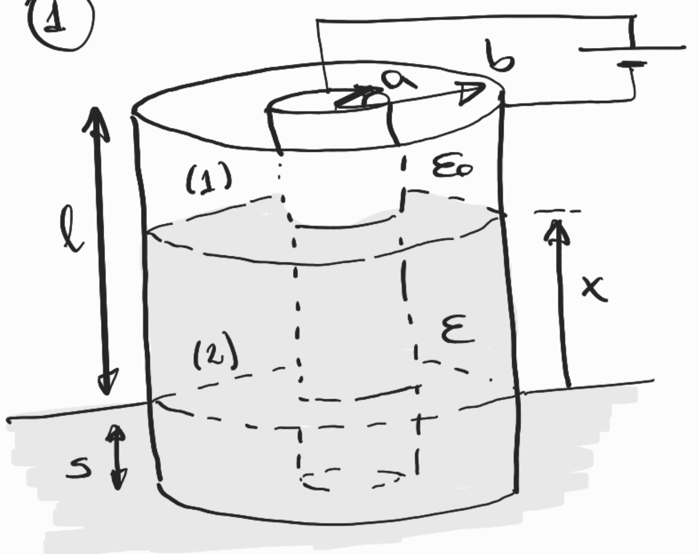
1



a) Para $a<r<b$, tanto en la región (1) – sin fluido – como en la región (2) – con fluido – ; $\nabla\cdot\vec{D} = \rho_L = 0$

$$\begin{cases} \nabla\cdot(\underbrace{\varepsilon_0}_{cte}\vec{E})=0 \rightarrow \nabla\cdot\vec{E}=0 & (\text{región } 1) \\ \nabla\cdot(\underbrace{\varepsilon}_{cte}\vec{E})=0 \rightarrow \nabla\cdot\vec{E}=0 & (\text{región } 2) \end{cases}$$

Por la simetría de la configuración $\vec{E} = E(r)\,\hat{e}_r$ (coords. cilíndricas)

$$\nabla\cdot\vec{E} \underset{cil}{=} \frac{1}{r}\frac{\partial(rE(r))}{\partial r} = 0 \rightarrow E(r) = \frac{\overset{cte}{A}}{r}$$

$$V_0 = \int_a^b E(r)\,dr = A\ln r\Big|_a^b = A\ln(b/a) \rightarrow A = \frac{V_0}{\ln(b/a)}$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{V_0}{\ln(b/a)}\left(\frac{1}{r}\right)\hat{e}_r}\;;\; a<r<b \quad (\text{tanto en (1) como en (2)})$$

$$\begin{cases} \boxed{\vec{D}(r) = \frac{\varepsilon_0 V_0}{\ln(b/a)}\left(\frac{1}{r}\right)\hat{e}_r}\;;\; a<r<b \quad (\text{región (1)}) \\ \boxed{\vec{D}(r) = \frac{\varepsilon V_0}{\ln(b/a)}\left(\frac{1}{r}\right)\hat{e}_r}\;;\; a<r<b \quad (\text{región (2)}) \end{cases}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \Rightarrow \begin{cases} \boxed{\vec{P}(r) = 0}\;;\; a<r<b \quad (\text{región (1)}) \\ \boxed{\vec{P}(r) = (\varepsilon-\varepsilon_0)\frac{V_0}{\ln(b/a)}\left(\frac{1}{r}\right)\hat{e}_r}\;;\; a<r<b \quad (\text{región (2)}) \end{cases}$$

b)

$$\boxed{\sigma_L^{(1)}\Big|_{r=a} = \vec{D}\cdot\underset{\hat{e}_r}{\hat{n}}\Big|_{r=a} = \frac{\varepsilon_0 V_0}{a\ln(b/a)}} \qquad \boxed{\sigma_L^{(1)}\Big|_{r=b} = \vec{D}\cdot\underset{-\hat{e}_r}{\hat{n}}\Big|_{r=b} = \frac{-\varepsilon_0 V_0}{b\ln(b/a)}}$$

$$\boxed{\sigma_P^{(1)}\Big|_{r=a} = 0} \qquad \boxed{\sigma_P^{(1)}\Big|_{r=b} = 0} \qquad \boxed{\rho_P^{(1)} = 0} \qquad \left(\vec{P} = 0 \text{ e } (1)\right)$$

$$\boxed{\sigma_L^{(2)}\Big|_{r=a} = \vec{D}\cdot\underset{\hat{e}_r}{\hat{n}}\Big|_{r=a} = \frac{\varepsilon V_0}{a\ln(b/a)}} \qquad \boxed{\sigma_L^{(2)}\Big|_{r=b} = \vec{D}\cdot\underset{-\hat{e}_r}{\hat{n}}\Big|_{r=b} = \frac{-\varepsilon V_0}{b\ln(b/a)}}$$

$$\boxed{\sigma_P^{(2)} = \vec{P}\cdot\underset{-\hat{e}_r}{\hat{n}}\Big|_{r=a} = \frac{-(\varepsilon-\varepsilon_0)V_0}{a\ln(b/a)}} \qquad \boxed{\sigma_P^{(2)}\Big|_{r=b} = \vec{P}\cdot\underset{\hat{e}_r}{\hat{n}}\Big|_{r=b} = \frac{(\varepsilon-\varepsilon_0)V_0}{b\ln(b/a)}}$$

$$\boxed{\rho_P^{(2)} = -\nabla\cdot\vec{P} \underset{\text{cil}}{=} -\frac{1}{r}\frac{\partial(\overset{\text{cte}}{r P(r)})}{\partial r} = 0}$$

c)

$$\boxed{C = \frac{q}{V_0}} = \frac{\left[\left(\sigma_L^{(1)}\Big|_{r=a}\right)(l-x) + \sigma_L^{(2)}\Big|_{r=a}(s+x)\right](2\pi a)}{V_0}$$

$$= \frac{\cancel{V_0}\, 2\pi \cancel{a}}{\cancel{a}\ln(b/a)\cancel{V_0}}\left[\varepsilon_0(l-x) + \varepsilon(s+x)\right] = \boxed{\frac{2\pi}{\ln(b/a)}\left[\varepsilon_0(l-x)+\varepsilon(s+x)\right]}$$

$$\boxed{U = \frac{1}{2}CV_0^2 = \frac{\pi V_0^2}{\ln(b/a)}\left[\varepsilon_0(l-x)+\varepsilon(s+x)\right]}$$

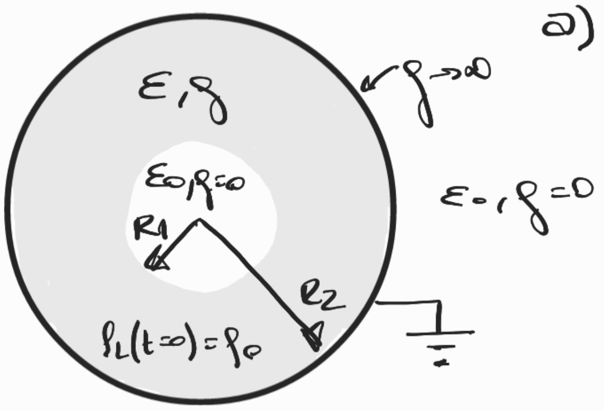
d)

$$\sum \vec{F} = 0 \rightarrow \frac{\partial U}{\partial x}\Big|_{V_0}\hat{x} - \rho_m g\left[\pi(b^2-a^2)x\right]\hat{x} = 0$$

$$\Rightarrow \frac{(\varepsilon-\varepsilon_0)\pi V_0^2}{\ln(b/a)} = \rho_m g\pi(b^2-a^2)x \Rightarrow \boxed{x = \frac{(\varepsilon-\varepsilon_0)V_0^2}{\ln(b/a)\rho_m g(b^2-a^2)}}$$

— x —

②



a) En la región $R_1<r<R_2$

$$\underbrace{\nabla\cdot\vec J_L}_{=\,g\,\underbrace{\nabla\cdot\vec E}_{=\,\frac{\rho_L}{\varepsilon}}} + \frac{\partial \rho_L}{\partial t} = 0 \quad \text{(Ec. Continuidad)}$$

$\vec J_L = g\vec E$

$\vec E = \vec D/\varepsilon$

$$\Rightarrow \frac{\partial \rho_L}{\partial t} + \frac{g}{\varepsilon}\rho_L = 0 \longrightarrow \rho_L(r,t) = \underbrace{\rho_L(r,t=0)}_{=\,\rho_0}\, e^{-\frac{g}{\varepsilon}t}$$

$$\boxed{\rho_L(r,t) = \rho_0 e^{-\frac{g}{\varepsilon}t}}$$

b) Gauss $\displaystyle\oint_S \varepsilon\vec E\cdot\hat n\, da = \int_V \rho_L \underbrace{dv}_{=\,(2\pi r)L\,dr}$ (no hay $\sigma_L|_{r=R_1}$)

sup. cilíndrica de radio r ($R_1<r<R_2$) y largo L → S

$$\varepsilon E(r,t)\,(\cancel{2\pi} r)\cancel{L} = \rho_0 e^{-\frac{g}{\varepsilon}t}(\cancel{2\pi}\cancel{L}) \underbrace{\int_{R_1}^{r} r\,dr}_{=\,\frac{1}{2}(r^2-R_1^2)}$$

$$\Rightarrow \boxed{\vec E(r,t) = \frac{(r^2-R_1^2)}{2\varepsilon r}\rho_0 e^{-\frac{g}{\varepsilon}t}\,\hat e_r} \quad R_1<r<R_2$$

$$\boxed{\vec J_L(r,t) = \frac{g\,(r^2-R_1^2)}{2\varepsilon r}\rho_0 e^{-\frac{g}{\varepsilon}t}\,\hat e_r} \quad \vec J_L = g\vec E$$

$$\boxed{\vec D(r,t) = \frac{(r^2-R_1^2)}{2r}\rho_0 e^{-\frac{g}{\varepsilon}t}\,\hat e_r} \quad \vec D = \varepsilon\vec E$$

c) $$\boxed{\sigma_L(r=R_2,t) = (\underbrace{\vec D_2}_{=\,0} - \underbrace{\vec D_1}_{=\,\varepsilon\vec E}) \cdot \underbrace{\hat n}_{=\,\hat e_r}\Big|_{r=R_2} = -\frac{(R_2^2-R_1^2)}{2R_2}\rho_0 e^{-\frac{g}{\varepsilon}t}}$$

dentro del conductor ideal ($g\to\infty$) — $\varepsilon\vec E$ hallado en (b)

d) $$\boxed{i_{total}(r,t)} = i_c(r,t) + i_d(r,t) = \int_S \vec J_L\cdot\hat n\, da + \int_S \frac{\partial\vec D}{\partial t}\cdot\hat n\, da$$

(S: cara lateral cilindro radio r y largo L)

$$= \frac{g\,(r^2-R_1^2)\rho_0 e^{-\frac{g}{\varepsilon}t}}{2\varepsilon \cancel{r}}(2\pi \cancel{r})L + \left(-\frac{g}{\varepsilon}\right)\cancel{\varepsilon}\frac{(r^2-R_1^2)}{2\cancel{\varepsilon}\cancel{r}}\rho_0 e^{-\frac{g}{\varepsilon}t}(2\pi\cancel{r})L = \boxed{0}$$

—×—

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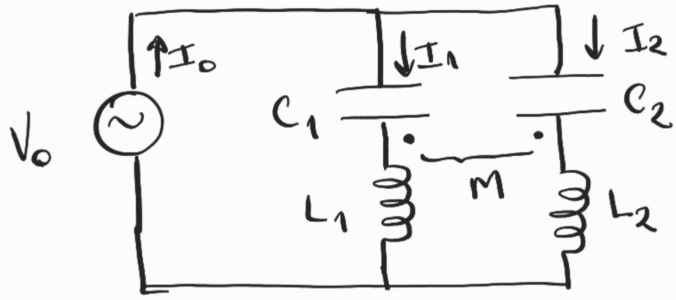

a)

$$\begin{cases} V_0 = \left(\frac{1}{j\omega C_1}\right) I_1 + (j\omega L_1) I_1 + (j\omega M) I_2 \quad (1) \\ V_0 = \left(\frac{1}{j\omega C_2}\right) I_2 + (j\omega L_2) I_2 + (j\omega M) I_1 \quad (2) \\ I_0 = I_1 + I_2 \end{cases}$$

b) igualando (1) y (2):

$$\left[\frac{1}{j\omega C_1} + j\omega L_1\right] I_1 + (j\omega M) I_2 = \left[\frac{1}{j\omega C_2} + j\omega L_2\right] I_2 + (j\omega M) I_1$$

$$I_2 \left[\frac{1}{j\omega C_2} + j\omega L - j\omega M\right] = \left[\frac{1}{j\omega C_1} + j\omega L_1 - j\omega M\right] I_1$$

$$I_2 = \frac{\left[\frac{1}{j\omega C_1} + j\omega L_1 - j\omega M\right]}{\left[\frac{1}{j\omega C_2} + j\omega L_2 - j\omega M\right]} I_1 \quad (*)$$

Sust. en (1):

$$V_0 = \left\{ \left(\frac{1}{j\omega C_1} + j\omega L_1\right) + j\omega M \left[\frac{\left(\frac{1}{j\omega C_1} + j\omega L_1\right) - j\omega M}{\left(\frac{1}{j\omega C_2} + j\omega L_2\right) - j\omega M} \right] \right\} I_1$$

$$= \left\{ \frac{\left(\frac{1}{j\omega C_1} + j\omega L_1\right)\left(\frac{1}{j\omega C_2} + j\omega L_2\right) + j\omega M \left(\cancel{\frac{1}{j\omega C_1}} + \cancel{j\omega L_1} - j\omega M - \cancel{\frac{1}{j\omega C_1}} - \cancel{j\omega L_1}\right)}{\frac{1}{j\omega C_2} + j\omega L_2 - j\omega M} \right\} I_1$$

$$= \left[\frac{\left(\frac{1}{j\omega C_1} + j\omega L_1\right)\left(\frac{1}{j\omega C_2} + j\omega L_2\right) + \omega^2 M^2}{\left(\frac{1}{j\omega C_2} + j\omega L_2 - j\omega M\right)} \right] I_1$$

$$\boxed{I_1 = \frac{\left(\frac{1}{j\omega C_2} + j\omega L_2 - j\omega M\right) V_0}{\left(\frac{1}{j\omega C_1} + j\omega L_1\right)\left(\frac{1}{j\omega C_2} + j\omega L_2\right) + \omega^2 M^2}}$$

Sust. I_1 en (*):

$$\boxed{I_2 = \frac{\left(\frac{1}{j\omega C_1} + j\omega L_1 - j\omega M\right) V_o}{\left(\frac{1}{j\omega C_1} + j\omega L_1\right)\left(\frac{1}{j\omega C_2} + j\omega L_2\right) + \omega^2 M^2}}$$

$$\boxed{I_o = I_1 + I_2 = \frac{\frac{1}{j\omega C_1} + j\omega L_1 + \frac{1}{j\omega C_2} + j\omega L_2 - 2j\omega M}{\left(\frac{1}{j\omega C_1} + j\omega L_1\right)\left(\frac{1}{j\omega C_2} + j\omega L_2\right) + \omega^2 M^2} V_o}$$

c) $I_1(\omega_1) = 0$

$$\frac{1}{j\omega_1 C_2} + j\omega_1 L_2 - j\omega_1 M = 0 \Leftrightarrow 1 - \omega^2 L_2 C_2 + \omega_1^2 M C_2 = 0$$

$$1 - \omega_1^2 (L_2 - M) C_2 = 0 \rightarrow \boxed{\omega_1 = \frac{1}{\sqrt{(L_2 - M) C_2}}} \quad (\text{Si } L_2 > M)$$

$$I_2(\omega_2) = 0 \Leftrightarrow 1 - \omega_2^2 L_1 C_1 + \omega_2^2 M C_1 = 0$$

$$1 - \omega_2^2 (L_1 - M) C_1 = 0 \rightarrow \boxed{\omega_2 = \frac{1}{\sqrt{(L_1 - M) C_1}}}$$

(Si $L_1 > M$)

d) $I_o(\omega_o) = 0$

$$\frac{1}{j\omega_o}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + j\omega_o (L_1 + L_2 - 2M) = 0$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \omega_o^2 (L_1 + L_2 - 2M) \rightarrow \boxed{\omega_o = \sqrt{\frac{C_1 + C_2}{C_1 C_2 (L_1 + L_2 - 2M)}}}$$

(Como $M \leq \sqrt{L_1 L_2} \Rightarrow L_1 + L_2 - 2M \geq L_1 + L_2 - 2\sqrt{L_1 L_2} = \left(\sqrt{L_1} - \sqrt{L_2}\right)^2 > 0$ si $L_1 \neq L_2$)

Obs:

Los valores para los ω_1, ω_2 y ω_o también pueden hallarse directamente a partir de las ecs. de mallas y nodos de (a), haciendo cero las respectivas corrientes.

Ej: $I_1(\omega_1) = 0 \rightarrow V_o = (j\omega_1 M) I_2 \quad (1)$

$$V_o = \left[\frac{1}{j\omega C_2} + j\omega L_2\right] I_2 \quad (2)$$

igualando (1) y (2):

$$j\omega_1 M = \frac{1}{j\omega_1 C_2} + j\omega_1 L_2 \rightarrow \omega_1 = \frac{1}{\sqrt{(L_2 - M) C_2}}$$