Hojas de fórmulas

(2021)

Transformada de Laplace ***F*(*s*)**	Función en el tiempo ***f*(*t*)**
1	Impulso unitario
$\dfrac{1}{s}$	Escalón unitario
$\dfrac{1}{s^2}$	t
$\dfrac{n!}{s^{n+1}}$	t^n (n = entero positivo)
$\dfrac{1}{s+a}$	e^{-at}
$\dfrac{1}{(s+a)(s+b)}$	$\dfrac{e^{-at}-e^{-bt}}{b-a}$
$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t\right) \qquad 0<\zeta<1$
$\dfrac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t-Arctg\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\dfrac{1}{(1+Ts)^n}$	$\dfrac{1}{T^n(n-1)!}t^{n-1}e^{-t/T}$
$\dfrac{\omega_n^2}{(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\dfrac{T\omega_n^2 e^{-t/T}}{1-2\zeta T\omega_n+T^2\omega_n^2}+\dfrac{\omega_n e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t-Arctg\left(\dfrac{T\omega_n.\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)\right)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$
$\dfrac{\omega_n}{s^2+\omega_n^2}$	$\mathrm{sen}(\omega_n t)$
$\dfrac{\beta}{(s+\alpha)^2+\beta^2}$	$e^{-at}\,\mathrm{sen}(\beta t)$
$\dfrac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$e^{-at}\cos(\beta t)$
$\dfrac{\omega_n}{(1+Ts)(s^2+\omega_n^2)}$	$\dfrac{T\omega_n e^{-t/T}}{1+T^2\omega_n^2}+\dfrac{sen\left(\omega_n t-Arctg(T\omega_n)\right)}{\sqrt{1+T^2\omega_n^2}}$
$\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t+Arctg\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\dfrac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos(\omega_n t)$
$\dfrac{1}{s(1+Ts)}$	$1-e^{-t/T}$

Transformada de Laplace *F*(*s*)	Función en el tiempo *f*(*t*)
$\frac{1}{s(1+Ts)^2}$	$1-\frac{t+T}{T}e^{-t/T}$
$\frac{\omega_n^2}{s(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{T^2\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}+\frac{e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t-\Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $donde\ \Phi=tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)+tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t+Arctg\left(\frac{1-2\zeta^2}{2\zeta\sqrt{1-\zeta^2}}\right)\right)$
$\frac{\omega_n^2}{s^2(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-T-\frac{2\zeta}{\omega_n}+\frac{T^3\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}+\frac{e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t-\Phi)}{\omega_n\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $donde\ \Phi=2tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)+tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{1}{s^2(1+Ts)^2}$	$t-2T+(t+2T)e^{-t/T}$
$\frac{\omega_n^2(1+as)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{1+2a\zeta\omega_n+a^2\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t+Arctg\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)\right)$
$\frac{\omega_n^2(1+as)}{s^2+\omega_n^2}$	$\omega_n\sqrt{1+a^2\omega_n^2}\,sen\left(\omega_n t+Arctg\left(a\omega_n\right)\right)$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\frac{\omega_n\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $+\frac{(T-a)\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}$ $donde\ \Phi=tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)-tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2+\omega_n^2)}$	$\frac{\omega_n^2(T-a)}{1+T^2\omega_n^2}e^{-t/T}+\frac{\omega_n\sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}}\mathrm{sen}(\omega_n t+\Phi)$ $donde\ \Phi=tan^{-1}\left(a\omega_n\right)-tan^{-1}\left(T\omega_n\right)$
$\frac{\omega_n^2(1+as)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\frac{\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $donde\ \Phi=tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)-tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$

Transformada de Laplace ***F*(*s*)**	**Función en el tiempo** ***f*(*t*)**
$\frac{\omega_n^2(1+as)}{s(1+Ts)(s^2+\omega_n^2)}$	$1+\frac{T\omega_n^2(a-T)}{1+T^2\omega_n^2}e^{-t/T}-\frac{\sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}}\cos(\omega_n t+\Phi)$ $donde\ \Phi = tan^{-1}(a\omega_n)-tan^{-1}(T\omega_n)$
$\frac{\omega_n^2(1+as)}{s(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\frac{\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $+\frac{T(a-T)\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}$ $donde\ \Phi = tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)$ $-tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)-tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\frac{1+as}{s^2(1+Ts)}$	$t+(a-T)(1-e^{-t/T})$
$\frac{s}{s^2+\omega_n^2}$	$\cos(\omega_n t)$
$\frac{s}{(s^2+\omega_n^2)^2}$	$\frac{1}{2\omega_n}t\operatorname{sen}(\omega_n t)$
$\frac{s}{(s^2+\omega_{n1}^2)(s^2+\omega_{n2}^2)}$	$\frac{1}{\omega_{n1}^2-\omega_{n2}^2}\left(\cos(\omega_{n1}t)-\cos(\omega_{n2}t)\right)$
$\frac{s}{(1+Ts)(s^2+\omega_n^2)}$	$\frac{-1}{1+T^2\omega_n^2}e^{-t/T}+\frac{1}{\sqrt{1+T^2\omega_n^2}}\cos\left(\omega_n t-Arctg(T\omega_n)\right)$
$\frac{1+as+bs^2}{s^2(1+T_1s)(1+T_2s)}$	$t+(a-T_1-T_2)+\frac{b-aT_1+T_1^2}{T_1-T_2}e^{-t/T_1}-\frac{b-aT_2+T_2^2}{T_1-T_2}e^{-t/T_2}$
$\frac{\omega_n^2(1+as+bs^2)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\frac{\sqrt{(1-a\zeta\omega_n-b\omega_n^2+2b\zeta^2\omega_n^2)^2+\omega_n^2(1-\zeta^2)(a-2b\zeta\omega_n)^2}}{\sqrt{(1-\zeta^2)}}$ $x\ e^{-\zeta\omega_n t}\operatorname{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $donde\ \Phi = tan^{-1}\left(\frac{\omega_n\sqrt{1-\zeta^2}(a-2b\zeta\omega_n)}{b\omega_n^2(2\zeta^2-1)+1-a\zeta\omega_n}\right)-tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\frac{s^2}{(s^2+\omega_n^2)^2}$	$\frac{1}{2\omega_n}\left(\operatorname{sen}(\omega_n t)+\omega_n t\cos(\omega_n t)\right)$

Respuesta al escalón de sistemas de 2º orden sin ceros

Tiempo de levantamiento 10%-90% :

$$t_R = \frac{0{,}366.e^{2.\zeta} + 0{,}6536}{\omega_n} \qquad 0 < \zeta \le 1$$

$$t_R = \frac{2.\ln(9).\zeta - 1{,}0364/\zeta}{\omega_n} \qquad 1 < \zeta$$

Tiempo de establecimiento n% :

$$t_S = \frac{-\ln\left(\frac{n\%}{100}\sqrt{1-\zeta^2}\right)}{\zeta\omega_n}; \qquad t_S^{5\%} = \frac{3}{\zeta\omega_n} \quad 0 < \zeta < 0{,}6; \qquad t_S^{2\%} = \frac{4}{\zeta\omega_n} \quad 0 < \zeta < 0{,}8$$

Máximo sobretiro y tiempo de máximo sobretiro:

$$M_p(\%) = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad \zeta^2 = \frac{\left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2}{\pi^2 + \left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2} \qquad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Transformadas Z

$\{e(k)\}$	$E(z)$
$\{1\}$	$\frac{z}{z-1}$
$\{k\}$	$\frac{z}{(z-1)^2}$
$\{k^2\}$	$\frac{z(z+1)}{(z-1)^3}$
$\{a^k\}$	$\frac{z}{z-a}$
$\{ka^k\}$	$\frac{az}{(z-a)^2}$
$\{\sin ak\}$	$\frac{z \sin a}{z^2 - 2z \cos a + 1}$
$\{\cos ak\}$	$\frac{z(z - \cos a)}{z^2 - 2z \cos a + 1}$
$\{a^k \sin bk\}$	$\frac{az \sin b}{z^2 - 2az \cos b + a^2}$
$\{a^k \cos bk\}$	$\frac{z^2 - az \cos b}{z^2 - 2az \cos b + a^2}$

TABLE 2-3 TABLE OF COMMONLY USED *z*-TRANSFORMS

Laplace transform, $E(s)$	Time function, $e(t)$	z-transform, $E(z)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$
$\frac{a}{s^2(s+a)}$	$t - \frac{1-\epsilon^{-at}}{a}$	$\frac{Tz}{(z-1)^2} - \frac{(1-\epsilon^{-aT})z}{a(z-1)(z-\epsilon^{-aT})}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z \sin(aT)}{z^2 - 2z\cos(aT) + 1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2 - 2z \cos aT + 1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2 - 2z\epsilon^{-aT}\cos(bT) + \epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2 - z\epsilon^{-aT}\cos bT}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}$

TABLA 3.1 Muestreo de un sistema continuo $G(s)$.

La tabla da el equivalente con mantenedor de orden cero del sistema continuo, $G(s)$, precedido por el mantenedor de orden cero. El sistema muestreado se describe por el operador de transferencia discreta. Para sistemas de segundo orden el operador de transferencia discreta viene dado en términos de los coeficientes de

$$H(q) = \frac{b_1 q + b_2}{q^2 + a_1 q + a_2}$$

$G(s)$	$H(q)$ o los coeficientes de $H(q)$
$\frac{1}{s}$	$\frac{h}{q-1}$
$\frac{1}{s^2}$	$\frac{h^2(q+1)}{2(q-1)^2}$
e^{-sh}	q^{-1}
$\frac{a}{s+a}$	$\frac{1-\exp(-ah)}{q-\exp(-ah)}$
$\frac{a}{s(s+a)}$	$b_1 = \frac{1}{a}(ah - 1 + e^{-ah}) \quad b_2 = \frac{1}{a}(1 - e^{-ah} - ahe^{-ah})$ $a_1 = -(1 + e^{-ah}) \quad a_2 = e^{-ah}$
$\frac{a^2}{(s+a)^2}$	$b_1 = 1 - e^{-ah}(1 + ah) \quad b_2 = e^{-ah}(e^{-ah} + ah - 1)$ $a_1 = -2e^{-ah} \quad a_2 = e^{-2ah}$
$\frac{ab}{(s+a)(s+b)}$	$b_1 = \frac{b(1 - e^{-ah}) - a(1 - e^{-bh})}{b-a}$ $b_2 = \frac{a(1 - e^{-bh})e^{-ah} - b(1 - e^{-ah})e^{-bh}}{b-a}$ $a_1 = -(e^{-ah} + e^{-bh})$ $a_2 = e^{-(a+b)h}$
$\frac{(s+c)}{(s+a)(s+b)}$	$b_1 = \frac{e^{-bh} - e^{-ah} + (1 - e^{-bh})c/b - (1 - e^{-ah})c/a}{b-a}$ $b_2 = \frac{c}{ab}e^{-(a+b)h} + \frac{b-c}{b(a-b)}e^{-ah} + \frac{c-a}{a(a-b)}e^{-bh}$ $a_1 = -e^{-ah} - e^{-bh} \quad a_2 = e^{-(a+b)h}$
$\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = 1 - \alpha\left(\beta + \frac{\zeta\omega_0}{\omega}\gamma\right) \quad \omega = \omega_0\sqrt{1-\zeta^2} \quad \zeta < 1$ $b_2 = \alpha^2 + \alpha\left(\frac{\zeta\omega_0}{\omega}\gamma - \beta\right) \quad \alpha = e^{-\zeta\omega_0 h}$ $a_1 = -2\alpha\beta \quad \beta = \cos(\omega h)$ $a_2 = \alpha^2 \quad \gamma = \operatorname{sen}(\omega h)$
$\frac{s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = \frac{1}{\omega}e^{-\zeta\omega_0 h}\operatorname{sen}(\omega h) \quad b_2 = -b_1 \quad \omega = \omega_0\sqrt{1-\zeta^2}$ $a_1 = -2e^{-\zeta\omega_0 h}\cos(\omega h) \quad a_2 = e^{-2\zeta\omega_0 h}$

q = variable compleja (z)
h = período de muestreo (T)