Coord T(1) en la base U

$$\, _U(T)_E = \begin{pmatrix} 1 & -2 & 5 \\ 0 & 3 & 3 \\ 1 & 0 & 1 \end{pmatrix}.$$

$E = \{1, x+1, (x+1)^2\}$

$T(1) = 1\cdot(1,1,0) + 0\cdot(1,2,3) + 1\cdot(3,2,1)$

$= (4,3,1) = 4(1,0,0) + 3(0,1,0) + 1(0,0,1)$

$x^2 + x - 1$

$\mathbb{R}_2[x] \xrightarrow{T} \mathbb{R}_3 \longrightarrow \mathbb{R}_3$

$E \quad {}_U(T)_E \quad U, \quad {}_{\mathcal{C}}(Id)_U \quad \mathcal{C}$

$x^2 + x - 1 = a\cdot 1 + b(x-1) + c(x+1)^2$

$= cx^2 + x[b + 2c] + a - b + c$

$\begin{cases} c = 1 \\ b + 2c = 1 \to \boxed{b = -1} \\ a - b + c = -1 \to a + 1 + 1 = -1 \end{cases}$

$\boxed{a = -3}$

$T(x^2 + x - 1) = T(-3\cdot 1 + (-1)(x-1) + 1(x-1)^2)$

$= -3\,T(1) + (-1)\,T(x-1) + 1\,T((x+1)^2)$

Cálculado

Conocemos $T(x^2 + x - 1)$ ...

$$_U(T)_E = \begin{pmatrix} 1 & -2 & 5 \\ 0 & 3 & 3 \\ 1 & 0 & 1 \end{pmatrix}$$

$\mathrm{Coord}_U\left(T((x+1)^2)\right)$

$\mathrm{Coord}_U\left(T(x-1)\right)$

$T(x-1) = -2(1,1,0) + 3(1,2,3) + 0(\ ,\ ,\ ) = (1,0,9)$

$T((x+1)^2) = 5(1,1,0) + 3(1,2,3) + 1.(3,2,1) = (11,13,10)$

$T(1) = (4,3,1)$

Escribir x^2+x-1 como combinación lineal de la base E.

$$x^2+x-1 = -3.1 - (x-1) + (x+1)^2$$

$$(-3,-1,1) = \mathrm{Coord}_E(x^2+x-1)$$

$$\begin{aligned} T(x^2+x-1) &= T(-3.1 - (x-1) + (x+1)^2) \\ &= -3\,T(1) - T(x-1) + T((x+1)^2) \\ &= -3(4,3,1) - (1,0,9) + (11,13,10) \\ &= (2,4,-2) = 2(1,0,0) + 4(0,1,0) - 2(0,0,1) \end{aligned}$$

Base canónica

Cambio de base de U a $\mathcal{E}$

$U = \{(1,1,0), (1,2,3), (3,2,1)\}$

$\mathcal{E} = \{(1,0,1), (0,1,0), (0,0,2)\}$

$$_{\mathcal{E}}(Id)_U = \begin{pmatrix} 1 & & \\ 1 & & \\ -1/2 & & \end{pmatrix}$$

$\hookrightarrow \text{Coord}_{\mathcal{E}}(Id(1,1,0)) = \text{Coord}_{\mathcal{E}}(1,1,0)$

$(1,1,0) = a(1,0,1) + b(0,1,0) + c(0,0,2)$

$$\begin{cases} a = 1 \\ b = 1 \\ a + 2c = 0 \end{cases} \rightarrow c = -1/2$$

producto de matrices

$$_U(T)_E \rightsquigarrow {}_{\mathcal{E}}(T)_E = {}_{\mathcal{E}}(Id)_U \cdot {}_U(T)_E$$

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9. Sean las funciones $S, T, U : \mathbb{R}^3 \to \mathbb{R}^3$ dadas por
   - $S(x, y, z) = (z, y, x + 8)$.
   - $T(x, y, z) = (x, y, z - 8)$.
   - $U(x, y, z) = (x + y, x - y, 0)$.

$(S + T)(x,y,z) = S(x,y,z) + T(x,y,z)$

$= (z + x, 2y, x + z)$

$S(x,y,z) = (0, 2x, y)$

$T(x,y,z) = (x^2, e^y, z^2)$

$S \circ T = S(x^2, e^y, z^2) = (0, 2x^2, e^y)$ No lineal

$S \circ S \circ T = S(0, 2x^2, e^y) = (0, 0, 2x^2)$

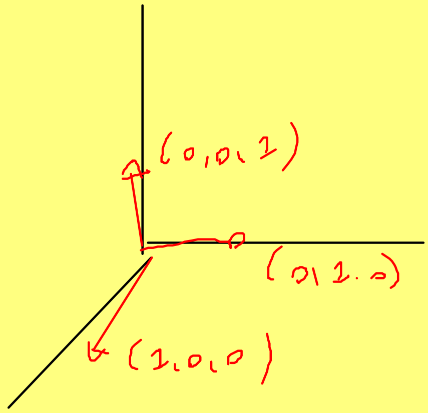
$S \circ S \circ S \circ T = S(0, 0, 2x^2) = (0, 0, 0) \rightarrow S^3 \circ T$ es la transformación (lineal)

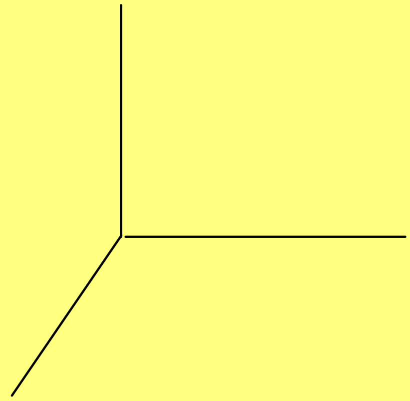
$S \circ T$ es no lineal, y dado que $S^3 \circ T$ sea lineal:

i) $S^3 \circ T = (0, 0, 0)$

ii) $S^2 \circ T = (x^2, 0, 0) \Rightarrow S^3 \circ T$ sea lineal

$\Rightarrow S$ en algún lugar aplica $\sqrt{\ }$. no es lineal,





$$\left.\begin{aligned} T(0,0,1) &= (0,0,0) \\ T(0,1,0) &= (0,0,0) \end{aligned}\right\} N(T) \text{ es el plano } yz$$

$$T(1,0,0) = (1,0,0) \rightarrow Im(T) \text{ es el eje } x$$