Figure 9.3 also shows another V_t due to gravity (assuming the axis of the circle is vertical). If, as shown previously, the centrifugal force is normally more than 100 times the gravitational force, then this gravitational settling velocity will be less than a hundredth of the centrifugal one and can be left out of consideration. V_t, the terminal settling velocity we calculate, is a velocity in the *radial direction* at right angles to the main circular motion of the particle.

Now substituting this centrifugal acceleration for the gravitational one in Eq. (8.4) and dropping the ρ_{fluid} term, we find

$$V_t = \frac{V_c^2 D^2 \rho_{\text{part}}}{18\mu r} \tag{9.15}$$

Example 9.3. Repeat the computation of the terminal settling velocity shown in Example 8.1 for a particle in a circular gas flow with velocity $V_c = 60$ ft/s (18.29 m/s) and radius 1 ft (0.3048 m). The density of the fluid can be ignored. By direct substitution in Eq. (9.15), we find

$$V_t = \frac{(18.29\ \text{m/s})^2(10^{-6}\ \text{m})^2(2000\ \text{kg/m}^3)}{(18)(1.8 \times 10^{-5}\ \text{kg/m}\cdot\text{s})(0.3048\ \text{m})} = 0.0068\ \frac{\text{m}}{\text{s}}$$

$$= 0.68\ \frac{\text{cm}}{\text{s}} = 0.022\ \frac{\text{ft}}{\text{s}} \qquad \blacksquare$$

This answer is 112 times as large as the value found in Example 8.1, indicating again that much greater settling velocities can be obtained this way. One may compute the particle Reynolds number here, finding that it is about 0.00046. Hence the assumption of a Stokes' law type of drag seems reasonable. In centrifugal devices the settling velocities are higher than those due to gravity, so that if we were to make up a centrifugal equivalent to Fig. 8.6 we would find that the drag-coefficient Reynolds number curve would begin at smaller particle diameters than for gravity settling. The Cunningham correction factor is unaffected by how fast the particles move, and thus that part of the curve would be unaffected by the switch from gravity settling to centrifugal settling.

At this point let us reconsider the Stokes' law assumption. If we consider the overall gas flow, with velocities on the order of 60 ft/s, the Reynolds numbers are on the order of a half million. The flow is highly turbulent. How can we apply Stokes' law, which requires that the particle Reynolds numbers be less than about 0.3 and that the fluid flow around the particle be laminar? If we take the view of a person riding on the particle, we can see that the patch of fluid surrounding us is in turbulent, rapid circular motion, with one turbulent eddy moving us toward the center, then another moving us away from the center, etc. However, in the immediate locality of the particle there is a small net movement of the particle relative to the surrounding gas caused by centrifugal force. This net movement is so slow that the gas molecules can easily move out of the particle's way in a laminar fashion. It is this net particle movement, superimposed on the overall turbulent gas flow, that causes the average radially outward movement of the particle and is the movement discussed in this section.

After all this theoretical discussion, how does one construct a practical centrifugal particle collector? There are many types, but the most successful is sketched in Fig. 9.4. It is universally called a *cyclone separator*, or simply a *cyclone*. It is probably the most widely used particle collection device in the world. In any industrial district of any city, a sharp-eyed student can find at least a dozen of these outside various industrial plants.

As the sketch shows, a cyclone consists of a vertical cylindrical body, with a dust outlet at the conical bottom. The gas enters through a rectangular inlet, normally

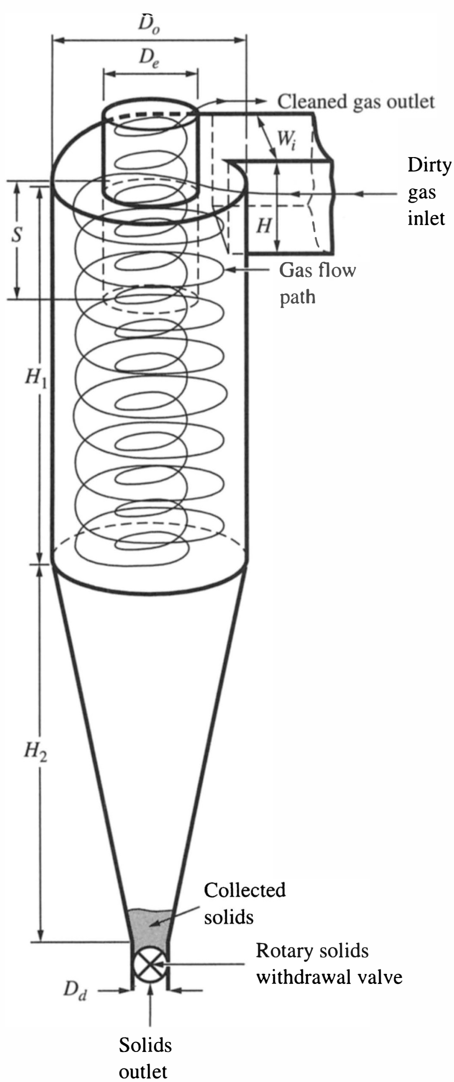

FIGURE 9.4
Schematic of a cyclone separator. Dimensions are typically based on the overall diameter D_o. Taken as ratios to that dimension, $W_i = 0.25$, $H = 0.5$, $H_1 = 2$, $H_2 = 2$, $D_e = 0.5$, $S = 0.625$, $D_d = 0.25$. For example, if $D_o = 1$ ft, then $W_i = 0.25$ ft, etc. Ashbee and Davis [2] show a table with six sets of values for these dimension ratios. The principal differences are that high-efficiency cyclones have smaller values of W_i whereas high-throughput cyclones have larger values of W_i and of D_e. The dimension ratios here are for the "conventional" design.

twice as high as it is wide, arranged tangentially to the circular body of the cyclone, so that the entering gas flows around the circumference of the cylindrical body, not radially inward. The gas spirals around the outer part of the cylindrical body with a downward component, then turns and spirals upward, leaving through the outlet at the top of the device. During the outer spiral of the gas the particles are driven to the wall by centrifugal force, where they collect, attach to each other, and form larger agglomerates that slide down the wall by gravity and collect in the dust hopper in the bottom.

Clearly the cyclone separator sketched in Fig. 9.4 is merely a gravity settler that has been made in the form of two concentric helices. Only the outer helix contributes to collection; particles that get into the inner helix, which flows upward to the gas outlet, escape uncollected. Thus the outer helix is equivalent to the gravity settler. The inlet stream has a height W_i in the radial direction, so that the maximum distance any particle must move to reach the wall is W_i (defined on Fig. 9.4). The comparable distance in a gravity settler is H (Fig. 9.1). The length of the flow path is $N\pi D_o$, where N is the number of turns that the gas makes traversing the outer helix of the cyclone, before it enters the inner helix, and D_o is the outer diameter of the cyclone. This length of the flow path corresponds to L in the gravity settler. Making these substitutions directly into the gravity settler equations, Eqs. (9.5) and (9.12), we find

$$\eta = \frac{N\pi D_o V_t}{W_i V_c} \qquad \text{block flow} \tag{9.16}$$

and

$$\eta = 1 - \exp - \left(\frac{N\pi D_o V_t}{W_i V_c} \right) \qquad \text{mixed flow} \tag{9.17}$$

If we then substitute the centrifugal Stokes' law expression, Eq. (9.15), into these two equations, and make the appropriate cancellations, we find

$$\eta = \frac{\pi N V_c D^2 \rho_{\text{part}}}{9 W_i \mu} \qquad \text{block flow} \tag{9.18}$$

and

$$\eta = 1 - \exp - \left(\frac{\pi N V_c D^2 \rho_{\text{part}}}{9 W_i \mu} \right) \qquad \text{mixed flow} \tag{9.19}$$

Here D is the particle diameter. The outside diameter of the cyclone, D_o, does not appear directly but only indirectly through W_i, which is proportional to it. Observe also that the right side of Eq. (9.18) is the Stokes' stopping distance (Section 8.2.4) divided by $W_i/2\pi N$.

Equations (9.18) and (9.19) contain a parameter N, which represents the number of turns the gas makes around the cyclone before it leaves the collecting area near the wall. There seems to be no satisfactory theoretical basis for calculating N from fluid mechanical principles. A value of $N = 5$ represents the experimental data best. Unless one has specific information to the contrary, one should assume that $N = 5$ throughout this book.

Example 9.4. Compute the efficiency-diameter relation for a cyclone separator that has $W_i = 0.5$ ft, $V_c = 60$ ft/s, and $N = 5$, for both the block and mixed flow assumptions, assuming Stokes' law.

Here, as in Example 9.1, we can get the result with one numerical computation, using ratios. First we compute the block flow efficiency for a 1-μ particle, viz.,

$$\eta = \frac{\pi N V_c D^2 \rho_{\text{part}}}{9 W_i \mu}$$

$$= \frac{(\pi)(5)(60 \text{ ft/s})(10^{-6} \text{ m})^2(3.28 \text{ ft/m})^2(124.8 \text{ lbm/ft}^3)}{(9)(0.5 \text{ ft})(1.8 \times 10^{-2} \text{ cp})[6.72 \times 10^{-4} \text{ lbm/(ft} \cdot \text{s} \cdot \text{cp})]} = 0.0232$$

Then, as we did in Example 9.1, we can use this number, plus the fact that the particle diameter enters the equation to the second power, to make up the following table:

Particle diameter, μ	η_{block}	η_{mixed}
0.1	0.000232	0.000232
1	0.0232	0.0230
2	0.0930	0.0888
3	0.209	0.189
4	0.372	0.311
5	0.582	0.441
6.559	1.00	0.632
10	—	0.902
15	—	0.995

■

Comparing this result to that for gravity settling chambers in Example 9.1, we see the form of the result is the same, but the maximum particle size for which the device is effective is much smaller. If we plotted these data as in Fig. 9.2, we would find an identical plot, but with the diameter scale multiplied by a factor of $(6.559/57.45) = 0.114$. This occurs because the models and their resulting equations are truly the same except for the substitution of centrifugal force for gravity, and the change in dimensions.

Next we introduce a new term, the *cut diameter,* which is widely used in describing particle collection devices. This definition gives us a measure of the size of particles caught and the size passed for a particle collector. A kitchen colander—a sheet metal dish with uniform, circular holes—has a cut diameter; all the particles that can pass through the holes in any direction will do so (if we shake long enough), whereas those larger than the holes will not. If we considered only spherical peas in a colander with uniform circular holes, then the cut diameter would be the diameter of the holes. For peas larger than the cut diameter the collection efficiency would be 100 percent, and for those smaller it would be 0 percent. For all practical particle collection devices the separation is not that sharp; there is no single diameter at which the efficiency goes suddenly from 0 percent to 100 percent. The universal convention in the air pollution literature (and the particle technology literature in general) is to define *cut diameter* as the diameter of a particle for which the efficiency curve has the value of 0.50, i.e., 50 percent.

We can substitute this definition into Eq. (9.18) and solve for the cut diameter that goes with Stokes' law, block flow model, finding:

$$D_{\text{cut}} = \left(\frac{9 W_i \mu}{2\pi N V_c \rho_{\text{part}}} \right)^{1/2} \qquad \text{block flow} \tag{9.20}$$

Although one might logically expect that Eq. (9.19), with its more realistic mixed flow model, would better represent experimental data, Eq. (9.18) appeared in the literature earlier [3] and has been more widely used. It is widely known as the *Rosin-Rammler equation* and is reasonably accurate in estimating the performance of cyclones.

Example 9.5. Estimate the cut diameter for a cyclone with inlet width 0.5 ft, $V_c = 60$ ft/s and $N = 5$.

$$D_{\text{cut}} = \left(\frac{(9)(0.5\ \text{ft})(1.8 \times 10^{-5}\ \text{kg/m} \cdot \text{s})}{2\pi(5)(60\ \text{ft/s})(2000\ \text{kg/m}^3)} \right)^{1/2} = 4.63 \times 10^{-6}\ \text{m} \approx 5\ \mu \qquad \blacksquare$$

This example shows that for a typical cyclone size and the most common cyclone velocity and gas viscosity, the cut diameter is about 5 μ. Comparing this calculation with that in Example 9.4 shows that the cut diameter we would calculate by the mixed model is somewhat larger, but not dramatically so. It is an industrial rule of thumb that if a gas stream contains few particles smaller than 5 μ then a cyclone is probably the only collector one should consider. It works well on most particles that size and larger (e.g., sawdust from wood shops and wheat grains from pneumatic conveyers), and is a low-cost, easy-maintenance device. It is not satisfactory for sticky particles, like tar droplets.

Suppose we wish to apply a cyclone separator for even smaller particles. What are our options? From Eq. (9.20) we can see that the alternatives are to make W_i smaller or V_c larger. (Generally we cannot alter the gas viscosity or the particle density.) Making V_c larger is generally too expensive because, as we shall see later in this section, the pressure drop across a cyclone is generally proportional to the velocity squared. To make W_i smaller, we must make the whole cyclone smaller if we are to keep the same ratios of dimensions. But the inlet gas volumetric flow is proportional to W_i squared, so that a small cyclone treats a small gas flow. Very small cyclones have been used to collect small particles from very small gas flows for research and gas-sampling purposes, but the industrial problem is to treat large gas flows. Several practical schemes have been worked out to place a large number (up to several thousand) small cyclones in parallel, so that they can treat a large gas flow, capturing smaller particles. The most common of these arrangements, called a **multiclone**, is sketched in Fig. 9.5.

The many small cyclones in the multiclone are mass-produced and inserted into sheet metal supporters. In the device shown, the circular gas motion in each

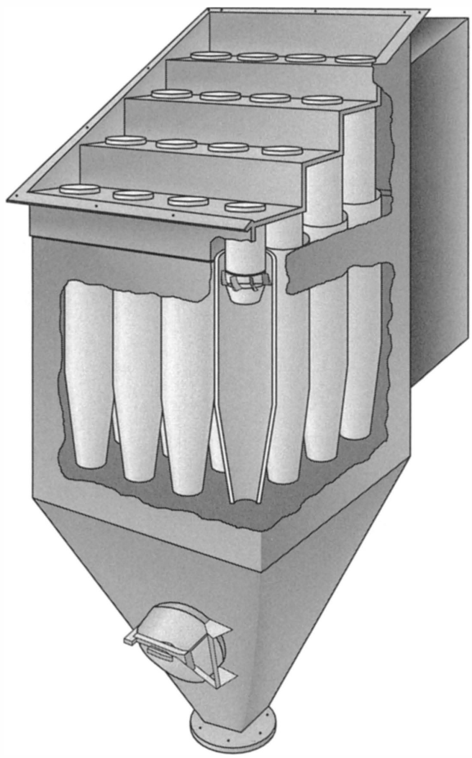

FIGURE 9.5
A multiclone, which places a large number of small cyclones in parallel. The dirty gas flows through an entrance duct, the edge of which is shown in the sketch at the rear, into the chamber shown in the cutaway, then flows downward into the individual tubes, getting its spiral motion from the turning vanes shown. The cleaned gas flows up the central tubes and out through the top of the device (through an outlet flue, not shown, which bolts to the slanting top of the device). The collected particles fall to the conical bottom. (Courtesy of Joy Environmental.)

cyclone is caused by a set of sheet metal turning vanes that replace the solid top of an ordinary cyclone. The gas outlet tubes are connected to a common gas outlet header. If the individual cyclone were one-half foot in diameter, the W_i in Eq. (9.20) would be about 0.125 ft. Repeating Example 9.5 for a W_i of 0.125 ft, we find a predicted cut diameter of 2.3 μ, which is about the actual cut diameter of these devices.

Although Eq. (9.20) is a fair predictor of cut diameters, Eq. (9.18), upon which it is based, is a poor predictor of the relation of collection efficiency to diameter. Equation (9.19), which takes mixing into account, is a better predictor, but neither is really good. Figure 9.6 on page 262 compares the predictions that Eqs. (9.18) and (9.19) make with a curve representing a summary of experimental data [4] that can be represented with satisfactory accuracy by the following totally empirical data-fitting equation:

$$\eta = \frac{(D/D_{\text{cut}})^2}{1 + (D/D_{\text{cut}})^2} \tag{9.21}$$

(See also Problem 9.6.)

Example 9.6. A gas stream contains particles with a particle size distribution by mass that is given by the log-normal distribution, with $D_m = 20\ \mu$ and $\sigma = 1.25$

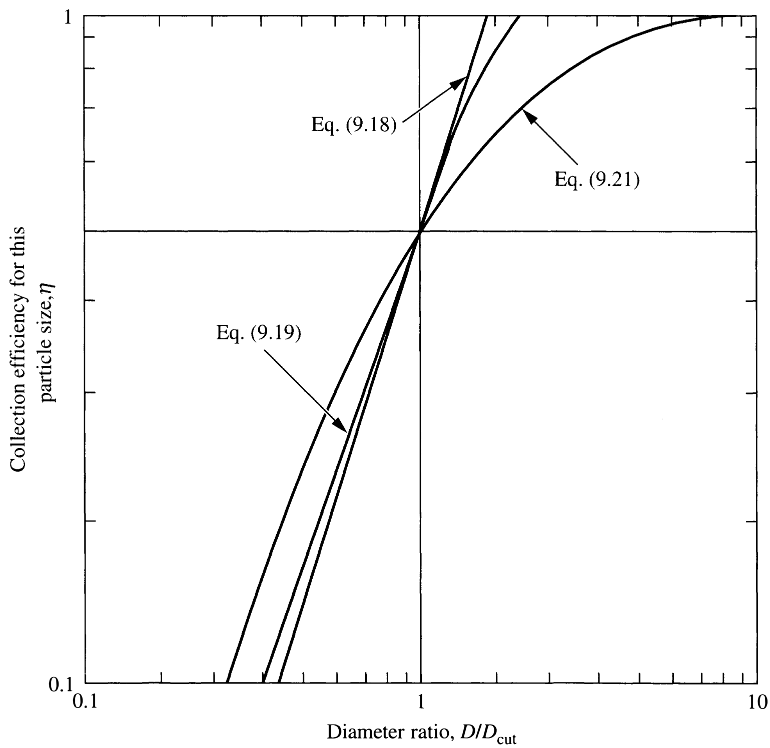

FIGURE 9.6
Collection efficiency vs. particle diameter curves for cyclones. Here, all three curves must pass through 0.5 at $D = D_{\text{cut}}$ because of the definition of D_{cut}. Equation (9.21) is very close to the experimental results for typical cyclones.

(see Fig. 8.10). We pass this through a cyclone separator whose cut diameter is 5 μ, and whose efficiency-diameter relation is given by Eq. (9.21) (and shown in Fig. 9.6). What is the percentage by weight of the particles caught? What is the mass mean diameter of the particles that pass through?

We cannot solve this problem analytically but must instead divide the particle distribution into size fractions and compute the penetration for each one, as illustrated in Section 7.8. The result is shown in Table 9.1. In the first column we have divided the distribution into 10 fractions, those from 0 to 0.1 of the mass of the particles, those from 0.1 to 0.2, etc. The second column shows the z corresponding to the Φ at the end of this interval, such as 0.1, 0.2, etc. These values are found from a table like Table 8.3, but arranged for even values of Φ instead of even values of z. The third column shows the value of (D/D_{mean}) at the end of the size interval, found by solving Eq. (8.20) for a log-normal distribution. The first value is

$$\frac{D}{D_{\text{mean}}} = \exp(z\sigma) = \exp(-1.282 \times 1.25) = 0.2014$$

This calculation shows that 0.1 = 10 percent of the particles have diameters less

TABLE 9.1
Performance computation for a cyclone separator

Φ	z	$\left(\frac{D}{D_{\text{mean}}}\right)_{\text{end}}$	$\left(\frac{D}{D_{\text{mean}}}\right)_{\text{mid}}$	η	$p\,\Delta\Phi$	$\sum p\,\Delta\Phi$
0.1	−1.282	0.2014	0.1007	0.1396	0.0860	0.0860
0.2	−0.842	0.3491	0.2752	0.5479	0.0452	0.1312
0.3	−0.524	0.5194	0.4343	0.7511	0.0249	0.1561
0.4	−0.253	0.7289	0.6242	0.8617	0.0138	0.1700
0.5	0	1.0000	0.8644	0.9228	0.0077	0.1777
0.6	0.253	1.3720	1.1860	0.9575	0.0043	0.1819
0.7	0.524	1.9251	1.6486	0.9775	0.0022	0.1842
0.8	0.842	2.8648	2.3950	0.9892	0.0011	0.1853
0.9	1.282	4.9654	3.9151	0.9959	0.0004	0.1857
1		1.0000	4.9654	0.9975	0.0003	0.1859

than $(0.2014 \times 20\ \mu) = 4.02\ \mu$. The average diameter of the smallest 10 percent of the particles is approximately half of this, or 2 μ. The fourth column shows this average diameter ratio, listed as $(D/D_{\text{mean}})_{\text{mid}}$. For the first entry this is the average of the end value and zero. For the next eight values it is the average of the value at the end of the range and at the end of the previous range. The final value is taken as the end value of the preceding range, which introduces only a small error.

The fifth column of Table 9.1 shows the collection efficiency η for the midrange diameter, computed by Eq. (9.21):

$$\begin{aligned}\eta &= \frac{(D/D_{\text{cut}})^2}{1+(D/D_{\text{cut}})^2}\\ &= \frac{[(D/D_{\text{mean}})(D_{\text{mean}}/D_{\text{cut}})]^2}{1+[(D/D_{\text{mean}})(D_{\text{mean}}/D_{\text{cut}})]^2} = \frac{(0.1007\cdot 20\ \mu/5\ \mu)^2}{1+(0.1007\cdot 20\ \mu/5\ \mu)^2} = 0.1396\end{aligned}$$

In the sixth column is $p\,\Delta\Phi$, the amount of mass in this size interval that passes through uncollected, e.g.,

$$p\,\Delta\Phi = (1-0.1396)(0.1-0) = 0.0860$$

We see that 86 percent by mass of the particles in this size range (8.6 percent of the total particle mass) pass through the cyclone uncollected. The final column is the sum of the values in column 6, showing the cumulative fraction uncollected. The lower right-hand value shows that 0.186 = 18.6 percent of the particles are not collected, so that the overall collection efficiency is $0.814 \approx 81$ percent.

The mass mean diameter of the particles that pass through the cyclone is the diameter that corresponds to half of the value at the bottom of column 7, or 0.0930. This is slightly more than the value at the end of the 0 to 0.1 weight fraction interval, so from the third column in Table 9.1 we know that it corresponds to a diameter of about $0.2014 \approx 0.2$ of the mean diameter or about 4 μ. At the end of this long example, the reader is encouraged to compare it with Example 7.5. This is simply that example repeated, using a real particle size distribution and a real collector efficiency relation. For all the devices discussed later in this chapter, final design calculations are made by the equivalent of this table.

One may repeat this example using 20 size intervals instead of the 10 here, and find that the final penetration value is 0.1836 instead of 0.1859. We rarely have size distribution or control efficiency data good enough to justify that extra computation. ■

The low collection efficiency, 81 percent, of Example 9.6 shows that a typical cyclone cannot meet modern control standards (usually > 95 percent required control efficiency) for any particle group that has a substantial fraction smaller than 5 μ in diameter.

Although Eq. (9.20) and Example 9.5 show that the practical cut diameter is limited, the physical reason is hidden in the mathematics. To get a high value of V_t, we need a high value of V_c; but a high value of V_c means that the gas stream is in the cyclone for only a very short time and has little time to be acted on by the high centrifugal force.

Example 9.7. In Example 9.5 how long does the gas spend in the high centrifugal force field near the wall where a particle has a good chance of being captured?

Here, following the assumptions leading to Eqs. (9.16) and (9.17),

$$t = \frac{L}{V} = \frac{N\pi D_o}{V_c} = \frac{5\pi \cdot 2\text{ ft}}{60\text{ ft/s}} = 0.525\text{ s}$$ ■

The distance the particle can move toward the wall is equal to the product of this time and V_t, but V_t is proportional to V_c squared, so that to get better collection efficiencies we must go to lower and lower times in the cyclone.

Previously we stated that the typical velocity at a cyclone inlet is 60 ft/s (18.29 m/s) and that this velocity is selected for pressure drop reasons. If one measures the pressure in the pipe leading the gas to the cyclone and the pressure in the pipe leaving the cyclone, one will find that the inlet pressure is higher. For a given cyclone one will generally find that the pressure drop, for various conditions, can be represented by an equation of the form

$$\text{Pressure drop} = P_{\text{in}} - P_{\text{out}} = K\left(\frac{\rho_g V_i^2}{2}\right) \tag{9.22}$$

where ρ_g is the gas density and V_i is the velocity at the inlet to the cyclone. (V_i is not the same as the velocity in the duct approaching the cyclone; typically it is about 1.5 times as high.) Designers who work regularly with air-conditioning or other piping systems have observed that most pressure drop data for their kinds of systems can be represented in the form of Eq. (9.22), with each particular kind of device having its own K. (All sudden expansions have a K of 1.0, all sudden contractions have a K of 0.5, etc. Tables of Ks for various types of pipes and fittings are widely published [5].) Most cyclone separators have Ks of about 8. It is also common in air-conditioning