This value is surprisingly small; the particle stops in 0.07 mm = 0.0027 inch. This makes clear that for particles of this size, and most particles of air pollution interest, the air is a very viscous fluid indeed. Intuitively this is comparably viscous to a baseball thrown into a pot of cold maple syrup.

In Chapter 9 we will see that the Stokes stopping distance is a natural distance scale for the behavior of particles. Several of the control-efficiency relations we develop there include one term that is the ratio of the Stokes stopping distance to some dimension of the piece of control equipment.

8.2.5 Aerodynamic Particle Diameter

Equation (8.12) also shows that any two particles that have the same value of $D^2 \rho_{\text{part}}\, C$ will have the same Stokes stopping distance for any initial velocity (in air with the same viscosity). We will see in Chapter 9 that any two particles with the same value of this set of properties will behave identically in several kinds of control devices. They have the same aerodynamic behavior. For that reason, we define a new property, the *aerodynamic particle diameter*:

$$\text{Aerodynamic particle diameter} = D_a = D(\rho_{\text{part}}\, C)^{1/2} \tag{8.13}$$

Often one sees this definition with the C omitted. This is a peculiar diameter, because it has the dimensions [(length) (mass/length^3)$^{1/2}$], e.g., [(m)(kg/m^3)$^{1/2}$]. It is strange to speak of a diameter with this kind of dimension, but that is the common usage. Thus the particle in Example 8.5 would have an aerodynamic particle diameter, D_a, of

$$D_a = 0.1\ \mu \left(2\ \frac{\text{g}}{\text{cm}^3} \cdot 2.21\right)^{0.5} = 0.21\ \mu \left(\frac{\text{g}}{\text{cm}^3}\right)^{0.5} = 0.21\ \mu_a$$

where the symbol μ_a stands for "microns, aerodynamic." In SI units this should be stated as 0.21 μm (1000 kg/m^3)$^{0.5}$, but that usage is seldom seen.

8.2.6 Diffusion of Particles

Small particles move by Brownian motion, which we describe according to the equations for diffusion. If a particle is large, then in any short period of time (e.g., 1 s) it will experience many collisions with the surrounding gas molecules that are hitting it from all sides, and the resulting net force will be quite small relative to the mass of the particle. Thus we do not see houses, desks, or marbles being moved about by Brownian motion. If the particle is small enough that it can only expect a few collisions per second and its inertia is small because of its small size, then the force of an individual collision is enough to make it move. Subsequent collisions, whose directions are random, will move it in other directions, so that its time-series path will be a series of short jumps in one direction and then another. One must use a microscope to observe such behavior, because the particle size at which it becomes important is too small for our eyes to distinguish.

In a uniform solution or suspension, Brownian motion does not cause any net change in the concentration with time in any part of the solution. As many particles move one way as move another. But if the concentration is not uniform, then Brownian motion tends to equalize the concentration. In so doing, it makes the particles move by diffusion, just as molecules in nonuniform solutions do. From diffusion theory, we know that for three-dimensional, nonsteady-state diffusion

$$\frac{\partial c}{\partial t} = \mathcal{D}\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right) \tag{8.14}$$

where $\mathcal{D}$ = diffusivity (normal units m^2/s)
c = concentration

For steady-state, one-dimensional diffusion Eq. (8.14) reduces to the well-known *Fick's law* of diffusion,

$$\text{Flux} = \frac{\text{Diffusive flow rate}}{\text{Unit area}} = -\mathcal{D}\frac{dc}{dx} \tag{8.15}$$

For spherical particles suspended in a perfect gas, $\mathcal{D}$ may be estimated from the kinetic theory of gases as

$$\mathcal{D} = \frac{kTC}{3\pi\mu D} \tag{8.16}$$

where k = Boltzmann constant
C = Cunningham correction factor from Eq. (8.8)

Example 8.6. Estimate the diffusivity of a 1-μ diameter particle in air at 20°C and 1 atm.

For a 1-μ diameter particle the Cunningham correction factor can be shown from Eq. (8.8) to be about 1.16, so

$$\mathcal{D} = \frac{(1.38 \times 10^{-23}\ \text{kg}\cdot\text{m}^2/\text{s}^2\cdot\text{K})(293.15\ \text{K})(1.16)}{(3\pi)(1.8 \times 10^{-5}\ \text{kg/m}\cdot\text{s})(10^{-6}\ \text{m})} = 2.8 \times 10^{-11}\ \frac{\text{m}^2}{\text{s}} \quad \blacksquare$$

Most gases diffuse in air with diffusivities of about 10^{-5} m^2/s, and diffusivities of solutes in liquids are typically about 10^{-9} m^2/s. Thus, particles on the order of a few microns do not diffuse rapidly. We may now turn back to Fig. 8.1 and observe that along the bottom of the page the values of $\mathcal{D}$ are shown for particles in air and water; the student may verify that the result in Example 8.6 is the same value shown on that figure.

We will see that for some collection devices diffusion plays a measurable role, and also that the coalescence behavior of fine particles in the atmosphere is governed by diffusion, with the diffusivity values shown in Example 8.6 and on Fig. 8.1.

8.3 PARTICLE SIZE DISTRIBUTION FUNCTIONS

So far we have considered a single particle, or a group of particles, all with the same size. But in particulate air pollution problems we are concerned with groups of particles having a variety of sizes (see Fig. 8.4). To discuss such groups and to make useful calculations about their behavior in collection devices, we need some way of describing the particle size distributions. This section discusses *distribution functions* and their application to groups of particles. Students who are familiar with distribution functions can skip this section.

8.3.1 A Very Simple Example: The Population of the United States

Table 8.2 shows the age distribution of the population of the United States, taken from the 1990 census. On the basis of the first line, 18.35 million people had not reached their fifth birthday by the date of the census. Dividing this number by the total at the bottom of the second column, we see that this was 7.38 percent of the total population. The next line shows a similar set of numbers; we also see in the rightmost column that the cumulative total population in the age range zero to nine was 14.66 percent. Every number in the rightmost column is the sum of the number above it (the cumulative percent up to the previous age group) and the number to its left (the incremental percent in this age group).

Figures 8.8 and 8.9 on pages 229 and 230, show the same information as Table 8.2, plotted in two different ways, integral and differential. Figure 8.8 shows a smooth curve drawn through the values in the rightmost column of Table 8.2 plotted vs. the age corresponding to the end of each interval. The resulting plot is a cumulative (integral) distribution of ages in the population of the United States.

TABLE 8.2
Population of the United States, 1990

Age range, years, Δn	Population, millions, ΔN	% in this age range, $\Delta\Phi$	Cumulative % of the total population up to age n, Φ
0–4	18.35	7.38	7.38
5–9	18.10	7.28	14.66
10–14	17.11	6.88	21.54
15–19	17.75	7.14	28.68
20–24	19.02	7.65	36.32
25–29	21.31	8.57	44.89
30–34	21.86	8.79	53.68
35–39	19.96	8.03	61.71
40–44	17.62	7.08	68.79
45–49	13.87	5.58	74.37
50–54	11.35	4.56	78.94
55–59	10.53	4.23	83.17
60–64	10.62	4.27	87.44
65–69	10.11	4.07	91.51
70–74	7.99	3.21	94.72
75–79	6.12	2.46	97.18
80–84	3.93	1.58	98.76
85+	3.08	1.24	100.00
Total	248.7 = N		

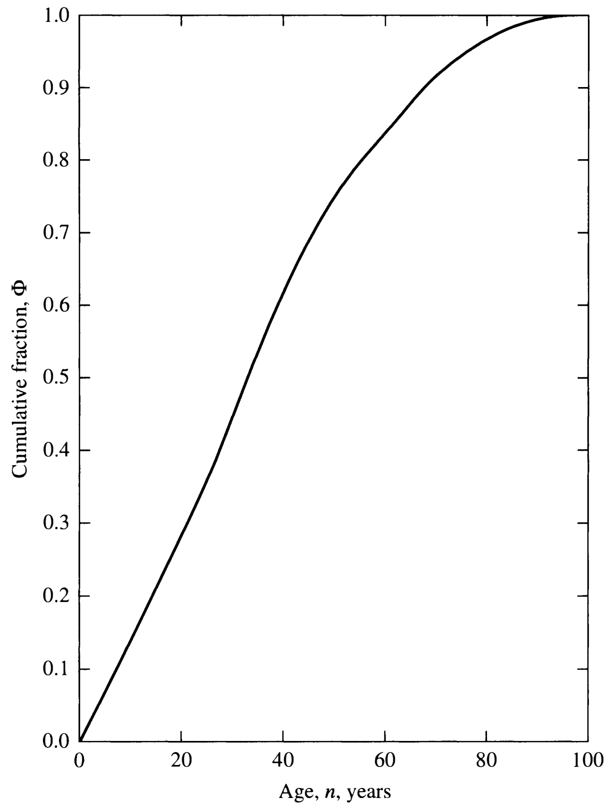

FIGURE 8.8
Age distribution for the United States in 1990 in integral form, based on Table 8.2.

If the total number of people in any age range, column 2 in Table 8.2, is ΔN and the total number of people in the whole population is N, and n represents some specific age, then the fraction of the population with ages n or less is Φ, defined by

$$\Phi = \frac{\sum_0^n \Delta N}{\sum_0^\infty \Delta N} \tag{8.17}$$

Φ has values from 0 (actually 1/248.7 million, practically zero, corresponding to the most recently born baby) to 1.00 ([actually 1.00 minus 1/248.7 million], practically 1.00, corresponding to the oldest person in the population). Figure 8.8 and Table 8.2 show Φ going from 0 to 1 (or 0 to 100 percent). In the language of statistics, the Φ curve is called a *normalized* curve, which means that all values have been divided by a suitable total so that the value of the variable ranges from 0 to 1. *Normalization* is common in statistics and practically universal in the study and use of distribution functions.

Figure 8.9 shows a plot of $d\Phi/dn$ vs. n. It is much more informative than Fig. 8.8. On it we can see that the birthrate reached a peak about 1960 (1990 minus 30) and a low about 1932 (1990 minus 58). These dates correspond to the baby boom following World War II and the birthrate decline corresponding to the Great

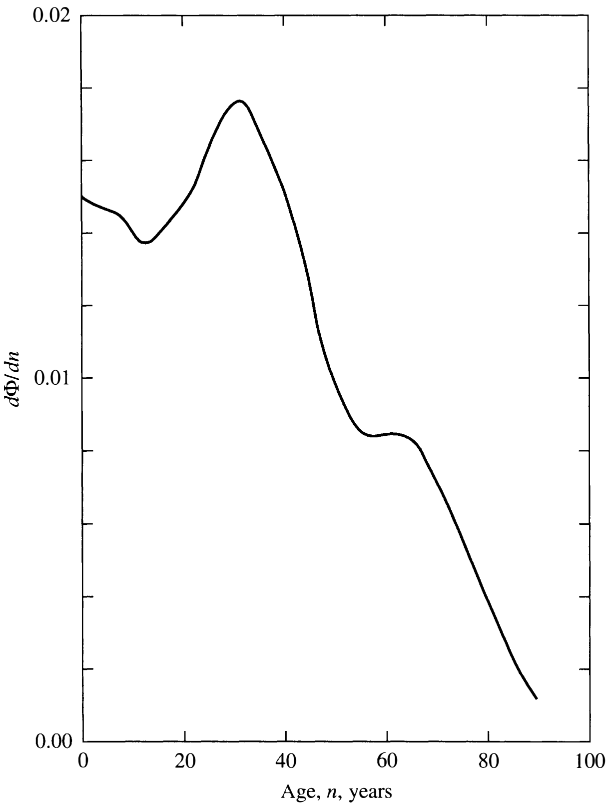

FIGURE 8.9
Age distribution for the United States in 1990 in differential form, based on Table 8.2.

Depression. This information could also be found from a careful examination of Fig. 8.8, but the information is much clearer on Fig. 8.9. (Plotting the integral in Fig. 8.8 smooths out most of the interesting details.) How can we relate these two figures? Table 8.2 shows the $\Delta\Phi$ values that correspond to each five-year period. If we divided these values by the time interval, five years, we would get the values of $\Delta\Phi/\Delta n$. So, for example, for the first five-year period, we would have

$$\frac{\Delta\Phi}{\Delta n} = \frac{\left(\dfrac{18.35 \text{ million}}{248.7 \text{ million}}\right)}{5 \text{ yr}} = 0.0148/\text{yr} = 1.48\%/\text{yr}$$

In making Fig. 8.9 this value (in decimal form) was plotted at 2.5 years, the corresponding values for each subsequent interval were plotted at the midpoint of those intervals, and a smooth curve was drawn through the points. For the last interval, 85+, Δn was arbitrarily selected as 15 years.

Since Fig. 8.8 is a plot of Φ vs. n, the slope of the curve at any value of n must be $d\Phi/dn$ at that n, and hence the smooth curve drawn through the values on Fig. 8.9 must represent the derivative of the curve in Fig. 8.8. From this it follows that the area under the curve on Fig. 8.9 (plotted as $d\Phi/dn$ vs. n) from n equals zero to infinity must have the value 1.0. Patient students might try counting squares to see if this is correct; it is.

Table 8.2 gives the most detailed information, but it is not very intuitive. The two figures are much more helpful for visualizing the situation. However, if we want to represent the data in the most compact form and to make mathematical manipulations with it, we would like to have some mathematical distribution function to represent the data. This would be of the form Φ = function of n. If two populations of different sizes have a similar distribution of ages (e.g., the United States and Canada), then they will have similar values of Φ as a function of n, even though the total populations' Ns are very different. If we know Φ as a function of n, and we know the total population N, then we can easily deduce the number of persons in any age segment of the population for either country.

Example 8.7. The simplest population distribution function we can think of is of the form $\Phi = (\text{constant}) \cdot n$, over the age range from 0 to n_{max}. We would expect this distribution in a population in which the birthrate was constant and everyone lived to the age n_{max}, but no longer. Such populations occasionally occur in science fiction, e.g., *Brave New World,* by Aldous Huxley.

Let us assume $n_{\text{max}} = 50$ years. Then, because we know that the Φ corresponding to n_{max} must be 1.0, we can determine the value of the constant, i.e.,

$$\text{Constant} = \frac{\Phi_{\text{max}}}{n_{\text{max}}} = \frac{1.00}{50\ \text{yr}} = \frac{0.02}{\text{yr}}$$

The figure analogous to Fig. 8.8 would be a straight line passing through the origin, with slope 0.02/yr, reaching 1.00 at 50 years. The figure corresponding to Fig. 8.9 would be a horizontal line with value $d\Phi/dn = 0.02/\text{yr}$ from 0 to 50 years, then dropping to 0 for all ages > 50 years. If we wanted to know the number of people in the age range 18 to 23 years we would need to know the total population N, which we here assume to be 500,000. Then

$$\begin{aligned}\Delta N &= N\ \Delta\Phi = N(\Phi_{\text{final}} - \Phi_{\text{initial}}) \\ &= 500\ 000 \left(\frac{0.02}{\text{yr}}\right)(23 - 18)\ \text{yr} = 50\ 000\ \text{people}\end{aligned}$$

Here we would have to be clear that "people in the age range 18 to 23" means those who have had their eighteenth birthday, and have not yet had their twenty-third birthday. This distinction can be important in more complex cases. ■

This example is given in much more detail than such a simple distribution function requires; but the manipulations are the same for the more mathematically complex distribution functions that follow. Returning to the U.S. population example, we see there is no simple mathematical equation for Φ as a function of n. By brute-force curve fitting we can compute

$$\begin{aligned}\frac{d\Phi}{dn} &= 1.544 \times 10^{-2} - 5.7907 \times 10^{-4}n + 5.9344 \times 10^{-5}n^2 - 1.9224 \times 10^{-6}n^3 \\ &\quad + 2.3849 \times 10^{-8}n^4 - 1.2072 \times 10^{-10}n^5 \qquad (8.18)\end{aligned}$$

which has no theoretical significance, but which represents the data fairly well (correlation coefficient = $R^2 = 0.966$).

Table 8.2, Figs. 8.8 and 8.9, and Eq. (8.18) are all different ways of representing the same set of experimental data. Given any one of them, and the value of N, one could reproduce all the others. The equation has the least intuitive content but is most satisfactory if we wish to use the data in a computer. The table also has little intuitive content, but is the most precise representation (although census data are only estimates of the true population). The two figures give the most intuitive picture of the data. In the following parts of this section, we look at the relation between distribution equations and their corresponding plots and tables.

The complexity of human behavior is great and so variable over time that one can seldom find a simple mathematical description of human behavior. (Equation 8.18 is brute-force and ugly!) However, for phenomena that do not involve individual human decisions, often we can find a satisfactory mathematical description of Φ as a function of some suitable variable. For example, if we measure the diameter of 1000 grains of beach sand and make a plot of Φ vs. diameter, we will probably find that the resulting curve can be satisfactorily represented by some relatively simple mathematical relation. Many distribution functions have been found to represent natural phenomena, e.g., the Gaussian or normal distribution, the log-normal, the gamma, the Weibull, the Poisson, etc. All of these are of the form Φ = some function of some parameter like age, or diameter, or wind speed, etc.

8.3.2 The Gaussian, or Normal, Distribution

The most famous and most widely used distribution function is the *Gaussian,* or *normal,* or *error distribution function.* It represents a great variety of observed distribution data well and is described by

$$\frac{d\Phi}{dx} = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left[\frac{(x - x_{\text{mean}})^2}{2\sigma^2}\right] \tag{8.19}$$

Here Φ has the same meaning as before (i.e., the fraction of the cumulative total in the size range of interest), x is some suitable dimension or measure (e.g., age, diameter, etc.), x_{mean} is the average value of x (a suitably chosen average; there are several choices), and σ^2 is a quantity called the variance, which can be considered as a constant for the purposes of Eq. (8.19).

Equation (8.19) shows that if we plot $d\Phi/dx$ vs. x as in Fig. 8.9, the plot must be symmetrical about x_{mean} because $(x - x_{\text{mean}})$ enters squared. The maximum value of $d\Phi/dx$ must occur at $x = x_{\text{mean}}$ because for that value the exponential term is 1.0, and for any other value of x it is smaller. When x is a large positive or negative number the exponential term will approach zero asymptotically, so the curve must approach zero in both directions moving from the center, producing a symmetrical, bell-shaped curve. A small value of σ makes the argument of the exponential term larger, so that the values are concentrated near x_{mean}; a large value of σ spreads the values out over a wide range of xs. Therefore, for the same x_{mean}, a small σ will give a tall, narrow bell, whereas a large σ will give a low, broad bell. For any value of σ the area under the bell-shaped curve from $x = -\infty$ to $x = +\infty$ is equal to 1.0. [The $(1/\sigma\sqrt{2\pi})$ term ahead of the exponential makes this integration come out right; it is called the *normalizing factor*].

So far, no one has found a way to integrate Eq. (8.19) analytically to get the explicit equation that we would like for Φ as a function of x, x_{mean}, and σ. (Many great mathematicians have tried; fame and fortune await the clever student who can do it!) But although there is no available analytical solution, the integration has been performed numerically, and tables of its values are widely available. Rather than treat x, x_{mean}, and σ as separate variables, all of these tables combine them by defining a new variable z as

$$z = \text{number of standard deviations from the mean} = \frac{(x - x_{\text{mean}})}{\sigma} \tag{8.20}$$

(z is sometimes called the number of *probits* from the mean.)

Substituting this definition into Eq. (8.19) and simplifying, we find

$$\frac{d\Phi}{dz} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \tag{8.21}$$

In this equation we can separate variables and integrate numerically to obtain a table of Φ as a function of z. Table 8.3 on page 234 presents the results of such a numerical integration. Much more detailed tables are available in mathematical handbooks. Table 8.3 does not contain any negative values of z because, as one can see from Eq. (8.21), the value of $d\Phi/dz$ is the same for a positive or negative value of z. It also shows that for $z = 0$ the value of Φ is 0.5; the distribution is symmetrical about $z = 0$, $\Phi = 0.5$.

Example 8.8. An investigator reports that the height of adult males in the United States is well represented by the normal, or Gaussian, distribution, with x = height, $x_{\text{mean}} = 5.75$ ft, and $\sigma = 0.8$ ft.

If this is correct, what fraction of this population is taller than 6 ft? shorter than 4 ft? Are there any men taller than 10 ft? Are there any shorter than 1 ft?

For 6 ft we have

$$z = \frac{6 \text{ ft} - 5.75 \text{ ft}}{0.8 \text{ ft}} = 0.31$$

From Table 8.3 we see that this value of z corresponds to a Φ of approximately 0.62, which indicates that $(1 - 0.62) = 0.38 = 38$ percent of this population is predicted to be taller than 6 ft. For 4 ft we find

$$z = \frac{4 \text{ ft} - 5.75 \text{ ft}}{0.8 \text{ ft}} = -2.19$$

Here we use the symmetry property of the normal distribution, shown at the bottom of Table 8.3, to calculate that

$$\Phi(-2.19) = 1 - \Phi(2.19) = 1 - 0.986 = 0.014 = 1.4\%$$

TABLE 8.3
Values of the cumulative frequency integral Φ as a function of z

z	Φ	z	Φ
0.0	0.5000	2.1	0.9821
0.1	0.5398	2.2	0.9861
0.2	0.5793	2.3	0.9893
0.3	0.6179	2.4	0.9918
0.4	0.6554	2.5	0.9938
0.5	0.6915	2.6	0.9953
0.6	0.7258	2.7	0.9965
0.7	0.7580	2.8	0.9974
0.8	0.7881	2.9	0.9981
0.9	0.8159	3.0	0.9986
1.0	0.8413	3.1	0.9990
1.1	0.8643	3.2	0.9993
1.2	0.8849	3.3	0.9995
1.3	0.9032	3.4	0.9997
1.4	0.9192	3.5	0.9998
1.5	0.9332	3.6	0.9998
1.6	0.9452	3.7	0.9999
1.7	0.9554	3.8	0.9999
1.8	0.9641	3.9	See Prob. 8.29
1.9	0.9713	4.0	See Prob. 8.29
2.0	0.9772		

Note: For negative values of z use $\Phi(-z) = 1 - \Phi(z)$. For example, $\Phi(-0.2) = 1 - \Phi(0.2) = 1 - 0.5793 = 0.4207$.

We would expect 1.4 percent of this population to have a height less than 4 ft. For 10 ft we compute $z = (10 - 5.75)/0.8 = 5.31$. Using the approximation in Problem 8.29, we find that $(1 - \Phi)$ is 6×10^{-8}. So, if this distribution truly represents the population, and if there are approximately 10^8 adult males in the United States, then we would expect to find about six men with a height above 10 ft. For the 1-ft-tall man, $z = -5.94$ and (see Problem 8.29) $\Phi = 0.14 \times 10^{-8}$, so we would expect to find about 0.14 adult male less than a foot tall in the population (or to find one 14 percent of the time). ■

This example shows how one uses the normal distribution function and Table 8.3. We also see that although the normal distribution is easy to use, it cannot be an absolutely correct description of this particular population, because we can be quite certain from observation that there are no adult males taller than 10 ft or shorter than 1 ft. One can carry this calculation out to even taller and shorter values, even to negative heights, and find a very small but nonzero probability that we will find a man with negative height. This should help the student realize that these mathematical distribution functions are *useful approximations* of experimental reality but not exact descriptions of nature. Generally, mathematical distribution functions like the normal, or others (log-normal, Weibull, gamma), do a satisfactory job of representing experimental data in the middle of the data range (where most of the data are) but become unreliable at representing the experimental data at the extreme values (tails) of the distributions.

Students are sometimes confused by the fact that x_{mean} and σ, which have exact and unambiguous definitions, appear in these distributions, which are approximations. For any sample with n members,

$$x_{\text{mean}} = \frac{1}{n}\sum x_i \quad \text{and} \quad s = \frac{1}{(n-1)}\left[\sum (x_i - x_{\text{mean}})^2\right]^{1/2} \tag{8.22}$$

These expressions are independent of whether the sample is best represented by the normal distribution function or some other distribution function, or is not well represented by any simple distribution function. In the limit, as n becomes large, the s in the preceding definition (the *sample standard deviation*) becomes σ, or (the *variance*)$^{0.5}$. Often people speak of statistics with the hidden assumption that the measurements we are discussing are taken from a population that is well represented by the normal distribution. That is generally a good guess, but not always right, as shown later.

Now we are ready to talk about particle size distributions. We can presumably obtain a sample of the particles in a gas stream by catching them on a filter or by some other technique; and we can count the particles of various sizes using a microscope and make up a table just like Table 8.2, with diameter replacing age range. However, we generally find that data obtained from this kind of experiment are *not* well represented by the normal distribution function of Eq. (8.19).

8.3.3 The Log-Normal Distribution

If we let x in Eq. (8.19) represent, not the particle diameter but its natural logarithm, we will obtain the following *log-normal distribution,* which is almost as widely used as the normal distribution:

$$\frac{d\Phi}{d\ln D} = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left[\frac{(\ln D - \ln D_{\text{mean}})^2}{2\sigma^2}\right] \tag{8.23}$$

or, alternatively,

$$\frac{d\Phi}{d\ln D} = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left[\frac{[\ln(D/D_{\text{mean}})]^2}{2\sigma^2}\right] \tag{8.24}$$

Many authors write Eqs. (8.23) and (8.24) with the σ replaced by $\ln \sigma_g$, i.e.,

$$\frac{d\Phi}{d\ln D} = \frac{1}{\ln\sigma_g\sqrt{2\pi}} \exp - \left[\frac{[\ln(D/D_{\text{mean}})]^2}{2(\ln\sigma_g)^2}\right] \tag{8.25}$$

Since σ is a constant for any particular distribution, this change makes no difference, except that the σ_g one finds using Eq. (8.25) is the exponential of the σ one finds using Eqs. (8.23) and (8.24). Typical values of σ in Eqs. (8.23) and (8.24) for particle distributions are 0.5 to 2, which correspond in Eq. (8.25) to σ_gs of 1.64 to 7.39. The latter are often called *logarithmic standard deviations* or *geometric standard deviations* and are written σ_g. The smallest possible value (σ_g) is 1.0, corresponding to a σ of zero.

The value of z that we defined in Eq. (8.20) for the normal distribution is converted to the log-normal distribution of particle diameters by replacing every x by ln D, or

$$z = \left[\frac{(x - x_{\text{mean}})}{\sigma}\right]_{\text{normal distribution}}$$

$$= \left[\frac{(\ln D - \ln D_{\text{mean}})}{\sigma} = \frac{\ln(D/D_{\text{mean}})}{\sigma}\right]_{\text{log-normal distribution}} \tag{8.26}$$

The student may verify that substituting Eq. (8.26) into Eq. (8.23) converts the latter into Eq. (8.21). Thus we can use Table 8.3 for the log-normal distribution, just as we did for the normal distribution, with the proper value of z from Eq. (8.26).

Returning now to the problem of the particle distribution function, we see that if Eq. (8.23) or (8.24) is a satisfactory representation of the distribution and if we could plot ln D vs. z (which is a function of D), we should obtain a straight-line plot. Fortunately, graph papers are available that make this easy. On them one simply plots D vs. Φ, and if the data are log-normally distributed, the result is a straight line. Figure 8.10 shows such a representation on log-normal paper (most often called *log-probability* paper) of particle sizes normally encountered in the exhaust gas from pulverized-coal furnaces. This paper is plotted so that z proceeds linearly across the bottom of the paper; the values of Φ corresponding to any z (looked up on Table 8.3 or its equivalent) are shown instead of z itself. As a result, the scale is compressed in the middle and expanded greatly near the right and left edges. This kind of representation is practically universal in the air pollution literature. No other way of presenting particle size data seems to be nearly as successful or as widely used.*

Because the representation in Fig. 8.10 is common in air pollution work, let us familiarize ourselves with its properties (which are explored in much more detail in Ref. 12). First, we observe that the axes are reversed compared to Fig. 8.9; diameter is plotted vertically and Φ horizontally. Log-probability paper is always laid out that way. The line, and any straight line on log-probability paper, is a representation of Eq. (8.23). That equation contains only two constants, D_{mean} and σ. Thus, if we specify the line, we have specified these two values, and conversely if we specify these two values, we have specified one and only one line on log-probability paper. To find the value of D_{mean} from the line on Fig. 8.10, we need only read the diameter that corresponds to the 50 percent "less than stated" size. On Fig. 8.10 this is

*Many other natural phenomena are also well represented by the log-normal distribution. Most weather data—e.g., distribution of hourly wind speeds over a year—are better represented by the log-normal distribution than by any other distribution function.

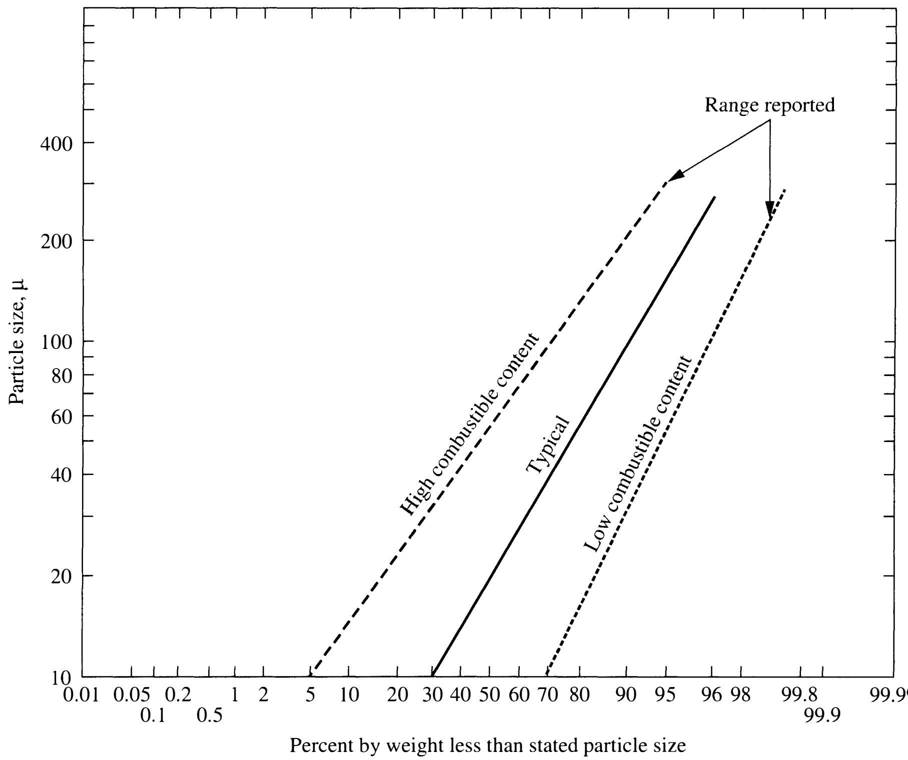

FIGURE 8.10
Example from the air pollution literature of the representation of particle size data in log-normal form. (From Ref. 13.) This is the distribution of the particles collected at the outlet of a pulverized-coal furnace. The particles in Fig. 8.4 would be approximately described by the "Typical" line on this figure.

approximately $D_{\text{mean}} = 20\ \mu$. Observe that this is not the arithmetic mean we are all used to. For a group of N particles with diameter D_i

$$\text{Arithmetic mean diameter} = \frac{1}{N}\sum D_i \tag{8.27}$$

and

$$\text{Log mean diameter} = \exp\left(\frac{1}{N}\sum \ln D\right) = (D_1 \cdot D_2 \cdot \cdots \cdot D_N)^{1/N} \tag{8.28}$$

The latter mean, called the *geometric* or *logarithmic mean,* is the value we obtained by reading the 50 percent point on Fig. 8.10. The reader may verify that these two means are not the same by considering a particle sample with only two particles, one with a diameter of 1 μ and the other with a diameter of 9 μ. Using Eqs. (8.27) and (8.28), we see that the arithmetic mean diameter is 5 μ and the log mean is 3 μ. For most particle size groupings encountered in nature this difference is not important, but when σ becomes large, it becomes more important.

To find σ from Fig. 8.10, we observe in Table 8.3 that $z = 1$ corresponds to $\Phi = 0.8413$, so that, in Eq. (8.20), we have

$$z = 1 = \frac{x_{0.84} - x_{\text{mean}}}{\sigma} \tag{8.29}$$

but in the distribution we are considering, the xs are the natural logs of the diameters, so we can solve for σ, writing

$$\sigma = \ln D_{0.84} - \ln D_{\text{mean}} = \ln \frac{D_{0.84}}{D_{\text{mean}}} \tag{8.30}$$

Reading the value of $D_{0.84}$ from Fig. 8.10 as about 70 μ, we find

$$\sigma = \ln \frac{70\ \mu}{20\ \mu} = 1.25; \qquad \sigma_g = \exp 1.25 = 3.49 \tag{8.31}$$

Thus, the complete characterization of the straight line drawn on Fig. 8.10 is $D_{\text{mean}} = 20$ μ, $\sigma = 1.25$.

Because of the symmetry about D_{mean}, we could just as well have found the value of σ from D_{mean} and $D_{0.16}$ if we had wished (except that it is off-scale in this figure). Because of the utility of the values at 16 percent and 84 percent for estimating σ, some log-probability papers have heavy lines drawn in at those percentages.

All the discussion so far has been in terms of natural logarithms, or ln. Since $\log_{10} x = (\ln x)/2.303$, we can convert all the formulae in this chapter that are in terms of ln to $\log_{10}$ by inserting 2.303 at the appropriate places.

8.3.4 Distributions by Weight and by Number

If we determine the distribution by catching the particles on a greased microscope slide and measuring the diameter of a suitable number of particles, our results will be presented as the percent by number at various size ranges. That is not the most common way of representing the data.

Example 8.9. A group of particles consists of three members, one with a diameter of 1 μ, one with a diameter of 4 μ, and one with a diameter of 10 μ. All three are spheres, and all have the same density. What percent by number of the particles have diameters less than 5 μ? What percent by length, by surface area, and by mass have diameters less than 5 μ?

Here, by number, we have $2/3 = 66.6$ percent of the particles have diameters less than 5 μ. By length we see that if we were to line the particles in a row, the length of those less than 5 μ would be $(1 + 4)$ μ, whereas the total length would be $(1 + 4 + 10)$ μ; so the percent by length less than 5 μ is $(5/15) = 33.3\%$. The surface area of each particle is πD^2, so the surface area of the particles less than 5 μ is $\pi(1^2 + 4^2)$ μ^2. Taking the ratio of this sum to the total, and noting that the πs cancel, we find the percentage of the surface area in particles less than 5 μ is $(1 + 16)/(1 + 16 + 100) = 14.5\%$. Proceeding the same way for mass we observe that the mass of any particle is $(\rho\pi/6)D^3$, and that the $(\rho\pi/6)$ terms will cancel, so that the fraction of the mass in particles less than 5 μ in diameter is $(1 + 64)/(1 + 64 + 1000) = 6.1\%$. ■

This example shows that if one asks what percent of the particles is smaller than some value, without specifying which percent one means, one can get widely varying answers, all correct. In Fig. 8.10 the axis label makes clear that the percent shown there is percent by weight. That is the most commonly used percent in such distributions. Percent by number is also common. Percent by area is widely used in discussing sprays (e.g., spray dryers and paint sprayers) and sometimes in air pollution work. The percent by length has no common application.

A general—and very useful—property of log-normal distributions is that if Φ of D^a is log normal, then Φ of D^b is also log normal, the values of σ are the same for both distributions, and the mean of the new distribution is

$$D_{\text{new mean}} = D_{\text{old mean}} \exp[(b - a)\sigma^2)] \tag{8.32}$$

Example 8.10. Compute the D_{mean} by number that corresponds to the distribution given in Fig. 8.10, for which we know that in the distribution by weight we have $D\text{mean} = 20\ \mu$, and $\sigma = 1.25$.

In the distribution by weight $a = 3$ (because the weight of a particle is proportional to D^3) and in the distribution by number $b = 0$ (because the number of a particle is independent of its diameter, $D^0 = 1$). Substituting into Eq. (8.32), we find

$$D_{\text{mean, number}} = 20\ \mu \exp[(0 - 3)1.25^2] = 0.18\ \mu$$ ■

This difference in mean diameters by weight and by number appears startling but is correct. The big particles have almost all the weight, so the mean by weight is close to the diameter of the largest-size particles that are present in significant numbers. But there are many more small particles than large, so the number mean is much smaller than the weight (or mass) mean.

Because both distributions have the same value of σ, the lines representing them on log-normal paper are parallel. Thus, once we have computed the logarithmic mean by number in the preceding example, we could in principle draw a line parallel to the line on Fig. 8.10, passing through the $D_{\text{mean by number}}$, and have the complete distribution by number. (In Fig. 8.10 that line would run off the plot at the bottom). The saving in time and effort afforded by using this set of properties of the log-normal distribution is very great and is one of the principal reasons why almost all workers in the air pollution field have selected this distribution to represent particle size data.

8.4 BEHAVIOR OF PARTICLES IN THE ATMOSPHERE

Much of what we discussed in this chapter is illustrated by Fig. 8.11 on page 240, which describes the behavior of particles in the atmosphere. This is a plot of $d\Phi_{\text{by area}}/dD$, similar to Fig. 8.9. If $d\Phi_{\text{by mass}}/dD$ were plotted, the peak to the

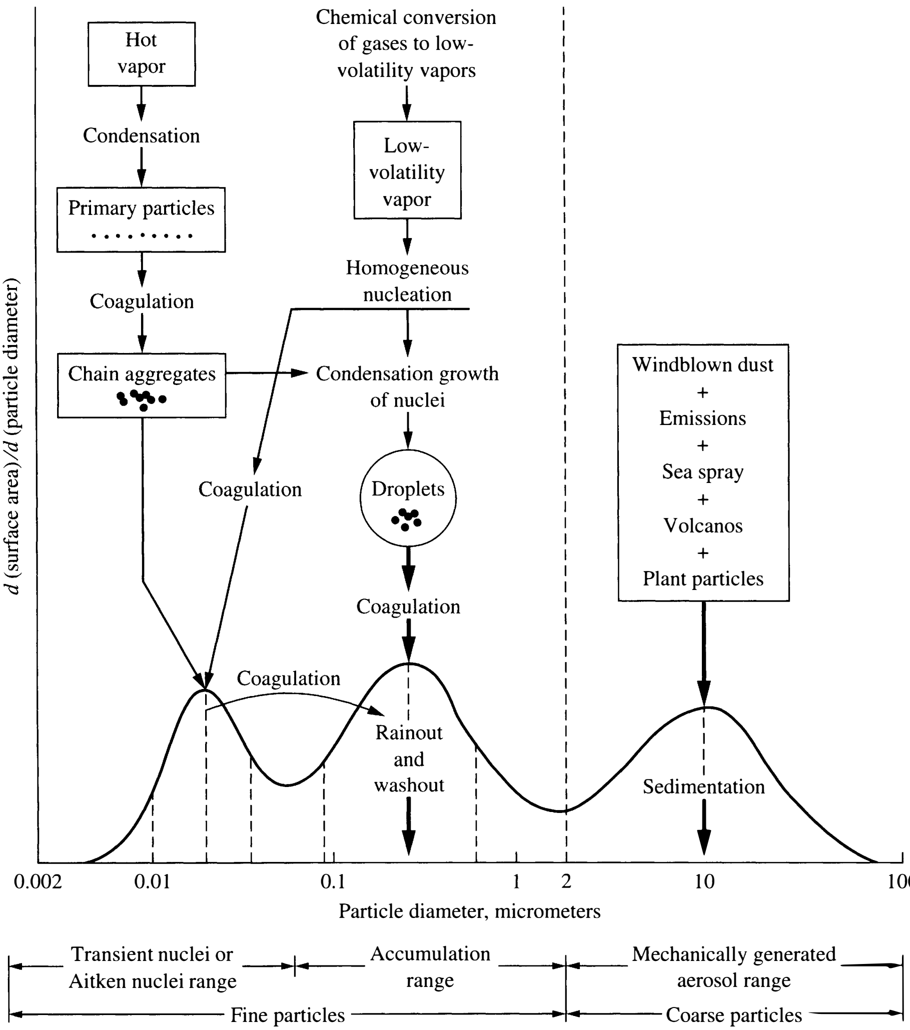

FIGURE 8.11
An estimate of the distribution of particles, *by surface area,* in an industrial atmosphere, after Whitby [14]. (Courtesy of EPRI.)

right would be much larger than the others, or if $d\Phi_{\text{by number}}/dD$ were presented, the peak at the left would be much larger than the others. It shows that the finest particles, with diameters 0.005 to 0.1 μ, enter the atmosphere mostly by condensation of hot vapors from combustion sources. Over time (usually several hours) these smallest particles grow, mostly by agglomeration onto each other. Some of this agglomeration occurs in the gas phase, caused by Brownian motion (diffusion) bringing them into contact; some occurs inside cloud or fog droplets.

Midsized particles (0.1 to 1 μ) are formed partly by the agglomeration of finer particles and partly by chemical conversion of gases and vapors to particles in the atmosphere. These particles are large enough to be removed by *rainout* (capture by drops in clouds) or *washout* (capture by falling raindrops). Although they do grow by agglomeration to form larger particles, this process is slow compared to rainout and washout. The larger particles (2 to 100 μ) are, as shown, mechanically generated; some are derived from industrial particle sources, whose control is discussed in the next chapter. These larger particles are mostly removed by gravity settling, with or without the action of clouds and rain.

The first two peaks in Fig. 8.11 represent almost exclusively secondary particles, formed in the atmosphere from gaseous precursors; the third peak represents for the most part primary particles, emitted to the atmosphere in particulate form. There is some deposition of smaller particles onto these primary particles, but it is not the major method of removal of these smaller particles.

The gaseous precursors of secondary particles are primarily SO_2, NO_x, NH_3, and hydrocarbons. The control of emissions of hydrocarbons, sulfur oxides, and nitrogen oxides are discussed in Chapters 10–12. Ammonia (NH_3) is widely distributed in the atmosphere, coming mostly from biological sources, rather than from human sources.

Figure 8.12 summarizes this chapter. On it we see a truck hauling sand down the road. Sand blows off the truck and falls to the ground, causing a local nuisance. The truck stirs up road dust and generates tire wear particles that are local air pollutants but that do not remain long in the atmosphere. The truck's exhaust contains fine

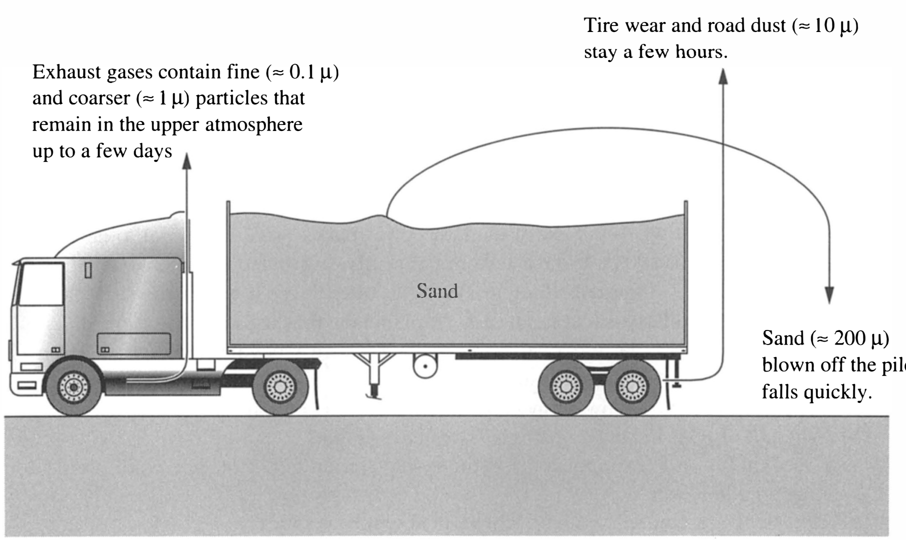

FIGURE 8.12
A truck, loaded with sand, puts three different sizes of particles into the atmosphere.

particles, generated by combustion, that remain in the atmosphere for several days and contribute to the regional fine particulate problem.

8.5 SUMMARY

1. The particles of air pollution interest are mostly in the size range 0.01 to 10 μ.
2. Particles smaller than about 2 μ are rarely produced by mechanical means; they are primarily produced by condensation or chemical reaction of gases or vapors.
3. These small particles behave quite differently from the particles with which we are familiar, like sand and gravel. Their high surface area per unit mass makes them adhere to one another if they are brought into contact.
4. Most particles of air pollution interest are in the size range where the Stokes' equation for the drag force on the particle can be used with satisfactory accuracy.
5. Because particles of air pollution interest are rarely present in the air or in a gas stream as a uniform particle size set, we normally have to deal with the distribution of particle sizes.
6. The fine particles in the atmosphere are largely secondary particles, formed in the atmosphere from gaseous precursors. Most of the coarser particles in the atmosphere are primary particles, which enter the atmosphere as particles.

PROBLEMS

See Common Units and Values for Problems and Examples, inside the back cover.

8.1. Determine the thickness of the pages in this textbook, in microns, by measuring the thickness of the text (excluding covers) and dividing that by the number of pages. Take into account the fact that page numbers go on both sides of the page.

8.2. One lbm of water is dispersed in droplets of diameter D. How many square feet of surface does the water have when $D = 1$ cm, 1 μ, 0.01 μ?

8.3. (*a*) If a solid material has a density of 1000 kg/m^3 and a particle of this material has a mass of 1 microgram and is a cube, how long is each side of the cube?
(*b*) Repeat part (*a*) for a particle of density 2000 kg/m^3.

8.4. A typical coal is 10 wt % ash. Most modern power plants grind their coal to an average particle size of about 100 μ. If the ash were uniformly distributed in the coal, what would be the expected size of the remaining ash particles after the coal was burned? Particles as small as 1 μ are regularly found in this ash. Explain how they are probably formed.

8.5. (*a*) Figure 8.1 shows that the smallest particles that are recognizable as particles have diameters of 0.01 μ. Suppose such a particle is pure carbon, atomic weight 12 g/mol, density 2000 kg/m^3. How many carbon atoms does it contain?
(*b*) Why does Fig. 8.1 show no particles smaller than this?

8.6. (*a*) Based on Fig. 8.1 (extrapolated!), estimate how far an SO_2 molecule would settle due to gravity in a year.
(*b*) How does this compare with typical vertical wind velocities?
(*c*) Is there any industrial process that separates gases by gravity?

8.7. Dustfall rates (sediment accumulations from the air) can be up to 100 tons/square mile · month.

(*a*) Is that number big or small?
(*b*) How many pounds per square foot per day is that?
(*c*) If the dust, in a settled condition, has a bulk density of 30 lbm/ft^3, how thick a layer will accumulate in a month?

8.8. In the U.S. air pollution literature and regulations, particle concentrations in gas streams are often expressed in grains/ft^3 (1 lbm = 7000 grains = 7000 gr; 1 gr = 0.065 g). These concentrations are normally abbreviated as gr/acf (grains per actual cubic ft) and often referred to as *grain loadings*.
(*a*) For a typical concentration of 100 gr/ft^3 in a dirty gas stream, what is the weight percentage of solids?
(*b*) What is the metric equivalent (g/m^3) of 100 gr/ft^3?
(*c*) If the particles are 10 μ spherical particles, how many are there in a ft^3?
(*d*) What is the most likely historical origin of the grain as a unit of mass?
(*e*) What other common materials normally have their masses expressed in grains?

8.9. Particles with a diameter of 1 mm (which corresponds roughly to coarse beach sand) are emitted from a tall stack. The wind is blowing at a velocity of 10 mi/h. The distance from the centerline of the stack to the plant's property line is five stack heights. What is the likelihood that most of the sand will fall on the plant's property?

8.10. To determine the diameter of a small spherical particle, we let it settle by gravity in air in the field of view of a microscope. The settling velocity was 0.001 ft/s. Estimate the diameter of the particle.

8.11. A particle is a hollow sphere of a metal oxide. The density of the metal oxide is 2000 kg/m^3. The hollow portion in the center of the sphere is full of air that has the same density as the surrounding air through which the sphere is falling at its terminal velocity. The outside diameter of the sphere is 10 μ and the thickness of its walls is 0.1 μ (i.e., the bubble in the center has a diameter of 9.8 μ). How fast is it falling?

8.12. (*a*) What value of C_d does Eq. (8.7) give for $\mathcal{R}_p = 0.3$?
(*b*) What is the percentage difference between this value and the Stokes' law value at this Reynolds number?

8.13. Example 8.3 shows the trial-and-error solution to a particle settling problem. Most students now have hand calculators with a "solve" routine that will do that trial-and-error calculation easily. Rework this problem on that kind of hand calculator:
(*a*) Combine Eqs. (8.2) and (8.5)–(8.7) and rearrange to

$$V + V^{1.7} \cdot 0.14 \left(\frac{D\rho_{\text{fluid}}}{\mu} \right)^{0.7} - \frac{D^2 g \rho_{\text{part}}}{18\mu} = 0$$

(*b*) Evaluate the constants, finding

$$V + V^{1.7} \cdot 0.8582 \text{ (s/m)}^{0.7} - 2.422 \text{ m/s} = 0$$

(*c*) Solve, using a solve routine, finding $V = 1.219$ m/s.

8.14. From the kinetic theory of gases we know that

$$\lambda = \frac{1}{\sqrt{2}\pi n \sigma^2} \tag{8.33}$$

where λ is the mean free path (the average distance a molecule travels between collisions), n is the number concentration of molecules (molecules/volume), and σ is the collision diameter of an individual molecule. This latter is determined by experimental viscosity measurements and has values in the range of 2 to $4 \cdot 10^{-10}$ m for common gases (see Fig. 8.1). For air the value is approximately $3.48 \cdot 10^{-10}$ m. The number concentration of molecules is Avogadro's number ($6.02 \cdot 10^{23}$ molecules/mol) times the molar density, which for ordinary gases at modest pressures is given by the idela gas law, $\rho = RT/P$.

Using these values, estimate the mean free path of air at 1 atm and 20°C.

8.15. A Crookes radiometer is an evacuated glass tube, with a vertical shaft, to which are attached small plates at a radius of a few centimeters. One side of each plate is polished like a mirror, and the other is painted flat black. The mirrors all point in one direction around the shaft, the black sides in the other direction. When the radiometer is placed in a bright light, the shaft rotates; the brighter the light, the faster it rotates.

(*a*) Which direction does it rotate, i.e., do the mirrored surfaces go forward or backward? Why?
(*b*) Would it behave the same way in a perfect vacuum? Why?
(*c*) How does this relate to the Cunningham correction factor?

8.16. A spherical particle with diameter 1 μ and specific gravity 4.0 is settling in still air.

(*a*) What is the terminal settling velocity of this particle, according to Stokes' law?
(*b*) What is the terminal settling velocity of this particle, according to Stokes' law, taking the Cunningham correction factor into account?

8.17. In Example 8.5 we saw that a particle with a 1-μ diameter, specific gravity of 2, and an initial velocity of 10 m/s would be stopped by air in a travel distance of 69 diameters.

(*a*) If we inject a baseball ($D = 2.9$ inches, $m = 0.32$ lbm) into a tank of some viscous fluid (molasses or honey or lube oil) at the same velocity and it is stopped in 69 diameters, what is the viscosity of the fluid? Assume Eq. (8.12) applies.
(*b*) Is the Reynolds number small enough for the Stokes' stopping distance to be applicable? If not, estimate what the observed stopping distance would be.

8.18. In Example 8.5.

(*a*) How long does it take the particle to come to zero velocity?
(*b*) How long does it take the particle to come to 1 percent of its initial velocity?
(*c*) What is the initial value of $\mathcal{R}_p$?
(*d*) If this is too large for the Stokes' drag force to be applicable, will the observed stopping distance be larger or smaller than that calculated in Example 8.5? By what percentage?

8.19. Figure 8.8 is a Φ vs. age plot for the United States in 1990. Assume that a childhood influenza had killed all the people in the United States born during the five-year period 1970 through 1975 and none of the subsequent immigrants to the United States had been born during that period.

(*a*) Sketch what Fig. 8.8 would look like in this circumstance.
(*b*) Sketch what Fig. 8.9 would look like in this circumstance.

8.20. Sketch the equivalents of Figs. 8.8, 8.9, and 8.10 for a particle group that is log-normally distributed, with each of the following sets of parameters (rough sketches with no numerical values will be satisfactory):

(*a*) $D_{\text{mean}} = 0.250$ in., $\sigma = 0.0001$
(*b*) $D_{\text{mean}} = 5.0$ in., $\sigma = 10$
(*c*) $D_{\text{mean}} = 10$ μ, $\sigma = 2$
(*d*) What physical systems might these distributions correspond to?

8.21. For the group of particles in Example 8.9:

(*a*) Sketch a plot of Φ *by mass* vs. particle diameter for this distribution, indicating all the important numerical values on your sketch. (The sketch can be quite rough and need not be to scale).
(*b*) Repeat (*a*) using Φ *by number*.

8.22. A group of particles is described by the log-normal distribution with $D_{\text{mean by weight}} = 5$ μ, and $\sigma = 0.8$.

(*a*) What fraction by weight of the particles have diameters less than 1 μ?
(*b*) What fraction by number of the particles have diameters less than 1 μ?

8.23. In Example 8.10 we computed the mass-mean diameter from the count-mean diameter by using a value of $b = 0$ in Eq. (8.32). What would be the physical significance of the distributions we would obtain if we had repeated the calculation in Example 8.10 using values of $b = 1$ and $b = 2$?

8.24. As described in Chapter 5, average wind velocities in the United States are about 10 mi/h. The highest values are about 100 mi/h, and the lowest about 1 mi/h. Assume for this problem only that 100 mi/h and 1 mi/h winds occur with equal frequency.
(*a*) Would this distribution of wind speeds be well represented by the normal distribution?
(*b*) Would it be well represented by the log-normal distribution?
(*c*) Sketch the equivalent of Fig. 8.9 for wind speeds, both in the normal and the log-normal form.

8.25. The emission factors table gives the following data for the particle size distribution in the waste gas from a mass-burn municipal waste incinerator [15]:

Particle diameter, μ	**Φ, cumulative weight % to this diameter**
0.625	14
1.0	18
2.5	24
5.0	32
10.0	37
15.0	47

No values are given for particles larger than 15 μ because they are of little air pollution interest.

Can these data be satisfactory represented by the normal distribution? by the log-normal distribution?

8.26. For the "Typical" line on Fig. 8.10, estimate the diameter that corresponds to 10 percent by weight and to 1 percent by weight.

8.27. (*a*) We now pass the "typical" particle group shown on Fig. 8.10 through a particle collector that is 100 percent efficient for particles larger than or equal to 10 μ in diameter, and zero percent efficient for particles with diameters less than 10 μ. For the particles that pass through this collector, sketch the equivalents of Figs. 8.8 and 8.9.
(*b*) Calculate the mass mean diameter and the number mean diameter of the particles that pass through.

8.28. If a population of particles is log normal with $D_{\text{mean by weight}} = 10$ μ and $\sigma = 1$, what is the diameter that has 99.9 percent of the weight smaller than it? What is the diameter that has 0.01 percent of the weight smaller than it?

8.29. Table 8.3 is easy to use with hand calculations, but not with a computer. Furthermore, it is not easily used for values of z greater than 3.8. For these purposes it is common to use the following algebraic approximation [16]:

$$z \approx t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2}$$

where $t = \sqrt{\ln[1/(1-\Phi)^2]}$

$a_0 = 2.30753$

$a_1 = 0.27061$

$b_1 = 0.99229$

$b_2 = 0.04481$

This approximation can only be used for $\Phi > 0.5$ and has a maximum error in z of ± 0.003.

(*a*) Test the accuracy of this approximation by computing the value of z corresponding to a Φ of 0.9772 and comparing that estimate with the value in Table 8.3.

(*b*) Using this approximation, estimate the Φ corresponding to $z = 6.0$.

8.30. A gas stream contains a group of particles whose size distribution is given by the "rectangular distribution," which is

$$d\Phi/dD = C_1 \text{ for particle diameters from 0 to } D_{max}$$

$$d\Phi/dD = 0 \text{ for particle diameters greater than } D_{max}$$

Here Φ is the cumulative fraction by mass of particles with diameter less than D, and D_{max} is the diameter of the largest particle. C_1 is equal to $(1/D_{max})$.

We pass this gas stream through a particle collector in which the collection efficiency is proportional to the particle diameter squared and is equal to 1.0 for a particle diameter of D_{max}. What is the overall collection efficiency of this collector?

8.31. A gas stream has particles whose distribution is represented by the "triangular distribution function," which is

$$\frac{d(\text{weight fraction})}{d(\text{particle diameter})} = b(\text{particle diameter})$$

for sizes 0 to 10 μ and

$$\frac{d(\text{weight fraction})}{d(\text{particle diameter})} = b(e - \text{particle diameter})$$

for sizes 10 to 20 μ. Here, $b = 0.01/\mu^2$ and $e = 20$ μ.

A particle collection device has collection efficiency represented by the equation

$$\text{Efficiency} = a(\text{particle diameter})^2$$

over the range of 0 to 20 μ. Here a has the value $0.0025/\mu^2$.

We now pass this gas stream through this collector. What fraction by weight of the particles is collected?

8.32. A gas stream contains a group of particles whose particle size distribution by weight is given by the "quadratic distribution function," which is

$$\text{Weight fraction with diameter less than } D = k_1 D^2 \quad \text{for} \quad 0 < D < \sqrt{1/k_1}$$

We now pass this gas stream through a collector whose efficiency as a function of particle size is given by these equations:

$$\text{Fraction collected} = k_2 D \quad \text{for} \quad 0 < D < 1/k_2$$

$$\text{Fraction collected} = 1.0 \quad \text{for} \quad 1/k_2 < D$$

What weight fraction of the particles in the gas stream is caught by this collector? Here, $k_1 = 0.01/\mu^2$ and $k_2 = 0.1/\mu$.

8.33. A contaminated air stream contains particles that follow the log-normal distribution by mass with $D_m = 10\ \mu$, $\sigma = 1.5$. We now pass this gas through a separator that removes all particles $D \geq 5\ \mu$. All particles $D < 5\ \mu$ pass through.
(*a*) What fraction by mass of the particles is removed?
(*b*) Sketch the equivalents of Figs. 8.8, 8.9, and 8.10 for the remaining particles (i.e., those still in the gas stream).
(*c*) What is the mass mean diameter of the particles that are captured? What is the mass mean diameter of the particles that pass through uncollected?

8.34. A contaminated air stream contains particles that follow the log-normal distribution by mass, with $D_m = 5\ \mu$ and $\sigma = 1.5$. We pass this contaminated air stream through a particle collector that removes all the particles larger than 4 μ, and which is 50% efficient for particles in the size range 2 to 4 μ. What is the overall weight percent collection efficiency of this collector for these particles?

8.35. The particles in an air stream are described by the log-normal distribution, with $D_{\text{mean by mass}} = 10\ \mu$ and $\sigma = 1.5$. We now pass this dirty air stream through a collector that is 100% efficient for particles with $D \geq 40\ \mu$, 50% efficient for particles 10 to 40 μ in diameter, and 0% efficient for particles smaller than 10 μ.
(*a*) What fraction by mass is collected by this collector?
(*b*) What is the mass median diameter of the particles that pass through uncollected?

8.36. Figure 8.11 suggests that in a typical atmosphere, about one-third of the surface area of the particles is contained in particles with diameter centered about 0.02 μ, about one-third in particles with diameter centered about 0.3 μ, and about one-third in particles with diameter centered about 10 μ. If the true situation were that the distribution by area was exactly one-third in each of these diameter ranges and if, instead of the broad distributions shown in Fig. 8.11, all of the particles were exactly either 0.02, 0.3, or 10 μ in diameter, then
(*a*) What would the fraction by weight be for each of the three particle sizes?
(*b*) What would the fraction by number be for each of the three particle sizes?

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