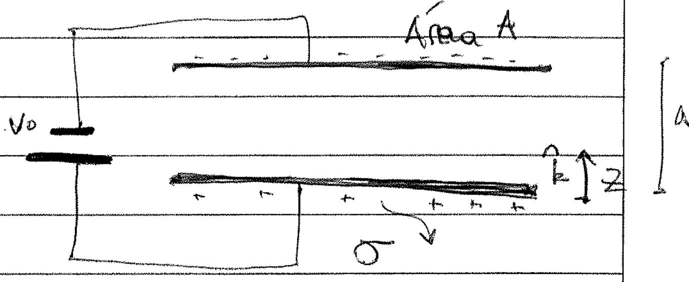
Problema 1 $\quad \varepsilon(z) = \dfrac{\varepsilon_1 \varepsilon_2 a}{\varepsilon_1 z + \varepsilon_2 (a-z)}$




a) Entre placas:

$$\vec{D} = \sigma \hat{k} = \varepsilon \vec{E}$$

$$V_0 = -\int \vec{E} \cdot d\vec{l} = \int_0^a \frac{\sigma}{\varepsilon(z)} dz =$$

$$V_0 = \int_0^a \sigma \left( \frac{a \varepsilon_1 \varepsilon_2}{\varepsilon_1 z + \varepsilon_2 (a-z)} \right)^{-1} dz = \frac{\sigma (\varepsilon_1 + \varepsilon_2) a}{2 \varepsilon_1 \varepsilon_2} \Rightarrow \sigma = \frac{2 V_0 \varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) a}$$

$$\vec{D} = \frac{2 V_0 \varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) a} \hat{k} \quad , \quad \vec{E} = \frac{\vec{D}}{\varepsilon(z)} \quad , \quad \vec{P} = \left( 1 - \frac{\varepsilon_0}{\varepsilon(z)} \right) \vec{D}$$

b) $\vec{P} \cdot \hat{n} \Big|_{sup} = \sigma_p$ ($\hat{n}$ normal saliente al material)

$$\sigma_p (z=0) = \left( 1 - \frac{\varepsilon_0}{\varepsilon(0)} \right) \vec{D} \cdot (-\hat{k}) = -\frac{(\varepsilon_1 - \varepsilon_0)}{(\varepsilon_1 + \varepsilon_2)} \frac{2 \varepsilon_2 V_0}{a}$$

$$\sigma_p (z=a) = + \frac{(\varepsilon_2 - \varepsilon_0)}{(\varepsilon_1 + \varepsilon_2)} \frac{2 \varepsilon_1 V_0}{a}$$

$$\rho_p = -\nabla \cdot \vec{P} = \frac{-\partial P}{\partial z} = -\frac{2 V_0 \varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) a} \frac{\partial}{\partial z} \left( 1 - \frac{\varepsilon_0}{\varepsilon(z)} \right)$$

$$= + \frac{2 V_0 \varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) a} \frac{\varepsilon_0}{\varepsilon(z)^2} \frac{\partial \varepsilon(z)}{\partial z}$$

$$\rho_p = \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2)} \frac{2 V_0 \varepsilon_0}{a^2}$$

Podemos tomar:

c) $C = \dfrac{Q}{V_0} = \dfrac{\sigma A}{V_0} = \dfrac{2 \varepsilon_1 \varepsilon_2 A}{(\varepsilon_1 + \varepsilon_2) a} \qquad U = \dfrac{C V_0^2}{2} = \dfrac{\varepsilon_1 \varepsilon_2 A V_0^2}{(\varepsilon_1 + \varepsilon_2) a}$

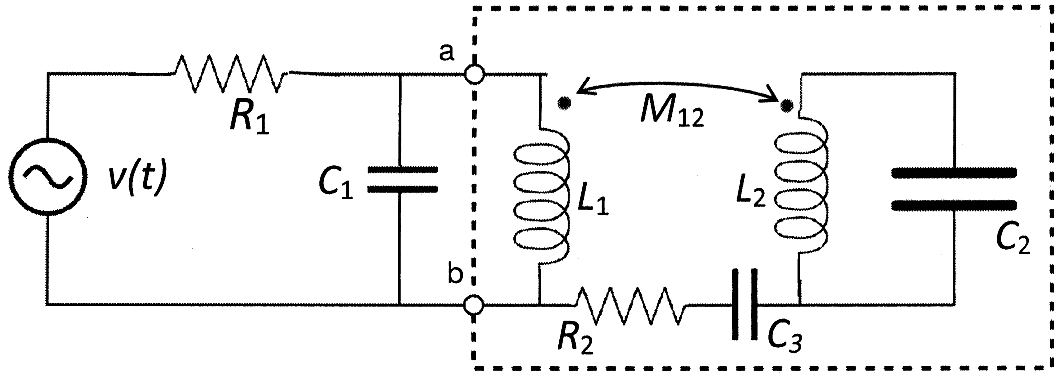
o $U = \displaystyle\int_V \frac{1}{2} \frac{D^2}{\varepsilon(z)} = A \int_0^a \frac{1}{2} D^2 \left( \frac{\varepsilon_1 z + \varepsilon_2 (a-z)}{\varepsilon_1 \varepsilon_2 a} \right) dz = \frac{A V_0^2 \varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) a}$

**Problema 2**

Considere el circuito de la figura, operando en régimen sinusoidal de frecuencia $\omega$, siendo *V(t)* la fem. Suponga que el coeficiente de inducción mutua vale $M_{12}$=$\sqrt{L_1 L_2}$ . Los valores de las inductancias, condensadores y resistencias se consideran conocidos.

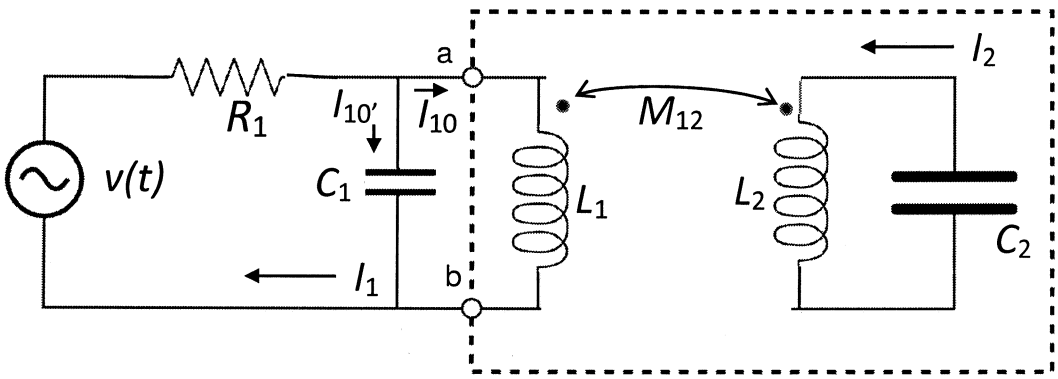
a) Calcule la impedancia compleja equivalente $Z(\omega)$ entre los bornes a y b.

b) Halle la frecuencia $\omega$ para que la caída de potencial en la resistencia $R_1$ sea nula.




**Solución**

Circuito real:




(a)

$$I_1 = I_{10} + I_{10'} \qquad (0)$$

$$V - R_1 I_1 \;(= V_{ab}) = i\,\omega\, L_1 I_{10} + \; i\,\omega\, M_{12}\, I_2 \qquad (1)$$

$$0 = i\,\omega\, L_2 I_2 + \; i\,\omega\, M_{12}\, I_{10} - \frac{i\, I_2}{\omega C_2} \qquad (2)$$

De (2) se obtiene;

$$I_2 = \frac{M_{12}}{\left(\frac{1}{\omega^2 C_2} - L_2\right)} I_{10} \qquad (2')$$

Sustituyendo (2') en (1);

$$V - R_1 I_1 = I_{10} i\,\omega \left[L_1 + \frac{M_{12}^2}{\left(\frac{1}{\omega^2 C_2} - L_2\right)}\right] = I_{10}\,\frac{i\,\omega L_1}{\omega^2 C_2 (\frac{1}{\omega^2 C_2} - L_2)}$$

$$= I_{10}\,\frac{i\,\omega L_1}{(1-\omega^2 C_2\, L_2)} \qquad (3)$$

Entonces la impedancia equivalente será $Z = \frac{i\,\omega L_1}{(1-\omega^2 C_2\, L_2)}$.

(b)

La caída de potencial en la resistencia $R_1$ es nula ($R_1 I_1 = 0$) para una cierta frecuencia $\omega$ entonces;

$$I_1 = I_{10} + I_{10'} = 0 \qquad (4)$$

De aquí obtenemos

$$V_{ab}(i\,\omega\, C_1) + \frac{V_{ab}(1-\omega^2 C_2\, L_2)}{i\,\omega L_1} = 0 \qquad (5)$$

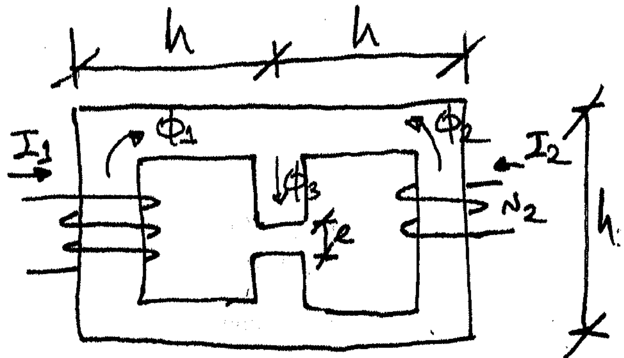
Entonces;

$$1 - \omega^2(C_1\, L_1 + C_2\, L_2) = 0 \qquad (6)$$

de donde resulta finalmente

$$\omega = \sqrt{\frac{1}{C_1\, L_1 + C_2\, L_2}} \qquad (7)$$

Problema 3.




$\mu=\alpha\mu_0$
sección S uniforme

a) $$\begin{cases} \Phi_3 = \Phi_1 + \Phi_2 \\ \Phi_1 \dfrac{3h}{\alpha\mu_0 S} + \Phi_3\left(\dfrac{h}{\alpha\mu_0 S} + \dfrac{e}{\mu_0 S}\right) = N_1 I_1 \\ (\Phi_1 - \Phi_2)\dfrac{3h}{\alpha\mu_0 S} = N_1 I_1 - N_2 I_2 \end{cases}$$

$$\Phi_1\, 3h + (\Phi_1 + \Phi_2)(h + \alpha e) = \alpha\mu_0 S N_1 I_1 =$$

$$= \Phi_1 (4h + \alpha e) + \Phi_2 (h + \alpha e) \Rightarrow \Phi_2 = \frac{\alpha\mu_0 S N_1 I_1 - \Phi_1 (4h + \alpha e)}{h + \alpha e}$$

$$\Rightarrow \Phi_1 (h + \alpha e) - \alpha\mu_0 S N_1 I_1 + \Phi_1 (4h + \alpha e) = \frac{\alpha\mu_0 S}{3h}(h + \alpha e)(N_1 I_1 - N_2 I_2)$$

$$\Rightarrow \Phi_1 (5h + 2\alpha e) = \frac{\alpha\mu_0 S}{3h}\left[(4h + \alpha e) N_1 I_1 - (h + \alpha e) N_2 I_2\right]$$

$$\Rightarrow \boxed{\Phi_1 = \frac{\alpha\mu_0 S}{3h(5h + 2\alpha e)}\left[(4h + \alpha e) N_1 I_1 - (h + \alpha e) N_2 I_2\right]}$$

$$\Rightarrow \boxed{\Phi_3 = \left[N_1 I_1 - \frac{3h}{\alpha\mu_0 S}\Phi_1\right]\frac{\alpha\mu_0 S}{h + \alpha e} = B_3 \cdot S} \Rightarrow \boxed{B_3 = \frac{\Phi_3}{S} = \frac{\mu_0 (N_1 I_1 + N_2 I_2)}{(5h/\alpha + 2e)}}$$

b) $$\boxed{M = N_1 \frac{\partial \Phi_1}{\partial I_2} = \frac{-(h + \alpha e)\mu_0 \alpha S N_1 N_2}{3h(5h + 2\alpha e)}}$$