

$$\begin{aligned}\text{Maximum possible rate} &= VD_{bc} = \left(1 \frac{\text{m}}{\text{s}}\right) (10^{-5} \text{ m})(10^{-3} \text{ g/m}^3) \\ &= 10^{-8} \frac{\text{g}}{\text{m} \cdot \text{s}} = 6.7 \times 10^{-12} \frac{\text{lbm}}{\text{ft} \cdot \text{s}}\end{aligned}$$

If we catch them all, we will collect  $10^{-8}$  g/s for every meter of fiber length. The actual amount caught will be this number times the target efficiency. The separation number is

$$N_s = \frac{(2000 \text{ kg/m}^3)(10^{-6} \text{ m})^2(1 \text{ m/s})}{(18)(1.8 \times 10^{-5} \text{ kg/m} \cdot \text{s})(10^{-5} \text{ m})} = 0.617$$

From Fig. 9.18, we see that for cylinders this value of  $N_s$  corresponds to a target efficiency of about 0.42, so we would expect to collect about  $0.42 \times 10^{-8}$  g/m · s. ■

**Example 9.19.** A filter consists of a row of parallel fibers across a flow, as described in Example 9.18, with the center-to-center spacing of the fibers equal to five fiber diameters. What collection efficiency will the filter have for the particles? Assume that the fibers are far enough apart that each one behaves as if it were in an infinite fluid, uninfluenced by the other fibers.

Here, we can use the preceding results to see that 42 percent of the particles that were traveling directly toward the fibers are collected. If the fibers are spaced five fiber diameters apart, then the open area is 80 percent  $[(5 - 1)/5]$ , and the blocked area is 20 percent (1/5). The target efficiency, as just described, applies only to those particles that were flowing toward the blocked area, so the overall collection efficiency is

$$\begin{aligned}\text{Collection efficiency} &= (\text{target efficiency}) (\text{percentage blocked}) \\ &= 0.42 \times 0.2 = 0.084 = 8.4\%\end{aligned}$$
 ■

**Example 9.20.** A filter consists of 100 rows of parallel fibers as described in Example 9.19, arranged in series. They are spaced far enough apart that the flow field becomes completely uniform between one row and the next (i.e., the rows do not interact). What is the collection efficiency of the entire filter?

Here, we calculate

$$\eta_{\text{overall}} = 1 - p_{\text{overall}} = 1 - (p_{\text{individual}})^n = 1 - (1 - 0.084)^{100} = 0.9998$$
 ■

These three examples show, in idealized form, what goes on within depth filters. Most such filters do not have an orderly array of parallel fibers; the filter medium consists of a tangled jumble of fibers in a random orientation, making up a thick mat. The mat resembles the felt material used to make hats, line pool tables, etc., or steel wool or fiberglass building insulation. (The student should examine a piece of any of these materials to see how different it is from woven cloth.) The idealization that individual fibers do not interact is clearly an approximation. But these thick fiber mats do operate almost entirely by impaction, sometimes called *impingement*, as

calculated here. The individual particles have many chances to contact an individual fiber on their path through the mat, and their likelihood of being caught on any one is shown in Fig. 9.18.

Such filters are often used where the particles to be caught are fine drops of liquids that are only moderately viscous. Such drops will coalesce on the fibers and then run off as larger drops, leaving the fibers ready to catch more fine drops. If the particles were solid, then this type of filter would require regular cleaning; for the liquid application it does not. The most widespread air pollution control use of depth filters is in the collection of very fine liquid drops, *sulfuric acid mist*, produced in sulfuric acid plants. Similar devices are used in many gas-liquid contacting devices to catch fine droplets; one brand uses the trade name Demister. This kind of device is also used for removing solid particles from gas streams that contain few of them, e.g., for cleaning the air of industrial clean rooms or hospital surgical suites and in personal protection dust masks. The filters are thrown away when they have collected enough particles that their pressure drop begins to increase. The depth filters used in those applications are normally called *high-efficiency, particle-arresting* (HEPA) or *absolute filters*. The air filters on household furnaces operate this way as well; typically the fibers are coated with a sticky substance to improve the retention of the collected dust and lint.

Depth filters collect particles mostly by impaction. Some older types of particle collectors also used impaction, to catch particles on solid walls, but they are seldom used now. Some size-specific particle analyzers (*impactors* or *cascade impactors*) use impaction on collecting surfaces to collect specific sizes of particles. In liquid scrubbers (discussed later), one of the principal collection mechanisms is the collision between the particle and a moving drop of liquid (usually water). We will have further use of Fig. 9.18 when we discuss scrubbers.

As discussed in Sec. 8.2.6, small particles move in gases by diffusion. In depth filters that diffusion leads to particle collection in addition to that computed above by impaction. We can use previously developed solutions for mass transfer in gases to compute the efficiency with which particles will diffuse to a collecting surface. In Fig. 9.17 consider the case of a very small particle, for which the separation number is so small that it has practically zero chance of impacting the target. If, however, it is in the stream of gas that passes close to the target and Brownian motion at right angles to the main flow moves it against the target, it will probably adhere. In this case, we would say that it was collected by diffusion (see Sec. 8.2.6) rather than by impaction.

Using this idea, Freidlander developed a theoretical equation, with constants determined by experiment, for the case of diffusional collection of particles from a gas stream flowing past a cylinder under circumstances where impaction was negligible [19]. Most of the published data could be represented by

$$\eta_i = \frac{6D^{2/3}}{\nu^{1/6} D_b^{1/2} V^{1/2}} + \frac{3D^2 V^{1/2}}{\nu^{1/2} D_b^{3/2}} \quad (9.46)$$

where all the terms are as defined previously, and  $\nu$  is the kinematic viscosity. The first term on the right is for diffusional collection, whereas the second is for collection

by noninertial contact. The calculations of Langmuir and Blodgett are based on point masses; the final term in Eq. (9.46) takes into account the fact that the particles have finite diameters and hence will contact the target if their center passes within  $D/2$  of it. This behavior is called *interception*.

**Example 9.21.** Repeat Example 9.18 for particles having a diameter of  $0.1 \mu$ . Take into account impaction, diffusion, and interception.

In this case  $N_s$  is  $(0.1)^2 = 0.01$  times the previous value, or 0.062, for which, from Fig. 9.18 we can read  $\eta_t =$  practically zero. Hence, a particle of this size will not be collected by impaction.

From Fig. 8.1 we can read that the diffusivity is about  $6 \times 10^{-6} \text{ cm}^2/\text{s}$  ( $6 \times 10^{-10} \text{ m}^2/\text{s}$ ). So

$$\begin{aligned} \eta_t &= \frac{6(6 \times 10^{-10} \text{ m}^2/\text{s})^{2/3}}{(1.49 \times 10^{-5} \text{ m}^2/\text{s})^{1/6}(10^{-5} \text{ m})^{1/2}(1 \text{ m/s})^{1/2}} \\ &\quad + \frac{3(10^{-7} \text{ m})^2(1 \text{ m/s})^{1/2}}{(1.49 \times 10^{-5} \text{ m}^2/\text{s})^{1/2}(10^{-5} \text{ m})^{3/2}} \\ &= 0.0086 + 0.00025 = 0.0088 \approx 0.9\% \quad \blacksquare \end{aligned}$$

The diffusion term is  $(0.0086/0.00025) = 34.4$  times the interception term. As the particles become smaller the diffusion term becomes relatively more important, whereas the interception term increases in importance as the particles increase in size. The interception and impaction mechanisms respond in the same general way to changes in velocity and particle diameter. The mechanisms are compared in Table 9.3.

There is some particle size at which there is a minimum collection efficiency (Problem 9.57). Typically, this size is in the range  $0.1$  to  $1 \mu$ , which is the size most likely to be deposited in the human lung. We would like to have a particle collection device that was most efficient for this size particle; no such device is known.

It has also been observed that if the particles are charged before they enter the filter, they will be collected with a higher efficiency than if they are not. This has led to the ESP–baghouse combination, in which an old ESP that does not meet new

**TABLE 9.3**  
**Comparison of collection mechanisms**

	Impaction and interception	Diffusion
Increasing particle size causes efficiency to	Increase	Decrease
Increasing gas velocity causes efficiency to	Increase	Decrease
Increasing target diameter causes efficiency to	Decrease	Decrease

emission standards has a baghouse attached to its downstream side. The particles passing from the ESP to the baghouse are mostly the smallest of the particles that entered the ESP, and many of them are charged. The measured performance of this combination is often better than one would predict for an ESP plus a baghouse treating uncharged particles.

### 9.2.3 Filter Media

Whether a filter behaves as a surface or a depth filter depends on the type of filter medium used. For shake-deflate baghouses (Fig. 9.13) the filter bags are made of tightly woven fibers, much like those in a pair of jeans. (The reader is invited to look at the sun through a single layer of such fabric, seeing that it has some pinholes, allowing light to come through, and to blow into such a fabric, observing that one can breathe in and out through one.) Pulse-jet baghouses (Fig. 9.14) use high-strength felted fabrics, so that they act partly as depth filters and partly as surface filters. This allows them to operate at superficial velocities (air-to-cloth ratios) two to four times those of shake-deflate baghouses; in recent years this higher capacity per unit size has allowed them to take market share away from the previously dominant shake-deflate type baghouses.

Filter fabrics are made of cotton, wool, glass fibers, and a variety of synthetic fibers. The choice depends on price and suitability for the expected service. Cotton and wool cannot be used above 180 and 200°F, respectively, without rapid deterioration, whereas glass can be used to 500°F (and short-term excursions to 550°F). The synthetics have intermediate service temperatures. In addition the fibers must be resistant to acids or alkalis if these are present in the gas stream or the particles as well as to flexing wear caused by the repeated cleaning. Typical bag service life is 3 to 5 years. Generally fibers that have many small microfibers sticking out their sides form better cakes than those that do not. The student should examine under a microscope a thread of cotton, which has such microfibers, and one of monofilament fishing line, which does not.

### 9.2.4 Scrubbers for Particulate Control

Just as filters work by separating the flow of particle-laden gas into many small streams, so also *scrubbers* effectively divide the flow of particle-laden gas by sending many small drops through it.

In air pollution control engineering, the term *scrubber* originally meant a device for collecting fine particles on liquid drops. Then when liquid drops were used to collect sulfur dioxide (see Chapter 11), the devices that did that were also called *scrubbers*. Recently, alas, some other types of devices have been marketed as *dry scrubbers*. In this chapter, we will use the original meaning of the term: a scrubber is a device that collects particles by contacting the dirty gas stream with liquid drops.

Most fine particles will adhere to a liquid drop if they contact it. So if we can make the drop and the particle touch each other, the particle will be caught on the

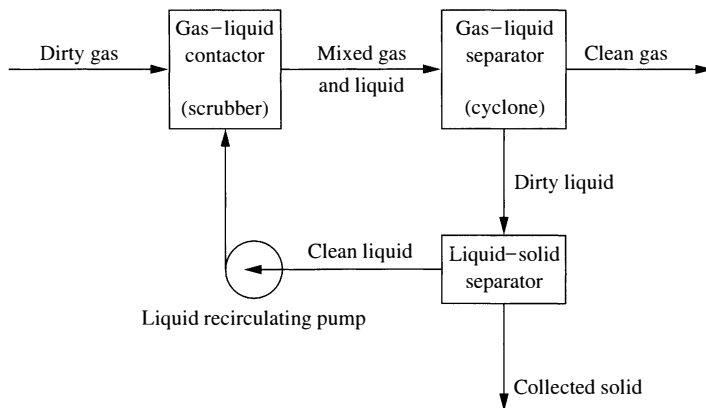
drop. Particles  $50\ \mu$  and larger are easily collected in cyclones. If our problem is to collect a set of  $0.5\text{-}\mu$  particles, cyclones will not work at all. However, if we were to introduce a large number of  $50\text{-}\mu$  diameter drops of a liquid (normally water) into the gas stream to collect the fine particles, then we could pass the stream through a cheap, simple cyclone and collect the drops and the fine particles stuck on them. This idea is the basis of almost all scrubbers for particulate control.

A complete scrubber has several parts, as sketched in Fig. 9.20. Most often, the gas–liquid separator is a simple cyclone of the type discussed in Sec. 9.1.2; water drops of the size encountered in most scrubbers pose few difficulties for such cyclones. The liquid–solid separator can be of many kinds although gravity settlers seem to be the most common. If possible, the engineer should try to save money by finding a place where the contaminated water stream can be recycled inside the plant without first removing the solids. There are many examples where that has been done successfully. Obviously, if there is no good way to deal with the contaminated water stream, then the scrubber has merely changed an air pollution problem into a water pollution problem.

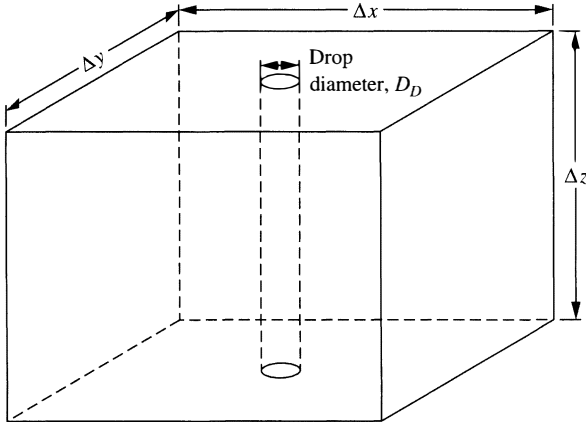
For the rest of this chapter, we will assume that the gas–liquid and the liquid–solid separations are relatively easy; we will only concern ourselves with the gas–liquid contactor, in which the particles are caught on the drops. Most of that capture takes place by impaction or impingement, as described in Figs. 9.18 and 9.19, to which we will refer often.

**9.2.4.1 Collection of particles in a rainstorm.** We will begin with a collection device that all students have witnessed—a rainstorm. From that we will work toward the more complex geometries of industrial interest.

Figure 9.21 on page 300 shows the geometry for which we will make a material balance on the particles and on the drops. We consider a space with dimensions  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ . The concentration of particles in the gas in this space is  $c$  (lbm/ft<sup>3</sup> or kg/m<sup>3</sup>).



**FIGURE 9.20**  
Component parts of a scrubber installation.



**FIGURE 9.21**  
Region considered in the material balance for a rainstorm.

Now we let one spherical drop of water of diameter  $D_D$  pass through this space. How much of the particulate matter in the space will be transferred to the drop? We can see that the volume of space swept out by the drop is the cylindrical hole shown in Fig. 9.21, whose volume is

$$V_{\text{swept by one drop}} = \frac{\pi}{4} D_D^2 \Delta z \tag{9.47}$$

The total mass of particles that was originally in that swept volume is that volume times the concentration  $c$ . The fraction of these that will be collected by the drop is the target efficiency  $\eta_t$ , which we can determine from Fig. 9.18 or its equivalent. So the mass of particles transferred from the gas to the drop is

$$\begin{aligned} \left( \begin{array}{c} \text{Mass transferred} \\ \text{to one drop} \end{array} \right) &= \left( \begin{array}{c} \text{swept} \\ \text{volume} \end{array} \right) (\text{concentration}) \left( \begin{array}{c} \text{target} \\ \text{efficiency} \end{array} \right) \\ &= \frac{\pi}{4} D_D^2 \Delta z c \eta_t \end{aligned} \tag{9.48}$$

Next we consider a region of space (still  $\Delta x \Delta y \Delta z$ ) that is large with respect to the size of any one individual raindrop through which a large number of raindrops are falling at a steady rate  $N_D$ , expressed as drops/time. Each of the drops stirs the region of gas around it so that there is no distinction between volume “swept by a drop” and volume “not swept by a drop,” as there would be for the foregoing single-drop example.

For the region  $\Delta x \Delta y \Delta z$  we wish to know how the concentration of particles in the air changes during the rainstorm. From a material balance on the particles in the space, we can say that

$$\begin{aligned} \frac{dc}{dt} &= - \frac{(\text{mass transferred to each drop})(\text{number of drops/time})}{(\text{volume of the region})} \\ &= - \frac{(\pi/4)(D_D^2 \Delta z c \eta_t N_D)}{\Delta x \Delta y \Delta z} = - \frac{\pi}{4} D_D^2 c \eta_t \left( \frac{N_D}{\Delta x \Delta y} \right) \end{aligned} \tag{9.49}$$

We multiply top and bottom of the equation by the volume of a single spherical drop and simplify to obtain

$$\frac{dc}{dt} = -\frac{\pi}{4} D_D^2 c \eta_t \left( \frac{N_D}{\Delta x \Delta y} \right) \frac{(\pi/6) D_D^3}{(\pi/6) D_D^3} = -1.5 \frac{c \eta_t}{D_D} \left( \frac{N_D (\pi/6) D_D^3}{\Delta x \Delta y} \right) \quad (9.50)$$

The final term in parentheses in Eq. (9.50) represents the volume of rain that fell per unit time (the number of drops per unit time times their individual volume) divided by the horizontal area through which they fell. (Weather reports often tell of rain falling at a rate of one inch per hour, a rapid rate indeed. One may also think of this as the rate at which the level in a container like a glass will rise if the rain falls in at its open top and none exits.) For the rest of this chapter the total liquid volumetric flow rate going to a scrubber (or to this region of space) will have the symbol  $Q_L$  ( $\text{m}^3/\text{s}$  or equivalent) so the rightmost term is  $Q_L/A$ , where  $A$  is the horizontal projection of the region of interest. Substituting this into Eq. (9.50), we can rearrange and integrate to find

$$\frac{dc}{dt} = -\frac{1.5}{D_D} c \eta_t \frac{Q_L}{A} \quad (9.51)$$

$$\frac{dc}{c} = -\frac{1.5}{D_D} \eta_t \frac{Q_L}{A} dt$$

$$\ln p = \ln \frac{c}{c_0} = -\frac{1.5}{D_D} \eta_t \frac{Q_L}{A} \Delta t \quad (9.52)$$

**Example 9.22.** A rainstorm is depositing 0.1 in./h, all in the form of spherical drops 1 mm in diameter. The air through which the drops are falling contains 3- $\mu$  diameter particles at an initial concentration 100  $\mu\text{g}/\text{m}^3$ . What will the concentration be after one hour?

Solving Eq. (9.52) for  $c$ , we find

$$c = c_0 \exp - \left( \frac{1.5 \eta_t Q_L \Delta t}{D_D A} \right)$$

We know all of the quantities on the right except  $\eta_t$ . From Fig. 8.7 we can read the terminal settling velocity of a 1-mm diameter drop of water in still air is about 14 ft/s = 4.2 m/s, so we can compute  $N_S$  from Eq. (9.45) as

$$N_s = \frac{\rho D_p^2 V}{18 \mu D_b} = \frac{(2000 \text{ kg/m}^3)(3 \times 10^{-6} \text{ m})^2 (4.2 \text{ m/s})}{(18)(1.8 \times 10^{-5} \text{ kg/m} \cdot \text{s})(10^{-3} \text{ m})} = 0.23$$

Here  $D_b$  (the barrier diameter in the definition of  $N_S$ ) =  $D_D$  (the drop diameter). From Fig. 9.18 we can read  $\eta_t \approx 0.23$ , so

$$c = 100 \frac{\mu\text{g}}{\text{m}^3} \exp \left[ -\frac{(1.5 \cdot 0.23)(0.1 \text{ in./h})(1 \text{ h})}{10^{-3} \text{ m}} \cdot \frac{\text{m}}{39.37 \text{ in.}} \right] = 43 \frac{\mu\text{g}}{\text{m}^3} \quad \blacksquare$$

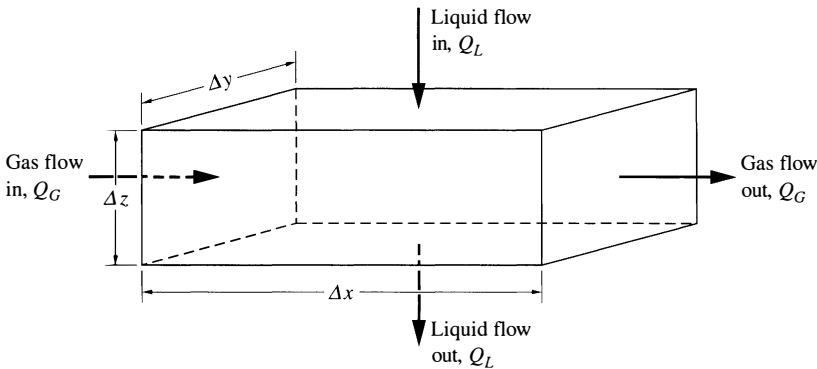
This example shows that the result depends on the total amount of rain that fell,  $Q_L \Delta t/A$ , which is 0.1 inch in this case, not on the time or rainfall rate separately.

The fractional removal is independent of the initial concentration;  $c/c_0$  does not depend on the value of  $c_0$ . Although rainfall collects large particles well, it does poorly for small particles. If the example had asked for the collection efficiency for particles of  $1\text{-}\mu$  diameter, we would have calculated an  $N_s$  one-ninth as large, and from Fig. 9.18 we would have computed an  $\eta_t$  of zero. Recall that Fig. 9.18 only describes the impaction mechanism, which would be zero in this case; the diffusional mechanism would have led to some collection, but the efficiency would have been very small.

This calculation suggests that, contrary to popular opinion, a rainstorm does not clean the air well. The rainstorm will remove large particles but have little effect on those smaller than  $1\text{-}\mu$ , which are of the greatest health concern and which are the most efficient light scatterers. It is a common observation in the northern and western United States that the air is much clearer after a rainstorm than before. The reason is not that the raindrops cleared the air, but that rainstorms in this region are normally followed by a flow of polar air, or air from over the Pacific Ocean; the incoming air is generally cleaner than the air it replaces.

**9.2.4.2 Collection of particles in crossflow, counterflow, and co-flow scrubbers.** To get good removal of small particles, we must find some way to increase the value of  $N_s$  for the drop-particle interaction to get a higher value of  $\eta_t$ . We will consider several scrubber geometries to see what the possibilities are.

**Crossflow scrubbers.** Consider the crossflow scrubber sketched in Fig. 9.22, which shows the overall dimensions and some of the notation. This is a large box with multiple spray nozzles that disperse the incoming liquid,  $Q_L$ , uniformly over the horizontal surface and a floor drain that collects the liquid at the bottom. The gas is assumed to move through the scrubber in uniform, blocklike flow at a total volumetric flow rate of  $Q_G$ .



**FIGURE 9.22**

Schematic of a crossflow scrubber.



A parcel of air moving through this scrubber behaves just like a parcel of air standing still in a rainstorm. If we can compute the time it takes such a parcel of air to travel through the scrubber, we can use it in Eq. (9.52) to compute the collection efficiency. The linear velocity of the gas is  $(Q_G/\Delta x \Delta y \Delta z)$ , and hence the time it takes a parcel of gas to pass through is the length of the scrubber divided by the linear velocity, or

$$\text{Travel time} = \Delta t = \frac{\Delta x \Delta y \Delta z}{Q_G} \quad (9.53)$$

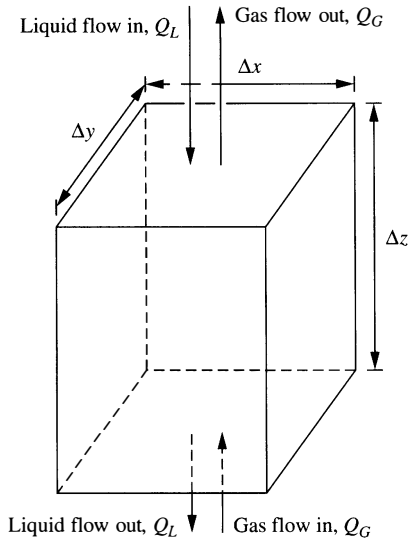
One may also think of this as the volume of the scrubber divided by the volumetric flow rate of gas, and hence as the quantity (1/the number of scrubber volumes of gas admitted per unit time). Substituting this value for travel time into Eq. (9.52), we find

$$\ln p = \ln \frac{c}{c_0} = -\frac{1.5\eta_t Q_L \Delta x \Delta y \Delta z}{D_D A Q_G} = -1.5 \cdot \frac{\eta_t}{D_D} \cdot \frac{Q_L}{Q_G} \cdot \Delta z \quad (9.54)$$

This equation says that the smaller the drop and the taller the scrubber, the more efficient it will be in removing particles. However, we must consider the path taken by an individual drop in the scrubber. A very large drop will fall almost straight down, because its vertical velocity due to gravity is much larger than the horizontal velocity of the gas. But a small drop has a much lower vertical velocity, so it will be carried along in the flow direction by the gas. If we try to get a good collection efficiency (a low value of  $p = c/c_0$ ) by increasing  $\Delta z$  or decreasing  $D_D$ , we see that the drops will pass out with the gas and not be collected in the scrubber. For this reason, this type of scrubber is not widely used. There are some applications; for example, one wishes to capture a valuable dust, of fairly large particle size, in an aqueous solution. In such cases it is common practice to locate the spray heads only in the most upstream part of the roof of the scrubber. The distance between the most downstream spray head and the outlet of the scrubber is calculated to allow most of the drops to reach the bottom of the scrubber before they reach the outlet.

**Counterflow scrubbers.** The next geometry to consider is the counterflow scrubber, sketched in Fig. 9.23 on page 304. Liquid enters the top of the scrubber through a series of spray nozzles that distribute it uniformly and falls by gravity. The gas enters the bottom of the scrubber and flows upward in uniform, blocklike flow.

We might be tempted to proceed as we did for the crossflow scrubber and simply compute the gas transit time and substitute it into Eq. (9.52). Alas, there is a complication. In the rainstorm problem and in the crossflow scrubber, the distance that a drop travels relative to fixed coordinates is the same distance it travels relative to the gas ( $\Delta z$  in both cases). Here that is no longer the case, because if the drop is at its terminal settling velocity  $V_t$  relative to the gas *that surrounds it*, but that gas is moving upward with velocity  $V_G = Q_G/\Delta x \Delta y$ , then the velocity of the drop relative to the fixed coordinates of the scrubber is  $V_{D\text{-Fixed}} = V_t - V_G$ .



**FIGURE 9.23**  
Schematic of a counterflow scrubber.

We remake our previous material balance as in the following:

$$\begin{aligned}
 \left( \begin{array}{l} \text{Mass of particles transferred} \\ \text{to drops per unit time} \\ \text{per unit volume} \end{array} \right) &= - \text{mass of particles transferred out of} \\
 &\quad \text{the gas per unit time per unit volume} \\
 &= (\text{volume swept/time})(\text{particle concentration}) \\
 &\quad \times (\text{target efficiency}) \\
 &= - (\text{gas volumetric flow rate}) \\
 &\quad \times (\text{change in particle concentration})
 \end{aligned} \tag{9.55}$$

To compute the quantity (volume swept by drops/time) we must compute the instantaneous number of drops per unit volume. The liquid flow into the system is  $Q_L$  ( $\text{m}^3/\text{s}$ ), and this consists of  $N_D$  drops/time, each of volume  $(\pi/6)D_D^3$ . The average time each such drop spends in the scrubber is the vertical distance divided by the vertical velocity *relative to fixed coordinates*, or

$$\text{Average time} = \frac{\Delta z}{(V_t - V_G)} \tag{9.56}$$

so at any time the number of drops in the system is

$$\text{Drops present at any time} = \frac{N_D \Delta z}{(V_t - V_G)} \tag{9.57}$$

The volume of gas that these drops sweep out per unit time is their number times their cross-sectional area times the velocity at which they move *relative to the gas*,

which is  $V_t$ . So we can compute

$$\begin{aligned} \frac{\text{Volume swept}}{\text{Time}} &= \left( \frac{N_D \Delta z}{V_t - V_G} \right) \left( \frac{\pi D_D^2}{4} \right) V_t \\ &= \left( \frac{Q_L}{\pi D_D^3/6} \right) \left( \frac{\Delta z \pi}{4} \right) \left( \frac{D_D^2 V_t}{V_t - V_G} \right) \\ &= Q_L \left( \frac{1.5}{D_D} \right) (\Delta z) \left( \frac{V_t}{V_t - V_G} \right) \end{aligned} \quad (9.58)$$

We substitute Eq. (9.58) into Eq. (9.55), finding

$$Q_L \left( \frac{1.5}{D_D} \right) (\Delta z) \left( \frac{V_t}{V_t - V_G} \right) c \eta_t = -Q_G \Delta c \quad (9.59)$$

If we now let the scrubber height be infinitesimally small, so that  $\Delta z$  and  $\Delta c$  become  $dz$  and  $dc$ , we can separate the variables and integrate, finding

$$\frac{dc}{c} = -1.5 \cdot \frac{\eta_t}{D_D} \cdot \frac{Q_L}{Q_G} \cdot \frac{V_t}{(V_t - V_G)} dz \quad (9.60)$$

$$\ln p = \ln \frac{c}{c_0} = -1.5 \cdot \frac{\eta_t}{D_D} \cdot \frac{Q_L}{Q_G} \cdot \frac{V_t}{(V_t - V_G)} \Delta z \quad (9.61)$$

Comparing Eq. (9.61), for counterflow scrubbers, to Eq. (9.54), for crossflow scrubbers, we see that the only difference is the addition of a  $[V_t/(V_t - V_G)]$  term, which accounts for the fact that each drop moves farther relative to the gas than it moves relative to the fixed geometry of the scrubber.

Equation (9.61) also allows us to see the limitation of this kind of scrubber. We can get 100 percent efficiency ( $c/c_0 = 0$ ) if we let  $V_t = V_G$ , because that makes the value of the right side negative infinity. Physically, that means that if the upward velocity of the gas equals the terminal settling velocity of the liquid, then the individual drop will stand still in the scrubber and will collect from an infinitely long column of gas as the gas passes. However, if we continue to put liquid into the scrubber ( $Q_L$  not equal to zero) and no liquid leaves, we will fill the scrubber with liquid. It will become *flooded* and will cease to operate as a scrubber. Since we want to use the smallest practical size drops in order to get high values of  $N_s$  and thus of  $\eta_t$ , flooding sets a very strong practical limitation on this kind of scrubber. There are some important applications where they are used (Chapter 11), but they do not play a major role in *particulate* air pollution control.

**Co-flow scrubbers.** Clearly we need a geometrical arrangement in which we can get very small drops to move at high velocities relative to the gas being scrubbed, to get a high  $N_s$  and high  $\eta_t$ , without blowing the drops out the side or top of the scrubber. The solution to this problem is the co-flow scrubber, shown schematically in Fig. 9.24 on page 306. In it, both gas and liquid enter at the left and exit at the right. However, the liquid enters at right angles to the gas flow; it comes in with