

and the volumetric flow through the section is

$$Q = HhV_{\text{avg}} \quad (9.28)$$

Making these substitutions in Eqs. (9.4) and (9.11), we find

$$\eta = \frac{wA}{Q} \quad \text{block flow} \quad (9.29)$$

and

$$\eta = 1 - \exp\left(-\frac{wA}{Q}\right) \quad \text{mixed flow} \quad (9.30)$$

In the literature, Eq. (9.29) is occasionally called the “theoretical laminar flow equation,” which would hold if we had block flow of gas with no mixing [6, 7]. It has no practical use. Equation (9.30) is the *Deutsch-Anderson equation*, the most widely used simple equation for design, analysis, and comparison of ESPs. It is the same equation we have used for gravity settlers and cyclones, with the terms renamed.

**Example 9.11.** Compute the efficiency-diameter relation for an ESP that has particles with a dielectric constant of 6 and  $(A/Q) = 0.2 \text{ min/ft}$  ( $\approx 0.060 \text{ s/m}$ ). We will use only the mixed flow equation.

Using the results of Example 9.10 we know that a  $1\text{-}\mu$  diameter particle will have a drift velocity of  $0.109 \text{ ft/s}$ , and that the drift velocity will be linearly proportional to the particle diameter. Thus for a  $1\text{-}\mu$  particle we may compute

$$\eta = 1 - \exp\left(-\frac{wA}{Q}\right) = 1 - \exp\left[-\left(0.109 \frac{\text{ft}}{\text{s}}\right)\left(0.2 \frac{\text{min}}{\text{ft}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\right] = 0.73$$

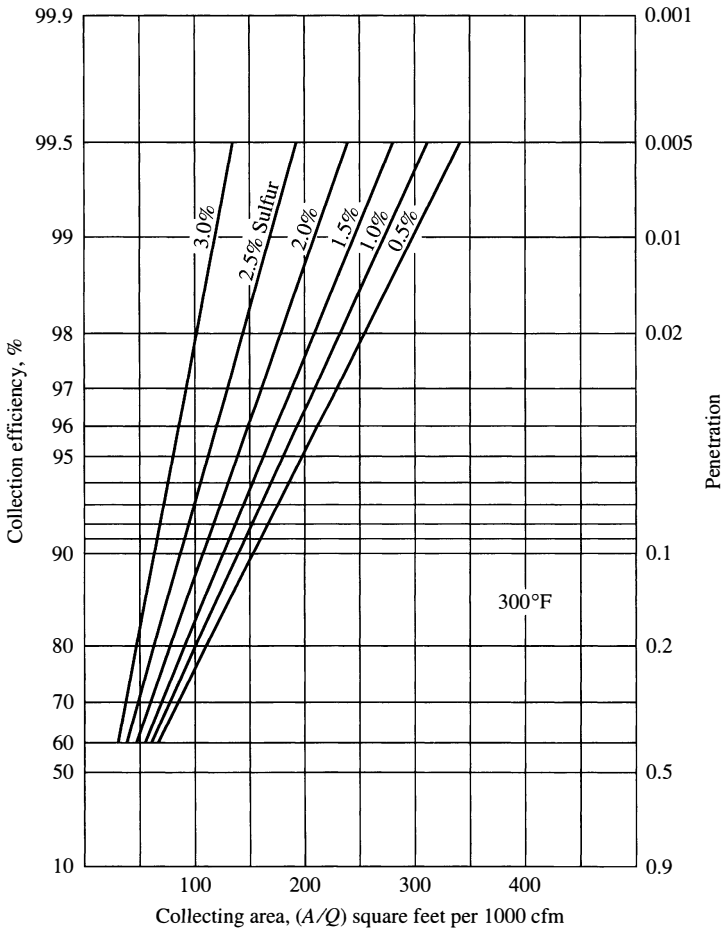
As in Examples 9.1 and 9.4, we make up a table using this one computed value by taking advantage of the fact that the computed drift velocity is proportional to the particle diameter to the first power.

Particle diameter, $\mu$	$\eta$
0.1	0.12
0.5	0.48
1	0.73
3	0.98
5	0.998

This example shows that this fairly typical precipitator has a cut diameter of about  $0.5 \mu$ , one-tenth of the cut diameter of a typical cyclone. If we plotted these values on an efficiency-diameter plot like Fig. 9.2, we would find a somewhat different shape, because the drift velocity in an ESP depends on  $D$ , whereas the terminal settling velocity in a gravity settler or cyclone depends on  $D^2$ .

One might hope to calculate the value of  $w$  from the theory previously presented and thus design precipitators by Eq. (9.30) with confidence. If every particle that got to the wall stayed there, then the performance calculated that way would be observed

in working precipitators. Unfortunately, the rapping that loosens the particles from the wall also re-entrains some of them in the gas, and various particles have various re-entrainment properties. However, Eq. (9.30) suggests that if we pass a particle-laden gas stream through various precipitators, all of the data for this stream will form a straight line on a plot of  $\log p$  vs.  $A/Q$ . Figure 9.9 is such a plot, in which the third variable is percent sulfur in the coal burned (explained later). Since the re-entrainment process is likely the same kind of random statistical process as the turbulence process, and since the amount re-entrained is likely to be proportional to the local cake thickness, which in turn should be a function of the local particle



**FIGURE 9.9** Summary of size-efficiency data for coal-fired power plant precipitators. Each line represents coal of a specified sulfur content. The precipitator size is specified in square feet of collecting area per 1000 cfm of gas flow; this is the common usage in the ESP industry. (From Ref. 8.) (Reprinted with permission of American Power Conference.)

concentration, we are not surprised that these experimental data for any fixed sulfur content fall on a straight line.

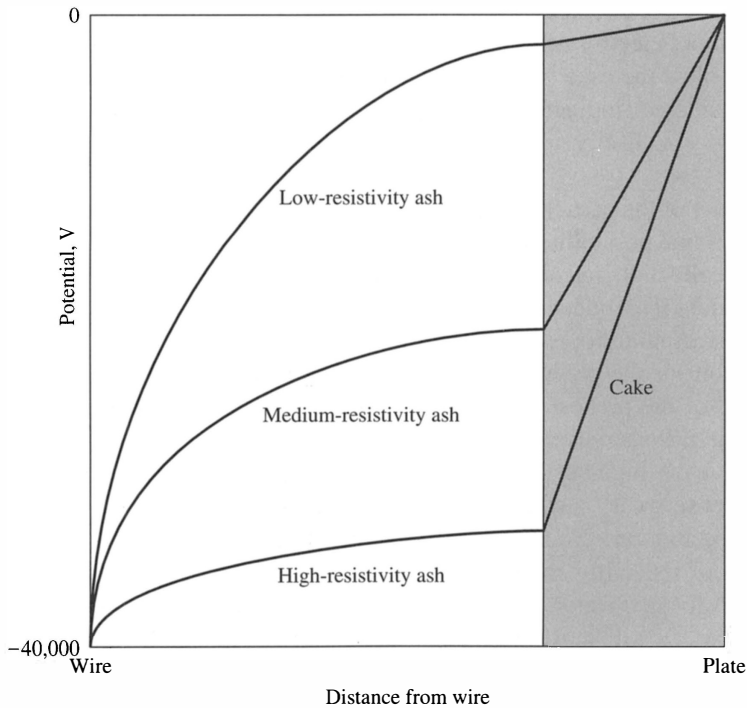
**Example 9.12.** From Fig. 9.9, estimate the value of  $w$  for coal containing 1 percent sulfur. From that figure at 99.5 percent efficiency we read that for 1 percent sulfur coal,

$$\frac{A}{Q} = \frac{310 \text{ ft}^2}{1000 \text{ ft}^3/\text{min}} = 0.31 \frac{\text{min}}{\text{ft}}$$

From Eq. (9.30), we calculate

$$w = -\frac{\ln p}{A/Q} = -\frac{\ln 0.005}{0.31 \text{ min/ft}} = 17.09 \frac{\text{ft}}{\text{min}} = 0.28 \frac{\text{ft}}{\text{s}} = 0.086 \frac{\text{m}}{\text{s}} \quad \blacksquare$$

The different lines for different coal sulfur contents on Fig. 9.9 are caused by sulfur's indirect effect on fly ash resistivity (discussed later). ESPs work well with medium-resistivity solids, but poorly with low-resistivity or high-resistivity solids. We can see why by referring to Fig. 9.10, which shows three situations. In each situation the voltage at the wire is  $-40 \text{ kV}$  and the voltage at the plate is zero; these



**FIGURE 9.10**

Voltage-distance relation between plate and wire for a low-, a medium-, and a high-resistivity ash.  $E = \partial V/\partial x$  is the slope of the voltage-distance curve.

are common conditions for precipitators. For the case of a low-resistivity solid, e.g., carbon black, the material forms a cake that is a good conductor of electricity. The voltage gradient in the cake is small. On reaching the plate the particles are discharged and hence there is very little electrostatic force holding the collected particles to the plate. The collected particles do not adhere and are easily re-entrained; the overall collection is poor. (In one instance an ESP was used to agglomerate fine carbon particles although it could not collect them; they were subsequently collected in a cyclone, which could not collect the unagglomerated particles [7].) Because of this low cake adhesion for low-resistivity particles, ESPs are generally not used to collect particles with a resistivity less than  $10^7$  ohm · cm.

Figure 9.10 also shows a cake of medium-resistivity particles on the collecting plate. The voltage gradient across the cake is adequate to provide electrostatic force to hold the cake in place, but not enough to cause trouble. Figure 9.10 also shows particles of very high resistivity, e.g., elemental sulfur, on the plate. Here most of the voltage gradient occurs through the cake, causing at least two problems. First, the voltage gradient near the wire has now fallen so much that it cannot produce a good corona discharge. Thus, the particles are not properly charged. Second, the voltage gradient inside the cake is so high that in the gas spaces between the particles stray electrons will be accelerated to high velocities and will knock electrons off of gas molecules and form a *back corona* inside the cake. This back corona is a violently energetic conversion of electrostatic energy to thermal energy that causes minor gas explosions, which blow the cake off the plate and make it impossible to collect the particles. It is considered impractical to collect particles with resistivities greater than  $2 \times 10^{10}$  ohm · cm. The practical resistivity range is greater than  $10^7$  and less than  $2 \times 10^{10}$  ohm · cm.

If the resistivity of the particles is too low, little can be done. If the resistivity is too high, there are some possibilities. The resistivity of many coal ashes is too high at 300°F for good collection, but satisfactorily low at 600°F. (The resistivity change is due to improvement in conduction of some minerals in the ash with temperature increase.) Thus a precipitator operating after the air preheater at 300°F (the normal power plant location for the precipitator) might not work well on this ash, but an ESP located ahead of the preheater at 600°F, called a *hot-side precipitator*, might work well. Hot-side ESPs are used in some coal-fired power plants.

An ash may have high resistivity because its surface is a poor conductor. If one could condense on its surface a hygroscopic, conducting material, the ash resistivity would be reduced. Such condensation is reflected by the various lines on Fig. 9.9. Some of the sulfur in coal is converted in the furnace to  $\text{SO}_3$ , which collects on the ash, absorbs water, and makes the ash more conductive. Hence low-sulfur coal produces an ash more difficult to collect than does high-sulfur coal, as seen in Fig. 9.9. One logical cure is to add  $\text{SO}_3$  to the gas stream approaching the precipitator to “condition” the ash. This works well sometimes. Coal ash is basic, so an acid conditioner seems best. Portland cement is acidic, and a basic conditioner like ammonia seems to work best for it. There are many proprietary

conditioners on the market, working in the area between art and science. ( $\text{SO}_3$  added as a conditioner increases the sulfur oxide emissions, but normally by a negligible amount.)

Another approach to the ash resistivity problem is to separate the charging and collecting functions. If the particles are charged in a separate charger, one can use a higher voltage and not worry much about the resulting sparks, because they do not pass through the cake and disrupt it. This idea has been tested on a pilot scale [9], with results positive enough that it is expected to be tried at full scale in the near future.

To calculate an appropriate value of  $w$  to use in Eq. (9.30), one would want to know the particle size distribution, the dielectric properties of the material, its resistivity, and whether the particles formed a coherent cake. Generally not all of that information is available in advance for a new material, so trials are made. Table 9.2 shows some representative values of  $w$  for industrial precipitators [6]. One should not think that these are truly the average particle velocities in the direction of collection. The rapping process re-entrains some fraction of the collected particles into the gas, so they must be collected again. Some of the gas bypasses the collecting zone in each section of the precipitator, in spite of the baffles that try to force it all through the collecting zone. The combination of these effects plus other effects discussed next causes the overall collection efficiency to be less than what we would calculate from Eq. (9.30) if we substituted the true drift velocity (if we could measure or calculate it). The values in Table 9.2 are those which, when substituted into Eq. (9.30), reproduce the observed ESP efficiencies in those industries.

**TABLE 9.2**  
**Typical values of the drift velocity encountered in industrial practice**

Application	Drift velocity $w$ , ft/s
Pulverized coal (fly ash)	0.33–0.44
Paper mills	0.25
Open-hearth furnace	0.19
Secondary blast furnace (80% foundry iron)	0.41
Gypsum	0.52–0.64
Hot phosphorus	0.09
Acid mist ( $\text{H}_2\text{SO}_4$ )	0.19–0.25
Acid mist ( $\text{TiO}_2$ )	0.19–0.25
Flash roaster	0.25
Multiple-hearth roaster	0.26
Portland cement manufacturing (wet process)	0.33–0.37
Portland cement manufacturing (dry process)	0.19–0.23
Catalyst particles	0.25
Gray iron cupola (iron-coke ratio = 10)	0.10–0.12

Source: Ref. 6.

**Example 9.13.** Our ESP has a measured efficiency of 90 percent. We wish to upgrade it to 99 percent. By how much must we increase the collecting area?

Using Eq. (9.30), we calculate

$$p_{\text{existing}} = 1 - \eta_{\text{existing}} = 0.1 = \exp\left(\frac{-wA_{\text{existing}}}{Q}\right)$$

$$p_{\text{new}} = 1 - \eta_{\text{new}} = 0.01 = \exp\left(\frac{-wA_{\text{new}}}{Q}\right)$$

$$\frac{\ln 0.1}{\ln 0.01} = 0.5 = \frac{(-wA_{\text{existing}}/Q)}{(-wA_{\text{new}}/Q)} = \frac{A_{\text{existing}}}{A_{\text{new}}}$$

$$\frac{A_{\text{new}}}{A_{\text{existing}}} = 2 \quad \blacksquare$$

This example shows that if the Deutsch-Anderson equation were obeyed exactly, then going from 90 percent to 99 percent efficiency requires that we double the collecting area; 90 percent to 99.9 percent, that we triple it, etc. Unfortunately life is harder than that. In Eq. (9.26) we found that the drift velocity is proportional to the particle diameter (down to the very small particles, where diffusion charging becomes important). Thus the big particles, which contain most of the mass, are removed first and, as the percentage efficiency by weight increases, the remaining particles become smaller and smaller and harder and harder to collect. To take this phenomenon into account, some designers use a modified Deutsch-Anderson equation with the form

$$p = 1 - \eta = \exp-(wA/Q)^k \quad (9.31)$$

where  $k$  is an arbitrary exponent, typically about 0.5 [10].

**Example 9.14.** Rework Example 9.13 using Eq. (9.31) instead of (9.30), taking  $k = 0.5$ .

Here for the existing unit  $(wA/Q) = (-\ln p)^{1/k} = (-\ln 0.1)^2 = 5.30$ . For the upgraded precipitator we need  $(wA/Q) = (-\ln 0.01)^2 = 21.20$ , so the new value of  $(A/Q)$ —assuming constant  $w$ —is  $(21.20/5.30) = 4.0$  times the old value of  $(A/Q)$ . We must quadruple the size of the precipitator instead of doubling it. ■

Equation (9.31) has no theoretical basis; it is a simple way to deal with the fact that as penetrations are reduced to lower and lower values the remaining particles become smaller and smaller, and collecting them becomes harder and harder. Theoretically we should make  $w$  a function of  $p$  ( $w$  gets smaller as  $p$  gets smaller) but using Eq. (9.31) is simpler. (See Problems 9.36 and 9.37.)

Uniform distribution of gas through a precipitator is very important. Nonuniform distribution of the gas flow in an ESP lowers its collection efficiency. We can see why by applying Eq. (9.30) to ESPs with uniform and nonuniform flows.

**Example 9.15.** A precipitator consists of two identical sections in parallel, each handling one-half of the gas. It is currently operating at 95 percent efficiency. We now hold the total gas flow constant, but maldistribute the flow so that two-thirds of the gas goes through one of the sections, and one-third through the other. What is the predicted overall collection efficiency?

For the existing situation, we calculate

$$p = 0.05 = \exp\left(\frac{-wA}{Q}\right); \quad \frac{wA}{Q} = -\ln 0.05 = 2.995$$

For the new situation, we have

$$p_1 = \exp\left[(-2.995)\left(\frac{1/2}{2/3}\right)\right] = 0.1057$$

$$p_2 = \exp\left[(-2.995)\left(\frac{1/2}{1/3}\right)\right] = 0.0111$$

$$Q_1 p_1 = \frac{2}{3} Q (0.1057) = 0.070 Q$$

$$Q_2 p_2 = \frac{1}{3} Q (0.0111) = 0.004 Q$$

Adding these two equations, we find

$$(Q_1 + Q_2)p = 0.074 Q$$

$$p = 0.074; \quad \eta = 1 - p = 92.6\% \quad \blacksquare$$

This example shows mathematically, for a very simple case, how maldistribution degrades precipitator performance. In a maldistributed flow, most of the gas passes through the high-velocity part, where it spends less than the average amount of time in the precipitator; and hence the collection efficiency is lower. Thus the gas passing through the high-velocity part contributes more to the penetration than it would in the uniform flow case. Considerable efforts are made to distribute the flow evenly through the precipitator. In Fig. 9.8 the gas enters through a set of perforated plates that even out the flow. In a new installation where there are long straight ducts to and from the ESP, this is normally satisfactory. In a retrofit, an ESP must often be fit into a plant near other big pieces of equipment and connected to the other equipment by short pieces of ducting that have frequent sharp bends. These bends introduce nonuniformities into the flow, which propagate through the ESP and result in poor performance. Inlet and outlet screens and baffles can even out the flow, but they cause a pressure drop, which is expensive for large gas flows. This trade-off between the desire to keep the pressure drop low (normally a few inches of water) and the need to have a uniform flow is important enough that for many large retrofit ESP installations in plants with limited space, a fluid mechanical model at 1/4 to 1/16 scale is built and lab tested with models of the associated ductwork to ensure that there will be adequate uniformity of flow without excessive pressure drop.

In a settling chamber or cyclone, all of the gas to be treated passes through the collecting zone. The same is not true for an ESP. In Fig. 9.8 we see that the wires and plates cannot reach completely to the top and bottom of the collecting volume; some space must be allowed for one or the other, because of the high voltage on one. Thus, some of the gas must pass through a region with poor collection. This is called *sneakage* in the ESP literature. Serious efforts are made to minimize it, using seals and baffles. The overall turbulence in the precipitator mixes the gas between the poor treatment regions and the major region, which has good treatment. Experts who are troubleshooting a poorly-operating precipitator always consider excessive sneakage due to worn or damaged seals as a likely cause.

The typical linear velocity of the gas inside an ESP is 3 to 5 ft/s, much lower than that in a cyclone. The typical pressure drop is 0.1 to 0.5 in. H<sub>2</sub>O, again much less than in a cyclone. The pressure drop in the ducts leading to and from the precipitator is generally more than in the ESP itself.

The ESP industry is now well established. Standard package units are available for small flows (down to the size of home air conditioners), and large power plants have precipitators costing up to \$30 million. The design shown in Fig. 9.8 is widely used, but other designs are widely used also. The collection requirements have been pushed from the 90–95% range typical in 1965 to the 99.5%-plus range now commonly specified. Faced with this challenge, and with the problem of upgrading existing precipitators to meet more stringent control requirements, ESP manufacturers have continued to use designs like Fig. 9.8 for collection of the first 90 or 95% of particles from large gas streams, but then often substitute other designs for the final collection stage. Vatauvuk [11] presents a table similar to Table 9.2, showing values of  $w$  for various industries, both for ordinary dry ESPs and for wet ESPs in which the collected particles are continually removed by a film of fluid (normally water) flowing down the collecting surface instead of by intermittent rapping. In his table the values of  $w$  for wet ESPs are two to three times those for dry precipitators, mostly reflecting that there is no particle re-entrainment by rapping. Wet ESPs are more complex, and the collected particles are not in the convenient form of a dry powder. But for the final 5% cleanup these problems seem a modest price to pay for the greatly improved collection efficiency. Another approach is to make the final 5% collection in a filter, as described next. Sometimes the ESP-filter combination is more economical than an equivalent-performance ESP or filter [12].

## 9.2 DIVIDING COLLECTION DEVICES

Gravity settlers, cyclones, and ESPs collect particles by driving them against a solid wall. Filters and scrubbers do not drive the particles to a wall, but rather divide the flow into smaller parts where they can collect the particles. In this section we shall first consider the two types of filters used in air pollution control, *surface filters* and *depth filters*. Then we shall discuss scrubbers.

The public often refers to any kind of pollution control device as a filter, giving the word *filter* the meaning “cleaning device.” Technically, a filter is one of