Ejercicos M-O

a. Sea $\langle , \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ un producto interno que cumple las siguientes propiedades:

- $\langle (1,0),(1,0)\rangle = 4$,
- $\langle (1,1),(1,1)\rangle = 1$ y
- $\langle (1,0),(3,3)\rangle = 0$. $\Rightarrow \langle (1,0),(1,1)\rangle = 0$

Indicar la opción correcta:

~~i)~~ $\{(1,0),(1,1)\}$ es una base ortonormal de $\mathbb{R}^2$. ☐
~~ii)~~ $\{(\frac{1}{2},0),(0,1)\}$ es una base ortonormal de $\mathbb{R}^2$. ☐
~~iii)~~ $\{\frac{1}{\sqrt{5}}(0,1),(-1,1)\}$ es una base ortonormal de $\mathbb{R}^2$. ☐
iv) $\{\frac{1}{\sqrt{5}}(0,1),\frac{1}{\sqrt{20}}(5,4)\}$ es una base ortonormal de $\mathbb{R}^2$. ☐

i) para que $\{(1,0),(1,1)\}$ sea bon $(1,0)$ tiene que tener norma $= 1$

pero $\langle (1,0),(1,0)\rangle = 4$

ii) $\langle (\frac{1}{2},0),(0,1)\rangle = \langle \frac{1}{2}(1,0),\ (1,1)-(1,0)\rangle$

$$= \frac{1}{2}\langle (1,0),\ (1,1)-(1,0)\rangle$$

$$= \frac{1}{2}\Big(\underbrace{\langle (1,0),(1,1)\rangle}_{0} - \underbrace{\langle (1,0),(1,0)\rangle}_{4}\Big) \neq 0$$

iii) $\langle \frac{1}{\sqrt{5}}(0,1),\ (-1,1)\rangle = \frac{1}{\sqrt{5}}\langle (1,1)-(1,0),\ (1,1)-2(1,0)\rangle$

$$= \frac{1}{\sqrt{5}}\Big(\langle (1,1),\ (1,1)-2(1,0)\rangle - \langle (1,0),\ (1,1)-2(1,0)\rangle\Big)$$

$$= \frac{1}{\sqrt{5}}\Big(\underbrace{\langle (1,1),(1,1)\rangle}_{1} - 2\underbrace{\langle (1,1),(1,0)\rangle}_{0} - \underbrace{\langle (1,0),(1,1)\rangle}_{0} + 2\underbrace{\langle (1,0),(1,0)\rangle}_{4}\Big)$$

$\langle \frac{1}{\sqrt{5}}(0,1),\ (-1,1)\rangle \neq 0$

iv) $\left\{\frac{1}{\sqrt{5}}(0,1),\ \frac{1}{\sqrt{20}}(5,4)\right\}$

$(0,1) = (1,1) - (1,0)$

$$(5,4) = 4(1,1) + (1,0)$$

$$\langle (0,1), (0,1) \rangle = \langle (1,1) - (1,0), (1,1) - (1,0) \rangle$$

$$= \langle (1,1), (1,1) - (1,0) \rangle - \langle (1,0), (1,1) - (1,0) \rangle$$

$$= \underbrace{\langle (1,1), (1,1) \rangle}_{=1} - \underbrace{\langle (1,1), (1,0) \rangle}_{=0} - \underbrace{\langle (1,0), (1,1) \rangle}_{=0} + \underbrace{\langle (1,0), (1,0) \rangle}_{=4}$$

$$\Rightarrow \langle (0,1), (0,1) \rangle = 5 \Rightarrow \|(0,1)\| = \sqrt{5}$$

$$\Rightarrow \left\| \frac{1}{\sqrt{5}} (0,1) \right\| = 1$$

$$\langle (5,4), (5,4) \rangle = \langle 4(1,1) + (1,0), 4(1,1) + (1,0) \rangle$$

$$= \langle 4(1,1), 4(1,1) + (1,0) \rangle + \langle (1,0), 4(1,1) + (1,0) \rangle$$

$$= \underbrace{\langle 4(1,1), 4(1,1) \rangle}_{= \underbrace{16 \langle (1,1), (1,1) \rangle}_{=16}} + \underbrace{\langle 4(1,1), (1,0) \rangle}_{=0} + \underbrace{\langle (1,0), 4(1,1) \rangle}_{=0} + \underbrace{\langle (1,0), (1,0) \rangle}_{=4}$$

$$\langle (5,4), (5,4) \rangle = 20 \Rightarrow \|(5,4)\| = \sqrt{20}$$

$$\Rightarrow \left\| \frac{1}{\sqrt{20}} (5,4) \right\| = 1$$

$$\langle (0,1), (5,4) \rangle = \langle (1,1) - (1,0), 4(1,1) + (1,0) \rangle$$

$$= \langle (1,1), 4(1,1) + (1,0) \rangle - \langle (1,0), 4(1,1) + (1,0) \rangle$$

$$= \underbrace{\langle (1,1), 4(1,1)\rangle}_{\underbrace{4\langle (1,1),(1,1)\rangle}_{=4}} + \underbrace{\langle (1,1),(1,0)\rangle}_{=0} - \underbrace{\langle (1,0), 4(1,1)\rangle}_{=0} - \underbrace{\langle (1,0),(1,0)\rangle}_{4}$$

$$= 0 \checkmark$$

Ejercicios: Múltiple opción (Total: 32 puntos)

Correctas: 8 puntos. Incorrectas: - 2 puntos. Sin responder: 0 punto.

1. En $\mathbb{R}^3$ se considera el producto interno

$x \quad y \quad z \quad 0 \quad 1 \quad 0$

$$\langle (u_1, u_2, u_3), (v_1, v_2, v_3)\rangle = u_1v_1 + u_2v_2 + 2u_3v_3 - u_1v_3 - u_3v_1.$$

Sea $T : \mathbb{R}^3 \to \mathbb{R}^3$ autoadjunta tal que:

- $T(u) = -2u$ para todo $u \in S = \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$
- $tr(T) = -1$

Entonces:

A. $T(2, 1, 1) = (1, -2, 3)$.

B. $T(2, 1, 1) = (-2, -2, 3)$.

C. $T(2, 1, 1) = (-2, -2, -1)$.

D. $T(2, 1, 1) = (1, -2, -1)$.

T autoadjunta $\Rightarrow$ T es diagonalizable y los subespacios propios son ortogonales

$T(u) = -2u$ para todo $u \in S = \{(x,y,z) \in \mathbb{R}^3 : z = 0\}$ $\Rightarrow$ -2 es un valor propio y $\dim S_{-2} \geq 2$

$Tr(T) = -1$

$$[T] = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \Rightarrow Tr(T) = -4 + \lambda \Rightarrow \lambda = 3$$

Entonces:

- -2 es valor propio y $\dim S_{-2} = 2$
- 3 es valor propio y $\dim S_3 = 1$

$S_{-2} = \{(x,y,z) \in \mathbb{R}^3 : z = 0\}$

base de S_{-2}: $\{(1,0,0), (0,1,0)\}$

$S_3 = S_{-2}^{\perp} = \{ (x,y,z) \in \mathbb{R}^3$ que son ortogonales a $(1,0,0)$ y a $(0,1,0) \}$

sea $(x,y,z) \in S_3$

$0 = \langle (x,y,z), (1,0,0) \rangle = x - z \quad \Rightarrow x = z$

$0 = \langle (x,y,z), (0,1,0) \rangle = y \quad \Rightarrow y = 0$

$S_3 = \{ (x,y,z) \in \mathbb{R}^3 : x = z , y = 0 \}$

base de S_3 : $\{ (1,0,1) \}$

Queremos escribir $(2,1,1)$ como combinación lineal de $\overbrace{(1,0,0), (0,1,0)}^{\in S_{-2}}$ y $\underbrace{(1,0,1)}_{\in S_3}$

$(2,1,1) = (1,0,0) + (0,1,0) + (1,0,1)$

$T(2,1,1) = T(1,0,0) + T(0,1,0) + T(1,0,1)$

$= -2(1,0,0) - 2(0,1,0) + 3(1,0,1)$

$= (1,-2,3)$

Sea $A \in M_{6\times 6}(\mathbb{R})$ tal que:

- $det(A - \lambda Id) = (\lambda^2 - 4\lambda + 4)^3$
- $\text{rango}(A - 2Id) = 4$
- $\dim(N((A - 2Id)^3)) = 4$

Indicar la opción correcta:

A) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

B) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

C) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

D) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

$B = \{v_1, v_2, v_3\}$

$${}_B[T]_B = \begin{pmatrix} \lambda & 0 & 0 \\ 1 & \lambda & 0 \\ 0 & 1 & \lambda \end{pmatrix}$$

v_3 es un vector propio $\quad v_3 \in N(T - \lambda Id)$

$T(v_2) = \lambda v_2 + v_3 \qquad v_2 \notin N(T - \lambda Id)$

$(T - \lambda Id)(v_2) = T(v_2) - \lambda v_2 = v_3$

$(T - \lambda Id)^2(v_2) = (T - \lambda Id)(v_3) = T(v_3) - \lambda v_3 = 0$

$\Rightarrow v_2 \in N((T - \lambda Id)^2)$

$(T - \lambda Id)(v_1) = T(v_1) - \lambda v_1 = \lambda v_1 + v_2 - \lambda v_1 = v_2 \neq 0$

$(T - \lambda Id)^2(v_1) = (T - \lambda Id)(v_2) = v_3 \neq 0$

$(T - \lambda Id)^3(v_1) = (T - \lambda Id)^2(v_2) = 0$

$\Rightarrow v_1 \in N((T - \lambda Id)^3)$

$$N(T-\lambda Id) \subseteq N((T-\lambda Id)^2) \subseteq N((T-\lambda Id)^3) = N((T-\lambda Id)^4)$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ v_3 & v_3, v_2 & v_1, v_2, v_3 \end{array}$$

Sea $A \in M_{6\times 6}(\mathbb{R})$ tal que:

- $det(A - \lambda Id) = (\lambda^2 - 4\lambda + 4)^3$
- rango$(A - 2Id) = 4$
- $\underbrace{\dim(N((A - 2Id)^3)) = 4}$

Indicar la opción correcta:

A) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$ ✓

B) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$ X

C) A es semejante a $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$ X

D) A es semejante a $\begin{pmatrix} \boxed{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$ X $\quad \dim N(A-2Id) = 3$

$$det(A-\lambda Id) = (\lambda^2 - 4\lambda + 4)^3 = (\lambda-2)^6$$

$\Rightarrow$ el único valor propio de A es 2

rango$(A-2Id) = 4 \Rightarrow \dim N(A-2Id) = 2$

$\Rightarrow \dim S_2 = 2$

$\Rightarrow$ J tiene dos subbloques asociados a 2

C) A es semejante a $\begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{matrix} \\ \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \end{array}$

$$\underset{\substack{\uparrow \\ v_3, v_6}}{N(A-2Id)} \subseteq \underset{\substack{\uparrow \\ v_3, v_6, v_5, v_2}}{N(A-2Id)^2} \subseteq \underset{\substack{\uparrow \\ v_3, v_6, v_5, v_2, v_1, v_4}}{N(A-2Id)^3} \Rightarrow \dim N(A-2Id)^3 = 6$$

$$(A-2Id)(v_2) = 2v_2 + v_3 - 2v_2 = v_3$$

$$(A-2Id)^2(v_2) = (A-2Id)v_3 = 0 \checkmark$$

$$\Rightarrow v_2 \in N(T-2Id)^2$$

B) A es semejante a
$$\begin{array}{c}
\begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\
\left(\begin{array}{cc|cccc}
2 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 0 & 0 & 0 & 0 \\ \hline
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 1 & 2
\end{array}\right)
\end{array}$$

$$\underset{\substack{\uparrow \\ v_2, v_6}}{N(A-2Id)} \subseteq \underset{\substack{\uparrow \\ v_2, v_6, v_1, v_5}}{N(A-2Id)^2} \subseteq \underset{\substack{\uparrow \\ v_2, v_6, v_1, v_5, v_4}}{N(A-2Id)^3} \Rightarrow \dim N(A-2Id)^3 = 5$$

A) A es semejante a
$$\begin{array}{c}
\begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\
\left(\begin{array}{c|ccccc}
2 & 0 & 0 & 0 & 0 & 0 \\ \hline
0 & 2 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 1 & 2
\end{array}\right)
\end{array}$$

$$\underset{\substack{\uparrow \\ v_1, v_6}}{N(A-2Id)} \subseteq \underset{\substack{\uparrow \\ v_1, v_6, v_5}}{N(A-2Id)^2} \subseteq \underset{\substack{\uparrow \\ v_1, v_6, v_5, v_4}}{N(A-2Id)^3} \quad \dim N(A-2Id)^3 = 4$$

Ejercicio 2: (12 puntos)

Considere el espacio vectorial $\mathcal{M}_{2x2}(\mathbb{R})$ con el producto interno $\langle A, B\rangle = \frac{1}{2}tr(AB^t)$. Considere además el operador lineal $T : \mathcal{M}_{2x2}(\mathbb{R}) \to \mathcal{M}_{2x2}(\mathbb{R})$ definido por:

$$T(A) = A^T + 2A.$$

Entonces, para $M = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$, $T^*(M)$ vale:

$$\boxed{\begin{pmatrix} 9 & 4 \\ 5 & 0 \end{pmatrix}}$$

$$\langle T(A), B\rangle = \langle A, T^*(B)\rangle$$

Vamos a buscar la matriz asociada a T^*

si B es bon $\quad {}_B[T]^*_B = {}_B[T]^t_B$

$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ base de $\mathcal{M}_{2x2}(\mathbb{R})$

$$\left\langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\rangle = \frac{1}{2} tr \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} tr \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}$$

$$\left\| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\| = \frac{1}{\sqrt{2}}$$

$$\left\langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right\rangle = \frac{1}{2} tr \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = 0$$

$B = \left\{ \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{pmatrix} \right\}$ bon de $\mathcal{M}_{2x2}(\mathbb{R})$

$$T(A) = A^t + 2A$$

$$T\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} + 2\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2\sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{2} \\ 2\sqrt{2} & 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{pmatrix} + 2\begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$$

$${}_B[T]_B = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$${}_B[T^*]_B = {}_B[T]_B^t = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$\Rightarrow T = T^*$ $\quad T(A) = A^t + 2A$

$T^*(A) = A^t + 2A$

$$T^*\begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ 5 & 0 \end{pmatrix}$$

Ejercicio 3: (12 puntos)

Consideremos $\mathbb{R}^3$ con el producto interno

$$\langle (x,y,z),(x',y',z')\rangle = xx' + yy' + 2zz' + zy' + yz'$$

y el subespacio $\quad 1\ {-1}\ 0$

$$S = \{(x,y,z) \in \mathbb{R}^3 : x + y = 0\}.$$

Entonces, la proyección ortogonal del vector $(1,2,1)$ sobre $S^\perp$ es:

$(1,2,-1)$

$S = \{ (x,y,z) \in \mathbb{R}^3 : x + y = 0 \}$

base de S : $\{ (0,0,1), (1,-1,0) \}$

$S^\perp = ?$

$(x,y,z) \in S^\perp$

$0 = \langle (x,y,z), (0,0,1) \rangle = 2z + y \Rightarrow y = -2z$

$0 = \langle (x,y,z), (1,-1,0) \rangle = x - y - z \Rightarrow x + 2z - z = 0$

$\Rightarrow x = -z$

base de $S^\perp$: $\{ (-1,-2,1) \}$

$$\text{proy}_{S^\perp}(1,2,1) = \text{proy}_{(-1,-2,1)}(1,2,1) = \underbrace{\frac{\langle (1,2,1), (-1,-2,1) \rangle}{\langle (-1,-2,1), (-1,-2,1) \rangle}}_{= -1} (-1,-2,1)$$

$\langle (1,2,1), (-1,-2,1) \rangle = -1 - 4 + 2 - 2 + 2 = -3$

$\langle (-1,-2,1), (-1,-2,1) \rangle = 1 + 4 + 2 - 2 - 2 = 3$

$\text{proy}_{S^\perp}(1,2,1) = -(-1,-2,1) = (1,2,-1)$