1er semestre 2017 Ej. 5

miércoles, 25 de noviembre de 2020 9:47

$T: P_2 \to M_{2\times2}(\mathbb{R})$

INYECTIVA : $N(T) = \{\vec{0}\}$

$p(t) = at^2 + bt + c$

$p'(t) = 2at + b$

$p''(t) = 2a$

$\Bigg\}\ \underline{T(p)} = \begin{pmatrix} \hat{b} & 2a \\ 2a+b & 2a \end{pmatrix}$

$\dim P_2 = 3 = \dim N(T) + \dim Im(T)$

SOBREYECTIVA ?

$Im(T) = \underline{M_{2\times2}}(\mathbb{R})$

$p \in N(T)$ si $T(p) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\downarrow$

$\left.\begin{matrix} b=0 \\ 2a=0 \\ 2a+b=0 \\ 2a=0 \end{matrix}\right\} \to \left.\begin{matrix} b = 0 \\ 2a = 0 \end{matrix}\right\} \{N(T): at^2+bt+c : a=b=0\}$

$\left(\begin{matrix} \dim N(T) = 1 \\ \dim Im(T) = 2 \end{matrix}\right)$

$T(p) = \begin{pmatrix} b & 2a \\ 2a+b & 2a \end{pmatrix} = a\begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix} + b\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

$\{\underbrace{\begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}}\} \xrightarrow{b} Im(T)$

$C = \left\{\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}\right\} \xrightarrow{b?} Im(T)$

$¿[C] = Im(T)?$ $\xrightarrow{si}$ $C \xrightarrow{b} Im(T) \to$ op. d

$\xrightarrow{NO}$ C NO ES BASE $\to$ OP. E

$\left\{\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}\right\} = C \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underbrace{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}}$

$\left(\begin{array}{cc|c} 1 & -1 & a \\ 1 & 1 & b \\ 2 & 0 & c \\ 1 & 1 & d \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & 0 & a+b \\ 1 & 1 & b \\ 2 & 0 & c \\ 0 & 0 & d-b \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & b \\ 2 & 0 & a+b \\ 0 & 0 & c-a-b \\ 0 & 0 & d-b \end{array}\right)$

$[C] = \{A \in M_{2\times2}(\mathbb{R}) : \underline{d-b=0\ ;\ c-a-b=0}\}$

$T(p) = \begin{pmatrix} b & 2a \\ 2a+b & 2a \end{pmatrix}$

$d-b=0? \to 2a-2a=0$

$c-a-b=0? \to 2a+b-b-2a=0$

$\uparrow$

$[C] = Im(T).$