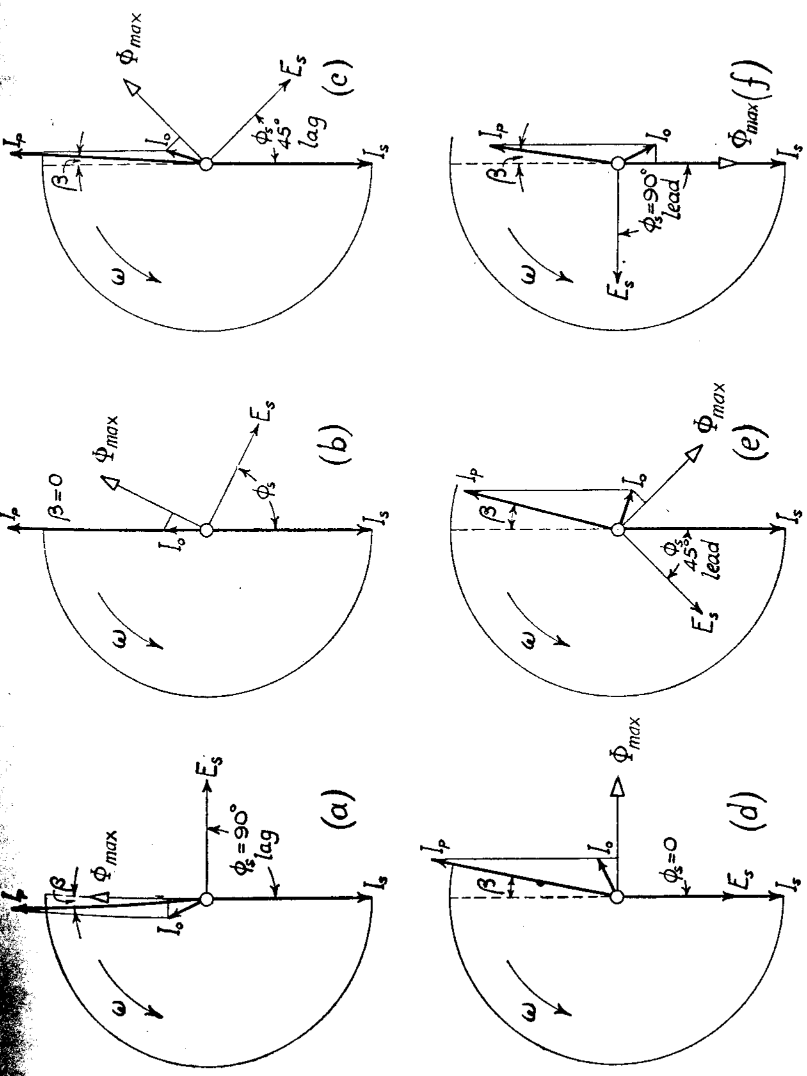

FIG. 14. VECTOR DIAGRAMS FOR A CURRENT TRANSFORMER WITH VARIOUS VALUES OF ϕ_s

[I_s, Z_s, E_s and Φ_{max} are constant. ϕ_s is varied. $K_T = 1$]

by the vector diagrams of Figs. 14 (*a*) and (*f*) respectively for $K_T = 1$. From the former $I_p > I_s$, i.e. $K_c > 1 > K_T$, and β lags; from the latter $I_p < I_s$, i.e. $K_c < 1 < K_T$, while β now leads.

In general, for all other secondary phase-displacements, the values of K_c and β are influenced by both components of I_0, as illustrated by Figs. 14 (*c*) and (*e*). It is clear that since K_c for certain values of ϕ_s exceeds and for other values falls short of K_T, I_s and Z_s being assumed constant, it may for one definite value attain a maximum; also since β is sometimes leading and sometimes lagging there must be a value of ϕ_s that will make it vanish.

3. **Conditions for maximum ratio and zero phase-angle.** Suppose I_s and Z_s to be constant; then since $E_s = Z_s I_s$ is proportional to Φ_{max}, the core-flux is constant. The exciting current I_0 and its two components I_m and I_w are, therefore, also constant. If X_s and R_s are the total reactance and resistance respectively of the secondary circuit, let $\phi_s = \arctan X_s/R_s$ be varied while $Z_s = \sqrt{(R_s^2 + X_s^2)}$ remains constant. Differentiating Equation 1 with respect to ϕ_s and equating to zero gives

$$\cos(\phi_s + \xi) = 0,$$

or

$$\phi_s = (\pi/2) - \xi;$$

i.e. the ratio will be a maximum when the secondary circuit has a positive phase-displacement differing from quadrature by the *hysteretic angle of lead*, ξ. The value of the ratio is then,

$$K_{c,max} \doteqdot K_T + (I_0/I_s)$$

Inserting the condition for maximum ratio into Equation 2, shows that when the ratio is greatest the phase-angle β is zero. This interesting case is illustrated by Fig. 14 (*b*), whence it is geometrically obvious that the transformer has its greatest ratio and zero phase-angle when, for a given value of I_s and E_s, I_0 may be arithmetically added to $K_T I_s = I'_p$; this occurs when the inductive secondary impedance has a phase-angle which is in defect of $\pi/2$ by the lead of I_0 on Φ_{max}.

4. **Conditions for maximum phase-angle.** Differentiating Equation 2 with respect to ϕ_s and equating to zero gives for the condition that β shall be a maximum, when I_0 and I_s are fixed,

$$\sin(\phi_s + \xi) = -I_0/K_T I_s$$

or

$$\phi_s = -\xi - \arcsin(I_0/K_T I_s).$$

The voltage E_s must lag on the current I_s by this angle, i.e. the secondary circuit must have an appropriate capacitive impedance. The maximum value of the phase-angle will be given by

$$\tan\beta_{max} = I_0/(K_T^2 I_s^2 - I_0^2)^{\frac{1}{2}}$$

and is a leading angle.

Referring now to Equation 1, K_c will be equal to K_T if the square bracket becomes unity. This involves either $I_0 = 0$, which is physically impracticable, or

$$\sin(\phi_s + \xi) = -I_0/2K_T I_s$$

i.e.

$$\phi_s = -\xi - \arcsin(I_0/2K_T I_s),$$

requiring a capacitive secondary load. Thus the maximum value of β occurs for a secondary phase-angle which is slightly greater than that which will cause K_c to be equal to K_T. The difference is, however, usually very small since $I_0/K_T I_s = I_0/I_p' \doteqdot I_0/I_p$, is a small quantity. Hence, with fair accuracy we can say that the ratio of a current transformer will be equal to its turns-ratio and its phase-angle will be a maximum when $\phi_s = -\xi$, i.e. when the secondary circuit is such that the current leads on the induced voltage by the hysteretic angle ξ.

5. **Numerical Example.** The properties discussed* in Sections 2, 3 and 4 are illustrated by the curves in Fig. 15, which have been calculated for a current transformer having the following particulars—

Nominal ratio, $K_{nc} = 5/5$;
Turns ratio, $K_T = 238/240 = 0{\cdot}9917_7$.
Secondary winding—
Resistance, $R_{ws} = 0{\cdot}346\ \Omega$ at 18° C.;
Leakage reactance,

$$X_{ws} = 0{\cdot}32\ \Omega \text{ at 50 cycles per sec.};$$

Total secondary impedance,

$$Z_s = 1\ \Omega,$$

Then with $I_s = 5$ A,

$$E_s = 5\text{ V}$$

for which $I_m = 34{\cdot}3 \times 10^{-3}$ A,

$$I_w = 10{\cdot}26 \times 10^{-3}\text{ A},$$
$$I_0 = 35{\cdot}8 \times 10^{-3}\text{ A},$$
$$\tan\xi = 10{\cdot}26/34{\cdot}3 = 0{\cdot}2990,$$

and $\xi = 16° 40'$.

The value of ϕ_s for maximum K_c and zero β is then $90° - 16° 40' = 73° 20'$. The maximum value of K_c is $0{\cdot}9988_3$. Further, since $I_0/K_T I_s = 35{\cdot}8 \times 10^{-3}/4{\cdot}9584 = 7{\cdot}220 \times 10^{-3}$ it follows that $\arcsin(I_0/K_T I_s) = 0{\cdot}00722$

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* For an interesting discussion of characteristic curves, see E. G. Reed, "Elementary theory of the current transformer," *Elect. J.*, vol. 20, pp. 182–186 (1923); "The phase-angle of the current transformer," *ibid.*, vol. 22, pp. 594–596 (1925); "Current transformer calculations," *ibid.*, vol. 23, pp. 67–71 (1926.)

radian or 24·8 minutes. Hence β will be a maximum when ϕ_s is $-16^\circ\ 40' - 25' = -17^\circ\ 5'$, while K_c will be equal to K_T when ϕ_s is $-16^\circ\ 40' - 12' = -16^\circ\ 52'$. The maximum value of β is very nearly 0·722 centiradian or 24·8 min.

6. The exciting current. Referring again to Equations 1 and 2 the values of K_c and β are functions of K_T, I_s, ϕ_s, I_0 and its components I_m and I_w. In practice K_T is fixed by the construction of a particular transformer, while ϕ_s and Z_s are fixed by the connection of a specified external burden to the terminals of the secondary winding, the frequency being constant. Changes in the primary current I_p result in corresponding changes in the value of the secondary current I_s and therefore in $E_s = Z_sI_s$, the voltage induced in the secondary winding by

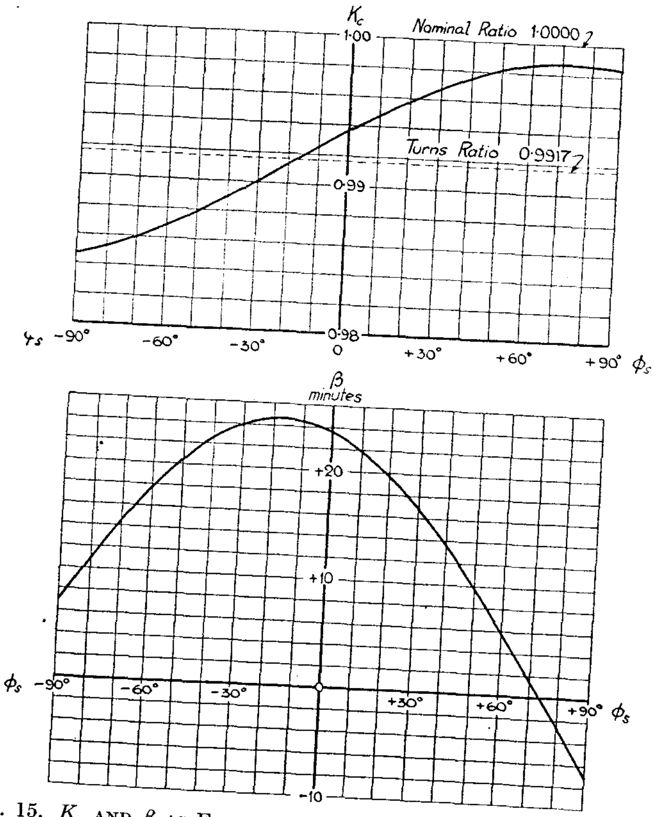

FIG. 15. K_c AND β AS FUNCTIONS OF ϕ_s; I_s CONSTANT AT 5 AMPERES

the core-flux. Thus, as the primary current is varied so also is the core-flux and with it the magnetizing and iron-loss components of the primary exciting current. These have an important influence upon the ratio and phase-angle of the transformer, as will be demonstrated in later sections of this chapter and in Chapter III.

The values of I_m and I_w are readily found from an "open-circuit" test on the transformer, full details of which are given on p. 411. The secondary circuit of the transformer is opened, measurements of current, voltage and power supplied to the primary being made for various values of the secondary voltage, the latter being measured by some electrostatic or potentiometric device. From the observations I_m and I_w are easily computed and are plotted* to a base of E_s, as in Fig. 16; further examples are given on p. 422, Fig. 209. It is interesting to observe (i) that the curve I_m = function (E_s) is the lower part of the alternating current magnetization curve for the iron core; and (ii) that the curve I_w = function (E_s) is, for the low densities encountered in current transformers over their working range, practically a straight line. These two facts have important consequences that will be adverted to later.

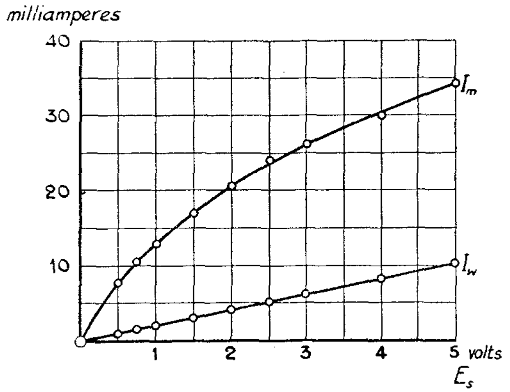

FIG. 16. VARIATION OF I_m AND I_w WITH E_s

The results of such a test can be displayed in an alternative form which brings out certain interesting conclusions. In Fig. 17 draw a line OI_m to represent the common direction of the vectors Φ_{max}, the main flux, and I_m, the magnetizing component

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* This diagram refers to tests made on the transformer described in Section 5 on p. 45.

of I_0; mark off a scale along this line suitable for the representation of I_m. Set off a similar scale on a line OI_w, perpendicular to OI_m, for the loss component I_w. For a given value of $E_s \propto \Phi_{max}$ in Fig. 16 read off I_m and I_w, setting them out as OC, CA in Fig. 17; then OA is the exciting current I_0 and the angle AOC is ξ. Repeating the construction for the entire open-circuit test gives the inflected curve OAB, which is the locus of the extremity of the vector representing I_0 as the flux changes with varying secondary currents.

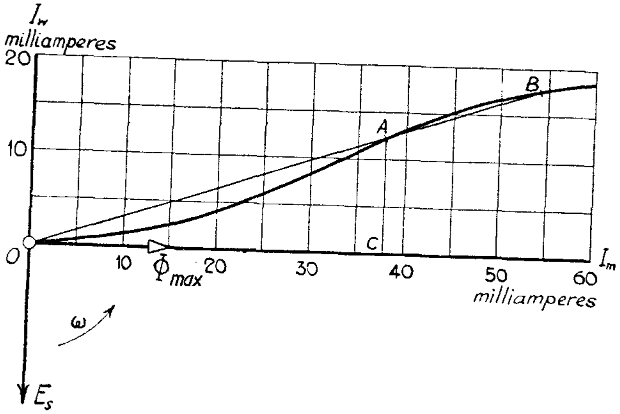

Fig. 17. The Excitation Triangle

It will be noticed that, in general, each value of ξ gives two values of I_0, such as OA and OB, corresponding with two different values of I_s. In the vector diagram of Fig. 14 (*b*) it is shown that $\beta = 0$ when $\phi_s = (\pi/2) - \xi$; this condition can be satisfied, however, by two widely different values of I_s for one of which the ratio will be much larger than for the other.

7. **Variation of K_c and β with I_s.** Since the current transformer in normal circumstances operates, at a constant frequency and with an external burden of fixed impedance, over a wide range of primary and corresponding secondary currents, it is important to see how its characteristics may be obtained under these conditions. In practice K_c and β are usually measured by one of the many methods described in Part 3, but these quantities may be readily calculated with the aid of Equations 1 and 2 if the following data has been provided: (i) the resistance and leakage reactance of the transformer secondary winding; (ii) the resistance and reactance of the external secondary burden; (iii) the turns-ratio; (iv) the variation of I_m and I_w with E_s. Knowing Z_s, the value of E_s can be found for various values of secondary current, the open-circuit curves yielding the corresponding values of I_m and I_w. Insertion of these in the equations, together with the given values of sin ϕ_s and cos ϕ_s give the desired K_c and β. Fig. 18 shows for two conditions the results for the transformer described in Section 5 on p. 45: (i) With an external burden of zero i.e. the secondary terminals short-circuited. Then $R_s = R_{ws} = 0{\cdot}346$ ohm, $X_s = X_{ws} = 0{\cdot}32$ ohm at 50 cycles per sec., $Z_s = 0{\cdot}472$ ohm, $\sin\phi_s = \sin\phi_{ws} = 0{\cdot}678$ and $\cos\phi_s = \cos\phi_{ws} = 0{\cdot}732$. (ii) With an external non-inductive burden of 0·6 ohm. Then $R_s = 0{\cdot}6 + 0{\cdot}346 = 0{\cdot}946$ ohm, $X_s = 0{\cdot}32$ ohm, $Z_s = 1$ ohm, $\sin\phi_s = 0{\cdot}32$, $\cos\phi_s = 0{\cdot}946$. The output to the external burden at full rated secondary current is $5 \times 5 \times 0{\cdot}6$ or 15 volt-amperes. A further discussion of the properties of these characteristic curves will be given in Chapter III and a critical

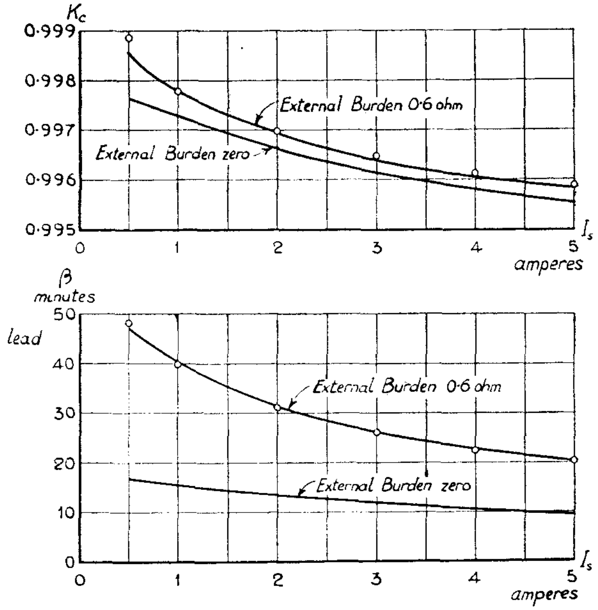

Fig. 18. Typical Characteristic Curves for a Current Transformer

[The circles are experimental values, see p. 54] treatment of the method of calculation in Chapter XVIII; they may be taken as of typical shape for a normal current transformer.

8. **The diagram of Möllinger and Gewecke.** The vector diagram of Fig. 13 may be transformed by a simple device into a very useful and instructive form. Suppose all the current and voltage vectors to be divided by I_s, taking the direction of the secondary current vector vertically downwards from O_1 in Fig. 19. Then since $E_s = Z_sI_s$ is compounded of the resistance drop R_sI_s and the reactance drop X_sI_s, this voltage triangle becomes an impedance triangle with sides Z_s, R_s, X_s which, for a given total secondary resistance and reactance, is fixed in size and shape. The vector $I'_p = K_TI_s$ becomes K_T, represented by the fixed length O_1O, while I_p becomes the line O_1A of a length which will denote to scale $K_c = I_p/I_s$. The line O_1A is compounded of $O_1O = K_T$ and the hypotenuse $O_1A_1 = I_0/I_s$ of the exciting-current triangle, the magnetizing and loss component sides of which are now I_m/I_s and I_w/I_s. Resolving K_c upon and perpendicular to K_T gives

$$K_c \cos\beta = K_T + A_1B_1 = K_T + AB$$

$$= K_T + \frac{I_m}{I_s}\sin\phi_s + \frac{I_w}{I_s}\cos\phi_s,$$

and $K_c \sin\beta = O_1B_1 = OB = \dfrac{I_m}{I_s}\cos\phi_s - \dfrac{I_w}{I_s}\sin\phi_s,$

which are equivalent to the equations on p. 41. Now in practice β rarely exceeds 2 degrees (0·03495 radian) for which extreme case $\cos\beta = 0{\cdot}9994 \doteqdot 1$ and $\sin\beta = 0{\cdot}0349_0 \doteqdot \beta$. Thus with a very high degree of precision

$$K_c \doteqdot K_T + AB \doteqdot K_T + OC, \text{ and } \beta \doteqdot OB/K_c$$

Hence OC is a measure to scale of the deviation of the actual ratio from the turns-ratio while OB represents the phase-angle of the transformer.

Suppose now that the secondary current is changed, R_s and X_s being unaltered. Then $E_s = Z_sI_s \propto \Phi_{max}$ varies in direct proportion to I_s and the new value of main flux will require new values of I_m and I_w. We have seen in Section 6 that I_w varies approximately in direct proportion to E_s; thus, I_w/I_s remains unchanged no matter what value the secondary current may have. Hence the locus of the extremity A of the line $OA = I_0/I_s$ for various values of I_s will be a straight line

inclined at an angle ϕ_s to the horizontal, parallel to the direction of the main flux vector and distant therefrom by an amount $OD = I_w/I_s$. This is the *Möllinger and Gewecke line.** The distance DA, being I_m/I_s can be readily set out for various values of secondary current with the aid of the magnetization curve, such as Fig. 16; from the points so obtained projection upon the ratio and phase-angle axes will give the ratio error and phase-angle of the transformer at any desired fraction of full rated secondary current.

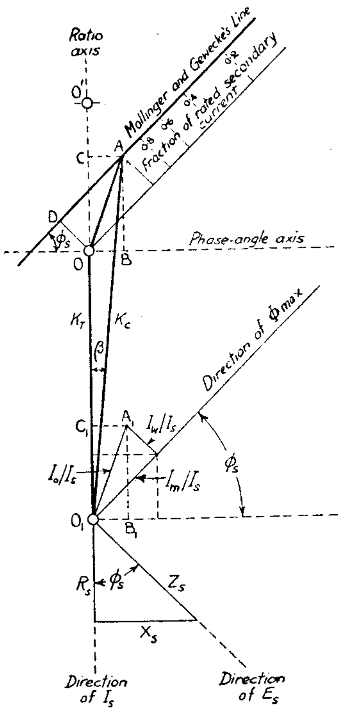

FIG. 19. MÖLLINGER AND GEWECKE'S FORM OF THE VECTOR DIAGRAM FOR A CURRENT TRANSFORMER

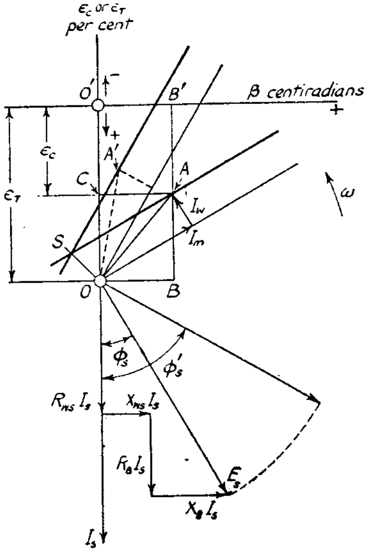

FIG. 20. CURRENT TRANSFORMER ERROR DIAGRAM

In practice it is usual to express the ratio error in terms of

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* J. Möllinger and H. Gewecke, "Zum Diagramm des Stromwandlers," *Elekt. Zeits.*, vol. 33, pp. 270–271 (1912). G. Keinath, *Arch. f. tech. Mess.*, Z21–1, T48 (1931). E. Dünner, "Messwandler, Einleitendes Referate," *Bull. Schw. Elekt. Verein*, vol. 24, pp. 85–88 (1933).

the nominal ratio K_{nc} for which the transformer is rated, the International definition for the ratio error being

$$\varepsilon_c = [(K_{nc}/K_c) - 1]100 \text{ per cent}$$

and is positive if $K_{nc} > K_c$, i.e. if the nominal ratio is greater than the actual ratio. Mark off the distance O_1O' to represent K_{nc} to the same scale as K_T; then $O'C$ is the deviation of the actual ratio from the nominal value, and from it ε_c can be calculated. It will be positive when measured downwards from O'. The distance $O'O$ is the deviation of the turns ratio K_T from the nominal ratio K_{nc}: expressed as a percentage we may write the turns error as

$$\varepsilon_T = [(K_{nc}/K_T) - 1]100 \text{ per cent.}$$

The essential parts of Möllinger and Gewecke's diagram may now be redrawn in simple form by omitting the lines for K_T and K_c, with their attendant construction lines, and moving up the impedance triangle to the origin O; this has been done in Fig. 20. Nor is it necessary to divide the vectors by I_s unless desired, the exciting current and the voltage vectors being set out to correspond with full-rated secondary current flowing in the given secondary impedance. The secondary voltage is shown in this diagram decomposed into resistance and reactance components due to the winding and the burden independently. The perpendicular distance from O to the Möllinger and Gewecke line is the value of I_w for full rated secondary current. The position of the points corresponding with any desired fraction, say $1/p$, of full current is obtained by reading from the open-circuit curve the value of I_m for the desired current and multiplying it by p. If the currents are expressed in terms of the secondary side, 50 milliamperes with a 5 ampere secondary rating corresponds with an error of 1 per cent in the ratio scale and may be represented by any convenient distance, e.g. 100 mm. A phase-angle error of 1 per cent is 1/100 of a radian, i.e. 1 centiradian, or 34·38 minutes, which is equivalent to a scale of 29·2 mm. for a phase-angle of 10 minutes. The two axes may in this way be easily graduated so that the percentage ratio error and the phase-angle in centiradians or in minutes may be read directly for any given secondary current, i.e. $O'C$ and $OB = O'B'$ respectively.

If the secondary power-factor is changed from $\cos\phi_s$ to $\cos\phi_s'$ while Z_s is unaltered, then E_s is the same as before. The

errors are now obtained by rotating the Möllinger and Gewecke line about O through the angle $\phi_s' - \phi_s$, so that A becomes A', and projecting the load points upon the axes as before. Alteration of Z_s for a given value of ϕ_s is accounted for by moving the line parallel with its original position through a distance equal to the change in I_w at full rated secondary current. In this way the diagram can be applied to show the behaviour of a given transformer under any desired conditions of operation.* The diagram is particularly useful for design purposes, since it enables the effect produced upon the ratio error and phase-angle by changes in the design to be quickly and accurately studied; indeed it can be made the basis of an entirely graphical process of design in the manner developed by Fleischhauer,† to whose work we shall refer again on p. 58.

In order to construct the diagram it is necessary to have the following data—

(i) Resistance R_B and reactance X_B of the external burden;
(ii) Resistance R_{ws} and leakage reactance X_{ws} of the secondary winding;
(iii) The open-circuit characteristics, giving I_m and I_w as functions of E_s.
(iv) The turns ratio K_T or its percentage deviation ε_T from the given nominal ratio K_{nc}.

In a projected design all these quantities may be regarded as known or readily calculable. The problem is rather more complicated when the diagram is to be constructed for an existing transformer. The resistances R_B and R_{ws}, and the reactance X_B are easily measured. The open-circuit data is obtained either by means of an a.c. potentiometer or some equivalent test-circuit (see p. 411 for examples) and presents no special difficulties. The turns-ratio is seldom precisely known and its direct measurement is somewhat tedious. By far the greatest difficulty, however, is concerned with the determination of X_{ws}. By short-circuiting the primary winding and measuring the impedance of the transformer at its secondary terminals it is possible to measure its apparent reactance $X_{ws} + K_T^2 \times X_{wp}$, where X_{wp} is the primary leakage reactance, but there is no way of separating from this the desired X_{ws} especially as K_T is not usually known. To get over the difficulty Möllinger and Gewecke assumed the secondary and reduced primary reactances to be equal, i.e. that X_{ws} is half the apparent reactance, as

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* For an interesting application to a transformer with a protective resistance, see R. Küchler, "Berechnung der Fehlergrössen von Messwandlern," *Elekt. Zeits.*, vol. 42, pp. 1418–1419 (1921).

† W. Fleischhauer, "Graphische Stromwandlerberechnung," *Wiss. Veröff. Siemens Konz.*, vol. 10, part 1, pp. 98–136 (1931); *Elekt. Zeits.*, vol. 53, pp. 691–693 (1932).

is frequently done in the case of power transformers. The leakage conditions in current transformers are, however, radically different and Schering* has shown that X_{ws} is usually much less than $K_T{}^2 X_{wp}$. Janvier† has attempted to separate the reactances by a graphical approximation which may, however, be very much in error, as Gocht‡ has shown; other processes are discussed in Chapter XVIII. On the whole, therefore, it must be admitted that the accurate determination of X_{ws} is the principal defect in applying Möllinger and Gewecke's diagram to a finished transformer.

9. The direct determination of X_{ws} and ε_T from the M. and G. diagram. The measurement of K_c (or ε_c) and β can be made with high accuracy by a great variety of convenient methods. Making use of this, the Möllinger and Gewecke diagram may be applied in an inverted sense to yield the values of X_{ws} and ε_T and to provide data from which the open-circuit curves may be constructed.

The method used by Berghahn § is illustrated by Fig. 21, which has been constructed from measurements made upon the transformer described in Section 5. The transformer has the following particulars: $K_{nc} = 5/5$, $R_{wp} = 0·454$ ohm and $R_{ws} = 0·346$ ohm at 18° C. A non-reactive burden ($R_B = 0·6$ ohm, $X_B = 0$) is put into the secondary circuit, the values of ε_c and β being measured, Schering and Alberti's method described on p. 469 being used in this example, with the following results—

I_p amperes . .	0·5	0·75	1·0	1·5	2·0	2·5	3	4	5
% Ratio error ε_c .	+ 0·118	+ 0·185	+ 0·222	+ 0·277	+ 0·302	+ 0·325	+ 0·352	+ 0·387	+ 0·41
Phase-angle β min. .	48·0	43·0	39·5	34·5	31·0	28·6	26·0	22·4	20·1

Choosing error and angle scales in accordance with p. 52 along horizontal and vertical axes through O', these results are plotted and give the lower M. and G. line SA. This line is parallel to the core flux vector; hence a perpendicular to it will be parallel to E_s, i.e. to Z_s. The total secondary resistance is $R_s = 0·6 + 0·346 = 0·946$ ohm. From any convenient point on the perpendicular, set off vertically downward to any scale a distance representing R_s; from the other extremity of this vertical draw the horizontal line to meet the perpendicular. Then the length of this horizontal to scale will be $X_s \equiv X_{ws}$, since the burden is non-reactive; in the present example the secondary reactance is $X_{ws} = 0·32$ ohm at the frequency of the test, i.e. 50 cycles per sec.

An air-cored inductance of 1·58 mH is then inserted in the secondary circuit, i.e. $X_B = 0·50$ ohm at 50 cycles per sec., making the total secondary reactance $X_s = 0·50 + 0·32 = 0·82$ ohm. The resistance of the burden is regulated to make the impedance Z_s the same as before, i.e. $0·946^2 + 0·32^2 = R_s{}^2 + 0·82^2$, where R_s is the new total resistance; hence, $R_s = 0·566$ ohm and R_B must be altered to 0·566 − 0·346 = 0·220 ohm. The values of ε_c and β are again measured as follows—

I_p amperes . .	0·5	0·75	1·0	1·5	2·0	2·5	3	4	5
% Ratio error ε_c . .	− 0·491	− 0·362	− 0·284	− 0·158	− 0·078	− 0·010	+ 0·035	+ 0·120	+ 0·190
Phase-angle β min. .	24·0	21·0	19·0	16·0	13·6	11·9	10·6	8·5	7

These results are plotted and give the second M. and G. line SA', intersecting the first in S; the bisector of the obtuse

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* H. Schering, "Zum Diagramm des Stromwandlers," *Arch. f. Elekt.*, vol. 7, pp. 47–56 (1919).

† W. Janvier, "Nouveau diagramme relatif au fonctionnement des transformateurs de courant," *Rev. Gén. de l'Él.*, vol. 24, pp. 619–623 (1928).

‡ K. Gocht, "Ein Messverfahren zur Bestimmung der sekundären Streuinduktivität, der Windungsabweichung und des Leerlaufstromes von Stromwandlern," *Elekt. Zeits.*, vol. 50, pp. 1653–1655 (1929).

§ A. Berghahn, "Ein einfaches Verfahren zur Ermittlung der Streureaktanz, der Windungsabweichung und der Leerlaufcharakteristik von Stromwandlern," *Elekt. Zeits.*, vol. 52, pp. 605–607 (1931). "Die Streureaktanzen eines Einphasentransformators," *Arch. f. Elekt.*, vol. 27, pp. 761–778 (1933).

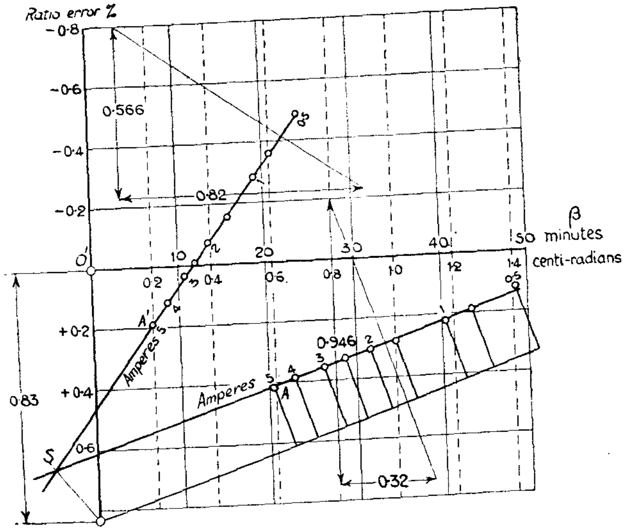

FIG. 21. USE OF FIG. 20 TO FIND SECONDARY LEAKAGE REACTANCE AND TURNS RATIO

angle at S will cut the ratio axis in O, as in Fig. 20. The distance $O'O = +\ 0{\cdot}83$ per cent is the turns-ratio error in terms of the nominal ratio. Since $\varepsilon_T = [(K_{nc}/K_T) - 1]100$ per cent, $K_T\,[1 + (\varepsilon_T/100)] = K_{nc}$; in the present case $K_{nc} = 1$ so that $K_T(1+0{\cdot}0083) = 1$ or $K_T = 0{\cdot}9917$. The primary had 240 turns, hence $T_s = 0{\cdot}9917 \times 240 = 238$ turns, which was actually the case, see p. 45.

Berghahn repeats the tests after interchanging the rôle of primary and secondary, finding $X_{wp} = 0{\cdot}55$ ohm and $\varepsilon_T = -\ 0{\cdot}83$ per cent.

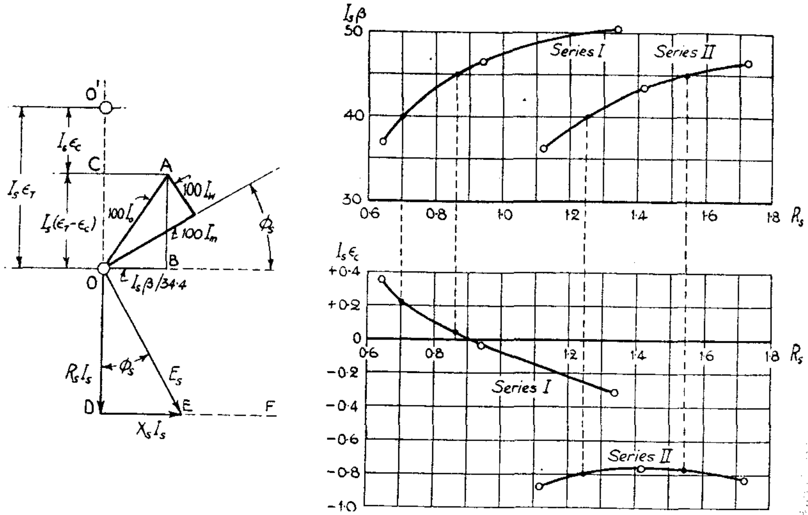

FIG. 22. GOCHTS' METHOD FOR FINDING X_{ws} AND ε_T

This verifies the statement already made that the secondary and reduced primary reactances are by no means equal. Further for the line SA, $Z_s = \sqrt{(0{\cdot}946^2 + 0{\cdot}32^2)} = 1$ ohm; for each value of primary current the secondary current can be readily calculated, since the value of ε_c is known, and hence $E_s = Z_sI_s$ becomes known also. For each point the values of I_m and I_w are easily read off the diagram and the open circuit curves plotted; this has been done in Fig. 16 where the circles show the results from the diagram in comparison with curves obtained with the aid of an a.c. potentiometer. The agreement is excellent.

In a method due to Gocht* the diagram is used in a rather different way. Fig. 22 shows the essential portions of the M. and G. diagram with scales of ratio error in per cent and phase-angle in minutes, but with all the sides of the exciting current and error triangles of Fig. 21 multiplied by the secondary current; I_0, I_m and I_w are expressed in terms of the secondary side. Two series of measurements of ε_c and β are made, by any suitable method, under the following conditions—

Series I. The secondary is loaded with a non-reactive burden so that the only reactance in the secondary circuit is X_{ws}. Increase R_s in steps in such a way that R_sI_s is constant, where R_s consists of R_{ws} plus the external variable burden, and observe ε_c and β. Then the locus of the extremity of E_s is the line DEF.

Series II. Add an inductance of known reactance X_B, so that the total reactance is $X_{ws} + X_B$ and again vary R_s keeping R_sI_s constant as before, observing ε_c and β. The locus of E_s is again DEF.

If E_s be supposed fixed, then ϕ_s, I_m, I_w, I_0 and with them $I_s\beta$ and $I_s(\varepsilon_T - \varepsilon_c)$ will also be fixed. Plot the curves of $I_s\beta = f(R_s)$ and $I_s\varepsilon_c = F(R_s)$; I_s is found by observing the primary current I_p and calculating I_s therefrom with the aid of the measured ratio. For a given value of E_s, i.e. of $I_s\beta$, the values of R_s in the two series of tests giving equal values of ϕ_s will be $R_{s\text{I}}$ and $R_{s\text{II}}$; then

$$\frac{X_{ws}}{R_{s\text{I}}} = \frac{X_{ws} + X_B}{R_{s\text{II}}}, \text{ or } X_{ws} = \frac{R_{s\text{I}}}{R_{s\text{II}} - R_{s\text{I}}}\,X_B.$$

For these values of R_s read off the corresponding values of $I_s\varepsilon_c$, $I_{s\text{I}}\varepsilon_{c\text{I}}$ and $I_s\varepsilon_{\text{II}c\text{II}}$ respectively and calculate $I_{s\text{I}}$ and $I_{s\text{II}}$ from the two values of R_s and the fixed R_sI_s. Then as the diagram shows,

$$I_{s\text{I}}(\varepsilon_T - \varepsilon_{c\text{I}}) = I_{s\text{II}}(\varepsilon_T - \varepsilon_{c\text{II}})$$

or

$$\varepsilon_T = \frac{I_{s\text{I}}\varepsilon_{c\text{I}} - I_{s\text{II}}\varepsilon_{s\text{II}}}{I_{s\text{I}} - I_{s\text{II}}}.$$

The method is much less direct than that of Berghahn, but yields good results, as the following example indicates.

Tests were made by Schering and Alberti's method on a transformer having the following particulars

$K_{nc} = 50/5$; $R_{ws} = 0{\cdot}64$ ohm; $X_B = 0{\cdot}52$ ohm at 50 cycles per sec.; resistance of added coil $= 0{\cdot}18$ ohm.

The resistance burden consists of a decade resistor variable in 0·1 ohm steps; denote this by R_0. In Series I $R_s = R_0 + 0{\cdot}64$; in Series II, $R_s = R_0 + 0{\cdot}64 + 0{\cdot}18 = R_0 + 0{\cdot}82$.

	R_0	R_s	I_s	ε_c in %	β in min.	$I_s\beta$	$I_s\varepsilon_c$
Series I	0 0·3 0·7	0·64 0·94 1·34	5·00 3·41 2·39	$+\ 0{\cdot}07_0$ $-\ 0{\cdot}01_0$ $-\ 0{\cdot}010_5$	7·4 13·7 21·1	37·0 46·6 50·5	+ 0·350 − 0·034 − 0·251
Series II	0·3 0·6 0·9	1·12 1·42 1·72	2·86 2·26 1·86	− 0·30 − 0·34 − 0·44	12·7 19·3 25·2	36·2 43·5 46·8	− 0·858 − 0·767 − 0·818

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* K. Gocht, "Ein Messverfahren zur Bestimmung der sekundären Streuinduktivität, der Windungsabweichung und des Leerlaufstromes von Stromwandlern," *Elekt. Zeits.*, vol. 50, pp. 1653–1655 (1929).

The value of $R_s I_s$ in these tests is 3·2. The curves $I_s\beta = f(R_s)$ and $I_s\varepsilon_c = F(R_s)$ are plotted in Fig. 22. From these curves the following values are taken for $I_s\beta = 40$ and 45.

$I_{sI}\beta$	R_{sI}	R_{sII}	I_{sI}	I_{sII}	$I_{sI}\,\varepsilon_{cI}$	$I_{sII}\,\varepsilon_{cII}$	X_{ws} ohms	ε_t %
40	0·70	1·25	4·57	2·56	+ 0·21	– 0·79	0·66	0·497
45	0·86	1·54	3·73	2·08	+ 0·04	– 0·76	0·64	0·484
						Mean	0·65	0·491

10. **Fleischhauer's graphical method for theory and design.** The Möllinger and Gewecke diagram gives a clear and convenient representation of the performance of a transformer working under normal conditions when the errors are small. If, however, the ratio error and phase-angle are to be found for currents far in excess of the rated values, or in low accuracy transformers, or in any other case where the errors become large, the diagram does not prove to be quite so useful. Again, the M. and G. diagram starts from a given secondary current and is built up therefrom to give the primary current; in actual practice, however, the transformer is operated over a given range of primary current derived from the supply network, and the determination of I_s from I_p by use of the diagram would only be possible by a tedious process of trial and error. Finally, the diagram is not in a form in which the influence of changes in the proportions of the transformer core and windings, variation in burden, or other features of design can be readily determined during the process of designing the transformer to yield the best performance under given practical conditions. To overcome these defects Fleischhauer* has developed an ingenious graphical method, based upon the M. and G. diagram, which is particularly useful for the solution of problems in design and for the investigation of overload conditions.

It has been shown on p. 44 that β will be zero if the secondary circuit is such that $\phi_s = (\pi/2) - \xi$, i.e. if the secondary power-factor is fairly low and the circuit inductive; this condition is approximately fulfilled in low accuracy transformers operating relays, trip-coils or similar burdens. In such cases, therefore, the vector diagram of currents assumes the simple form of Fig. 14 (*b*), in which I_p, $-I_s$ and I_0 are in the same straight line. Hence numerically,

$$T_p I_p = T_s I_s + T_p I_0$$

is the equation of ampere-turns. Dividing each term by l_i, the mean length of path in the iron core, gives

$$at_p = at_s + at_0,$$

where at_p, at_s and at_0 are the primary, secondary and exciting effective or r.m.s. ampere-turns per cm. In this equation at_p is given by the specified primary current; at_s and at_0 are undetermined functions of at_p and constitute two unknowns, necessitating a second relation in order to find them.

Let Z_s be the total secondary impedance of the transformer; then assuming sine waves,

$$E_s = Z_s I_s = 4{\cdot}44\, T_s f A_i B_{max}\, 10^{-8} \text{ volts},$$

where f is the frequency, A_i the iron section of the core in sq. cm., and B_{max} the maximum value of the core induction. Again,

$$T_s I_s = at_s \,.\, l_i;$$

eliminating I_s gives

$$B_{max}/at_s = Z_s\, l_i\, 10^8/4{\cdot}44 f A_i T_s^2$$

which is defined as the "impedance factor" of the core, since it is directly proportional to the secondary impedance. This factor can be given a simple graphical meaning and is the desired second relation.

Referring to Fig. 23 (*a*) the alternating current magnetization curve for the core plates is shown, giving B_{max} as an experimentally determined function of at, the effective ampere-turns per cm. Let at_p be a given value of the primary ampere-turns per cm.; then with the secondary circuit open ($at_s = 0$) the induction in the core is $A'P'$, and P' may be called the open-circuit point for this value of at_p. Keeping at_p constant let the secondary be supposed ideally short-circuited ($R_s = 0$) and free from leakage; then $at_0 = 0$, $B_{max} = 0$ and O is the short-circuit point. With any other condition of secondary load between these limits the operating point must fall on the curve between O and P', say at the point P, for which the equation $at_p = at_s + at_0$ must hold as shown; then

$$B_{max}/at_s = PA/PB = PA/AA' \propto \tan\theta \propto Z_s$$

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* W. Fleischhauer, "Graphische Stromwandlerberechnung," *Wiss. Veröff. Siemens Konz.*, vol. 10, part 1, pp. 98–136 (1931); *Elekt. Zeits.*, vol. 52, p. 1257 (1931); *ibid.*, vol. 53, pp. 691–693 (1932); *Arch. f. tech. Mess.*, Z221–2, Aug. (1932).

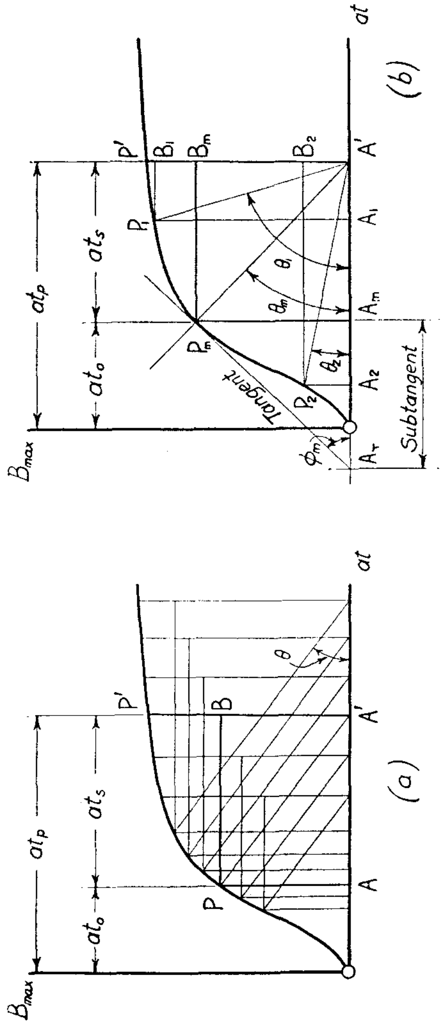

FIG. 23. FLEISCHHAUER'S METHOD FOR DETERMINING CURRENT TRANSFORMER ERRORS

For a fixed value of Z_s, i.e. of tan θ, and various values of P' all that is necessary is to draw parallels to PA' and from their intercepts on the axis of abscissae to read off at_p, at_s and at_0. Then using the formula

$$K_{c,\,max} = K_T + (I_0/I_s) = K_T + (at_0/at_s)\,.\,K_T$$
$$= K_T\,[1 + (at_0/at_s)]$$

the ratio can be calculated at once and the error is

$$\varepsilon_{c,\,max} = [(K_{nc}/K_{c,\,max}) - 1]\ 100 \text{ per cent}$$

for the value of I_p given by $l_i\,at_p$.

Fleischhauer has facilitated the process by calculating once for all, with a given quality of core material, a family of curves giving at_0 as a function of at_p for constant values of B_{max}/at_s lying between 1 and 10 000 in a finely-graded series of steps; a typical group of such curves in logarithmic co-ordinates is illustrated in Fig. 24 (*a*). In practice B_{max}/at_s is calculated from the preceding equation in terms of the burden, frequency, core dimensions and secondary turns, all of which are supposed fixed; at_0 is read off from the appropriate curve for a series of values of at_p and the ratio-error for given primary currents is thus determined. By the use of an additional group of charts Fleischhauer provides for the solution of the problem when $\beta \neq 0$, but the case we have examined is adequate in practice for the operating conditions usual in relay transformers.

The utility of such a graphical method in routine design* can be easily appreciated, since it is easy to find at once the effect produced on the accuracy of the transformer by a change in its dimensions, its secondary burden, etc., especially in dealing with transformers operating under extreme conditions such as occur in protective systems. Though the labour involved in preparing the charts is very considerable it is well worth while in design offices where a wide range of problems must be rapidly solved.

11. **Ratio error under overload conditions.** Current transformers intended for the operation of differential protective devices are required to work under conditions quite different from those imposed upon transformers working current-measuring instruments. Usually their secondaries are connected in some form of bridge or other balancing circuit which

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* For other graphical processes of design, see E. Billig, "Zur Vorausberechnung der Fehler von normalen Stromwandlern," *Elekt. Zeits.*, vol. 54, pp. 374–377 (1933); H. Schunck, "Zur graphischen Berechnung von Stromwandlern," *E.u.M.*, vol. 51, pp. 241–245 (1933).

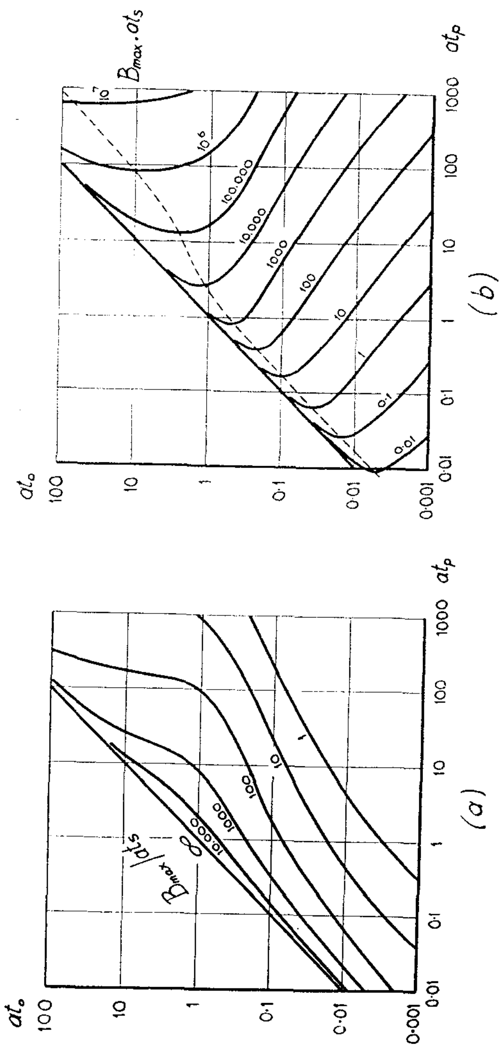

FIG. 24. FLEISCHHAUER'S CURVES FOR ERROR CALCULATIONS

is balanced under normal conditions: when overload occurs the balance of the secondary currents is disturbed and a relay device is actuated, resulting in the operation of the main switch and opening of the overloaded circuit. It is clear that current transformers used for such a purpose must be similar in their characteristics,* i.e. their secondary currents must vary in the same way with changes in the primary current up to many times the normal rated primary value; and they must retain reasonable accuracy of ratio up to the limit of overload at which the relay is to operate. The question will be reverted to in its practical aspects on p. 155; we merely note here that the ratio of a current transformer remains sensibly constant, i.e. $I_s \mathrel{\dot{\propto}} I_p$, so long as the core remains magnetically unsaturated. Once the knee of the magnetization curve is passed B_{max}, and consequently E_s and I_s, increase but slowly even when large increases are made in I_p; the ratio breaks away from constancy, and suddenly increases rather rapidly, with a consequent sudden growth in ε_c.

Fleischhauer's method is very convenient for the calculation of the performance of a current transformer in the region of saturation. Typical curves of ε_c found by this method are given in Fig. 25 for a transformer having the following particulars: $T_p = 2 \times 5$ turns for series or parallel connection for 150 or 300 amperes, $T_s = 298$ turns, $K_{nc} = 150/5$ or $300/5$, $K_T = 29{\cdot}8$ or $59{\cdot}6$, $I_pT_p = 1500$, Secondary burden 2·4 ohms, Rated volt-amperes 60, Power-factor of burden 0·5 and 1, $A_i = 21{\cdot}6$ sq. cm., $l_i = 49$ cm. The portion of the curves between 0 and 1 times I_{np} is the usual ratio-error curve for the normal rated range; the remainder of the curves illustrates the effect of overload on the ratio. For quite a considerable overload ε_c remains small, K_c approximately constant, and I_s grows in proportion to I_p until the iron enters the region of saturation, when ε_c increases rapidly. Since the point at which the break-away from proportionality occurs depends upon the degree of magnetization of the core it is under the control of the designer, who, by suitably proportioning the core-section, can arrange for a small value of ε_c up to any desired overload. In solving this design problem Fleischhauer's charts provide a quick and reliable method.

It is to be noted that the situation of the break-away point has a considerable influence upon the size and cost of the

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* *See* M. Walter, "Ulber die Eigenschaften der Stromwandler für Schutzrelais," *Elekt. Zeits.*, vol. 55, pp. 483–487 (1934).

transformer. The German Standard Rules contain a definition (see p. 21) which is intended to act as a means of comparing protective transformers in this respect, viz. the overload factor (*Überstromziffer*), which is the multiple of I_{np} which with rated secondary burden produces an error ε_c of 10 per cent irrespective of the secondary power-factor.* For example, in the transformer to which Fig. 25 refers the normal rating is 60 VA and with a secondary burden having a power-factor of 0·5 the value of the overload factor is about 18 for an error of 10 per cent. Hence at this overload the rating which the transformer must safely endure for a short period is 18 × 18 × 60 = 19 440 VA, which influences the weight of active material and the cost of the transformer.

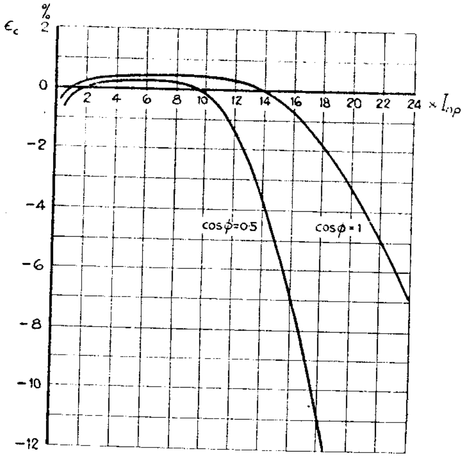

FIG. 25. RATIO ERROR OF CURRENT TRANSFORMER ON OVERLOAD

12. **Mathematical Treatment of Overload Conditions.** The graphical method for calculating the ratio error under overload conditions is particularly useful in design, requiring for its application only the magnetization curve of the core material. If the shape of this curve could be expressed by some empirical equation it should be possible to eliminate the graphical constructions and to determine the characteristics by arithmetical methods. The problem has recently been

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* Methods for computing and measuring the tripping burden and overload factor, definitions for which are given on p. 21, are given by E. Billig, "Auslösebürde und Überstromziffer von Stromwandler," *Bull. Schw. Elekt. Verein*, vol. 25, pp. 370–374 (1934).

attacked from this angle by Buchholz* who has shown that by assuming the magnetization curve to be a polynomial of the third degree, the ratio-error curve can be derived analytically in terms of the hypergeometric series of Gauss†; a brief sketch of his method follows.

Let x, B and Φ be corresponding instantaneous values of ampere-turns, flux density and flux, while x_n, B_n, Φ_n are given points on the magnetization curve. Then if δ is a parameter, it is assumed that the curve may be approximately fitted by a cubic of the form

$$x/x_n = (B/B_n)\cos^2\delta + (B/B_n)^3\sin^2\delta$$

or, more shortly,

$$\chi = \phi\cos^2\delta + \phi^3\sin^2\delta.$$

The parameter can be chosen so that in addition to the origin and the point (χ_n, ϕ_n) a third point is fitted, defined by the co-ordinates $\chi_1 = x_1/x_n$, $\phi_1 = \Phi_1/\Phi_n = B_1/B_n$; the value of the parameter will be

$$\sin^2\delta = (\phi_1 - \chi_1)/\phi_1(1-\phi_1)^2$$

Typical forms of the empirical curve are plotted in Fig. 26 (a).

In the current transformer, if i_p, i_s are the instantaneous primary and secondary currents, $x = i_pT_p + i_sT_s$ and

$$(i_pT_p + i_sT_s)/x_n = \phi\cos^2\delta + \phi^3\sin^2\delta \quad . \quad . \quad . \quad (1)$$

If R_s, L_s are the resistance and inductance of the secondary circuit and it is assumed that currents vary sinusoidally with pulsatance ω

$$R_si_s + L_s\frac{di_s}{dt} = -T_s\frac{d\Phi}{dt},$$

or

$$R_s\frac{i_sT_s}{x_n} + \omega L_s\frac{d}{d\,.\,\omega t}\left(\frac{i_sT_s}{x_n}\right) = -\frac{\omega T_s^2}{x_n}\,\frac{d\Phi}{d\,.\,\omega t} = -\frac{\omega T_s^2\Phi_n}{x_n}\cdot\frac{d\phi}{d\omega t} \quad (2)$$

Now write $L_{ns} = T_s^2\Phi_n/x_n$, $\eta_L = \omega L_s/\omega L_{ns}$ and suppose that the resistance is negligible in comparison with the reactance, as may be nearly true with a large reactive burden such as a relay. Putting $R_s = 0$ gives

$$-\phi = \frac{i_sT_s}{x_n}\eta_L \quad . \quad . \quad . \quad . \quad (3)$$

which on insertion in Equation 1 gives

$$\eta_L^3\sin^2\delta\left(\frac{i_sT_s}{x_n}\right)^3 + (1 + \eta_L\cos^2\delta)\frac{i_sT_s}{x_n} + \frac{i_pT_p}{x_n} = 0 \quad . \quad (4)$$

This cubic equation may be formally solved by Cardan's method, as explained in any treatise on higher algebra; the radicals in the solution are then expanded in infinite series which can be identified with the hypergeometric function

$$F(\alpha, \beta, \gamma, \theta) = 1 + \frac{\alpha\beta}{1\,.\,\gamma}\theta + \frac{\alpha(\alpha+1)\,\beta(\beta+1)}{1\,.\,2\,.\,\gamma(\gamma+1)}\theta^2, \text{ etc.}$$

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* H. Buchholz, "Übersetzungsverhältnis von Stromwandlern im Sättigungsgebiet," *A.E.G. Mitt.*, part 8, pp. 548–555 (Aug., 1930).

† See A. R. Forsyth, *A Treatise on Differential Equations*, 2nd edn., Ch. VI, pp. 185–216 (1888); or Gauss's *Gesammelte Werke*, vol. 3, pp. 123–163.

in the form

$$\frac{I_sT_s}{I_pT_p}(1+\eta_L\cos^2\delta)=\frac{3\cot\lambda}{\sqrt[3]{\cos\lambda}}\sqrt{[F(-\tfrac{1}{3},\tfrac{2}{3},1,\sin^2\lambda)-\cos^{\frac{2}{3}}\lambda]}\equiv H(\lambda), \quad (5)$$

where $$\tan^2\lambda=\frac{27}{2}\sin^2\delta\frac{\eta_L{}^3}{(1+\eta_L\cos^2\delta)^3}\left(\frac{I_pT_p}{x_n}\right)^2, \quad . \quad . \quad (6$$

and I_p, I_s are the r.m.s. values of the currents. The values of H (λ) are given in the following table and in Fig. 26 (b).

$\lambda°$ . .	0	5	10	15	20	25	30	35	40
H (λ) .	1·000	0·999	0·997	0·991	0·985	0·977	0·967	0·954	0·937
$\lambda°$ . .	45	50	55	60	65	70	75	80	81
H (λ) .	0·917	0·891	0·860	0·824	0·777	0·717	0·639	0·532	0·505
$\lambda°$ . .	82	83	84	85	86	87	88	89	90
H (λ) .	0·476	0·445	0·410	0·372	0·328	0·279	0·219	0·143	0

The method may be illustrated by a numerical example in which $T_s/T_p=50/7$, the burden being purely inductive, $L_s=10$ millhenrys, $R_s=0$. The magnetization curve was fitted by taking $\delta=78{\cdot}33$ degrees, the value of x_n being 1 000 ampere-turns and of Φ_n being $0{\cdot}0805\times10^6$ lines. From these, $L_{ns}=2$ millihenrys and $\eta_L=5$. Using Equation 6, $\tan\lambda=0{\cdot}214I_p$. For various values of λ the corresponding values of I_p are calculated and the figures for H (λ) are taken from the table or graph; using these in Equation 5 gives the ratio $K_c=I_p/I_s$ at once. The result is plotted in Fig. 26 (c).

13. **The output of a current transformer.** Consider a current transformer with a constant alternating current in its primary winding, its secondary circuit being closed through a variable external burden. If the secondary circuit be open, the core is subjected to the unopposed primary ampere-turns; the core flux and the e.m.f. induced in the secondary winding will have their greatest value but, since the secondary current is zero, the output is also zero. If now the secondary terminals be connected by a burden of zero resistance the current in the secondary circuit will have its largest value; the primary ampere-turns are almost completely annulled by the secondary ampere-turns, so that the flux is very small, and the induced voltage is just sufficient to circulate the secondary current through the impedance of the winding. Since the p.d. across the burden is zero, the output again vanishes. As the output is zero for the limiting burdens corresponding

with open and ideal short-circuit respectively, it follows that there must be some intermediate burden for which the output attains a maximum.

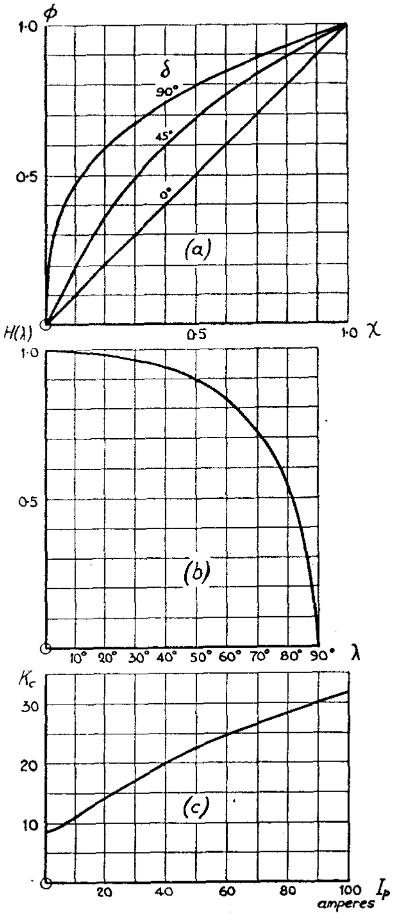

FIG. 26. BUCHHOLZ'S METHOD FOR CALCULATING RATIO ERROR ON OVERLOAD

Fleischhauer's graphical method* provides a ready means for investigating the variation of output with secondary burden. Confining attention to the case of a transformer in which the power-factor of the burden is maintained constant at the value giving $\beta=0$ and the maximum ratio, we may use the equations on p. 59. The *total* apparent power in volt-amperes generated in the secondary circuit is

$$P_s=Z_sI_s{}^2$$
$$=4{\cdot}44\,T_sf\,A_i\,B_{max}\,I_s\,10^{-8}.$$

Substituting $T_s=at_s\,.\,l_i/I_s$,

$$B_{max}\,.\,at_s=Z_sI_s{}^2 10^8/4{\cdot}44f\,A_il_i.$$

which is defined as the "output factor" of the core since it is proportional to the total secondary output.

Turning to the magnetization curve in Fig. 23 (a), if P is the operating point when the applied primary ampere-turns are OA' with a certain burden then, since $A'B=B_{max}$ and $AA'=at_s$, it follows that $B_{max}\,.\,at_s$ is proportional to the area of the rectangle $PBA'A$. With a fixed value of at_p it will be possible to draw two rectangles, such as $P_1B_1A'A_1$ and $P_2B_2A'A_2$ in Fig. 23 (b), of equal area; i.e. there are two

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* See also E. Wirz, "Berücksichtigung der Beglaubigungsvorschriften bei der Vorausberechnung der Stromwandler," *Bull. Schw. Elekt. Verein*, vol. 10, pp. 13–25 (1919). H. Neugebauer, "Stromwandler für Schützsysteme," *Siemens Zeits.*, vol. 11, pp. 147–151, 192–198 (1931). Also G. W. Stubbings, "The maximum volt-ampere output of a series transformer," *World Power*, vol. 5, pp. 131–135 (1926).

conditions of equal output corresponding with different values of B_{max}, namely, $A'B_1$ and $A'B_2$, different induced voltages proportional to these flux densities, and different secondary currents, proportional to A_1A' and A_2A' respectively. The burdens in ohms in the two cases are proportional to tan θ_1 and tan θ_2 respectively.

As P_1 moves downward and P_2 upward there will be some point such as P_m at which the two equal rectangles coincide; at this point they have the maximum area and the total secondary output has its greatest value. The output $Z_sI_s^2$ has its maximum value when $B_{max}at_s \equiv B_{max}\ (at_p - at_0)$ is greatest. Differentiating with respect to at_0, remembering that B_{max} is a function thereof and that at_p is fixed, on equating to zero,

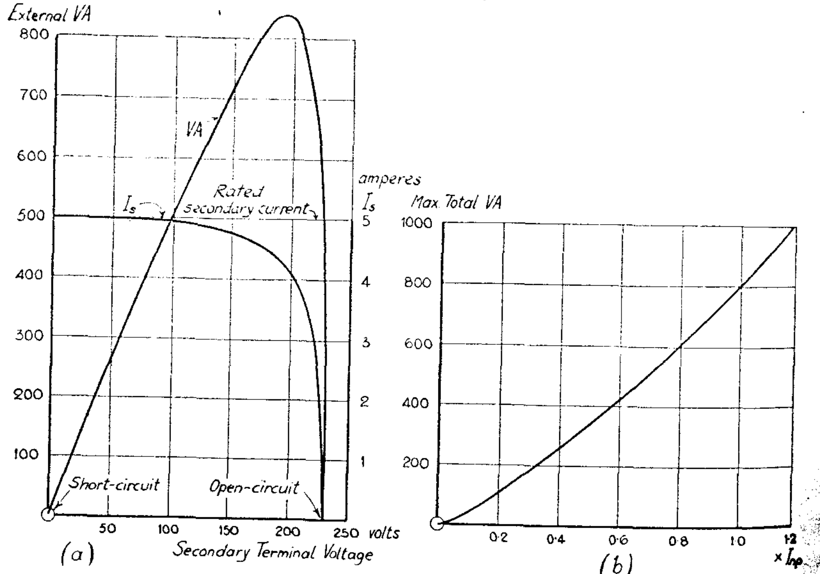

FIG. 27. THE OUTPUT CURVES FOR A CURRENT TRANSFORMER

$$B_{max}/at_s = B_{max}/at_0.$$

The left-hand side is the ratio $P_mA_m/A_mA' \propto \tan\theta_m$; the right-hand side is the slope of the tangent at P_m, i.e. the ratio $P_mA_m/A_mA_T \propto \tan\phi_m$. Hence at the point of maximum output $\theta_m = \phi_m$, or at_s equals the subtangent at P_m. This can be readily found by simple graphical trial.

Given the magnetization curve, the output curve for a fixed primary current is easily constructed by taking off a series of values of B_{max} with the corresponding values of at_s, calculating $B_{max}at_s$ and hence $P_s = Z_sI_s^2$ from the preceding formula. Knowing B_{max}, E_s is calculated from the e.m.f. equation and I_s from the relation $I_s = P_s/E_s$. Then if Z_B is the burden impedance, the volt-amperes expended in it will be $Z_BI_s^2$ and the voltage across it Z_BI_s. Finally $Z_BI_s^2$ and I_s can be plotted as a function of the voltage at the terminals of the burden, Z_BI_s. Typical curves are shown in Fig. 27 (*a*), plotted from measurements made on the transformer described in Section 11 when the primary ampere-turns are maintained constant at the full rated value of 1 500, the burden being varied from zero to infinite impedance with a constant power-factor of 0.8. The measured maximum output is 841 VA; the calculated value based on the assumption of $\beta = 0$ is 804 VA, which shows the order of approximation to be expected of Fleischhauer's method.

The calculation of the power curve is facilitated by the aid of curves, such as those given in Fig. 23 (*b*), which show in double logarithmic co-ordinates the values of at_0 as a function of at_p for constant values of $B_{max}at_s$ for a given quality of core steel. In use $B_{max}\,at_s$ is computed for chosen values of the total output $Z_sI_s^2$ and the corresponding values of at_0 are read off from the appropriate curves, for a given constant value of $at_p = I_pT_p/l_i$; the rest of the calculation then proceeds as described above. To each value of at_p there are two values of at_0, i.e. of at_s; the relation of at_0 to at_p for maximum total output is given by the dotted locus of the points of contact of vertical tangents to the curves, since at these points the two values of at_0 coincide. With the aid of this locus it is easy to find the maximum output for specified conditions. Fig. 27 (*b*) gives the calculated curve for the same transformer as before, showing the maximum total volt-amperes in the secondary circuit as a function of I_p/I_{np}, the impedance of the secondary burden being adjusted with each value of primary current to make the output a maximum.

CHAPTER III

CHARACTERISTICS OF THE CURRENT TRANSFORMER

1. Introductory. In Chapter II it has been shown that the ratio and phase-angle of a current transformer are affected by a considerable number of conditions, and the like is true of many of its additional characteristics. Some of these conditions, such as the magnetic properties of the core material, the proportioning of copper and iron in the transformer, magnetic leakage, etc., are internal features of construction and design; others of these conditions, such as the nature of the secondary burden, the frequency and wave-form of the primary current, etc., are external to the transformer. Each of these groups of conditions is responsible for certain influences upon the ratio, phase-angle and other characteristics of the transformer; internal or constructional conditions are considered in Sections 2 to 12, while external or operating conditions receive attention in Sections 13 to 26. We shall consider principally at first those conditions which affect the ratio and phase-angle and the methods used for making the errors small.

Referring to p. 41 it has been shown that to a sufficiently high degree of approximation for all practical purposes,

$$K_c \doteqdot K_T\left(1 + \frac{I_m}{K_T I_s}\sin\phi_s + \frac{I_w}{K_T I_s}\cos\phi_s\right),$$

$$\tan\beta \doteqdot \beta \doteqdot \frac{I_m}{K_T I_s}\cos\phi_s - \frac{I_w}{K_T I_s}\sin\phi_s.$$

Now the secondary current I_s is proportional to the induced voltage E_s in the secondary winding, and hence to the core flux Φ_{max}. The magnetizing and iron-loss components of the exciting current I_0, namely, I_m and I_w, are not, in general proportional to Φ_{max}. Hence as the secondary current changes the ratio and phase-angle cannot remain constant, which has been seen in the preceding chapter.

In an ideal current transformer the ratio would have a value equal to the nominal or rated ratio and the phase-angle would be zero for all conditions of operation. It is clear from these expressions that a practical iron-cored transformer can never

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conform to the ideal, since this would involve both I_m and I_w simultaneously vanishing, i.e. the use of an infinitely-permeable core material from which eddy currents and hysteresis are absent, which is obviously impracticable. The efforts of transformer designers have been largely directed, therefore, to a reduction of the imperfections to which I_m and I_w give rise, either by compensating for their effects or by the use of improved magnetic materials.

It is interesting to derive the general conditions* that should be fulfilled by the design of a current transformer. In practice the secondary circuit has a burden which is either a pure resistance or an inductance combined with resistance; hence ϕ_s is a positive angle lying between 0 and $\pi/2$, and both $\cos\phi_s$ and $\sin\phi_s$ are positive. Let l_i be the mean length of flux path in the iron core, A_i the cross-section of iron in the core, and B_{max} the maximum flux-density; then we have the relations

$$0{\cdot}4\pi\,.\,(\sqrt{2})\,I_m T_p = \frac{B_{max}}{\mu}l_i,$$

and

$$E_s = 4{\cdot}44\,T_s f\,B_{max}A_i\,10^{-8} = Z_s I_s,$$

since $\Phi_{max} = B_{max}A_i$ and μ is the permeability of the core.

Remembering that $\sin\phi_s = 2\pi f L_s/Z_s$ and $\cos\phi_s = R_s/Z_s$, we have

$$\frac{I_m}{K_T I_s}\sin\phi_s = \frac{10^8}{0{\cdot}4\pi}\cdot\frac{l_i}{T_s^2\mu A_i L_s}$$

and

$$\frac{I_m}{K_T I_s}\cos\phi_s = \frac{10^8}{0{\cdot}8\pi^2}\,\frac{l_i}{T_s^2\mu A_i}\cdot\frac{R_s}{f}$$

Let w_i be the total loss due to eddy currents and hysteresis per cubic centimetre of iron at the given frequency and flux density; then since I_w is referred to the primary side,

$$I_w = \frac{A_i l_i w_i}{E_s/K_T}$$

so that

$$\frac{I_w}{K_T I_s}\sin\phi_s = \frac{10^{16}}{\pi}\cdot\frac{l_i}{T_s^2 A_i}\left(\frac{w_i}{f\,B_{max}^2}\right)L_s$$

and

$$\frac{I_w}{K_T I_s}\cos\phi_s = \frac{10^{16}}{2\pi^2}\cdot\frac{l_i}{T_s^2 A_i}\left(\frac{w_i}{f^2 B_{max}^2}\right)R_s.$$

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* E. Wirz, "Berücksichtigung der Beglaubigungsvorschriften bei der Vorausberechnung der Stromwandler," *Bull. Schw. Elekt. Verein*, vol. 10, pp. 13–25 (1919).

Substituting in the expressions for K_c and $\tan\beta$ gives, if $a = 10^8/\pi$

$$K_c \doteqdot K_T\left[1 + \frac{l_i a}{T_s^2 A_i}\left(\frac{L_s}{0{\cdot}4\mu} + \frac{a w_i R_s}{2f^2 B^2_{max}}\right)\right].$$

$$\tan\beta \doteqdot \frac{l_i a}{T_s^2 A_i}\left[\frac{R_s}{0{\cdot}8\pi\mu} - \frac{\pi a\, w_i\, L_s}{B^2_{max}}\right]\frac{1}{f}.$$

From these equations it is easily seen that in order to keep the ratio error and the phase-angle small the design of the transformer should have the following features—

(i) A magnetic circuit of short mean path l_i and large cross-section A_i.

(ii) A large number of secondary turns. Since the rated secondary current is fixed at 5 amperes this means that the rated secondary ampere-turns must be as large as possible. Again, since the primary and secondary ampere-turns are approximately equal it follows that the rated primary ampere-turns must also be as large as possible. The importance of this condition is readily seen when it is remembered that the exciting ampere-turns, $T_p I_0$, are the vector resultant of the primary and secondary ampere-turns; hence if $T_p I_p$ and $T_s I_s$ are large in comparison with $T_p I_0$ the influence of the latter on K_c and β will be small. It is to be noted, however, that the accommodation of a large number of turns influences the dimensions of the window in the core plates and hence the size of the core; thus, condition (ii) is to some extent in conflict with (i).

(iii) R_s and L_s should be small, a condition which cannot usually be satisfied since some considerable part of R_s and L_s is contributed by the external burden. That portion due to the secondary winding itself can only be kept small by the use of a short mean length of copper turn, i.e. a small perimeter for the core limb; this is in conflict with the requirement of a large value of core section, A_i.

(iv) The core material should have a high permeability and small total loss per unit volume at not too low a flux density.

Just as in the design of a power transformer, it is necessary to make a compromise between these conflicting requirements in such a way that the allocation of copper and iron will give a current transformer which can be manufactured at a sufficiently low cost and has the desired accuracy. In addition there are often other considerations, not cited above, which profoundly influence the design; for example, for economic manufacture transformers may be designed not individually but as a "line" or "series."

Then again there may be special conditions, such as safe operation at extra-high voltage or under severe short circuit, that must be taken into account.

Current transformers are, broadly speaking, of two kinds, *measuring transformers* and *relay* or *protective transformers*, differing somewhat in constructional features but chiefly distinguished by their operating ranges and their accuracy. Measuring transformers are used for making current measurements over the normal working range of the circuits in which they are employed; they comprise (i) transformers of the highest accuracy and perfection used for the most precise laboratory measurements; (ii) rather less accurate transformers used in switchboard energy metering; and (iii) relatively rough transformers for the lowest grade of switchboard ammeters. Relay transformers operate in the region of overload on their primary circuits and are usually of a lower grade of accuracy than measuring transformers. The appropriate limits of error for all types are laid down in the national standards summarized in Chapter I (*q.v.*). Apart from the question of purpose and accuracy, current transformers are distinguished constructionally, notably by the arrangement of the primary winding and the type of iron core. There are two main types, the *bar type* with a single conductor primary, usually combined with a toroidally-wound ring core, and the *wound primary type* with core- or shell-type core. Then in addition there are variations of design imposed by operating voltage, mechanical and thermal safety and the like. Numerous illustrative examples of the various distinguishing types will be found in the course of this chapter; see for example Figs. 74 to 93.

2. **Simple methods for reducing ratio error and phase-angle.** In practice the burden is usually inductive with a power-factor between 0·3 and 1; hence all the terms in the expression for K_c on p. 70 are positive. If the transformer is wound so that $K_T = K_{nc}$, the actual ratio K_c will exceed the nominal value K_{nc} over the whole range of I_s and ϕ_s. If, however, K_T is made slightly less than K_{nc} it is possible to ensure that $K_c = K_{nc}$ for one particular value of secondary current with a given power-factor. Such turn adjustment is without appreciable effect on β.

The adjustment is made in practice by providing the secondary winding with slightly fewer turns than would make $K_T = K_{nc}$. We have seen on p. 72 that for a specified current rating T_s should be maintained as large as possible. When this is the case the alteration of T_s by a single turn makes only a small change in K_T and provides an adequate fine adjustment for the ratio; the percentage reduction in ratio, all else being unchanged, should be equal to the percentage change in T_s. This is not quite true, however, because the increased secondary current requires a larger induced secondary voltage and, as the turns are fewer, the flux must increase as the square of the increase of current. The greater flux requires more exciting current for its production, with the result that the change of ratio is slightly less than that in T_s.

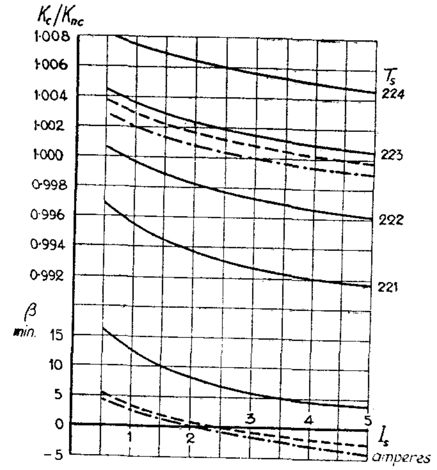

FIG. 28. THE EFFECT OF TURNS ADJUSTMENT ON RATIO ERROR AND PHASE-ANGLE

These conclusions may be well illustrated by the following figures* for a transformer having $K_{nc} = 5/5$ loaded with a secondary burden taking 25 volt-amperes at rated current of 5 amperes with a power-factor of 0·875; the frequency was 60 cycles per sec. and $T_p = 224$ turns.

Change in sec. turns	224 to 223	223 to 222	222 to 221
Per cent change in T_s	0·448	0·450	0·452
,, ,, K_c, $I_s = 1$ A.	0·400	0·402	0·404
,, ,, K_c, $I_s = 5$ A.	0·430	0·432	0·434

The characteristics for each value of T_s are plotted in Fig. 28 in full lines.

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[*] H. W. Price and C. K. Duff, "Minimizing the errors of current transformers by means of shunts," *Univ. of Toronto Eng. Res. Bull.*, No. 2, pp. 216–231 (1921).

In certain important cases a limit is set to the possible number of secondary turns, e.g. in current transformers of the bar type or similar construction with a single primary turn, when designed for a relatively low nominal ratio. For example, if in such a transformer the nominal ratio is 100/5 the secondary will have only about 20 turns, so that single-turn adjustment changes K_T by 5 per cent, which is altogether too coarse. It is necessary, therefore, to have some way of obtaining the equivalent of a fraction of a turn; this can be done in a variety of ways.* In principle these consist in arranging one or more of the secondary turns to enclose only a fraction of the total cross-sectional area of the core. The secondary coils of bar-type transformers are usually wound toroidally upon a laminated ring composed of a pile of silicon-iron stampings or of coiled-up nickel-iron strip; the fractional turn is passed through a hole in the laminations, as indicated diagrammatically in Fig. 29 (*a*). Several holes may be provided if desired, staggered round the ring as shown in Fig. 29 (*b*) so that the iron width is reduced by not more than the diameter of a single hole. A much better construction is shown in Fig. 29 (*c*), the total iron section being divided between two packets of

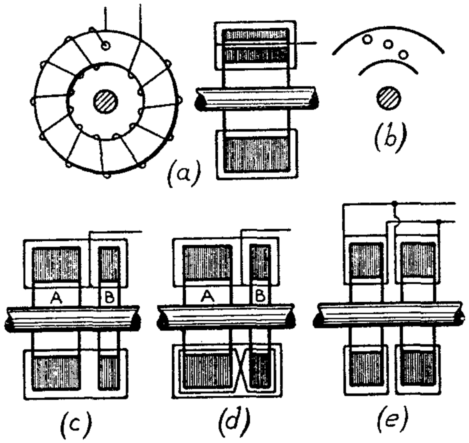

FIG. 29. METHODS FOR ADJUSTMENT OF TURNS-RATIO

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[*] For a discussion of the care to be taken to avoid unbalanced m.m.f.'s. in shell-type transformers with fractional turns, see R. H. Chadwick, "Transformer windings with fractional turns," *Gen. Elec. Rev.*, vol. 30, pp. 342–345 (1927).

laminations; the nearest whole number of secondary turns is wound uniformly round A and B together while the fractional adjustment turns link round B only. One of the features of a uniform ring-type winding is that it has a low leakage reactance (p. 131); the dissymmetry introduced by the adjusting turns gives to the secondary a certain leakage flux which may, in certain cases be beneficial to the value of β (see p. 44). If, however, the core is of nickel-iron this leakage reactance may be excessive but can be annulled by an ingenious device described by Wellings and Mayo.* Suppose, for example, that the core is in two equal parts with 20 secondary turns wound over the whole core and 2 turns over B only. This would be equivalent to winding 21 turns over the whole core, an additional turn on B only and a reversed turn on A only, all in series. The effect of the auxiliary excitation superposed on A and B in this way can be annulled by a closed figure-of-eight winding applied to the cores as in Fig. 29 (*d*). The extension of this idea to unequal cores will be obvious. Another device is shown in Fig. 29 (*e*), and consists in winding two equal rings each with double the normal number of turns to form secondaries rated at 2·5 amperes each, the two being joined in parallel. A single-turn adjustment on either section will then correspond with one-quarter of that resulting from a single-turn adjustment on a simple ring winding, since each section has twice the number of turns as the latter wound over half the area of iron. Edgcumbe and Ockenden† have described a simple arrangement not dependent upon the principle of dividing up the core area. The secondary of the current transformer is connected to its burden through an auxiliary transformer with a large number of primary and secondary turns and such a turns-ratio as will correct for the ratio error of the main transformer. If, for example, the auxiliary transformer has a ratio of 1 000/1 005, one turn represents an adjustment of 0·1 per cent; the additional phase-angle error introduced by the auxiliary transformer is usually negligible.

None of these devices has any useful effect in reducing β. It is often more important in practice that β should be small than that K_c should have the exact nominal value, as for

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* J. G. Wellings and C. G. Mayo, "Instrument Transformers," *Journal I.E.E.*, vol. 68, pp. 704–719, Discussion pp. 719–735 (1930).

† K. Edgcumbe and F. E. J. Ockenden, "Some recent advances in alternating current measuring instruments," *Journal I.E.E.*, vol. 65, pp. 553–586, Discussion pp. 586–599 (1927).

example when power is measured with a dynamometer wattmeter connected to the main circuit through instrument transformers, especially when the measurement is to be made at a low power-factor (see Chapter VI). By designing the transformer with a liberal cross-section of iron and the largest possible ampere-turns, β can be kept within reasonable limits. It is possible, as we have shown on p. 44, to make $\beta = 0$ by arranging $\phi_s = (\pi/2) - \xi$. This may be done by providing the secondary winding with a suitable amount of magnetic leakage, e.g. by putting the primary and secondary windings on opposite sides of the core.* In general, the presence of leakage reactance may, if not excessive, have a beneficial effect in reducing β.

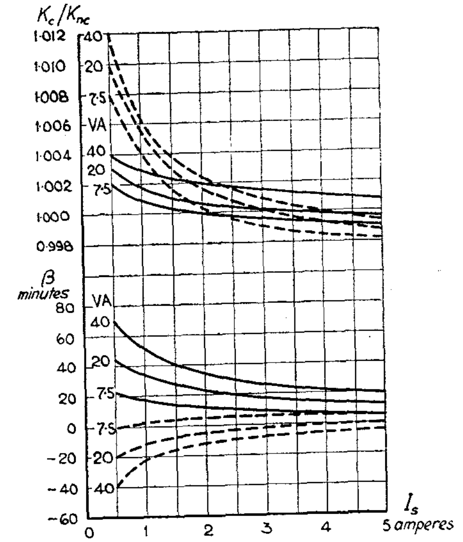

FIG. 30. THE EFFECT OF SECONDARY LEAKAGE REACTANCE ON RATIO AND PHASE-ANGLE

Edgcumbe and Ockenden (loc. cit. ante) give results for a ring core wound with 1 200 primary ampere-turns, plotted in Fig. 30. The full lines give the characteristics for outputs of 7½, 20 and 40 volt-amperes when the primary and secondary are uniformly wound over the ring; the dotted lines indicate the effect of putting the windings on opposite sides of the core. At the lowest burden the value of β does not exceed

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* The principle seems first to have been described by A. C. Heap in *British Patent*, No. 47, 1st Jan. (1903). Also see L. W. Wild, "Series transformers for wattmeters," *Elecn.*, vol. 56, pp. 705–706 (1906).

5 minutes (0·15 centiradian) when the leakage reactance is increased, while it amounts to about 23 minutes (0·7 centiradian) in the normal transformer. The ratio error, however, is considerably increased by the separation of the windings, as theory would lead one to expect, and varies much more with secondary current than in the normal transformer, i.e. over a range of about 1 per cent as compared with about 0·3 per cent.

All the arrangements described in this section can only be regarded as partial solutions of the problem of reducing the imperfections of a current transformer. It is not sufficient that the ratio error and phase-angle should be small; they should not vary appreciably with current, burden, frequency, or other operating conditions, i.e. the shape of the characteristic curve should be as flat as possible and as much under control as the magnitude of the errors. To attain these ends two main methods are available: (i) The use of auxiliary circuits and compounding devices. (ii) The employment of core materials magnetically better than the usual silicon-iron alloys. Some of the more important of these methods will now be considered.

3. **The use of secondary and primary shunts.** Almost all the methods of turns adjustment correct the ratio error by definite steps while leaving the phase-angle practically unchanged. Moreover, methods for improving the value of β, such as by increasing the magnetic leakage, often have a bad effect of K_c. Price and Duff (loc. cit.) have shown that continuous fine adjustment of the ratio and radical modification of β can be obtained by applying suitable shunts* either to the secondary or to the primary side of the transformer, the shunts consisting either of resistances, capacitances or combinations of these. In all cases the ratio is considered to mean the ratio of primary line current to the current in the useful secondary burden and not the ratio of the currents in the windings.

3*a*. SECONDARY NON-REACTIVE SHUNTS. Fig. 31 (*a*) shows the circuit and vector diagrams for this case. Starting with a given current I_s in the burden B, requiring a terminal voltage V_s and an induced voltage E_s, the core flux is Φ_{max}, to produce which necessitates a primary exciting current I_0. Adding this to the component $I'_p = K_s I_s$ which balances the secondary load current gives I''_p, the value the primary current would have if the shunt were absent. The shunt, however, carries a current I_{sh} in phase with V_s, and this is balanced by a component $I'_{sh} = K_T I_{sh}$ in the primary winding. Adding I'_{sh} to I''_p gives I_p, the actual primary current. It will be seen that since I_p is greater than I''_p and more nearly in phase with I'_p, the addition of the shunt *increases* the ratio and *reduces* the phase-angle; hence the transformer should first be compensated by turns adjustment until its ratio is a little too low before adding the shunt. The amount of increase in ratio is about 100 R_B/R_{sh} per cent and the diminution in angle about arctan (X_B/R_{sh}), where R_B and X_B are the resistance and reactance of the burden and R_{sh} is the shunt resistance, which is large in comparison with R_B and X_B.

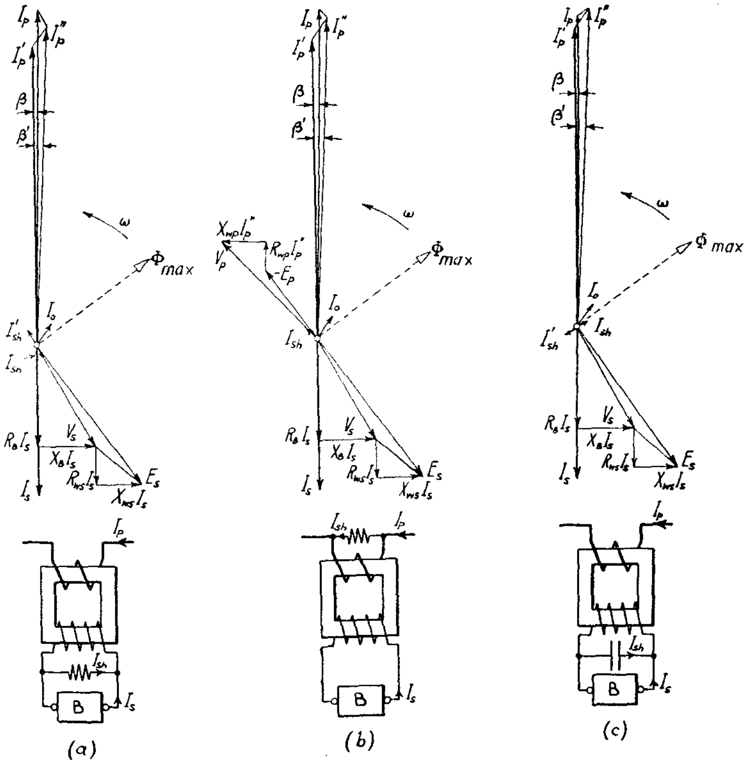

FIG. 31. THE EFFECT OF SHUNTS ON RATIO AND PHASE-ANGLE

For the transformer considered in Fig. 28, $R_B = 0{\cdot}875$ ohm and $X_B = 0{\cdot}487$ ohm at 60 cycles per sec. With $T_s = 222$ and $R_{sh} = 220$

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* This appears to have been put into practice by Siemens Brothers as far back as 1899, see *British Patent*, No. 17263 (1899), for the use of resistance shunts.

ohms the dotted curve in Fig. 28 was obtained, showing the notable improvement, principally in the value of β.

Clark* has recently pointed out that the corrective current taken by R_{sh} should vary with I_s in the same way as the flux, which is not exactly the case when it is shunted across the voltage V_s. In order to supply it at a voltage in phase with and proportional to E_s, Clark suggests that the resistor should be joined to a tertiary winding instead of applying it as a shunt to the burden; further, by adding reactance in series with the burden it should be possible to reduce β to zero and by these simple means to produce a highly accurate transformer.

3*b*. Primary Non-reactive Shunt. An impedance in the secondary circuit of a transformer can always be replaced in effect by an impedance in its primary circuit, and vice versa; consequently, ratio and phase-angle correction may be effected by applying a non-reactive shunt to the primary winding. The vector diagram for this case is given in Fig. 31 (*b*), where the vectors I_s, V_s, E_s, Φ_{max} and I_0 remain unchanged while I''_p is now the current in the primary winding, compounded of I'_p and I_0. The induced e.m.f. in the primary is $E_p = E_s/K_T$ and the p.d. across the primary terminals, V_p, will be the vector sum of $-E_p$ and the winding resistance and leakage reactance drops $R_{wp}I''_p$ and $X_{wp}I''_p$. In phase with V_p will be the shunt current I_{sh}; this added to I''_p gives the primary line-current I_p. As before $I_p > I''_p$ and $\beta < \beta'$, so that the ratio is increased and the phase-angle diminished. Indeed, the effect on the phase-angle is usually greater than with a secondary shunt. If R_t and X_t are the total resistance and reactance of the transformer and its burden measured at the primary terminals the increase in ratio is approximately 100 R_t/R_{sh} per cent and the reduction in angle is roughly arctan (X_t/R_{sh}).

The primary shunt has the advantage that it protects the transformer and its burden against high-frequency surges set up by transient conditions in the line. This aspect of the use of shunts will be considered on page 208.

3*c*. Secondary Capacitive Shunt. In Fig. 31 (*c*) the current I_{sh} taken by a condenser shunted across the burden leads by $\pi/2$ on V_s, the balancing current I'_{sh} in the primary being in a most advantageous position almost opposing I_0. It will be seen that $I_p < I''_p$ and $\beta < \beta'$, so that the ratio is *reduced* and the angle also *diminished*; the amount of change will be roughly $100\omega CX_B$ per cent and arctan ωCR_s respectively. The transformer should have such a turns adjustment as will make its ratio somewhat too high before adding the shunt.

In Fig. 28, for the same burden as before, the chain-dotted curves show the effect of a secondary shunt of 7 μF used with 223 secondary turns, verifying the above conclusions.

Since the impedance of a condenser changes with frequency while that of a resistor does not, it follows that ratio and phase-angle compensation will vary much less with frequency when effected by a resistive shunt than by a capacitive shunt. Similarly, condensers augment the susceptibility of the transformer to variation of errors with wave-form.

Ockenden* has recently pointed out that a condenser joined to a tertiary winding is even more effective than when used as a shunt, since the p.d. across it will vary with the flux instead of with the burden. By winding the tertiary with many turns only a small condenser is required. Using this device on a transformer with a nickel-iron core, Ockenden has succeeded in obtaining a value of $\beta = 0{\cdot}8$ minute (0·025 centiradian) from full to one-tenth of rated primary current, the ratio variation over this range being less than 1 in 10 000.

3*d*. Primary Capacitive Shunt. A condenser shunting the primary is similar in its effect to a secondary shunt. It is, moreover, an effective by-pass for high-frequency current surges since its impedance falls with increase of frequency; its presence may, however, give rise to undesirable resonance effects.

3*e*. Inductive Shunts. An inductive shunt, applied either to the primary or to the secondary, makes the ratio error and phase-angle worse, since the shunt current is in the wrong phase to effect any cancellation of I_o.

While these shunting devices are useful in certain cases they are admittedly imperfect; in particular, they do not give any appreciable control over the shape of the ratio and phase-angle curves, nor do they readily enable the ratio to be corrected independently of the phase-angle. We shall now examine some methods in which these defects are removed and other advantages are obtained.

4. **Two-stage current transformers.** An ingenious device for the improvement of the characteristics of a current transformer is the "two-stage" principle of transformation intro-

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* E. V. Clark, "Current Transformers. Reduction of phase-angle error," *Elecn.*, vol. 109, pp. 191–192 (1932).

* F. E. J. Ockenden, *Journal I.E.E.*, vol. 68, p. 720 (1930).

duced by Brooks and Holtz*; transformers of this type are made by the Sangamo Electric Co., the General Electric Co. of America, and others. Since the error of a current transformer is approximately inversely proportional to the square of the rated ampere-turns (see p. 72), the difficulty of making a highly accurate transformer with a single-turn low-current primary is very considerable; the two-stage principle enables this difficulty to be overcome and was specially developed for this purpose, though it is applicable to all other types in a precisely similar way.

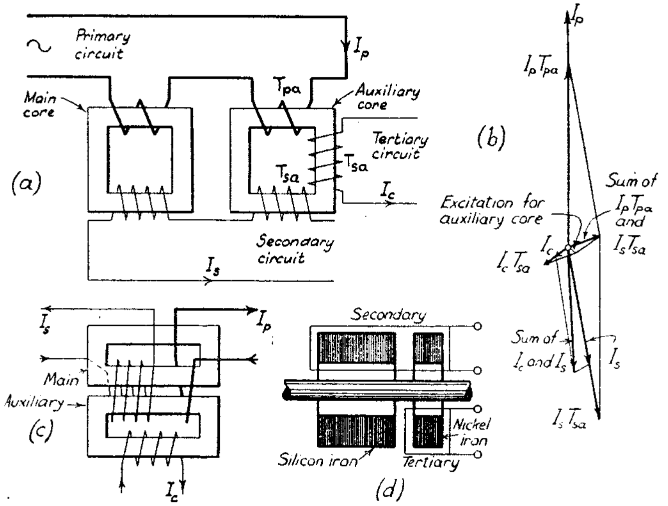

FIG. 32. TWO-STAGE CURRENT TRANSFORMERS

In Fig. 32 (*a*) the main core is wound as an ordinary transformer to give approximately correct ratio; the primary and secondary currents pass also through windings on an auxiliary core, the ratio of turns in the windings T_{sa}/T_{pa} being equal to the desired nominal ratio K_{nc}. The magnetizing effects of these windings on the auxiliary core are nearly in opposition, so that the resultant ampere-turns acting on the core will be the vector sum of I_pT_{pa} and I_sT_{sa} as shown in Fig. 32 (*b*). If the main transformer were perfect, i.e. if its ratio were correct and its phase-angle zero, the primary and secondary ampere-turns on the auxiliary core would balance and the core would

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* H. B. Brooks and F. C. Holtz, "The two-stage current transformer," *Trans. Amer. I.E.E.*, vol. 41, pp. 382–391 (1922). W. K. Dickinson and M. S. Wilson, "Two-stage current transformers," *Gen. Elec. Rev.*, vol. 31, pp. 656–659 (1928). See also Edgcumbe and Ockenden, *loc. cit.*, p. 576.

carry no flux. When the ratio and phase-angle deviate from the ideal, I_pT_{pa} and I_sT_{sa} will have a resultant which can be regarded as the effective applied ampere-turns for the auxilary core and is approximately equal to the exciting ampere-turns on the main core. The auxiliary core is provided with a tertiary winding, also of T_{sa} turns, connected to an external burden in which a current I_c will flow, the ampere-turns I_cT_{sa} being approximately equal and opposite to the resultant of I_pT_{pa} and I_sT_{sa}; their vector sum is the excitation required to set up flux in the auxiliary core. The corrective current I_c will clearly be such that when added vectorially to I_s their sum will be very closely in exact ratio and phase opposition with respect to I_p.

The compensation would be perfect if the auxiliary transformer required no exciting ampere-turns and wasted no energy in its burden; the errors in the second stage are, however, only of the second order of magnitude compared with those in the main transformer or first stage. Hence, in order that the arrangement should work to the best advantage (i) the burden on the second stage should be low and (ii) the magnetizing current and iron losses in the auxiliary core should be as small as possible, an advantage that may readily be gained by the use of nickel-iron alloy core plates. In practice the two stages are combined for convenience in manufacture in the way* shown by Fig. 32 (*c*) and (*d*).

The curves in Fig. 33 show in a striking way the improvement that can be obtained by applying the two-stage principle to a bar-type transformer for 60 cycles per sec. having a single primary conductor for 150 amperes. The secondary burden is 2·25 volt-amperes and the tertiary 1·9 volt-amperes, both at unity power-factor. The curves marked "single-stage" are obtained by using the total core and the main secondary, and show the very large ratio and angle errors commonly found in these transformers. Using the two stages effects a remarkable improvement and renders the bar-type transformer of adequate accuracy for metering purposes.

The main disadvantage of the two-stage transformer is that the instruments used with it must be provided with two independent current coils, one for the secondary current I_s and the other for the corrective current I_c. Moreover, for the two stages to operate independently of each other, the main

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* Fig. 32 (*c*) may represent the actual arrangement of a shell-type transformer; or it may be regarded as a diagrammatic representation of a core-type transformer with two packets of plates shown developed in the plane of the paper in order that the windings may be more clearly indicated.

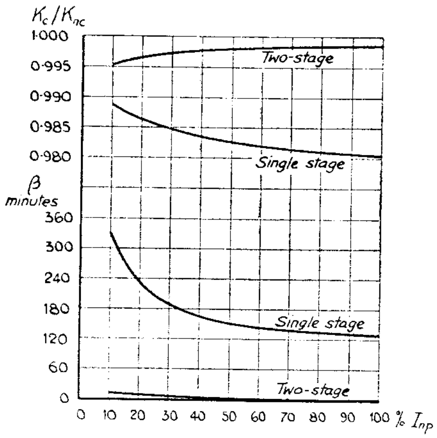

FIG. 33. CHARACTERISTIC CURVES FOR TWO-STAGE TRANSFORMER

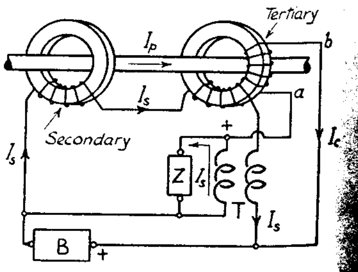

FIG. 34. THE TWO-STAGE TRANSFORMER OF BOYAJIAN AND SKEATS

and auxiliary circuits must have no appreciable mutual impedance, even such as would arise from the stages using one portion of a circuit in common; this obviously necessitates rather special design of the meters, etc., supplied by the transformer. These defects have been overcome by Boyajian and Skeats* in a very ingenious way, shown diagrammatically in Fig. 34 for a transformer with a bar primary. The burden B is the current element of an ordinary meter or other instrument of normal construction and is connected to the secondary and the tertiary in parallel; the mutual impedance between the two stages is the impedance of B and it is, therefore, necessary to provide another impedance equal and opposite to B common to the two circuits. This may be accomplished by the auxiliary impedance Z and the transformer T; Z is equal to B and the transformer, effecting the desired reversal of sign, has 1/1 ratio. If the tertiary circuit be opened at a, b, it is clear from the diagram that the secondary current flowing in the primary of T will set up a corresponding current in Z. Tracing the circuit from a through Z and B to b the impedance drops in Z and B cancel, so that the first-stage current sets up no p.d. between the points a and b in the tertiary circuit. Thus the total mutual impedance between the secondary and tertiary circuits has been reduced to zero; hence if the tertiary be closed, the corrective current I_c from the second stage flows into B and is superposed therein upon I_s without any interference, enabling the two-stage principle to be used with normal instruments. The two-stage transformer cores may be mounted in a bushing or otherwise linked over the high-voltage primary conductor; the transformer T may be mounted at any convenient place on the switchboard, near the burden B. It can easily be made of high accuracy so that any errors it may have can be regarded as negligible.

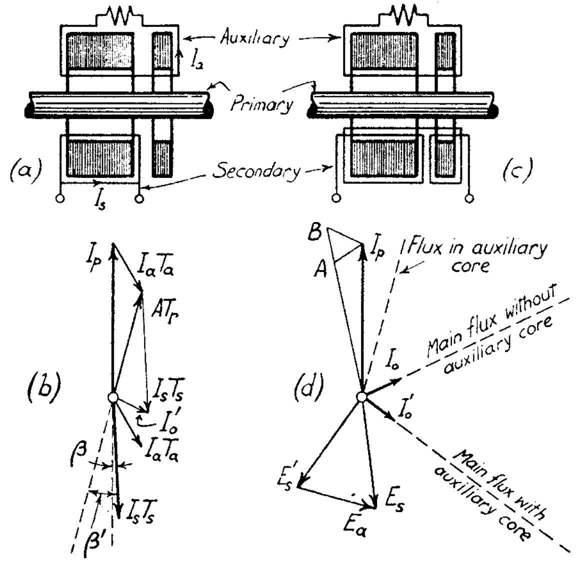

FIG. 35. WELLINGS AND MAYO'S COMPENSATED TRANSFORMER

5. **Wellings and Mayo's compensated transformer.** The two-stage principle aims to correct the errors in a transformer by injecting into the burden a corrective current which, when added to the secondary current, allows for the effect of the primary exciting current. The problem may, however, be approached from the primary side. If the transformer could be excited from an auxiliary source with exactly the correct magnitude and phase, then the transformer would operate without error. This principle has been developed by Wellings and Mayo (loc. cit.) at the B.T.-H. Co. particularly for bar-primary transformers. Fig. 35 (a) shows the arrangement for

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* A. Boyajian and W. F. Skeats, "Bushing-type current transformers for metering," *Journal Amer. I.E.E.*, vol. 48, pp. 308–310 (1929). Also see J. Goldstein, *Elekt. Zeits.*, vol. 53, pp. 428–429 (1932), and D. Garelli, *L'Elettro*, vol. 20, pp. 539–545 (1933), for discussion of this and other methods for improving the accuracy of current transformers.

correcting the phase-angle. A small auxiliary core operated at a relatively high flux-density energizes a winding of T_a turns wound upon the main and auxiliary cores. Since the main core and its secondary burden may be regarded as an inductive load upon the auxiliary winding, the current I_a has a fairly considerable phase-angle with respect to I_p, its magnitude and phase being regulated by the resistor shown. The resultant ampere-turns AT_r on the main core are the sum of I_p and I_aT_a and act thereon as its effective primary excitation; a current I_s will be set up in the secondary, having a phase-angle β' with respect to the resultant primary ampere-turns AT_r, the sum of AT_r and I_sT_s being the excitation ampere-turns required to set up the flux in the main core as shown in Fig. 35 (b). This excitation may be represented by an exciting current I_0' supposed to flow in the single-turn primary winding. It will be seen that I_s is brought much more nearly into opposition with I_p, i.e. β is made very small. By suitable design of the auxiliary core it is easy to arrange for a relatively greater auxiliary excitation at low loads than at high loads, so that where the phase-angle would be greatest the correction is also greatest. By this means the angle can be reduced and at the same time given a flat characteristic, as illustrated by Fig. 36.

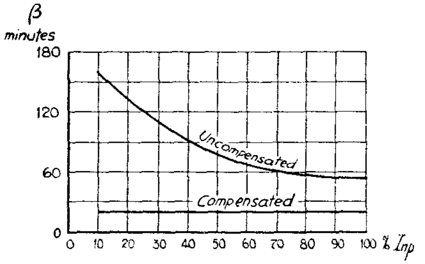

Fig. 36. The Effect of Fig. 35 (a) on the Value of β

This device makes little difference to the ratio error; Fig 35 (c) shows the modification required to reduce the ratio error and flatten its characteristic. It consists in taking a few turns of the secondary round the auxiliary core, the effect of which may be explained as follows. Reference to Fig. 35 (b shows that the flux in the auxiliary core (approximately in phase with AT_r) leads very considerably upon the flux in the main core and induces in the auxiliary secondary turns a correspondingly leading voltage E_a in Fig. 35 (d). This, together with E'_s, the voltage induced in the main secondary, gives E the total voltage available to circulate I_s through the impedance of the secondary winding and the external burden. It will be seen that, in comparison with the normal transforme diagram, E'_s and the main flux lag considerably behind the

normal positions, so that the relative positions of the primary exciting components have been altered from I_0 to I'_0 by the addition of the auxiliary secondary turns. Hence the available secondary current has been changed in the proportion of OA to OB, i.e. the ratio error has been reduced. As before, a suitable degree of saturation in the auxiliary core results in a flat ratio characteristic, and the phase-angle (intentionally exaggerated in Fig. 35 (d)) can be independently dealt with, as previously described, by the upper auxiliary winding; test figures will be given on p. 101.

6. **The use of magnetic shunts and similar devices.** Wilson[*] has described some interesting transformers, made by the G.E.C. of America, in which the errors are compensated by magnetic shunts and, in improved designs, by short-circuited windings. The absolute value of K_c can be regulated by suitable adjustment of the secondary turns; the ratio usually falls, however, as the secondary current increases. If it were possible to increase, by some artificial means, the value of K_T as the secondary current becomes greater, then the ratio-current curve could be appreciably flattened, i.e. K_c would tend to be constant in value for all currents. One method of securing this result artificially is shown in Fig. 37 (a) where the secondary is wound in two portions, of which S_1 contains the greater proportion of the total turns while S_2, having about 1 to 2 per cent of the total, is wound on another part of the core. If a part of the magnetic flux could be shunted past the smaller number of turns at the lower values of secondary current and caused to pass through them as the current increases, the desired change in turns ratio could be secured. The main core is composed of silicon-iron plates and is provided with a small packet of nickel-iron plates in parallel with the limb on which S_2 is wound. As the current increases the nickel-iron saturates more quickly than the main core, so that a relatively increasing amount of flux will pass through S_2. The device works well but is rather troublesome to construct. The relative change of reluctance between two portions of the magnetic circuit can be secured with silicon-iron alone if differential magnetization is arranged so that one part operates on the increasing and one on the decreasing branch of the permeability/flux-density

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[*] M. S. Wilson, "A new high-accuracy current transformer," *Journal Amer. E.E.*, vol. 48, pp. 179–182 (1929). See also "New developments in electrical easuring instruments," *Proc. N.E.L.A.*, vol. 86, pp. 813–818 (1929); and Keinath, *Elekt. Zeits.*, vol. 52, pp. 1564–1565 (1931).

curve. Fig. 37 (*b*) shows a convenient way of doing this. Each core limb is divided by a hole into two parts of unequal section, S_2 being wound round the narrower section in such a way that the secondary current flowing in these turns tends to strengthen the flux through this section and thus to divert a larger proportion of the main flux through it.

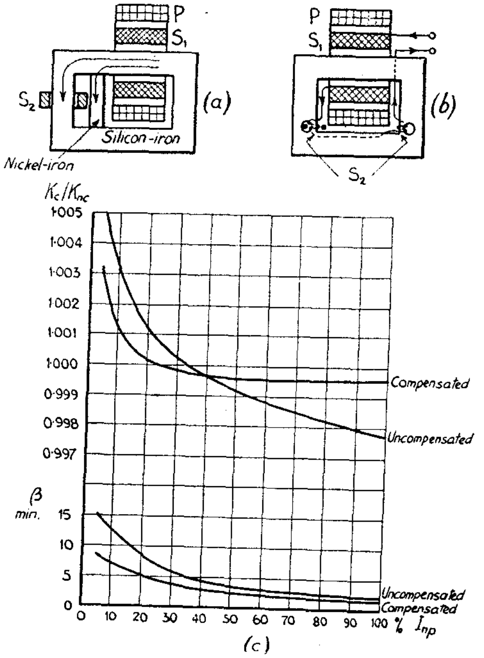

FIG. 37. WILSON'S COMPENSATED TRANSFORMERS

Neither of these arrangements has much effect on β. As a rule β is positive, i.e. $-I_s$ leads on I_p. Consequently, if I could be caused to lag slightly the value of β would be reduced; and if the amount of the lagging effect could be made greater with smaller values of I_s the β-current curve would be flattened. These conditions can be fulfilled in Fig. 37 (*b*) by putting a short-circuited loop round the section of the core upon which S_2 is wound. In the same way as the "lag band" in an a.c. meter, the loop causes the main flux to lag in phase upon its normal position and, with the lag of flux, E_s and I_s lag also, thus decreasing β; the differential saturation of the various parts of the core corrects for the falling of β with increased values of I_s.

The curves in Fig. 37 (*c*) show the excellent results that can be obtained; these relate to a 15 000 volt, 60 cycles per sec., 20/5 transformer loaded with an external burden of 0·68 ohms and 980 millihenrys (19·4 volt-amperes at a power-factor of 0·88).

Except for the provision of the holes to accommodate S_2 and the lag band, the transformer is of normal construction; the corrective devices are very simple and in no way impair the insulation, mechanical strength, or other properties of the transformer.

A further ingenious method making use of differential saturation, without requiring any modification in the construction of a bar-type transformer with a ring-type core, has recently been introduced by Schwager,* and is based on the following principle. Variations of K_c and β with I_s are due to the fact that I_0 and B_{max} ($\propto E_s \propto I_s$) do not vary in linear proportion, i.e. to non-linearity of the magnetization curve or, in other words, to variable μ. If μ were constant for all values of I_s, then all the remaining sources of error in K_c and β could be annulled by turns adjustment and by the shunting of a resistance or a capacitance across the secondary burden. Schwager attempts to secure the equivalent of constant μ by combining the exciting current required for an iron core with low saturation (concave upwards) with that necessary for a highly saturated core (concave downwards), the combination giving a closely linear magnetization curve, Fig. 38 (*a*). The result will be more nearly perfect if a number of cores is used with the saturation graded among them. This effect is ingeniously secured by having a low-saturated main core for the transformer itself and connecting across its secondary terminals a small higher-saturated core in which the saturation is graded from point to point by an adjustable air-gap, as in Fig. 38 (*b*) and (*c*). Residual compensation is made by a condenser across the secondary, while an auxiliary burden AB in series with the working burden B makes the characteristics practically independent of the latter.

The degree of precision secured is very remarkable, as the following figures show. They refer to a bar-type transformer with a ring core weighing about 270 lb., designed for 115 kV, $K_{nc} = 400/5$, Burden 50 volt-amperes with a power-factor of 0·5 at 60 cycles per sec. The

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* A. C. Schwager, "Der fehlerlose Stromwandler," *Bull. Schw. Elekt. Verein*, vol. 23, pp. 514–532 (1932), *E.u.M.*, vol. 51, pp. 12–13 (1933). A. C. Schwager and V. A. Treat, "Shaping of magnetization curves and the zero error current transformer," *Trans. Amer. I.E.E.*, vol. 52, pp. 45–52 (1933). Also see K. Rottsieper, *A.E.G. Mitt.*, part 2, pp. 38–40 (Mar., 1933).

small auxiliary choking coil weighed only a few pounds and bears to the main core the approximate proportions shown in Fig. 38 (*d*).

I_s amperes	0·5	1·0	2·0	3·0	4·0	5·0
K_c/K_{nc}	1·0006	1·0001	1·0007	1·0004	1·0000	0·9999
β minutes	− 11	− 7	0	+ 1	0	+ 1

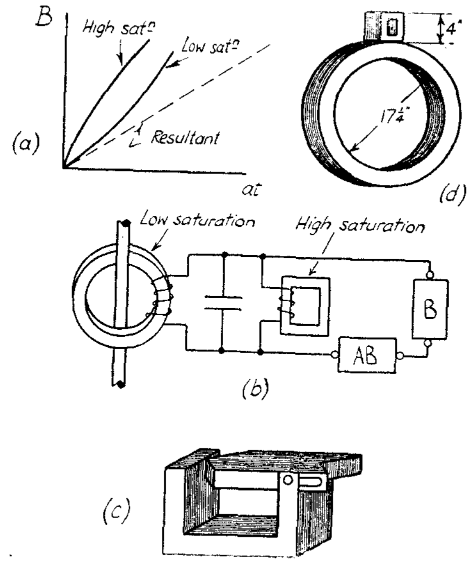

FIG. 38. SCHWAGER'S COMPENSATED TRANSFORMER

7. **The influence of core characteristics (general).** The methods described in the previous sections aim at improvement of the characteristics of a current transformer by compensating, with more or less completeness, the imperfections arising from given values of I_m and I_w. It is possible to proceed, however, from an entirely different standpoint by endeavouring from the outset to make I_m and I_w so small that the imperfections due to them become negligible and compensation is unnecessary, except perhaps in transformers of the very highest precision.

The material most commonly used for current transformer cores is a silicon-iron alloy,* containing 3 to 4 per cent of silicon, in the form of punchings 0·35 to 0·5 mm. in thickness. The high resistivity and very narrow hysteresis loop possessed by this material results in a low value of the inherent eddy-current and hysteresis losses per unit volume, though these may be increased in an assembled core by injudicious mechanical treatment and assembly of the plates. To assist in keeping down the total iron loss, the punching tools must be sharp, and local strain produced by the punching operation should be relieved by appropriate annealing; all burrs should be removed and the laminae thoroughly insulated from each other by varnish, lacquer, thin paper, etc The core should be assembled upon insulated clamping bolts or rivets, the holes for which are punched in the initial stamping operation. In general the process of assembling current transformer cores follows much the same lines as for power transformers but with the exercise of even greater care; by taking all precautions it is possible to make I_w quite small.

The reduction of I_m is much more troublesome. The prime essential is to reduce the core reluctance by the choice of a suitably large cross-section of iron with a short flux path, the material having a high permeability in the range of flux densities used in practice, say 500 to 1 500 lines per sq. cm. The iron circuit should be closed, but if joints are unavoidable they should be arranged to have the least possible reluctance by interleaving of the plates and secure clamping. Butt joints are inadmissible in current transformers; even when carefully faced-up and assembled under considerable pressure their presence greatly augments I_m. They are, moreover, a source of additional eddy currents at the metal-to-metal contact through which the main flux passes; even the thinnest paper interposed to remove this evil puts a gap into the magnetic circuit sufficient to raise I_m to a prohibitive value.

The simplest type of core* is the ring core, used in bar-type transformers, crossed-coil transformers and other designs, consisting of a pile of ring-shaped stampings securely bolted or riveted together, Fig. 39 (*a*). The punching waste is fairly great and may be entirely avoided by the modern method of using narrow strip wound up spirally into a ring of appropriate depth, Fig. 39 (*b*). This method is particularly valuable for modern core materials of the nickel-iron class, the magnetic properties of which are very sensitive to the effects of punching

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* Hadfield constructed a small instrument transformer with a silicon-iron core in 1903, this being the first transformer in which the material was used. It is exhibited in the Science Museum at South Kensington, but no particulars of its performance are given.

* For a discussion of core construction, see G. Keinath, "Stromwandler. Aufbau des Wandlersystems," *Arch. f. tech. Mess.*, Z281–1, T31 (Aug., 1931).