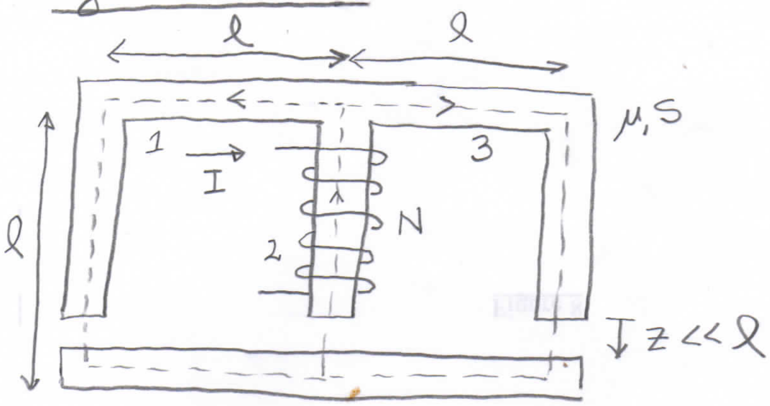
Ejercicio Nº 1



parte a:

$$\oint \vec{H} \cdot d\vec{l} = I_{total}$$

$$H_{1\mu} 3l + H_{1a} z + H_{2\mu} l + H_{2a} z = NI$$

$H_{i\mu} \rightarrow$ H en núcleo rama i

$H_{ia} \rightarrow$ H en entrehierro rama i

$$H_{3\mu} 3l + H_{3a} z + H_{2\mu} l + H_{2a} z = NI$$

$$\Phi_1 + \Phi_3 = \Phi_2 \qquad \Phi_{i\mu} = \Phi_{ia} \Rightarrow B_{i\mu} = B_{ia}$$

$$\Phi = BS \qquad \mu H_{i\mu} \qquad \mu_0 H_{ia}$$

$$B_{1a}\left(\frac{3l}{\mu} + \frac{z}{\mu_0}\right) + B_{2a}\left(\frac{l}{\mu} + \frac{z}{\mu_0}\right) = NI$$

$$B_{3a}\left(\frac{3l}{\mu} + \frac{z}{\mu_0}\right) + B_{2a}\left(\frac{l}{\mu} + \frac{z}{\mu_0}\right) = NI$$

Restando ambas ecuaciones: $B_{1a} = B_{3a}$ (lógico x simetría)

$\Rightarrow B_{2a} = 2B_{1a}$

$$B_{1a}\left(\frac{5l}{\mu} + \frac{3z}{\mu_0}\right) = NI \Rightarrow \boxed{B_{1a} = \frac{NI}{\frac{5l}{\mu} + \frac{3z}{\mu_0}} = B_{3a}}$$

$$\boxed{B_{2a} = \frac{2NI}{\frac{5l}{\mu} + \frac{3z}{\mu_0}}}$$

parte b: $\Phi_{total} = N\Phi_2 = NSB_{2\mu} = NSB_{2a} = \dfrac{2N^2SI}{\frac{5l}{\mu} + \frac{3z}{\mu_0}}$

$$\Rightarrow \boxed{L = \frac{2N^2S}{\frac{5l}{\mu} + \frac{3z}{\mu_0}}}$$

parte c: $U = \int \dfrac{\vec{B} \cdot \vec{H}}{2} dV = \dfrac{1}{2}\left[\left(\dfrac{B_{1a}^2}{\mu} 3lS + \dfrac{B_{1a}^2}{\mu_0} zS\right) \cdot 2 + \right.$

$$\left. + \frac{B_{2a}^2}{\mu} lS + \frac{B_{2a}^2}{\mu_0} zS\right] = \frac{1}{2} B_{1a}^2 \left[\frac{6lS}{\mu} + \frac{2zS}{\mu_0} + \frac{4lS}{\mu} + \frac{4zS}{\mu_0}\right] =$$

$$= \frac{1}{2} B_{1a}^2 \left(\frac{10lS}{\mu} + \frac{6zS}{\mu_0}\right) = \frac{1}{2} \frac{N^2 I^2}{\left(\frac{5l}{\mu} + \frac{3z}{\mu_0}\right)^{\cancel{2}}} 2S \cancel{\left(\frac{5l}{\mu} + \frac{3z}{\mu_0}\right)}$$

$$\boxed{U = \frac{1}{2}\,\frac{2SN^2I^2}{\frac{5\ell}{\mu} + \frac{3z}{\mu_0}} = \frac{LI^2}{2}}$$

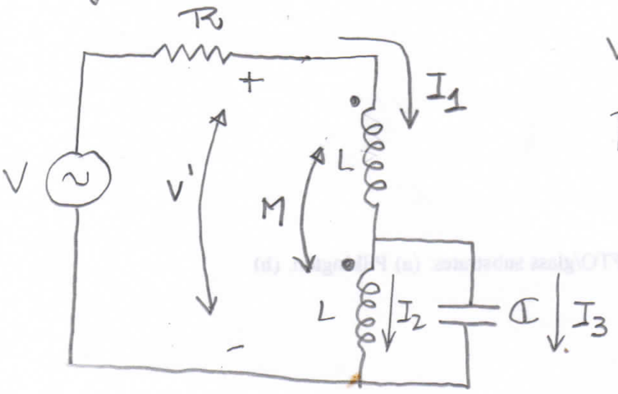
parte d: $$\boxed{F = \left.\frac{\partial U}{\partial z}\right|_I = -\frac{SN^2I^2}{\left(\frac{5\ell}{\mu} + \frac{3z}{\mu_0}\right)^2}\,\frac{3}{\mu_0}}$$

La fuerza es atractiva (por el signo de − que la hace ir en la dirección opuesta a z)

Ejercicio Nº 2

Segundo Parcial de Electromagnetismo (1128)

11/12/2020



$V(t) = V\cos\omega t$

parte a:

$$Z_{eq} = \frac{V'}{I_1}$$

$$V' = j\omega L I_1 + j\omega M I_2 + \frac{I_3}{j\omega C} = j\omega L I_1 + j\omega M I_2 + j\omega L I_2 + j\omega M I_1$$

$$\Rightarrow \frac{I_3}{j\omega C} = j\omega L I_2 + j\omega M I_1 = j\omega(L+M)(I_1+I_2)$$

$$I_1 = I_2 + I_3 \Rightarrow \left(\frac{1}{j\omega C} - j\omega M\right) I_3 = j\omega(L+M) I_2$$

$$\left(-\frac{1}{\omega^2 C} - M\right) I_3 = (L+M) I_2$$

$$I_3 = -\frac{L+M}{M+\frac{1}{\omega^2 C}} I_2$$

$$I_1 = I_2\left(1 - \frac{L+M}{M+\frac{1}{\omega^2 C}}\right) = I_2\left(\frac{\frac{1}{\omega^2 C} - L}{M + \frac{1}{\omega^2 C}}\right)$$

$$I_2 = \frac{M + \frac{1}{\omega^2 C}}{\frac{1}{\omega^2 C} - L} I_1$$

$$Z_{eq} = j\omega(L+M)\left[1 + \frac{M+\frac{1}{\omega^2 C}}{\frac{1}{\omega^2 C} - L}\right]$$

$$\boxed{Z_{eq} = j\omega(L+M)\left[\frac{\frac{2}{\omega^2 C} - (L-M)}{\frac{1}{\omega^2 C} - L}\right]}$$

parte b: $P_R = R\, I_1(t)^2 \qquad \langle P_R\rangle = R\langle I_1(t)^2\rangle$

$$I_1(t) = |I_1|\cos(\omega t - \varphi) \Rightarrow \langle P_R\rangle = R|I_1|^2 \underbrace{\langle\cos^2(\omega t - \varphi)\rangle}_{=\frac{1}{2}}$$

$$\langle P_R\rangle = \frac{R|I_1|^2}{2}$$

$\langle P_R \rangle \geq 0 \Rightarrow \langle P_R \rangle_{min} = 0$

$|I_1| = 0$

$V = (R + Z_{eq}) I_1 \Rightarrow I_1 = \dfrac{V}{R + Z_{eq}}$ $\quad I_1 = 0$ si $Z_{eq} \to \infty$

$Z_{eq} \to \infty$ si $\dfrac{1}{\omega^2 C} = L \Rightarrow \boxed{\omega = \dfrac{1}{\sqrt{LC}}}$

parte c: $\langle P_R \rangle$ es máxima si $|I_1|$ es máxima

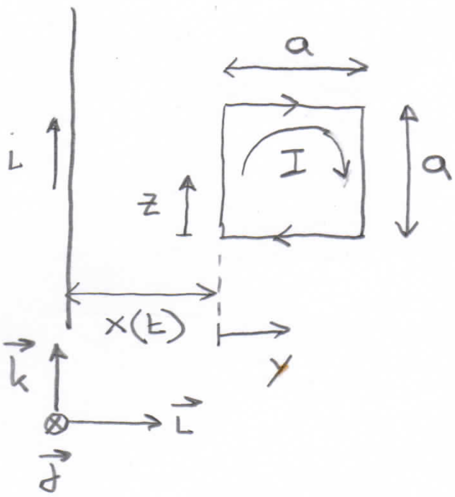
$|I_1| = \dfrac{V}{|R + Z_{eq}|} = \dfrac{V}{\sqrt{R^2 + |Z_{eq}|^2}} \Rightarrow |I_1|$ es máxima cuando $|Z_{eq}| = 0$

Z_{eq} es imaginaria

$\Rightarrow \dfrac{2}{\omega^2 C} = L - M \Rightarrow \boxed{\omega = \sqrt{\dfrac{2}{C(L-M)}}}$

$M^2 < L_1 L_2 = L^2 \Rightarrow M < L$ y $\omega \in \mathbb{R}$.

Ejercicio Nº 3 — Segundo Parcial Electromagnetismo (1128) 11/12/2020 (5/6)



parte a:

Oriento la espira en sentido horario

$$\mathcal{E} = R I + L \frac{dI}{dt}$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\Phi = \int \vec{B}\cdot\hat{n}\, da \qquad \hat{n} = \vec{j}$$

$$\vec{B} = B\vec{j} \qquad \text{Ampere: } B 2\pi(x+y) = \mu_0 i$$

$$\vec{B} = \frac{\mu_0 i \vec{j}}{2\pi(x+y)}$$

$$\Phi = \frac{\mu_0 i}{2\pi}\int_0^a\int_0^a \frac{dy\,dz}{x+y} = \frac{\mu_0 i}{2\pi} \ln(x+y)\Big|_0^a a = \frac{\mu_0 i a}{2\pi}\ln\frac{x+a}{x}$$

$$\frac{d\Phi}{dt} = \frac{\mu_0 i a}{2\pi}\left(\frac{\dot{x}}{x+a} - \frac{\dot{x}}{x}\right) = \frac{\mu_0 i a \dot{x}}{2\pi}\,\frac{x-x-a}{x(x+a)} = -\frac{\mu_0 i a^2 \dot{x}}{2\pi x(x+a)}$$

$$\mathcal{E} = \boxed{\frac{\mu_0 i a^2 \dot{x}}{2\pi x(x+a)} = R I + L\frac{dI}{dt}}$$

Si $\dot{x} > 0$ la espira se aleja del cable $\Rightarrow$ el flujo de $\vec{B} = B\vec{j}$ disminuye y la $\mathcal{E} > 0$ tenderá a hacer que aumente en esa dirección. Se verifica la ley de Lenz.

parte b: $\vec{F} = \oint I\, d\vec{l} \wedge \vec{B}$

$$L = 0 \Rightarrow I = \frac{\mu_0 i a^2 \dot{x}}{2\pi R x(x+a)}$$

$$\vec{F} = I\left[\int_0^a dz\,\vec{k}\wedge\vec{B} + \int_0^a dy\,\vec{i}\wedge\vec{B} + \int_a^0 dz\,\vec{k}\wedge\vec{B} + \int_a^0 dy\,\vec{i}\wedge\vec{B}\right] =$$

$$= I\left[\int_0^a dz\left(-\frac{\mu_0 i \vec{i}}{2\pi x}\right) + \cancel{\int_0^a dy\frac{\mu_0 i \vec{k}}{2\pi(x+y)}} + \int_0^a dz\frac{\mu_0 i \vec{i}}{2\pi(x+a)} - \cancel{\int_0^a dy\frac{\mu_0 i \vec{k}}{2\pi(x+y)}}\right] = I\frac{\mu_0 i a}{2\pi}\left(-\frac{1}{x} + \frac{1}{x+a}\right)\vec{i} = \frac{I\mu_0 i a}{2\pi}\,\frac{-x-a+x}{x(x+a)}\,\vec{i}$$

$$\vec{F} = -\frac{I\mu_0 i a^2 \vec{i}}{2\pi x(x+a)} = \boxed{-\frac{\mu_0^2 i^2 a^4 \dot{x}\,\vec{i}}{4\pi^2 R x^2(x+a)^2} = \vec{F}}$$

Segundo Parcial Electromagnetismo (1128)

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parte c:

$$\frac{\mu_0 i a^2 \dot{x}}{2\pi x (x+a)} = L \frac{dI}{dt} \qquad dI = \frac{\mu_0 i a^2 \dot{x}\, dt}{2\pi L\, x(x+a)} = \frac{\mu_0 i a^2 dx}{2\pi L\, x(x+a)}$$

$$I - I(0) = \frac{\mu_0 i a^2}{2\pi L} \int_{x(0)}^{x} \frac{dx}{x(x+a)} = \frac{\mu_0 i a}{2\pi L} \int_{x(0)}^{x} \frac{a+x-x}{x(x+a)} dx =$$

$$= \frac{\mu_0 i a}{2\pi L} \int_{x(0)}^{x} \left(\frac{1}{x} - \frac{1}{x+a} \right) dx = \frac{\mu_0 i a}{2\pi L} \Big[\ln x - \ln(x+a) \Big]_{x(0)}^{x}$$

$$\boxed{I = I(0) + \frac{\mu_0 i a}{2\pi L} \ln \left[\frac{x}{x(0)} \, \frac{x(0)+a}{x+a} \right]}$$

$$\ln \frac{x}{x+a} - \ln \frac{x(0)}{x(0)+a}$$ ← los 2 términos son negativos

$$\frac{d}{dx}\left(\ln \frac{x}{x+a} \right) = \frac{1}{x} - \frac{1}{x+a} = \frac{x+a-x}{x(x+a)} = \frac{a}{x(x+a)} > 0$$

La función $\ln \frac{x}{x+a}$ es creciente $\Rightarrow$ si x aumenta (e $I(0) = 0$)

$\Rightarrow I(t) > 0$ de acuerdo con el razonamiento de la ley de Lenz de la parte a.