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# 8 Soil Pollution and Its Control

## 8.1 INTRODUCTION

Contamination of soil was recognized only in the past few decades as a significant pollution problem. The primary paths of exposure to pollutants are via ingestion of food or water or airborne inhalation. Soil does not generally represent a direct path of exposure. Some portion of soil and the contaminants it may contain may be suspended in the air or in water potentially providing some exposure. In addition, contaminants may desorb into an air or water phase from the soil particles to which it is associated and provide a path of exposure.

In this chapter we will examine the basic characteristics of soil, the contaminant fate and transport characteristics in soil, and processes and approaches to remove or otherwise render harmless contaminants in soils. In the previous chapter we touched on the transport of soils by flowing water, so we will focus here on stable soils, that is soils that are largely immobile. In such soils, contaminants can move only by release into the pore fluid (either air or water) and by subsequent pore fluid transport processes. Significant groundwater flows occur in soils that can lead to advection and transport of contaminants. In addition, there is the possibility of unsaturated soils, that is, soils in which the pore space is only partially filled with water. Both of these issues will be addressed in detail in this chapter.

## 8.2 SOIL CHARACTERISTICS

Soils are of interest as a medium for the transport of fluids because they are *porous* and *permeable.* Porous refers to the pore spaces that constitute the gaps between the soil grains. The permeability of a soil refers to its ability to translate hydraulic forces into fluid flow and it is a function of the total volume and structure of this pore space. The permeability of a soil to movement of a particular fluid also depends on the fraction of the pore space filled by that fluid. If the pore space is *saturated* as in the bottom sediment considered in the previous chapter, the permeability of a medium is at a maximum. If the fluid only fills a portion of the pore space as in the upper layers of soil, the permeability to the fluid decreases and *wettability* becomes important. Wettability is the characteristic of certain fluids to adhere to the surface of the soil grains and it significantly influences the mobility and retention of a fluid by the media.

The basic system of soil and water that is of interest is shown in Figure 8.1. Near the surface, the pore spaces of most soils are partially filled with both air and water. This is the *unsaturated* or *vadose zone* of the soil. Immediately after a rainfall, the water that infiltrates into the soil drains into the deeper soil. Due to the fact that water tends to wet most soils, however, a portion of the water is retained. The quantity of water that remains, typically measured on a volume water per volume total soil basis, is termed the *field capacity* of the soil. Below this zone is often an *aquifer* in which the pore spaces are *saturated,* or filled, with water. This water can either be *confined* by layers of low permeability soil or *unconfined* and free to rise and fall according to the water that infiltrates from above. An unconfined aquifer also is called a *water table* aquifer and the water table refers to the height of the column of water in the aquifer as measured by a well. Due to the wetting of the soil by water, there is additional water above the level that would show in a well, but this water is held by the soil in a *capillary fringe* and not generally available for flow or removal. The water level as measured in a well is a measure of the *hydraulic head* which is the driving force

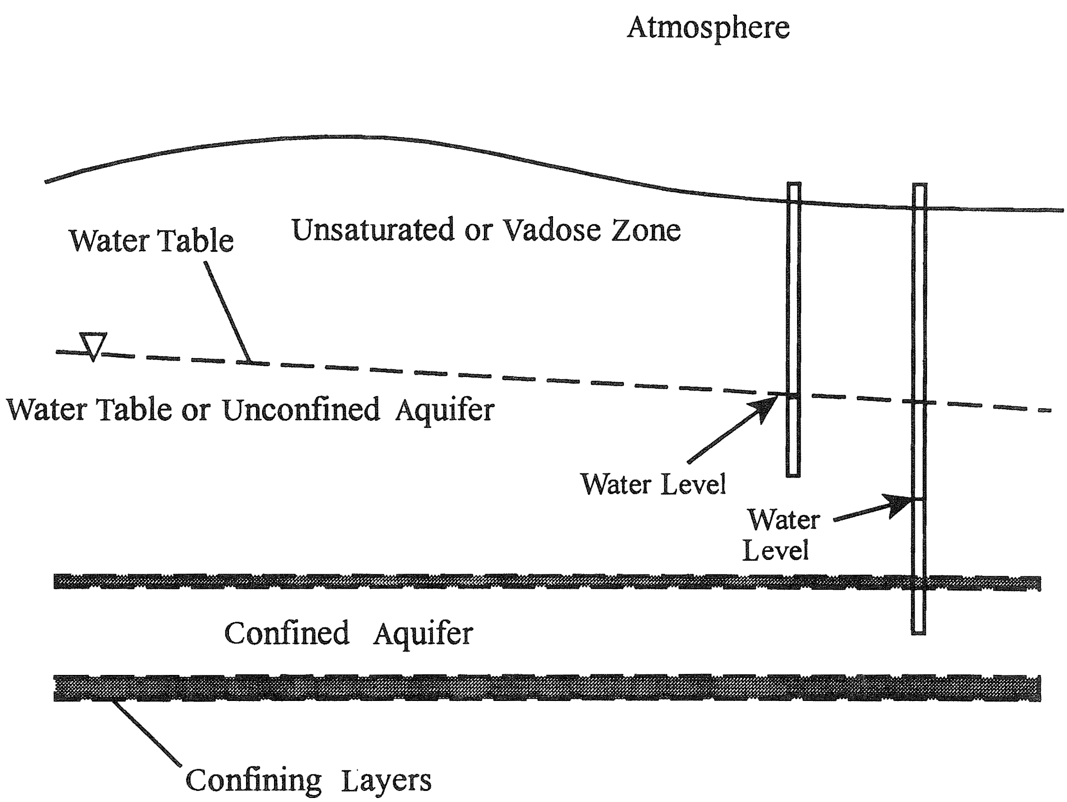


FIGURE 8.1 Depiction of soil-water system of interest showing vadose zone and both an unconfined and confined aquifer. Note that the head or water level in the well in the confined and unconfined aquifer are not equal unless there is a hydraulic connection between the two aquifers.

for water flow. A well used to measure the hydraulic head is termed a *piezometer.* The water in a confined aquifer may be under pressure causing the water level in a well to rise toward the surface. A pressurized confined aquifer is referred to as an *Artesian aquifer.* Both a confined and unconfined aquifer also are depicted in Figure 8.1.

The study of fluid motion in a porous medium will start with a discussion of the soil grains that make up the medium, the pore spaces formed by the space between these grains, and the permeability and wettability of the medium that results. Each of these will be discussed in turn.

### 8.2.1 Solid Phase Characteristics

The basic composition of soils has been discussed previously in Chapters 4 and 7. Soil is composed of a mixture of sand, silt, and clay in varying proportions. Sand is the description of soil particles of diameter greater than 20 μm (International Soil Science Society) to 50 μm (U.S. Department of Agriculture). It is of high permeability or low resistance to flow and contains minimal organic matter and has a low sorption capacity. Silt (2 to 20–50 μm in size) and clay (<2 μm) exhibit more resistance to flow and account for most of the sorption capacity of a soil. All soils are some mixture of sand, silt, and clay and Figure 8.2 shows a classification of various soil types by their composition. The basic characterization of soils is thus by their particle size distribution. In addition to the textural classes depicted in Figure 8.2, a soil might be characterized by its mean particle size, $d_{50}$. The notation $d_{50}$ means that 50% of the particles in a soil are equal to or smaller than this diameter. Another common indicator of particle size distribution is the *uniformity coefficient*, which is the

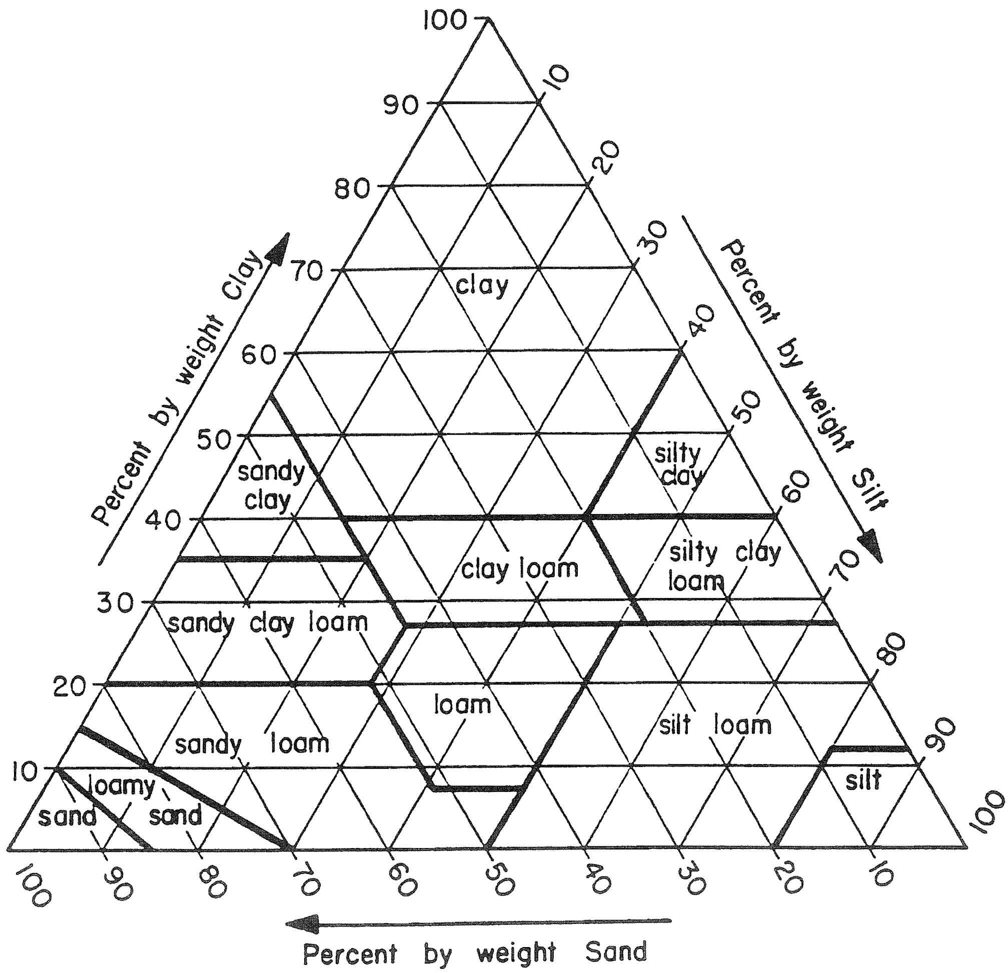


**FIGURE 8.2** Soil classification system. (From Hillel, D. (1980) *Fundamentals of Soil Physics,* Academic Press, New York. With permission.)

ratio of $d_{60}$ to $d_{10}$, or the ratio of the particle size greater than 60% of the particles to that greater than 10% of the particles.

Soils develop via two basic mechanisms, deposition by winds (aeolian soils) and water (alluvial soils) or by weathering of parent bedrock. Soils that form via deposition processes tend to be layered. For example, during high water flow conditions only coarser material may settle while under low flow conditions settling of finer grained material may occur. These layers of differing soil type exhibit different permeabilities and result in the development of preferential flow channels. Soils formed by weathering of bedrock may be layered as well but often not as distinctly. The fine clays in this soil can be washed out of upper layers (the process of eluviation) and washed in to lower layers (the process of illuviation). The upper layers of soil typically contain the bulk of the organic matter in soil due to the presence of decaying plant and animal matter. A surface soil might contain 2 to 4% organic matter although peat and some other high organic matter surface materials may contain considerably more. In deeper soils, the organic matter may be less than a few tenths of a percent.

Of course our primary interest is not in the solid particles but in the void or pore spaces between particles where the mobile fluid, either air or water, resides. The most basic characterization of the void space of a soil is its volume, the voidage or porosity of a soil. The porosity as we have used it previously is defined simply as the ratio of the volume of void space to the total volume of the soil,

$$\varepsilon = \frac{V_v}{V_t} \tag{8.1}$$

The bulk or dry density of a soil, $\rho_b$, can be related to the porosity through the solid grain density, $\rho_s$,

$$\rho_b = \rho_s(1-\varepsilon) \tag{8.2}$$

The porosity of most soils does not vary a great deal, remaining in the range of 25 to 50% and typically between 35 and 45%. Only a portion of this porosity may be available for penetration by a mobile fluid but the total porosity typically remains within the range of 25 to 50%. A random packing of identical spheres exhibits a porosity of about 0.40 to 0.44. A regular packing of identical spheres produces a porosity as high as 0.48 (a cubic lattice packing) or as low as 0.26 (a rhombohedral lattice packing). Of course, the presence of different particle sizes can reduce these theoretical porosities as a result of the smaller particles filling the void space between larger particles.

The shape of the void space between particles also is important but it is not easily estimated except in uniform packings of identical spheres. In any real media, the pores are a distribution of sizes just as the solid grains exhibit a distribution of sizes. The pore size distribution is usually measured by the pressure required to force a volume of liquid into the pores and is a function of *wettability*. The pore size distribution is an important consideration in flow through porous media with the finest pores representing the greatest resistance to flow. Thus, as will be indicated later, the permeability of a medium may be dominated by the effect of a relatively small fraction of large pores. As is often the case with irregular flow paths, the hydraulic diameter, 4 times the ratio of the pore volume to the wetted surface area, is often used to characterize the flow through the pore space. Considering a single sphere and the void space surrounding it

$$\begin{aligned}
\text{Volume of soil grain } &= \frac{1}{6}\pi d_p^3 \\
\text{Volume of void space} &= \frac{\varepsilon}{1-\varepsilon}\frac{1}{6}\pi d_p^3 \\
\text{Wetted grain surface area} &= \pi d_p^2 \\
\text{Hydraulic diameter } d_h &= \frac{4\dfrac{\varepsilon}{1-\varepsilon}\dfrac{1}{6}\pi d_p^3}{\pi d_p^2} = \frac{2}{3}\frac{\varepsilon}{1-\varepsilon}d_p
\end{aligned} \tag{8.3}$$

Often the pore space is not filled with the flowing fluid. *Saturation* is the indicator of the relative proportion of the pore space filled with a particular fluid. For example, if $\varepsilon_w$ is the fraction of the total volume filled with water (the *volumetric content*) then the water saturation, $\varphi_w$, is

$$\varphi_w = \frac{\varepsilon_w}{\varepsilon} \tag{8.4}$$

The hydraulic diameter, as a ratio of (four times) flow volume to wetted perimeter depends on the saturation. This quantity also depends upon the wetting characteristics of the flowing fluid for the soil because some fluids (typically water) may wet the entire perimeter of the pore space at very

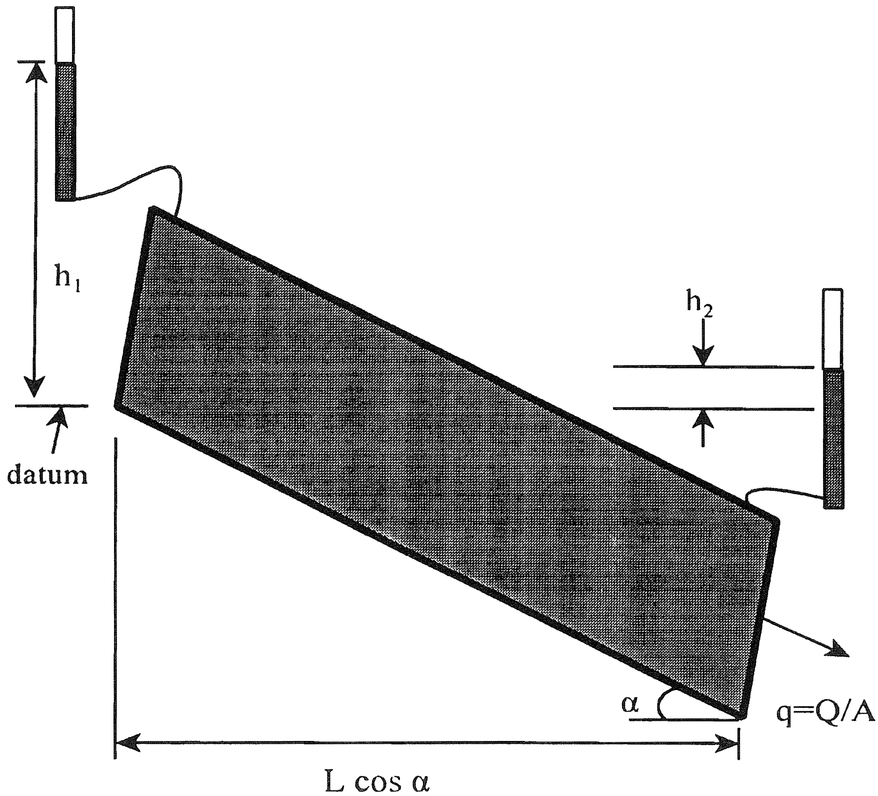


FIGURE 8.3 Soil column for the development of Darcy's Law.

low saturations. Because of their very different behavior, let us discuss flow in water-saturated pore spaces and partially saturated pore spaces separately.

### 8.2.2 Flow of Fluid in Fully Saturated Media

Fully saturated media are media in which the pore spaces are filled with the flowing fluid of interest, typically water. Permeability is defined by Darcy's Law which states that the volumetric flowrate of the fluid, Q, through an area, A, of a porous medium is proportional to the hydraulic head applied across the medium. Hydraulic head is the height that a column of liquid would reach in a manometer if connected to the porous medium and is measured by the water levels in the wells in Figure 8.1. For analysis of Darcy's Law, let us consider the soil column in Figure 8.3. The different heights of liquid in the manometers at each end of the soil column indicate that a gradient in hydraulic head across the medium equal to $\Delta h = h_2 - h_1$ exists. Darcy's Law then states

$$\frac{Q}{A} = q = -k_p \frac{\Delta h}{L}$$

or more generally (8.5)

$$q = -k_p \frac{dh}{dx}$$

$k_p$ is the hydraulic conductivity or, using a term that is largely out of fashion, the coefficient of permeability. The negative sign recognizes that the change in liquid column height (or head) must be negative (i.e., $h_2 - h_1 < 0$, or $h_2 < h_1$) for a positive flow, that is a flow from left to right in

Figure 8.3. The conductivity has units of velocity and the quantity Q/A or q is often called the Darcy velocity, a superficial velocity that represents the average fluid velocity in the porous medium based on the entire cross-sectional area of flow. In reality, of course, only a portion of the cross-sectional area given by the porosity ε is actually available for flow. Thus, the actual average fluid velocity in the interstitial spaces, $u_i$, is given by

$$u_i = \frac{q}{\varepsilon} \tag{8.6}$$

Before discussing Darcy's Law further, let us examine the meaning of the driving force, the hydraulic head. The driving force for flow of the fluid in the pores of a soil is its potential energy. Referring back to Chapter 4, the potential of a fluid is the energy it contains relative to some reference state. The contributions to its total energy are enthalpy, potential energy and kinetic energy. If we consider a fluid moving at velocity v, elevation z, and pressure P, its energy content (per unit mass) relative to an unmoving fluid at the reference elevation ($z = z_0$) and reference pressure ($P = P_0$) is

$$\hat{E} = \hat{E}_I + (P - P_0)\hat{V} + g(z - z_0) + \frac{U^2}{2} \tag{8.7}$$

For flow of a fluid in soils under natural gradients, the changes in internal and kinetic energy are generally negligible compared to the effects of pressure differences and gravity. Recognizing that the specific volume, $\hat{V}$ is just the inverse of density, ρ, and using the reference elevation $z_0 = 0$ where the pressure is $P_0 = 0$, the energy per unit mass becomes

$$\hat{E} = \frac{P}{\rho} + gz \tag{8.8}$$

As discussed in Chapter 3, the pressure measured by the manometers in Figure 8.3 is related to the height of liquid in the manometer relative to the connection point, $h_p$, through $P = \rho g h_p$. Thus, the hydraulic head can be seen as the sum of a *pressure head*, $h_p$, and an *elevation head*, $h_e$, and through Equation 8.8 is related to the energy content or potential driving flow through the medium.

$$h = h_p + h_e = \frac{\hat{E}}{g} \tag{8.9}$$

hydraulic head = pressure head + elevation head = flow potential

In a static column of fluid, the hydraulic head is a constant. As one moves down the column of fluid, the pressure head increases to exactly offset the reduction in elevation head, a consequence of the hydrostatic equation, $\Delta P/\rho g = \Delta h_p = \Delta h_e$. Only when these heads do not balance can flow occur. In a water table aquifer where the free surface is at atmospheric pressure, the pressure head can be taken as zero and flow will occur when the elevation head changes with position, that is, when there is a slope to the free surface. In a confined aquifer, the pressure head may be nonzero everywhere in the aquifer and flow may occur due to pressure variations even if the elevation head is constant. In such a case, the variations in pressure head result in the variations in total head required for flow.

The total head in a groundwater aquifer can be measured by the water level in a well. If a well is installed solely for the purpose of measuring elevation and total head, it is termed a *piezometer.*

At the free surface in a well open to the atmosphere, the pressure head relative to atmospheric pressure is zero and the elevation head at the free surface then equals the total head. In a water table aquifer, the water level in the well and the water table or level in the surrounding media will be the same. Measurement of the water level elevation in a well, that is, the total hydraulic head, is then as simple as measuring the depth to water in the well minus the elevation of the measuring point to a common reference datum, such as sea level or any locally convenient reference point. Note that the ground surface does not provide a suitable reference or *datum* since its level will change leading to misinterpretation of the changes in elevation in head of the groundwater aquifer. In a confined aquifer, the water level also can be measured by a piezometer. The water level must continue to rise until the elevation head is equal to the total head since the pressure head at the free surface is zero relative to atmospheric pressure.

This form of Darcy's Law can be understood if one models the porous medium as a bundle of capillary tubes. This is a crude model of the pore structure of a porous medium but it does indicate the basic fluid flow behavior. Considering a single cylindrical pore in the bundle, Newton's second law, or the law of conservation of momentum, says that the time rate of change of momentum in a given direction is balanced by the net force acting in that direction,

$$\frac{d(mv)_i}{dt} = \sum F_i \tag{8.10}$$

Considering steady flow through the pore, then the net force acting on the control volume shown in Figure 8.4 is zero. The forces acting on that control volume in the axial direction include the viscous force associated with the shear stress from Newton's law of viscosity and the pressure force. Flow within a porous medium is one of the few situations in the environment where turbulence is not generally important and viscous effects dominate. Under such conditions, it is possible to derive analytical expressions between velocity and pressure.

The viscous force is the product of the shear stress ($\tau_{xr}$, force per unit area) and the area on which it acts ($2\ \pi\ r\ dx$). At radial position r, this is

$$\text{Viscous force}\big|_r = \tau_{xr} 2\pi r dx\big|_r = \mu \frac{\partial u}{\partial r} 2\pi r dx\big|_r \tag{8.11}$$

The viscous force is thus the "pull" associated with the movement of fluid outside the cylindrical shell transferred to the fluid within the shell via viscosity. As shown in Figure 8.4, the net viscous force is the difference between the force tending to accelerate the fluid in the cylindrical shell from below and the force tending to slow the fluid in the cylindrical shell from above.

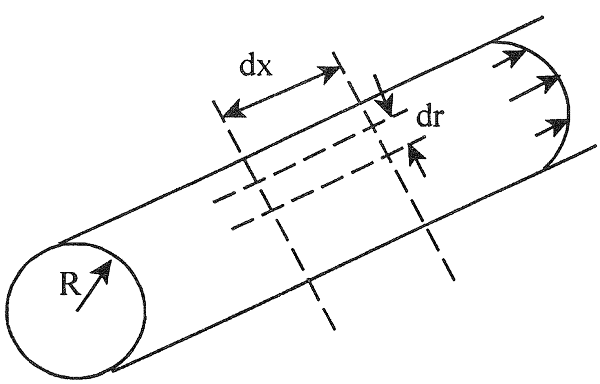


**FIGURE 8.4** Control volume for flow in a single capillary in a bundle of capillary tubes model of porous medium.

$$F_v = \tau_{xr}(2\pi r)dx\big|_r - (-\tau_{xr})(2\pi r)dx\big|_{r+dr}$$

$$= -\mu\frac{\partial u}{\partial r}(2\pi r)dx\big|_r - \left(\mu\frac{\partial u}{\partial r}\right)(2\pi r)dx\big|_{r+dr} \tag{8.12}$$

Similarly the pressure force is the product of the area and the pressure (also a force per unit area and equal to the hydraulic head, h, times ρg). The pressure force acts on the area 2πr dr and thus the net pressure force acting on the cylindrical shell in Figure 8.4 is

$$F_p = \rho g h\, 2\pi r\, dr\big|_x - \rho g h\, 2\pi r\, dr\big|_{x+dx} \tag{8.13}$$

Summing the pressure and viscous forces, dividing by the product (dr dx) and collecting terms gives

$$\frac{\mu\frac{\partial u}{\partial r}(2\pi r)\big|_{r+dr} - \mu\frac{\partial u}{\partial r}(2\pi r)\big|_r}{dr} = \frac{\rho g h(2\pi r)\big|_{x+dx} - \rho g h(2\pi r)\big|_x}{dx} \tag{8.14}$$

In the limit as the cylindrical shell becomes a differential element, that is when dr dx → 0.

$$\frac{\partial}{\partial r}(2\pi r)\mu\frac{\partial u}{\partial r} = \frac{\partial(\rho g h)}{\partial x}(2\pi r) \tag{8.15}$$

Dividing both sides by 2πr, the constant 2 π cancels but r cannot be moved across the differential. Taking also the viscosity as a constant gives

$$\frac{\mu}{r}\frac{\partial}{\partial r}r\frac{\partial u}{\partial r} = \rho g\frac{\partial h}{\partial x} \tag{8.16}$$

as the equation governing the relationship between the velocity in the capillary tube and the pressure gradient along its length. Note that the length of the capillary is typically longer by a factor $L_c/L$ than the length of the medium, and the pressure gradient is smaller than that based on the length of the medium by the same factor. The factor $L_c/L$ is termed the tortuosity factor, τ, and the pressure gradient is given by

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial L}\frac{L}{L_c} = \frac{1}{\tau}\frac{\Delta h}{L} \tag{8.17}$$

Taking this gradient as constant, the velocity will only depend upon the radius r and the equation becomes the ordinary differential equation

$$\frac{d}{dr}r\frac{du}{dr} = \left(\frac{\rho g \Delta h}{\mu L \tau}\right)r \tag{8.18}$$

which can be integrated once to give

$$r\frac{du}{dr} = \frac{\rho g \Delta h}{\mu L \tau}\frac{r^2}{2} + C_1 \tag{8.19}$$

where $C_1$ is a constant of integration. Dividing by r and integrating once again

$$u = \frac{\rho g \Delta h}{\mu L \tau}\frac{r^2}{4} + C_1 \ln r + C_2 \tag{8.20}$$

where $C_2$ is the second constant of integration. Between r = 0 (the center of the capillary) and r = R (the capillary wall), the velocity must be everywhere finite. That is, $C_1$ must be zero or else the logarithm term would not be bounded at r = 0. In addition, friction at the tube wall causes the velocity to approach 0 there. Therefore

$$\begin{aligned} 0 &= \frac{\rho g \Delta h}{\mu L \tau}\frac{R^2}{4} + C_2 \\ C_2 &= -\frac{\rho g \Delta h}{\mu L \tau} R_2 \end{aligned} \tag{8.21}$$

This gives the velocity profile in the capillary tube as

$$u = \frac{\rho g \Delta h}{4\mu L \tau}(r^2 - R^2) \tag{8.22}$$

This implies that the velocity is positive when Δh (= $h_2 - h_1$) is negative (since R > r). This simply means that the pressure at the downstream end of the capillary tube is less than the pressure at the upstream end when the flow is positive (i.e., from upstream to downstream).

Although Equation 8.22 does describe the velocity profile in the capillary tube, rarely are we interested in this level of detail. Instead we are more interested in the relationship between the volumetric flowrate and the pressure drop, as in Darcy's Law. The volumetric flowrate is the product of the velocity and the flow area or

$$\begin{aligned} Q &= \int_0^R u(r)\varepsilon 2\pi r\, dr \\ &= \int_0^R \frac{\varepsilon 2\pi \rho g \Delta h}{4\mu L \tau}(r^3 - rR^2)dr \\ &= \frac{\varepsilon 2\pi \rho g \Delta h}{4\mu L \tau}\left(\frac{r^4}{4} - R^2\frac{r^2}{2}\right)\Bigg|_0^R \\ &= -\frac{\varepsilon \pi \rho g \Delta h}{8\mu L \tau}R^4 \end{aligned} \tag{8.23}$$

This approach recognizes that the flow area is on average equal to ε times the total area. The superficial velocity, or Darcy velocity is then

$$\frac{Q}{A} = \frac{Q}{\pi R^2} = -\frac{\varepsilon R^2 \rho g}{8\mu\tau}\frac{\Delta h}{L} \tag{8.24}$$

or, by comparison to Darcy's Law,

$$k_p = -\frac{\varepsilon R^2 \rho g}{8\tau\mu} \tag{8.25}$$

Thus, the fluid hydraulic conductivity is seen to encompass fluid dependent properties (ρ and μ) as well as media-dependent properties ($\varepsilon R^2/\tau$). The factor of $^1/_8$ depends on the geometry of the pore through which the fluid is moving. For example, in a wide rectangular pore, the factor is $^1/_3$. If we assume that the pore space is formed from identical spheres and replace the capillary radius in Equation 8.25 with the hydraulic radius from Equation 8.3, the hydraulic conductivity is given by

$$k_p = \frac{1}{72\tau}\frac{\varepsilon^3}{(1-\varepsilon)^2} d_p^2 \frac{\rho g}{\mu} \tag{8.26}$$

This can be compared to the Kozeny-Carmen equation which predicts the permeability in a bed of uniform spheres

$$k_p = \frac{1}{180}\frac{\varepsilon^3}{(1-\varepsilon)^2} d_p^2 \frac{\rho g}{\mu} \tag{8.27}$$

This is equivalent to Equation 8.26 with a tortuosity of 2.5. Saffman (1959) developed a random pore model that arrived at the result that the tortuosity is 3. In practice, neither model can be of much use in predicting permeability except in a clean and uniform sand bed. In a real soil or a fouled sand filter, there is a large variation in pore shapes and sizes which leads to ambiguity, at best, in the analysis of simple permeability relationships.

The separation of fluid and media-dependent properties in the hydraulic conductivity is often explicitly recognized in an alternative formulation of Darcy's Law.

$$\frac{Q}{A} = q = -\kappa_p \frac{\rho g}{\mu}\frac{\Delta h}{L} \tag{8.28}$$

Here, $\kappa_p$ is the intrinsic permeability of the medium and is, in principle, solely dependent upon the characteristics of the medium. In reality, some fluids may alter the medium such as water flowing through swelling clays and thus there may also be a dependence of the intrinsic permeability on the type of flowing fluid. By comparison to Equation 8.24, the intrinsic permeability is seen to be proportional to the square of the capillary radius ($\kappa_p = 1/180[\varepsilon^3/(1-\varepsilon)^2]d_p^2$in the Kozeny-Carmen equation) suggesting that finer grained soils exhibit far lower permeabilities. As a result of this phenomenon, groundwater flows and contaminant migration tends to be far greater in coarse fill zones in soil. As examples, old abandoned creek beds, tree root zones, and any sort of man-made construction from mines to foundation piles would tend to contain and be surrounded by more loosely consolidated and coarser grained material than the soil outside these areas. As a result, these zones offer paths for preferential groundwater and contaminant flow. The assessment of contamination at a site can often be reduced to a problem of finding these preferential flow paths.

Intrinsic permeability and hydraulic conductivity are used in a number of different systems of units. Among the most common and their relationships are

**TABLE 8.1**
**Typical Values of Permeability and Hydraulic Conductivity of Soils**

| | Approximate Range of Intrinsic Permeability or Hydraulic Conductivity (Water) | | | |
|---|---|---|---|---|
| **Soil Type** | $\kappa_p$ D ~ μm² | $\kappa_p$ cm² | $k_p$ m/day | $k_p$ gal/day/ft² |
| Gravel | $100–10^5$ | $10^{-6}–10^{-3}$ | $10^2–10^5$ | $2000–10^7$ |
| Sand | $1–10^3$ | $10^{-8}–10^{-5}$ | $1–10^3$ | 20–20000 |
| Silt and sand | 0.01–100 | $10^{-10}–10^{-6}$ | 0.01–100 | 0.2–2000 |
| Silt | $10^{-4}–1$ | $10^{-12}–10^{-8}$ | $10^{-4}–1$ | $2(10)^{-2}–20$ |
| Silty clay | $10^{-6}–0.1$ | $10^{-14}–10^{-9}$ | $10^{-6}–0.1$ | $2(10)^{-5}–2$ |
| Clay | $10^{-7}–10^{-4}$ | $10^{-15}–10^{-12}$ | $10^{-7}–10^{-4}$ | $2(10)^{-6}–0.002$ |
| Sandstone | $10^{-5}–0.1$ | $10^{-13}–10^{-9}$ | $10^{-5}–0.1$ | $2(10)^{-4}–2$ |
| Fractured rocks | $10^{-4}–10$ | $10^{-12}–10^{-7}$ | $10^{-4}–10$ | 0.002–200 |

$$\text{Hydraulic Conductivity} \quad 1 \text{ m/day} = 24.54 \text{ USgal/day/ft}^2$$

$$\text{Intrinsic Permeability} \quad 1 \text{ D} = 9.87 \times 10^{-9} \text{ cm}^2 \tag{8.29}$$

$$1 \text{ D} \approx 0.835 \text{ m/day (water)} \approx 1 \text{ μm}^2$$

The darcy (D) is the permeability that gives a darcy velocity, q, of 1 cm/s for a fluid with a viscosity of 1 cp (approximately that of water) under a pressure gradient of 1 atm/cm. As indicated above, a soil with a permeability of 1 D would provide a water flow velocity of 0.835 m/day if the water level dropped 1 m per m of length and 0.835 cm/day if the water level dropped 1 m per 100 m of length. Table 8.1 summarizes the range of permeabilities of various soil types

As can be seen in the above table, the observed values of hydraulic conductivity or intrinsic permeability vary over a very wide range. It becomes very difficult to assess, even on the basis of extensive measurements, an estimate of the effective permeability of a subsurface formation. Zones of low and high permeability that differ not by factors of 2 or 3 but by as much as 10 orders of magnitude are commonplace. This is the fundamental problem of understanding fluid flow in the subsurface. It influences the rate and direction of contaminant migration in the subsurface in a profound manner.

It remains to emphasize that the conductivity or permeability must still be multiplied by the gradient in the hydraulic head. These are often linked quantities. If a large head gradient existed across a very permeable medium, a large flow would result that would tend to raise the water level and the hydraulic head downstream. The net effect is that the range of groundwater flowrates is typically less than the range of hydraulic permeabilities. Flowrates tend to be higher in more permeable media but the hydraulic head gradients tend to be smaller than in less permeable media. Typical groundwater flowrates vary from a few centimeters per year to a few hundred meters per year.

Darcy's Law also defines the manner in which the head varies in a groundwater system. If we consider steady one-dimensional flow of groundwater in a zone of constant cross-sectional area, a material balance on the differential element of length gives

$$qA\,|_x - qA\,|_{x+dx} = 0$$

dividing by $dx$ (8.30)

$$\frac{q\,|_x - q\,|_{x+dx}}{dx} = 0$$

as $dx \to 0$

$$\frac{d}{dx}q = 0$$

Which simply means that the velocity q is a constant. Substituting the general form of Darcy's Law,

$$\frac{d}{dx}Ak_p\frac{dh}{dx} = 0$$
$$\frac{d}{dx}k_p\frac{dh}{dx} = 0 \tag{8.31}$$

Note that both Equations 8.30 and 8.31 should contain minus signs that can be divided out since the terms are set equal to zero. If the conductivity is uniform in a region, Equation 8.31 can be further written

$$\frac{d^2h}{dx^2} = 0$$

or (8.32)

$$\frac{dh}{dx} = \text{constant} = \frac{\Delta h}{L}$$

$$h = \frac{\Delta h}{L}x + h_0$$

That is, the hydraulic head varies linearly in a steady-state groundwater flow in a homogeneous medium. Example 8.1 illustrates the relationship between head gradient and flow. Note that it is the fact that the head gradient is a constant that allows us to replace the gradient with the difference in hydraulic head over length of travel. It should be emphasized that this is only true in steady (i.e., not time-dependent) flow in a medium with uniform permeability without sources or sinks such as withdrawal or injection wells. If any of these other conditions apply, the material balance must be modified and Equation 8.32 no longer holds.

### *Example 8.1: Relationship between flow and permeability and variations in hydraulic head*

Consider two piezometers that indicate water levels of 2 m and 1 m above sea level, respectively, in a deep water table aquifer. Between the two piezometers is 9 m of sand with a conductivity of 1 m/day and 1 m of silt with a conductivity of 0.01 m/day. What would a piezometer indicate if it were placed at the interface between the sand and the silt? What is the groundwater velocity?

Since the change in height is negligible in the deep water table, the cross-sectional area of groundwater flow is constant and the velocity is also constant. Thus, the difference between the velocity calculated across the sand and silt must be zero.

$$q_{sand} - q_{silt} = 0 = \left(k_p\frac{\Delta h}{L}\right)_{sand} - \left(k_p\frac{\Delta h}{L}\right)_{silt}$$

$$= 1\text{ m/day}\frac{2\text{ m} - h}{9\text{ m}} - 0.01\text{ m/day}\frac{h - 1\text{ m}}{1\text{ m}}$$

Solving this equation for the head at the sand-silt interface, h, h = 1.917. Almost 92% of the total change in head is across the less permeable silt.

The velocity can then be calculated in either the sand or the silt

$$q = \left(k_p \frac{\Delta h}{L}\right)_{sand} = \left(k_p \frac{\Delta h}{L}\right)_{silt}$$

$$= (1 \text{ m/day}) \frac{0.083 \text{ m}}{9 \text{ m}} = (0.01 \text{ m/day}) \frac{0.917 \text{ m}}{1 \text{ m}}$$

$$= 0.00917 \text{ m/day} = 3.35 \text{ m/year}$$

Alternatively, the two resistances in series approach employed to derive the overall mass transfer coefficient in the two-film theory can be employed.

$$\left(\frac{L}{k_p}\right)_{overall} = \left(\frac{L}{k_p}\right)_{sand} + \left(\frac{L}{k_p}\right)_{silt}$$

$$\left(\frac{L}{k_p}\right)_{overall} = 0.00917 \text{ day}^{-1}$$

And the velocity is then again

$$q = \left(\frac{k_p}{L}\right)_{overall} (\Delta h) = (0.00917 \text{ day}^{-1})(1 \text{ m}) = 0.00917 \text{ m/day}$$

Equation 8.31 must be modified when the area through which the groundwater flow changes. This commonly occurs in a shallow water table aquifer as might occur in seepage through a dike or levee. If the depth of the aquifer is not much larger than the change in elevation in the water table, the change in flow area must be considered.

If the flow area is the height of the water table above an impermeable layer, h, times its width, w, Equation 8.31 becomes

$$\frac{d}{dx} qhw = 0$$

$$\frac{d}{dx} k_p wh \frac{dh}{dx} = 0 \qquad (8.33)$$

$$\frac{k_p}{2} w \frac{dh^2}{dx} = Q = \text{constant}$$

Integrating between two groundwater levels, $h_1$ and $h_2$, and defining $q_1 = Q/(h_1 w)$, this becomes

$$\frac{Q}{h_1 w} = q_1 = \frac{K_p}{2L} \frac{h_1^2 - h_2^2}{h_1} \qquad (8.34)$$

Note that although the volumetric flowrate is constant, the groundwater velocity changes. Note also that unlike the linear relationship of Equation 8.32, there is now a parabolic relationship between the volumetric flowrate, Q, and head, h.

Equations 8.31 and 8.34 can be compared by writing $h_1 = h_2 + \Delta h$. Then 8.34 can be written

$$\begin{aligned}\frac{Q}{h_1 w} = q_1 &= \frac{k_p}{2L}\frac{h_1^2 - (h_1 - \Delta h)^2}{h_1} \\ &= \frac{k_p}{2L}\frac{h_1^2 - (h_1^2 - 2h_1\Delta h + \Delta h^2)}{h_1} \\ &= \frac{k_p}{2L}\frac{(2h_1\Delta h - \Delta h^2)}{h_1}\end{aligned} \tag{8.35}$$

Which for $\Delta h << h_1$, $\Delta h^2 << 2h_1\Delta h$, and

$$\frac{Q}{h_1 w} = q_1 = q_2 = k_p\frac{\Delta h}{L} \tag{8.36}$$

That is, Equations 8.30 through 8.32 are now seen as the limit of Equation 8.34 when the change in elevation of a water table is small compared to the depth of the aquifer. Example 8.2 reworks Example 8.1 assuming a shallow aquifer.

### *Example 8.2: Flow and head gradients in a shallow unconfined aquifer*

Repeat Example 8.1 assuming that the piezometers indicate the water levels relative to an impermeable strata and thus represent the entire depth of the aquifer at that point.

In this case the volumetric flow per unit width of aquifer is constant,

$$\frac{Q_{sand}}{w} = \frac{Q_{silt}}{w}$$

Setting the difference between these two equal to zero and solving for the water level at the interface, h, using 1 to represent the values in sand and 2 to represent the values in silt, the positive root is the only physically meaningful solution and it gives

$$h = \left(\frac{k_{p1}h_1^2 L_2 + k_{p2}h_2^2 L_1}{k_{p1}L_2 + k_{p2}L_2}\right)^{1/2} = 1.937 \text{ m}$$

Thus, the water level is 2 m higher at the interface between the sand and silt. The volumetric flowrate per unit width of aquifer is then given by Equation 8.34,

$$\frac{Q}{w} = 1 \text{ m/day}\frac{(2 \text{ m})^2 - (1.937 \text{ m})^2}{2(9 \text{ m})} = 0.014\frac{\text{m}^3}{\text{m}^2 \cdot \text{day}}$$

Dividing by $h_1$ to get the velocity at the upgradient edge of the sand gives 0.0068 m/day while dividing by $h_2$ to get the velocity at the downgradient edge of the silt gives 0.014 m/day. Note that the velocity calculated in Example 8.1 lies between these two values. The velocity calculated by Equations 8.30 through 8.32 provides an "average" for the velocity in a shallow water table aquifer.

### 8.2.3 Flow of Fluid in a Partially Saturated Media

The hydraulic conductivity and intrinsic permeability discussion above assumes that water or some other liquid fills all of the pore space available to it. Near the surface, water only tends to fill part of the pore space with the remainder filled by air. This has two immediate consequences. First, the effective permeability of the media with respect to water is reduced in that the total flow area is no longer available. As long as the water, or other partially saturating fluid, is continuous through the media, this is often handled by correcting the conductivity with a relative permeability that is a function of the saturation of the flowing fluid. That is, the Darcy velocity of a fluid present at pore volume fraction, or saturation, $\phi_f$ is given by

$$q = -\kappa_r(\phi_f)\kappa_p \frac{\rho g}{\mu}\frac{\Delta h}{L} \tag{8.37}$$

The relative permeability, $\kappa_r$, ranges between 0 and 1. A common and relatively simple model for the relative permeability is that given by Brooks and Corey (1964)

$$\kappa_r = \left[\frac{\varphi_f - \varphi_{ir}}{1-\varphi_{ir}}\right]^{\frac{2+3b}{b}} \tag{8.38}$$

Here, $\varphi_f$ is the saturation of the flowing liquid, $\varphi_{ir}$ is the saturation beyond which the saturation cannot be reduced (the irreducible residual), and b is a grain-size distribution parameter that varies from about 2.8 in a uniform sand to more than 10 in clays. Typical relative permeability vs. saturation curves are shown in Figure 8.5.

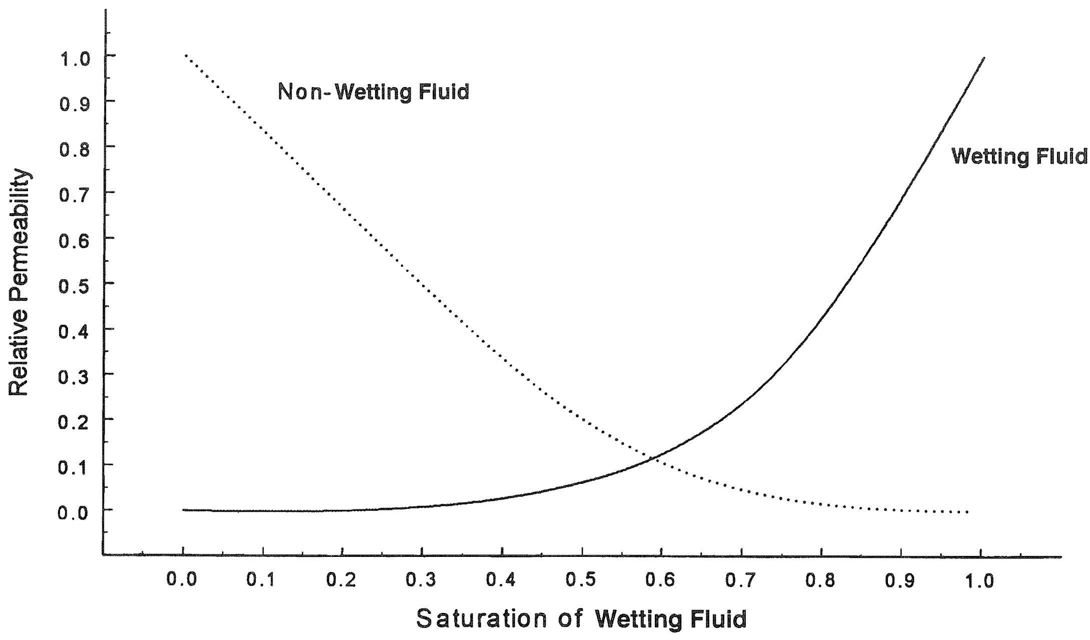


**FIGURE 8.5** Relative permeability vs. fluid saturation.