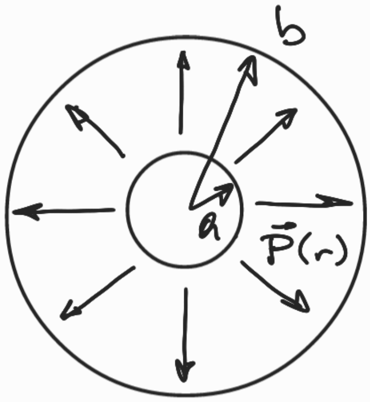
Solución examen Electromagnetismo (1128)     4/2/22

(1)




$$\vec{P}(r) = \frac{k}{r}\hat{e}_r \quad a \le r \le b$$

a) coords. esféricas ; $\vec{P} = P(r)\hat{e}_r$

$$\boxed{\rho_p = -\nabla\cdot\vec{P} = -\frac{1}{r^2}\frac{d(r^2P(r))}{dr} = -\frac{1}{r^2}\frac{d}{dr}\left(r^{\cancel{2}}\frac{k}{\cancel{r}}\right) = -\frac{k}{r^2}}$$

$\rho_p(r)$ para $a<r<b$

$$\boxed{\sigma_p\big|_a = P\cdot\underset{-\hat{e}_r}{\hat{n}}\Big|_{r=a} = -\frac{k}{a}} \qquad \boxed{\sigma_p\big|_b = P\cdot\underset{\hat{e}_r}{\hat{n}}\Big|_{r=b} = \frac{k}{b}}$$

$$\boxed{Q_p = \underbrace{\int_V \rho_p dV}_{\int_a^b -\frac{k}{\cancel{r^2}}4\pi \cancel{r^2}dr = -4\pi k(b-a)} + \underbrace{\int_{S_a}\sigma_p\big|_a dS}_{-\frac{k}{\cancel{a}}4\pi a^{\cancel{2}}} + \underbrace{\int_{S_b}\sigma_p\big|_b dS}_{\frac{k}{\cancel{b}}4\pi b^{\cancel{2}}} = -4\pi k(b-a) - 4\pi k a + 4\pi k b = 0}$$

b) $\boxed{\vec{D} = 0,\ \forall r}$ , no hay cargas libres $(\nabla\cdot\vec{D} = \rho_L = 0)$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \text{con } \begin{cases} P = 0 \ ; r<a \text{ y } r>b \\ P = \frac{k}{r}\hat{e}_r \ ; a \le r \le b \end{cases}$$

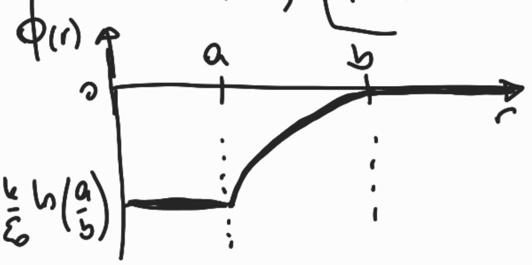
$$\vec{E} = E(r)\hat{e}_r$$

$$\Rightarrow \boxed{\vec{E} = \begin{cases} 0 & r<a \\ -\frac{P(r)}{\varepsilon_0} = -\frac{k}{\varepsilon_0 r}\hat{e}_r & a \le r \le b \\ 0 & r>b \end{cases}}$$

Para $r<a$ ; $\boxed{\phi(r) = -\int_\infty^r E\,dr = -\underbrace{\int_\infty^b E\,dr}_{0} - \int_b^a -\frac{k}{\varepsilon_0 r}dr - \underbrace{\int_a^r E\,dr}_{0} = \frac{k}{\varepsilon_0}\ln\left(\frac{a}{b}\right)}$

$r>b$ ; $\boxed{\phi(r) = -\underbrace{\int_\infty^r E\,dr}_{0} = 0}$

$a<r<b$ ; $\boxed{\phi(r) = -\int_\infty^r E\,dr = -\underbrace{\int_\infty^b E\,dr}_{0} - \int_b^r -\frac{k}{\varepsilon_0 r}dr = \frac{k}{\varepsilon_0}\ln\left(\frac{r}{b}\right)}$



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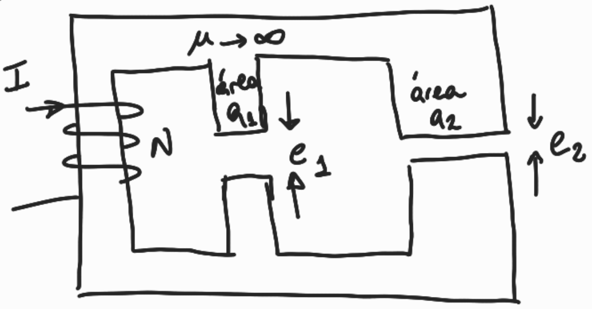


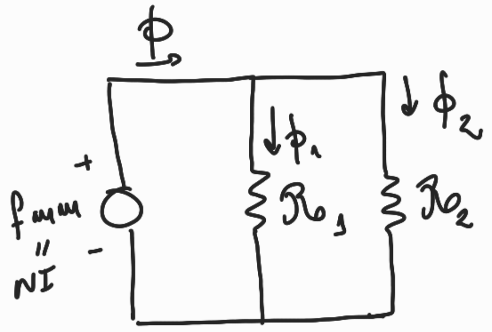


En los tramos donde la permeabilidad es infinita ($\mu \to \infty$), la reluctancia tiende a cero ya que es proporcional al inverso de $\mu$.

Entonces, en la rama izquierda, donde está el enrollado, la reluctancia es nula $\boxed{\mathcal{R}_{o\mu}=0}$; mientras que en las ramas central y derecha sólo aportan a las reluctancias las regiones con entrehierros.

- rama central $\boxed{\mathcal{R}_{o_1} = \dfrac{e_1}{\mu_0 a_1}}$ y rama derecha $\boxed{\mathcal{R}_{o_2} = \dfrac{e_2}{\mu_0 a_2}}$

Ampère $\to$

$$NI = \phi \overbrace{\mathcal{R}_{o\mu}}^{=0} + \phi_1 \mathcal{R}_{o_1}$$ (recorro rama izquierda y central)

$$NI = \phi \underbrace{\mathcal{R}_{o\mu}}_{=0} + \phi_2 \mathcal{R}_{o_2}$$ ( " " " " derecha)

$$\phi = \phi_1 + \phi_2$$

$$\boxed{\phi_1 = \frac{NI}{\mathcal{R}_{o_1}}} \qquad \boxed{\phi_2 = \frac{NI}{\mathcal{R}_{o_2}}}$$

$$\boxed{\phi = \phi_1 + \phi_2 = NI\left(\frac{1}{\mathcal{R}_{o_1}} + \frac{1}{\mathcal{R}_{o_2}}\right) = NI\left(\frac{\mathcal{R}_{o_1}+\mathcal{R}_{o_2}}{\mathcal{R}_{o_1}\mathcal{R}_{o_2}}\right)}$$

$$\boxed{L = N\frac{d\phi}{dI} = N^2\left(\frac{\mathcal{R}_{o_1}+\mathcal{R}_{o_2}}{\mathcal{R}_{o_1}\mathcal{R}_{o_2}}\right) = N^2\left(\frac{\frac{e_1}{\cancel{\mu_0} a_1} + \frac{e_2}{\cancel{\mu_0} a_2}}{\frac{e_1 e_2}{\mu_0^2 a_1 a_2}}\right) = \mu_0 N^2\left(\frac{a_2}{e_2} + \frac{a_1}{e_1}\right)}$$

b) $$\boxed{B_1 = \frac{\phi_1}{a_1} = \frac{NI}{\mathcal{R}_{o_1} a_1} = \frac{NI\mu_0 \cancel{a_1}}{e_1 \cancel{a_1}} = \frac{\mu_0 NI}{e_1}}$$

$$\boxed{B_2 = \frac{\phi_2}{a_2} = \frac{NI}{\mathcal{R}_{o_2} a_2} = \frac{\mu_0 NI}{e_2}}$$

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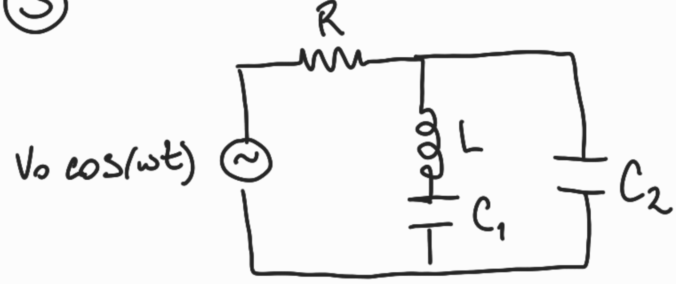



$$\tilde{Z}_T = R + \cfrac{1}{\underbrace{\cfrac{1}{j\omega L + \cfrac{1}{j\omega C_1}}}_{=\ \frac{j\omega C_1}{1-\omega^2 L C_1}} + \underbrace{\cfrac{1}{\cfrac{1}{j\omega C_2}}}_{=\ j\omega C_2}}$$

$$\boxed{\tilde{Z}_T = R + \frac{1-\omega^2 L C_1}{j\omega C_1 + j\omega C_2(1-\omega^2 L C_1)}} = \boxed{\underbrace{R}_{Re(Z)} + j\underbrace{\frac{(\omega^2 L C_1 - 1)}{\omega(C_1+C_2) - \omega^3 L C_1 C_2}}_{Im(Z)}}$$

$$\tilde{I}(t) = \frac{V_0 e^{j\omega t}}{\tilde{Z}_T} = \frac{V_0 e^{j\omega t}}{|\tilde{Z}_T| e^{j\phi}} = \frac{V_0}{|\tilde{Z}_T|} e^{j(\omega t - \phi)} = \tilde{I}_0 e^{j\omega t} \quad \text{con } \tilde{I}_0 = \frac{V_0}{|\tilde{Z}_T|} e^{j(-\phi)}$$

$$\Rightarrow \boxed{I(t) = Re\left\{\frac{V_0}{|\tilde{Z}_T|} e^{j(\omega t - \phi)}\right\} = \frac{V_0}{|\tilde{Z}_T|}\cos(\omega t - \phi)}$$

donde $|\tilde{Z}_T| = \sqrt{Re^2(\tilde{Z}_T) + Im^2(\tilde{Z}_T)} = \sqrt{R^2 + \left(\frac{\omega^2 L C_1 - 1}{\omega(C_1+C_2) - \omega^3 L C_1 C_2}\right)^2}$

y $\phi = \operatorname{arctg} \frac{Im(\tilde{Z}_T)}{Re(\tilde{Z}_T)} = \operatorname{arctg}\left\{\frac{\omega^2 L C_1 - 1}{R\omega\left[(C_1+C_2) - \omega^2 L C_1 C_2\right]}\right\}$

$$\boxed{\langle P \rangle = \frac{1}{2} Re\{V \tilde{I}_0^*\}} = \frac{1}{2} Re\left\{V_0^2 \frac{e^{j\phi}}{|\tilde{Z}_T|}\right\} = \frac{1}{2}\frac{V_0^2}{|\tilde{Z}_T|} \underbrace{\cos\phi}_{=\ \frac{R}{|\tilde{Z}_T|}} = \boxed{\frac{1}{2}\frac{V_0^2 R}{|\tilde{Z}_T|^2}}$$

$$= \boxed{\frac{1}{2} V_0^2 \frac{R}{R^2 + \left(\frac{\omega^2 L C_1 - 1}{\omega(C_1+C_2) - \omega^3 L C_1 C_2}\right)^2}}$$

b)

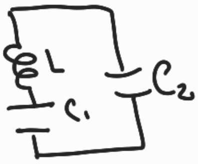
$\omega_0$ tal que $I_0(\omega_0) = I_{0\,máx}$, la corriente máxima corresponde a $|\tilde{Z}_T|$ mínimo y esto ocurre cuando la $Im(\tilde{Z}_T) = 0$ : $\omega^2 L C_1 - 1 = 0$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{L C_1}}}$$

$\omega^*$ tal que $I_0(\omega^*) = 0$, la corriente por la resistencia se anula cuando $|\tilde{Z}_T| \to \infty$ : $(C_1 + C_2 - \omega^{*2} L C_1 C_2)\omega^* = 0 \to \omega^* = 0$ $(\omega > 0)$ ✗

$\to \omega^* = \sqrt{\frac{C_1 + C_2}{L(C_1 C_2)}}$

$\omega^* = \frac{1}{\sqrt{L C_{eq}}}$ con $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ (serie)




Se comporta como circuito $LC_{eq}$ serie