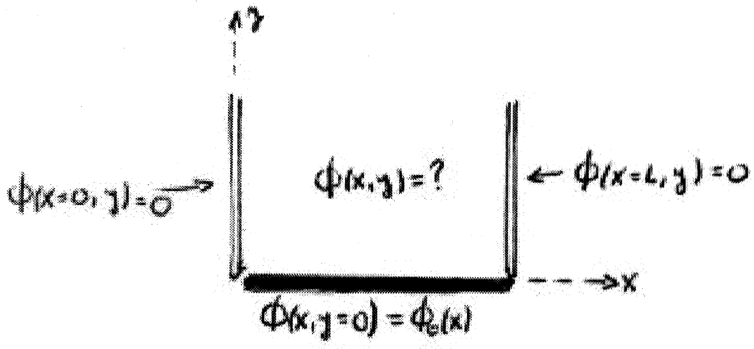
Solución al Problema 1:



a) Ansatz : $\phi(x,y) = \sum_n b_n \operatorname{sen}\left(\frac{n\pi}{L}x\right) e^{\pm\frac{n\pi}{L}}$

$\Rightarrow \boxed{\phi(x,y) = V_0 \operatorname{sen}\left(\frac{3\pi}{L}x\right) e^{-\frac{3\pi}{L}y}}$

b) $\boxed{\phi(x,y) = V_0 \operatorname{sen}\left(\frac{3\pi}{L}x\right)\left[\frac{e^{-\frac{3\pi}{L}y} - e^{+\frac{3\pi}{L}y} e^{-\frac{6\pi}{L}h}}{1-e^{-\frac{6\pi}{L}h}}\right]}$

2) a) $\vec{E} = \begin{cases} \dfrac{Q}{2\pi r l \varepsilon_1}\hat{e}_r, & a<r<b \\ \dfrac{Q+\sigma_0 2\pi b l}{2\pi r l \varepsilon_2}\hat{e}_r, & b<r<c \\ \dfrac{Q+\sigma_0 2\pi b l}{2\pi r l \varepsilon_0}\hat{e}_r, & r>c \end{cases}$ $\vec{P} = \begin{cases} (\varepsilon_1-\varepsilon_0)\vec{E}, & a<r<b \\ (\varepsilon_2-\varepsilon_0)\vec{E}, & b<r<c \\ 0, & r>c \end{cases}$

$\boxed{\sigma_{P1a} = -\frac{(\varepsilon_1-\varepsilon_0)Q}{2\pi a l \varepsilon_1}}$ $\boxed{\sigma_{P1b} = \frac{(\varepsilon_1-\varepsilon_0)Q}{2\pi b l \varepsilon_1}}$ $\boxed{\sigma_{P2b} = -\frac{[Q+\sigma_0 2\pi b l][\varepsilon_2-\varepsilon_0]}{2\pi b l \varepsilon_2}}$

$\boxed{\sigma_{P2c} = \frac{[Q+\sigma_0 2\pi b l][\varepsilon_2-\varepsilon_0]}{2\pi c l \varepsilon_2}}$ $\boxed{\rho_{P1} = \rho_{P2} = 0}$

b) • $\dot{\sigma}_{La} = -\frac{g_1}{\varepsilon_1}\sigma_{La} \Rightarrow \boxed{\sigma_{La}(t) = \frac{Q}{2\pi a l}e^{-\frac{g_1}{\varepsilon_1}t}, \quad t \geq 0}$

• $\begin{cases} D_2 - D_1|_{r=b} = \sigma_{Lb} = \varepsilon_2 E_2 - \varepsilon_1 E_1 \\ J_1 - J_2|_{r=b} = \dot{\sigma}_{Lb} = g_1 E_1 - g_2 E_2 \end{cases}$

$\Rightarrow \dot{\sigma}_{Lb} = \left(\frac{g_1}{\varepsilon_1} - \frac{g_2}{\varepsilon_2}\right)\varepsilon_1 E_1 - \frac{g_2}{\varepsilon_2}\sigma_{Lb} = \left(\frac{g_1}{\varepsilon_1} - \frac{g_2}{\varepsilon_2}\right)\frac{Qe^{-\frac{g_1}{\varepsilon_1}t}}{2\pi b l} - \frac{g_2}{\varepsilon_2}\sigma_{Lb}$

- Homogénea: $\sigma_h(t) = Ae^{-g_2/\varepsilon_2 t}$
- Particular: $\sigma_{Part}(t) = Be^{-g_1/\varepsilon_1 t}$

$\Rightarrow -\frac{g_1}{\varepsilon_1}B = \left(\frac{g_1}{\varepsilon_1} - \frac{g_2}{\varepsilon_2}\right)\frac{Q}{2\pi b l} - \frac{g_2}{\varepsilon_2} \Rightarrow \boxed{B = \left(\frac{g_2}{g_1}\cdot\frac{\varepsilon_1}{\varepsilon_2} - 1\right)\frac{Q}{2\pi b l} + \frac{g_2}{\varepsilon_2}\frac{\varepsilon_1}{g_1}}$

$\Rightarrow \sigma_{Lb}(t) = Ae^{-\frac{g_2}{\varepsilon_2}t} + Be^{-\frac{g_1}{\varepsilon_1}t}$

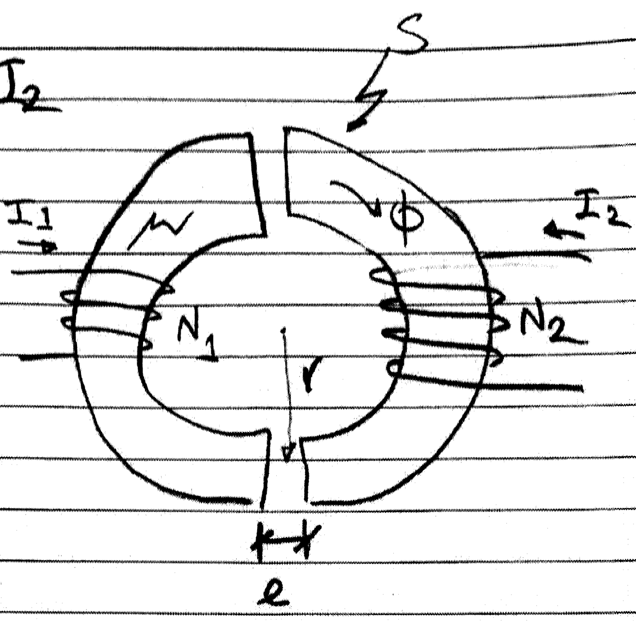
$\sigma_{Lb}(t=0) = \sigma_0 = A + B \Rightarrow \boxed{A = \sigma_0 - B}$

$\Rightarrow \boxed{\sigma_{Lb}(t) = \sigma_0 e^{-\frac{g_2}{\varepsilon_2}t} + B\left(e^{-\frac{g_1}{\varepsilon_1}t} - e^{-\frac{g_2}{\varepsilon_2}t}\right)}$

$\boxed{\rho_{La}(t) = \rho_{Lb}(t) = 0, \quad \forall t}$

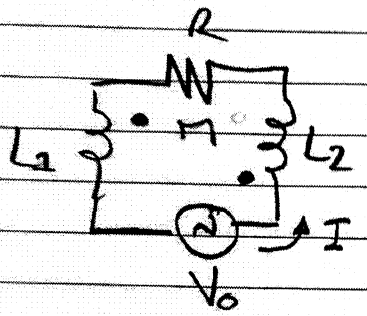
$\sigma_{Lc}(t) = \frac{1}{2\pi c l}\left[Q + \sigma_0 2\pi b l - \sigma_{Lb}(t) 2\pi b l - \sigma_{La}(t) 2\pi a l\right]$

3) a) $$\underbrace{\left[\frac{2\pi r}{\mu S} + \frac{2e}{\mu_0 S}\right]}_{\mathcal{R}}\phi = N_1 I_1 - N_2 I_2$$



$$\Rightarrow \boxed{L_1 = \frac{N_1^2}{\mathcal{R}}}\ ;\ \boxed{L_2 = \frac{N_2^2}{\mathcal{R}}}\ \boxed{M = \frac{-N_1 N_2}{\mathcal{R}}}$$

b)



$$V_0 = j\omega(L_2 + M)I + IR + j\omega(L_1 + M)I$$

$$\Rightarrow I = \frac{V_0}{R + j\omega(L_1 + L_2 + 2M)} \Rightarrow \boxed{\begin{array}{l} I(t) = \dfrac{V_0 \cos(\omega t - \varphi)}{\sqrt{R^2 + \omega^2(L_1 + L_2 + 2M)^2}} \\ \varphi = \text{Arctg}\left[\dfrac{\omega(L_1 + L_2 + 2M)}{R}\right] \end{array}}$$

c) $$U = (L_1 + L_2 + 2M)\frac{I^2}{2}$$

$$\boxed{\begin{array}{l} F = \left.\dfrac{\partial U}{\partial e}\right|_I = (N_1^2 + N_2^2 + 2N_1N_2)\dfrac{I^2}{2}\cdot\left(\dfrac{-1}{\mathcal{R}^2}\right)\cdot\dfrac{\partial \mathcal{R}}{\partial e} \\ \dfrac{\partial \mathcal{R}}{\partial e} = \dfrac{2}{S}\left(\dfrac{1}{\mu_0} - \dfrac{1}{\mu}\right) = 2\cdot\dfrac{\mu - \mu_0}{\mu\mu_0 S} \end{array}}$$

$$\boxed{F_{máx} = F\Big|_{I = I_{máx}}} \qquad I_{máx} = \frac{V_0}{\sqrt{R^2 + \omega^2(L_1 + L_2 + 2M)^2}}$$