SOLUCIÓN V1 :

Ejercicio 1 :

Sumas:

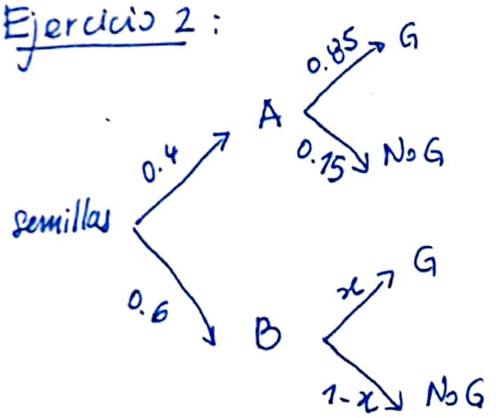
$R_1 \backslash R_2$	6	7	9
3	9	10	12
4	10	11	13
5	11	12	14
8	14	15	17

$$\mathbb{P}(\text{"la suma da un número par"}) = \frac{6}{12} = \frac{1}{2}$$

$$\mathbb{P}(\text{"1ª vez suma par"}; \text{"2ª vez suma par"}) \underset{\text{indep}}{=} \mathbb{P}(\text{"1ª vez suma par"}) \times \mathbb{P}(\text{"2ª vez suma par"})$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4} \longrightarrow \text{E}$$

Ejercicio 2 :



$$\boxed{\mathbb{P}(B \mid \text{No}G) = 0.7143} \quad (*)$$

$$\mathbb{P}(A \mid G) = \frac{\mathbb{P}(G \mid A)\,\mathbb{P}(A)}{\mathbb{P}(G)} = \frac{0.85 \times 0.4}{0.85 \times 0.4 + \underbrace{\mathbb{P}(G \mid B)}_{?}\,\underbrace{\mathbb{P}(B)}_{0.6}}$$

Tenemos que hallar $x = \mathbb{P}(G \mid B)$. Usamos $(*)$

$$\underset{0.7143}{\mathbb{P}(B \mid \text{No}G)} = \frac{\overbrace{\mathbb{P}(\text{No}G \mid B)}^{1-x}\,\overbrace{\mathbb{P}(B)}^{0.6}}{\mathbb{P}(\text{No}\,G)} = \frac{(1-x)\,0.6}{0.4 \times 0.15 + 0.6\,(1-x)} \Longrightarrow$$

Resolver :
$$0.7143 = \frac{(1-x)\,0.6}{0.4 \times 0.15 + 0.6(1-x)} \longrightarrow \boxed{x = 0.75}$$

$$0.7143 \times 0.4 \times 0.15 = (1-x)\,0.6\,(1-0.7143)$$

$$\frac{0.7143 \times 0.4 \times 0.15}{0.6\,(1-0.7143)} = 1-x$$

Entonces $\mathbb{P}(A|G) = \dfrac{0.85 \times 0.4}{0.85 \times 0.4 + 0.75 \times 0.6} = 0.43 \longrightarrow$ B

Ejercicio 3:

Sea $p = \mathbb{P}(\text{"el jugador acierta"}) = \dfrac{C^{30}_3}{C^{40}_3} \longleftarrow$ las 3 cartas son de cualquier palo menos oro.

Si $X = \#$ de aciertos $\sim Bin(5,p)$

$$\mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X=1) - \mathbb{P}(X=0) = 1 - C^5_1\, p(1-p)^4 - C^5_0\, p^0 (1-p)^5$$

$$\approx 0.68 \quad \text{D}$$

Ejercicio 4:

Sea $X = \#$ de informes leídos (incluye el informe en el que se detecta un incumplimiento)

$$\Longrightarrow X \sim Geo(p) \qquad p = \mathbb{P}\left(\begin{array}{l}\text{"el accidente lo provoca el} \\ \text{error de un empleado"}\end{array}\right) \underset{\substack{\uparrow \\ \text{según el} \\ \text{prevencionista}}}{=} 0.35$$

$$\mathbb{P}(X \geq 3) = 1 - \mathbb{P}(X=2) - \mathbb{P}(X=1)$$

$$= 1 - (1-p)p - p$$

$$\approx 0.42 \longrightarrow \text{A}$$

Ej 5:

$$1 = \int_{-\infty}^{+\infty} f_X(x)\,dx = \int_0^1 kx^2\,dx + \int_1^{+\infty} \frac{k\,dx}{x^3} = k\left[\frac{x^3}{3}\Big|_0^1 - \frac{1}{2x^2}\Big|_1^{+\infty}\right]$$

$$= k\left[\frac{1}{3} - 0 + \frac{1}{2}\right] = k\left[\frac{5}{6}\right] \Longrightarrow \boxed{k = \frac{6}{5}}$$

$$\mathbb{P}(0 \leq X \leq 4 \mid X \geq 1) = \frac{\mathbb{P}(0 \leq X \leq 4 \,;\, X \geq 1)}{\mathbb{P}(X \geq 1)} = \frac{\mathbb{P}(1 < X \leq 4)}{\mathbb{P}(X \geq 1)} = \frac{(*)}{(**)}$$

$$(*) = \int_1^4 \frac{k}{x^3}\,dx = k\left(\frac{-1}{2x^2}\right)\Big|_1^4 = k\left(\frac{-1}{2.16} + \frac{1}{2}\right) = k\left(\frac{-1+16}{32}\right) = k\,\frac{15}{32}$$

$$(**) = \int_1^{+\infty} \frac{k}{x^3}\,dx = \frac{-k}{2x^2}\Big|_1^{+\infty} = 0 + \frac{k}{2}$$

$$\longrightarrow \frac{(*)}{(**)} = \frac{\cancel{k}\,\frac{15}{32}}{\frac{\cancel{k}}{2}} = 2 \times \frac{15}{32} = \frac{30}{32} = \boxed{\frac{15}{16}} \quad y\ k = \frac{6}{5} \longrightarrow (C)$$

Ejercicio 6,

33 X
66 Y

Sea T = duración de la bombita:

Si la bombita es de X: $T \sim Exp(\lambda = \frac{1}{4})$

" Y: $T \sim Exp(\lambda = \frac{1}{5})$

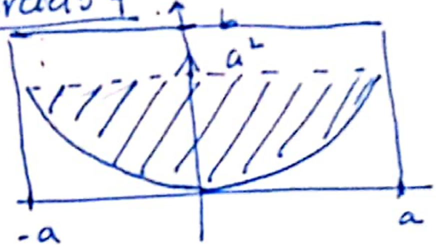
Recordar: $F_T(x) = \mathbb{P}(T \leq x) = 1 - e^{-\lambda x}$

$$\mathbb{P}(T \geq 5) = \underbrace{\mathbb{P}(T \geq 5 \mid X)}_{\substack{1 - F_T(5) \\ \text{con } T \sim Exp(\lambda = \frac{1}{4})}} \underbrace{\mathbb{P}(X)}_{\frac{33}{99}} + \underbrace{\mathbb{P}(T \geq 5 \mid Y)}_{\substack{1 - F_T(5) \\ \text{con } T \sim Exp(\lambda = \frac{1}{5})}} \underbrace{\mathbb{P}(Y)}_{\frac{66}{99}}$$

$$= e^{-\frac{1}{4}.5}\,\frac{33}{99} + e^{-\frac{1}{5}5}\,\frac{66}{99}$$

$$\approx 0.3408 \longrightarrow (E)$$

Ejercicio 7



$$f_X(x) = \begin{cases} \frac{1}{2a} & \text{si } x \in [-a, a) \\ 0 & \text{si no} \end{cases}$$

$$f_Y(x) = \begin{cases} \frac{1}{b} & \text{si } x \in [0, b] \\ 0 & \text{si no} \end{cases}$$

$$\mathbb{P}(X^2 \leq Y \leq a^2) = \iint_{\{(x,y):\ x^2 \leq y \leq a^2\}} f_{XY}(x,y)\,dx\,dy \underset{\text{indep}}{=} \iint_{\{(x,y):\ x^2 \leq y \leq a^2\}} f_X(x)\, f_Y(y)\,dx\,dy$$

$$= \int_{-a}^{a} dx \int_{x^2}^{a^2} \frac{1}{2a}\,\frac{1}{b}\,dy$$

$$= \frac{1}{2ab} \int_{-a}^{a} \Big(\underbrace{y \Big|_{x^2}^{a^2}}_{a^2 - x^2} \Big) = \frac{1}{2ab} \int_{-a}^{a} (a^2 - x^2)\,dx$$

$$= \frac{1}{2ab} \left(a^2 x - \frac{x^3}{3} \right) \Big|_{-a}^{a}$$

$$= \frac{1}{\cancel{2}ab} \Big[\underbrace{a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3} \right)}_{\cancel{2}\left(a^3 - \frac{a^3}{3} \right)} \Big]$$

$$= \frac{1}{b} \left(a^2 - \frac{a^2}{3} \right) = \frac{2a^2}{3b}$$ B

Ejercicio 8:

Se pide $\mathbb{P}(|X-Y|=1)$

Valores de $|X-Y|$:

$Y \backslash X$	1	2	3
0	1	2	3
1	0	1	2
2	1	0	1

$$\mathbb{P}(|X-Y|=1) = \mathbb{P}(X=1; Y=0) + \mathbb{P}(X=1; Y=2) + \mathbb{P}(X=2; Y=1) + \mathbb{P}(X=3; Y=2)$$

$$= a + 0.1 + b + c$$

$$= a+b+c+0.1$$

Pero como se tiene qe cumplir qe las prob. dentro de la tabla sumen 1 $\Rightarrow$

$$\underbrace{a+b+c+0.1}_{\text{despejo de acá}} + (0.2 + 0.05 + 0.03 + 0.1 + 0.05) = 1 \Rightarrow$$

$$a+b+c+0.1 = 1 - 0.43 \Rightarrow a+b+c+0.1 = \boxed{0.57} \quad \text{D}$$

(No es necesario determinar el valor de a, b y c)