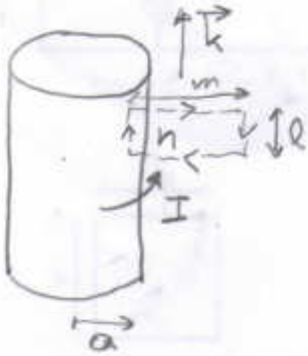
parte a: $\vec{B} = \nabla \wedge \vec{A}$

Teorema de Stokes: $\underbrace{\int_S \overbrace{(\nabla \wedge \vec{A})}^{}_{\overset{\shortparallel}{\vec{B}}} \cdot \vec{n}\, da}_{} = \oint_{\mathcal{C}} \vec{A} \cdot d\vec{l}$

$$\boxed{\phi = \oint_{\mathcal{C}} \vec{A} \cdot d\vec{l}}$$

parte b: $\vec{B} = B\vec{k}$



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{total}$

$(B_{int} - B_{ext})\, l = \mu_0 n l I$

Esta expresión es independiente de m por lo que puedo hacer $m \to \infty$ en donde $B_{ext} \to 0$. Luego $B_{ext} = 0 \;\; \forall r$

$\vec{B} = \mu_0 n I \vec{k} \quad r < a$

$\vec{B} = 0 \quad r > a$

Considero circunferencia radio r coaxial con eje del solenoide y $\vec{A} = A\vec{e}_\varphi$ ($A = A(r)$ es única forma de obtener $\vec{B}$ según $\vec{k}$)

$2\pi r A = \phi \quad r < a \quad \phi = \mu_0 n I \pi r^2$

$\qquad\qquad\qquad r > a \quad \phi = \mu_0 n I \pi a^2$

$$\Rightarrow \boxed{\vec{A} = \begin{cases} \dfrac{\mu_0 n I r}{2}\vec{e}_\varphi & r < a \\ \dfrac{\mu_0 n I a^2}{2r}\vec{e}_\varphi & r > a \end{cases}}$$

parte c: $\vec{A} = A(r)\vec{e}_\varphi \Rightarrow \vec{B} = \dfrac{1}{r}\dfrac{\partial (rA)}{\partial r}\vec{k}$

$r < a \quad \vec{B} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(\dfrac{\mu_0 n I r^2}{2}\right)\vec{k} = \mu_0 n I \vec{k}$ ✓

$r > a \quad \vec{B} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(\dfrac{\mu_0 n I a^2}{2}\right)\vec{k} = 0$ ✓

parte d: $\nabla\wedge\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\frac{\partial\nabla\wedge\vec{A}}{\partial t} = -\nabla\wedge\left(\frac{\partial\vec{A}}{\partial t}\right)$

$$\nabla\wedge\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0 \Rightarrow \vec{E}+\frac{\partial\vec{A}}{\partial t} = \nabla\varphi$$

Elijo $\vec{E} = -\frac{\partial\vec{A}}{\partial t} \Rightarrow \boxed{\begin{array}{ll}\vec{E} = -\frac{\mu_0 n r}{2}\frac{dI}{dt}\vec{e}_\varphi & r<a \\ \vec{E} = -\frac{\mu_0 n a^2}{2r}\frac{dI}{dt}\vec{e}_\varphi & r>a\end{array}}$

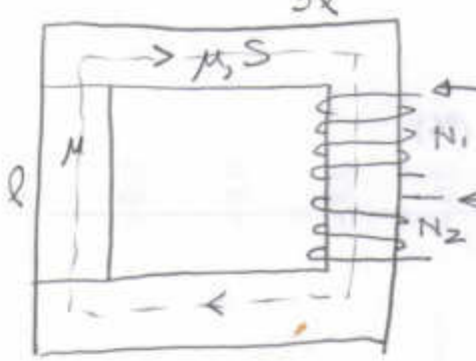
A menos del gradiente de un potencial

Verificación: $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} \Rightarrow \vec{E} = E\vec{e}_\varphi$

$r<a \quad E\,2\pi \cancel{r} = -\mu_0 n\frac{dI}{dt}\pi r^{\cancel{2}} \Rightarrow E = -\frac{\mu_0 n r}{2}\frac{dI}{dt}$

$r>a \quad E\,2\pi r = -\mu_0 n\frac{dI}{dt}\pi a^2 \Rightarrow E = -\frac{\mu_0 n a^2}{2r}\frac{dI}{dt}$

Ejercicio Nº2



parte a: $4\ell H = N_1 I_1 - N_2 I_2$, con $H = \frac{B}{\mu}$

$$\Rightarrow \boxed{B = \frac{\mu}{4\ell}\left(N_1 I_1 - N_2 I_2\right)}$$

parte b: Flujo a) $\Phi_a = \frac{\mu S}{4\ell}\left(N_1 I_1 - N_2 I_2\right)$

$\cdot\ \Phi_b = B_{3\ell} S = B_\ell S'$

$\ell H_\ell + 3\ell H_{3\ell} = N_1 I_1 - N_2 I_2$

$\ell \frac{B_\ell}{\mu} + 3\ell \frac{B_{3\ell}}{\mu} = N_1 I_1 - N_2 I_2 = \frac{\ell}{\mu}\left(1 + 3\frac{S'}{S}\right) B_\ell = N_1 I_1 - N_2 I_2$

$$B_\ell = \frac{\mu}{\ell}\frac{S\left(N_1 I_1 - N_2 I_2\right)}{S + 3S'} \Rightarrow \Phi_b = \frac{\mu}{\ell}\frac{SS'\left(N_1 I_1 - N_2 I_2\right)}{S + 3S'}$$

$$\Phi_b = \frac{\Phi_a}{2} \Rightarrow \frac{\cancel{S}S'}{S + 3S'} = \frac{\cancel{S}}{8} \Rightarrow 8S' = S + 3S' \Rightarrow \boxed{S' = \frac{S}{5}}$$

$$\boxed{B_\ell = \frac{5\mu}{8\ell}\left(N_1 I_1 - N_2 I_2\right)}$$

parte c: $\Phi_{tot_1} = N_1 \Phi_b = \frac{\mu}{\ell}\frac{SS'\left(N_1^2 I_1 - N_1 N_2 I_2\right)}{S + 3S'}$

$$\boxed{L_1 = \frac{\mu}{\ell}\frac{SS' N_1^2}{S + 3S'}} \qquad \frac{SS'}{S + 3S'} = \frac{S^2}{5\left(S + \frac{3S}{5}\right)} = \frac{S}{8} \qquad \boxed{L_1 = \mu\frac{S N_1^2}{8\ell}}$$

$$\boxed{M = -\frac{\mu}{\ell}\frac{SS' N_1 N_2}{S + 3S'} = -\mu\frac{S N_1 N_2}{8\ell}}$$

$$\Phi_{tot_2} = -N_2 \Phi_b = -\frac{\mu}{\ell}\frac{SS'\left(N_1 N_2 I_1 - N_2^2 I_2\right)}{S + 3S'}$$

$$\Rightarrow \boxed{L_2 = \frac{\mu}{\ell}\frac{SS' N_2^2}{S + 3S'} = \mu\frac{S N_2^2}{8\ell}}$$

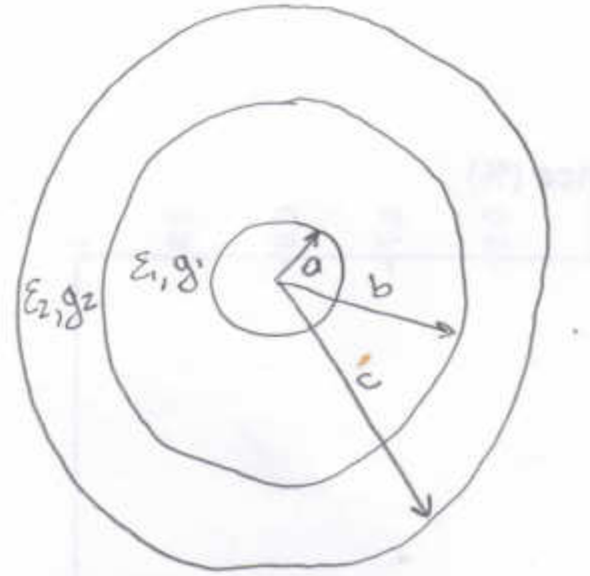
parte d: $W_b = \int \frac{\vec{H}\cdot\vec{B}}{2}\, dV = \frac{B_\ell^2}{2\mu} S'\ell + \frac{B_{3\ell}^2}{2\mu} S 3\ell = \frac{B_\ell^2 \ell}{2\mu}\left(S' + \frac{S'^2}{S^2} 3S\right)$

$$= \frac{B_\ell^2 \ell}{2\mu}\frac{S'}{S}\left(S + 3S'\right) = \frac{\mu}{2\ell}\frac{SS'\left(N_1 I_1 - N_2 I_2\right)^2}{S + 3S'}$$

$$S' = \frac{S}{5} \quad \boxed{W_b = \frac{\mu}{2l}\frac{S^2}{5}\frac{\left(N_1 I_1 - N_2 I_2\right)^2}{S + \frac{3S}{5}} = \frac{\mu}{2l}\frac{S}{8}\left(N_1 I_1 - N_2 I_2\right)^2}$$

$$S' = S \quad \boxed{W_a = \frac{\mu}{2l}\frac{S}{4}\left(N_1 I_1 - N_2 I_2\right)^2}$$

$$\boxed{\frac{W_a}{W_b} = \frac{1/4}{1/8} = 2}$$



parte a:

$$D_1 2\pi r \cancel{L} = \sigma_a 2\pi a \cancel{L}$$

$$\vec{D}_1 = \frac{a\sigma_a}{r}\vec{e}_r \Rightarrow \vec{E}_1 = \frac{a\sigma_a}{\varepsilon_1 r}\vec{e}_r$$

$$V_a - V_b = \Delta V = -\int_b^a \vec{E}_1 \cdot d\vec{r} = \int_a^b \frac{a\sigma_a\, dr}{\varepsilon_1 r}$$

$$= \frac{a\sigma_a}{\varepsilon_1}\ln r\Big|_a^b = \frac{a\sigma_a}{\varepsilon_1}\ln\frac{b}{a}$$

$$Q_a = 2\pi a L \sigma_a = 2\pi \cancel{a} L \frac{\varepsilon_1 \Delta V}{\cancel{a}\ln\frac{b}{a}}$$

$\boxed{C_{ab} = \frac{2\pi L \varepsilon_1}{\ln\frac{b}{a}}}$ Análogamente $\boxed{C_{bc} = \frac{2\pi L \varepsilon_2}{\ln\frac{c}{b}}}$

parte b: $\vec{E}_1 = \frac{\Delta V}{\ln\frac{b}{a}\, r}\vec{e}_r \Rightarrow \vec{J}_1 = g_1 \vec{E}_1 = \frac{g_1 \Delta V}{\ln\frac{b}{a}}\frac{\vec{e}_r}{r}$

$$I = \oint \vec{J}\cdot\vec{n}\, ds = \int 2\pi \cancel{r} L g_1 \frac{\Delta V}{\cancel{r}\ln\frac{b}{a}} \Rightarrow \Delta V = \frac{\ln b/a}{2\pi L g_1} I$$

$\Rightarrow \boxed{R_{ab} = \frac{\ln b/a}{2\pi L g_1}}$ Obs: $R_{ab} C_{ab} = \frac{\varepsilon_1}{g_1}$ Análogamente: $\boxed{R_{bc} = \frac{\ln c/b}{2\pi L g_2}}$

parte c: $\vec{E}_1 = \frac{a\sigma_a}{\varepsilon_1 r}\vec{e}_r$ $\sigma_a = \frac{Q_a}{2\pi a L} \Rightarrow \vec{E}_1 = \frac{Q_a \vec{e}_r}{2\pi\varepsilon_1 L r} \Rightarrow \vec{J}_1 = \frac{g_1 Q_a \vec{e}_r}{2\pi\varepsilon_1 L r}$

$\vec{E}_2 = -\frac{a\sigma_c \vec{e}_r}{\varepsilon_2 r}$ $\sigma_c = \frac{Q_c}{2\pi c L} \Rightarrow \vec{E}_2 = -\frac{Q_c}{2\pi\varepsilon_2 L r}\vec{e}_r \Rightarrow \vec{J}_2 = -\frac{g_2 Q_c \vec{e}_r}{2\pi\varepsilon_2 L r}$

$$V_0 = V_a - V_c = -\int_c^a \vec{E}\cdot d\vec{r} = \int_a^c \vec{E}\cdot d\vec{r} = \int_a^b \frac{Q_a\, dr}{2\pi\varepsilon_1 L r} -$$

$$-\int_b^c \frac{Q_c\, dr}{2\pi\varepsilon_2 L r} = \frac{Q_a}{2\pi\varepsilon_1 L}\ln b/a - \frac{Q_c}{2\pi\varepsilon_2 L}\ln\frac{c}{b} = \frac{Q_a}{C_{ab}} - \frac{Q_c}{C_{bc}}$$

En régimen estacionario no se acumula carga

$$\Rightarrow \oint \vec{J}\cdot\vec{n} = 0 \quad -\frac{g_1 Q_a 2\pi L r_{int}}{2\pi\varepsilon_1 L r_{int}} - \frac{g_2 Q_c 2\pi L r_{ext}}{2\pi\varepsilon_2 L r_{ext}} = 0$$

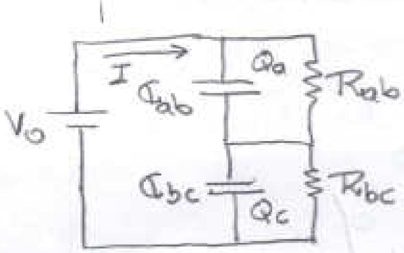
$$-\frac{g_1}{\varepsilon_1}Q_a - \frac{g_2}{\varepsilon_2}Q_c = 0 \Rightarrow Q_c = -\frac{g_1}{\varepsilon_1}\frac{\varepsilon_2}{g_2}Q_a$$

$$V_0 = \frac{Q_a}{C_{ab}} + \frac{g_1}{\varepsilon_1}\frac{\varepsilon_2}{g_2}\frac{Q_a}{C_{bc}} = Q_a\left(\frac{1}{C_{ab}} + \frac{1}{R_{ab}C_{ab}}\frac{R_{bc}\cancel{C_{bc}}}{\cancel{C_{bc}}}\right) = \frac{Q_a}{C_{ab}}\frac{R_{ab}+R_{bc}}{R_{ab}}$$

$$\boxed{Q_a = \frac{R_{ab}C_{ab}V_0}{R_{ab}+R_{bc}}} \qquad \boxed{Q_c = -\frac{1}{\cancel{R_{ab}C_{ab}}}\frac{R_{bc}C_{bc}\cancel{R_{ab}C_{ab}}V_0}{R_{ab}+R_{bc}} = -\frac{R_{bc}C_{bc}V_0}{R_{ab}+R_{bc}}}$$

$$\boxed{\vec{E}_1 = \frac{1}{r\ln b/a}\frac{R_{ab}V_0}{R_{ab}+R_{bc}}\vec{e}_r \ (a<r<b)} \qquad \boxed{\vec{E}_2 = \frac{1}{r\ln c/b}\frac{R_{bc}V_0}{R_{ab}+R_{bc}}\vec{e}_r \ (b<r<c)}$$

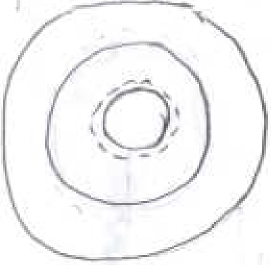
Verificación: El sistema se puede ver como la serie de dos sistemas de una resistencia y un condensador en paralelo.



$$V_0 = \frac{Q_a}{C_{ab}} - \frac{Q_c}{C_{bc}}$$

$$\frac{Q_a}{C_{ab}} = R_{ab}I = R_{ab}\frac{V_0}{R_{ab}+R_{bc}} \Rightarrow Q_a = \frac{R_{ab}C_{ab}V_0}{R_{ab}+R_{bc}}$$

parte d:



$$\oint \vec{J}\cdot\hat{n}\,dS = -\frac{dQ}{dt} \Rightarrow \frac{g_1 Q_a}{\cancel{2\pi}\varepsilon_1 \cancel{Lr}}\cancel{2\pi r L} = -\frac{dQ_a}{dt}$$

$$\frac{dQ_a}{dt} + \frac{g_1}{\varepsilon_1}Q_a = 0 \Rightarrow \boxed{Q_a(t) = Q_a(0)\,e^{-\frac{g_1}{\varepsilon_1}t}}$$

Análogamente para el conductor de radio c:

$$\frac{g_2 Q_c}{\cancel{2\pi}\varepsilon_2 \cancel{Lr}}\cancel{2\pi r L} = -\frac{dQ_c}{dt} \Rightarrow \boxed{Q_c(t) = Q_c(0)\,e^{-\frac{g_2}{\varepsilon_2}t}}$$

$Q_a(0)$ y $Q_c(0)$ son los valores hallados en la parte c