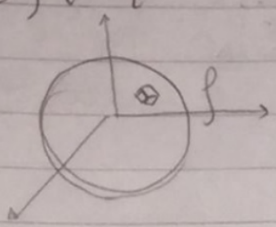
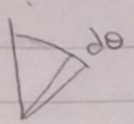
d) i

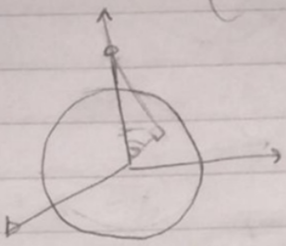


$dv = dr\, r^2 d\theta\, \text{sen}\theta\, d\varphi$

$$\phi = \left[\iiint \frac{r^2\, \text{sen}(\theta)\, d\theta\, d\varphi\, dr}{\sqrt{r^2+z^2-2rz\cos(\theta)}}\right] \frac{\sigma}{4\pi\varepsilon_0}$$

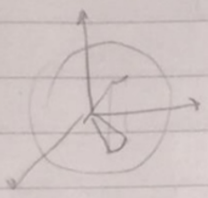
ρ





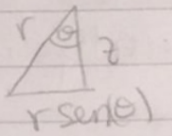
$$d = \sqrt{(r\,\text{sen}(\theta))^2 + (z - r\cos(\theta))^2}$$

$$\text{sen}\theta\, d\theta = \frac{du}{2rz}$$



$$\phi = \frac{\sigma}{2\varepsilon_0} \int_0^a \int_0^{\pi} \frac{r^2\, \text{sen}(\theta)\, dr\, d\theta}{\sqrt{r^2+z^2-2rz\cos\theta}}$$

falta dθ



ojo que el cambio de variable es de θ a u ⇒ hay que evaluar u(θ=0) y u(θ=π)

$$du = \text{sen}(\theta)\, 2rz\, d\theta$$

$$r^2+z^2-2rz\cos(\theta) = u$$

$$\phi = \frac{\sigma}{2\varepsilon_0} \int_0^{\pi} \int_{u(0)}^{u(a)} \frac{r\, du}{2z\, \sqrt{u}} = \frac{\sigma}{4z\varepsilon_0} \int_0^{\pi} r \cdot 2\sqrt{u}$$

$$= \frac{\sigma}{2z\varepsilon_0} \int_0^a r\left[\sqrt{r^2+z^2+2rz} - \sqrt{r^2+z^2-2rz}\right]$$

$z>r$

$$= \frac{\sigma}{2z\varepsilon_0} \int_0^a r\left[|r+z| - |z-r|\right] =$$

Para $z>a \rightarrow |z-r| = z-r$

$z<a \rightarrow |z-r| = r-z$