$V = P_2(\mathbb{R})$ $\quad \langle p, q \rangle = \int_{-1}^{1} p(t)\,q(t)\,dt$

$S = [t, t^2]$

Hallar una base de $S^\perp$

$B_V = \{1, t, t^2\}$ $\quad \dim V = 3$

$V = S \oplus S^\perp$

$\Rightarrow \dim V = \dim S + \dim S^\perp$

$\Rightarrow \dim S^\perp = 1$

$S^\perp : v \in S^\perp \Longleftrightarrow \langle v, s \rangle = 0$

$\forall s \in S$

$\Longleftrightarrow \langle v, s_i \rangle = 0$ donde

$\{s_1, s_2\} \xrightarrow{b} S.$

$\langle v, t \rangle = 0$

$\langle v, t^2 \rangle = 0$

$v \in V \quad v = a + bt + ct^2$

$$\langle \underbrace{a + bt + ct^2}_{\prime}, t \rangle = 0$$

$$\int_{-1}^{1} (\quad) \, t \, dt = \int_{-1}^{1} at + bt^2 + ct^3$$

$$= \frac{at^2}{2}\Big|_{-1}^{1} + \frac{bt^3}{3}\Big|_{-1}^{1} + \frac{ct^4}{4}\Big|_{-1}^{1} = \frac{2b}{3} = 0$$

$\Rightarrow b = 0$

$$\underbrace{\langle a+ct^2, t^2 \rangle} = 0$$

$$\int_{-1}^{1} at^2 + ct^4 \, dt = \left(a\frac{t^3}{3} + c\frac{t^5}{5} \right)\Big|_{-1}^{1}$$

$$= \frac{2}{3}a + \frac{2}{5}c = 0$$

$$\Rightarrow \boxed{a = -\frac{3}{5}c}$$

$$S^{\perp} = \left\{ -\frac{3}{5}c + ct^2 : c \in \mathbb{R} \right\}$$

$$= \left[-\frac{3}{5} + t^2 \right]$$

$\mathcal{P}_1(\mathbb{R}) \quad \{1, 1-t\}$

$$p_1(t) = 1$$

$$p_2(t) = 1 - t - \frac{\langle 1-t, 1 \rangle}{\|1\|^2} \cdot 1$$

$$\langle 1-t, 1 \rangle = \int_{-1}^{1} 1 - t = t - \frac{t^2}{2} \Big|_{-1}^{1}$$

$$= 2$$

$$\|1\|^2 = \langle 1, 1 \rangle = \int_{-1}^{1} 1 \, dt = t \Big|_{-1}^{1} = 2$$

$$\Rightarrow p_2(t) = 1 - t - \frac{2}{2} \cdot 1 = -t$$

$$\{1, -t\}$$

$$\|1\| = \sqrt{2}$$

$$\|-t\| = ?$$

$$\langle -t, -t \rangle = \int_{-1}^{1} t^2 = \frac{t^3}{3} \Big|_{-1}^{1} = \frac{2}{3}$$

$$\Rightarrow \quad \| -t \| = \sqrt{\frac{2}{3}}$$

$$\left\{ \frac{1}{\sqrt{2}}, -\sqrt{\frac{3}{2}}\, t \right\}$$

T diag. no nulo : $T^2 = 0$

$\Updownarrow$

$\exists\ B = \{v_1, \dots, v_n\} \xrightarrow{b} V$ formada

por vect. prop. $\exists$

$$_B(T)_B = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$T(v_i) = \lambda_i v_i \Rightarrow T^2(v_i)$$

$$= T(\lambda_i v_i) = \lambda_i^2 v_i \overset{T^2=0}{=} 0$$

$$\Rightarrow \lambda_i^2 \overset{\neq \vec{0}}{v_i} = 0$$

$$\Rightarrow \boxed{\lambda_i = 0}$$

$\Rightarrow \lambda_i = 0 \quad i = 1, \ldots, n$

$\Rightarrow {}_B(T)_B = \begin{pmatrix} 0 & & \\ & \ddots & \\ & & 0 \end{pmatrix}$

$\Rightarrow T = 0$

$C_1 = C(-2, r)$ $\quad r = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

$C_1 = C(-2, 1)$ $\quad r_i = \sum_{j \neq i} |a_{ij}|$

$C_2 = C(-4 + i, \frac{1}{2})$ $\quad r = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$

$C_3 = C(-4 - i, \frac{5}{8})$ $\quad r = \frac{1}{8} + \frac{1}{4} + \frac{1}{4} = \frac{1}{8} + \frac{1}{2}$

$C_4 = C(2, 1)$ $\quad = \frac{5}{8}$

T no invertible $\Rightarrow 0$ vap

$\dim(N(T+4I)) = 2 \Rightarrow mg(-4) = 2$

$\dim(Im(T-I)) = 3 \Rightarrow \dim(N(T-I)) = 2$

$\Rightarrow mg(1) = 2$

$\Rightarrow \chi_T(\lambda) = \lambda(\lambda+4)^2(\lambda-1)^2$

$\Rightarrow T$ es diag.

$$D = \begin{pmatrix} 0 & & & & 0 \\ & -4 & & & \\ & & -4 & & \\ & & & 1 & \\ 0 & & & & 1 \end{pmatrix}$$

$tr(T) = -6$.

$$A^n = (P^{-1}JP)^n = P^{-1}J\cancel{P P^{-1}}JP \cdots P^{-1}JP.$$

$$= P^{-1}J^nP.$$

$$- T(x,y) = (-2x+y, -x)$$

$$_B(T)_B = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\chi_T(\lambda) = \begin{vmatrix} -2-\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = (2+\lambda)\lambda + 1$$

$$= \lambda^2 + 2\lambda + 1$$

$$= (\lambda+1)^2$$

$\lambda = -1$, $m_a = 2$

$$S_{-1}: \left(\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right)$$

$$\Rightarrow -x + y = 0 \Rightarrow x = y$$

$$S_{-1} = [(1,1)] \qquad m_g = 1$$

$$\Rightarrow J_T = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$J^{27} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}^{27} = \begin{pmatrix} -1^{27} & 0 \\ 27(-1)^{26} & -1^{27} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 27 & -1 \end{pmatrix}$$

Base de Jordan $\{v_1, v_2\}$

$v_2 \in S_{-1}$, $T(v_1) = -v_1 + v_2$.

$\Rightarrow (T + I)(v_1) = v_2$.

$v_2 \in S_{-1} \Rightarrow$ Tomamos $v_2 = (1,1)$

$(T + I)(v_1) = v_2$ $\qquad P = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$v_1 = (x, y)$

$$\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow -x + y = 1$$

$\Rightarrow y = 1 + x$

$v_1 = (0, 1)$

Calcular P^{-1}

$$P^{-1} J^{n} P(1,0) = J^{27}(1,0)$$