① a) $\rho(r, t=0) = \rho_0$

$$\left.\begin{aligned} &\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \\ &\vec{J} = g\vec{E} = g\frac{\vec{D}}{\varepsilon_0} \end{aligned}\right\} \Rightarrow \frac{\partial \rho}{\partial t} + \frac{g}{\varepsilon_0}\nabla\cdot\vec{D} = \frac{\partial \rho}{\partial t} + \frac{g}{\varepsilon_0}\rho = 0$$

$$\Rightarrow \boxed{\rho(r,t) = \rho_0 e^{-\frac{gt}{\varepsilon_0}}} \quad t>0, \; r\in(a,b)$$

$$\int_V \nabla\cdot\vec{J}\, dV = -\int_V \frac{\partial \rho}{\partial t} dV = \oint_V \vec{J}\cdot\hat{n}\, dA = J\cdot 4\pi r^2$$

$$-\int_V \frac{\partial \rho}{\partial t} dV = \frac{g\rho_0}{\varepsilon_0} 4\pi\left(\frac{r^3 - a^3}{3}\right) e^{-\frac{gt}{\varepsilon_0}} \Rightarrow \boxed{\vec{J}(r,t) = \frac{g\rho_0}{3\varepsilon_0}\left(\frac{r^3-a^3}{r^2}\right) e^{-\frac{gt}{\varepsilon_0}}\hat{e}_r}$$

$$\dot{\sigma}\Big|_{r=a} = -J(r=a,t) = 0 \Rightarrow \boxed{\sigma(r=a,t) = \sigma(r=a,0) = 0}$$

$$\dot{\sigma}\Big|_{r=b} = J(r=b,t) \Rightarrow \sigma(r=b,t) = \frac{g\rho_0}{3\varepsilon_0}\frac{b^3-a^3}{b^2}\int_0^t e^{-\frac{gt}{\varepsilon_0}} dt$$

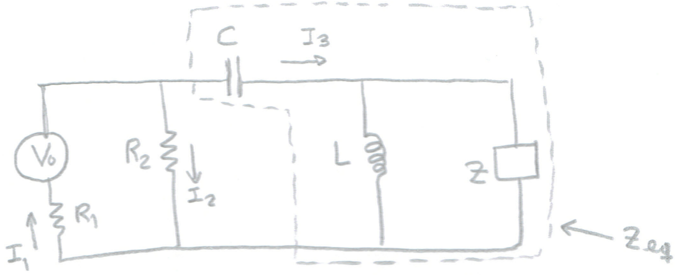
$$\Rightarrow \boxed{\sigma(r=b,t) = \frac{\rho_0}{3}\left(\frac{b^3-a^3}{b^2}\right)\left(1-e^{-\frac{gt}{\varepsilon_0}}\right)}$$

b)
$$\left.\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{g}{\varepsilon_0}\nabla\cdot(\vec{D}-\vec{P}) = 0 \\ &\nabla\cdot\vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho_0) = \frac{2\rho_0}{r} \end{aligned}\right\} \Rightarrow \frac{\partial \rho}{\partial t} + \frac{g}{\varepsilon_0}\rho = \frac{2g\rho_0}{\varepsilon_0 r}$$

$$\left.\begin{aligned} &\Rightarrow \rho(r,t) = \rho^* e^{-\frac{gt}{\varepsilon_0}} + \frac{2\rho_0}{r} \\ &\rho(r,0) = \rho_0 = \rho^* + \frac{2\rho_0}{r} \end{aligned}\right\} \Rightarrow \boxed{\rho(r,t) = \rho_0 e^{-\frac{gt}{\varepsilon_0}} + \frac{2\rho_0}{r}\left(1 - e^{-\frac{gt}{\varepsilon_0}}\right)}$$

$t>0, \; r\in(a,b)$

2)



$$\boxed{Z_{eq} = \left(\frac{1}{i\omega L} + \frac{1}{Z}\right)^{-1} - \frac{i}{\omega C}}$$

a) $R_2 I_2 = Z_{eq} I_3$

Si $Z_{eq} = \infty \Rightarrow \boxed{I_3 = 0}$

$\Rightarrow I_1 = I_2$

$\Rightarrow$ Potencia media disipada en R_1 $= \frac{|I_1|^2 R_1}{2}$

$$= \frac{V_0^2 \, R_1}{2(R_1+R_2)^2}$$

Si $Z = -\frac{i}{\omega C'} \Rightarrow Z_{eq} = \left(\frac{1}{i\omega L} - \frac{\omega C'}{i}\right)^{-1} - i/\omega C$

Cuando $Z_{eq} = \infty \Rightarrow \frac{1}{\omega L} = \omega C' \Rightarrow \boxed{C' = \frac{1}{\omega^2 L}}$

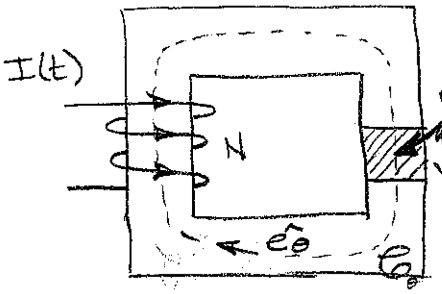
b) Potencia disipada en $R_2 = 0 \Rightarrow I_2 = 0 \Rightarrow Z_{eq} = 0$

Si $Z = i\omega L' \Rightarrow Z_{eq} = \left(\frac{1}{i\omega L} + \frac{1}{i\omega L'}\right)^{-1} - \frac{i}{\omega C} = 0$

$\Rightarrow \omega^2 C = \frac{1}{L} + \frac{1}{L'} \Rightarrow \boxed{L' = \frac{L}{\omega^2 CL - 1}}$ asumiendo $\omega^2 CL > 1$.

(3)

$I(t) = I_0 \cos(\omega t)$



a) $\nexists$ Ampère (recorro $\mathcal{C}_0$ en stdo. horario): según $\hat{e}_\theta$

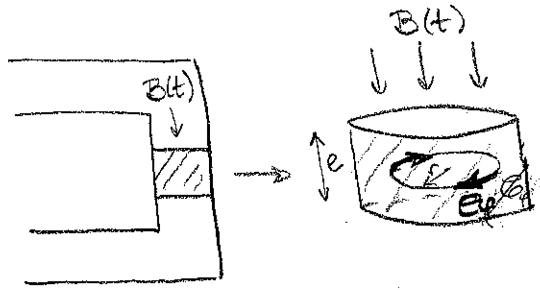
$$\oint_{\mathcal{C}_0} H \cdot \vec{dl} = I_{\mathcal{C}_0}$$

despreciando el flujo de $\vec{J}$ a través de $\mathcal{C}_0$ en el Cu

$$\underbrace{H_\mu}_{= \frac{B}{\mu}}(l-e) + \underbrace{H_{Cu}}_{= \frac{B}{\mu_0}} e = NI$$

$$B\left[\frac{l-e}{\mu} + \frac{e}{\mu_0}\right] = NI \rightarrow B = \frac{NI}{\frac{l-e}{\mu} + \frac{e}{\mu_0}} \rightarrow \boxed{B(t) = \underbrace{\frac{NI_0}{\left(\frac{l-e}{\mu}\right) + \frac{e}{\mu_0}}}_{= B_0} \cos(\omega t)\, \hat{e}_\theta}$$

b)



$$\nexists\ \nabla \times \vec{E}_{ind} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_{\mathcal{C}_\varphi} \vec{E}_{ind} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$$

$$E_{ind}(2\cancel{\pi} \cancel{r}) = +\omega B_0 \operatorname{sen}(\omega t)(\cancel{\pi} r^{\cancel{2}})$$

$$\Rightarrow \boxed{\vec{E}_{ind} = \frac{\omega B_0 r}{2} \operatorname{sen}(\omega t)\, \hat{e}_\varphi}$$

$$\boxed{\vec{J} = g\vec{E} = \frac{g\omega B_0 r}{2} \operatorname{sen}(\omega t)\, \hat{e}_\varphi}$$

c)

$$P(t) = \int_V \vec{J} \cdot \underbrace{\vec{E}}_{= \frac{\vec{J}}{g}} dV = \frac{1}{g}\int_V J^2 \underbrace{dV}_{= 2\pi r (dr) e} = \frac{B_0^2 \omega^2 g^{\cancel{2}}}{2^{\cancel{2}4}\, \cancel{g}} \operatorname{sen}^2(\omega t)\, e\, \cancel{2}\pi \underbrace{\int_0^R r^3 dr}_{= R^4/4}$$

$$= \frac{e g B_0^2 \omega^2 \pi R^4}{8} \operatorname{sen}^2(\omega t)$$

$$P_{media} = \frac{1}{T}\int_0^T P(t)\, dt = \frac{e g B_0^2 \omega^2 \pi R^4}{8} \underbrace{\frac{1}{T}\int_0^T \operatorname{sen}^2(\omega t)\, dt}_{= \frac{1}{2}} = \quad (T = 2\pi/\omega)$$

$$\boxed{P_{media} = \frac{e g B_0^2 \omega^2 \pi R^4}{16}}$$

— x —