Ejercicio 1:

a) $\cdot\ \nabla\cdot\vec{D} = \rho \longrightarrow D(4\pi r^2) = Q \longrightarrow \vec{D}(\vec{r}) = \dfrac{Q}{4\pi r^2}\,\hat{e_r}$ (1)

$\cdot\ \vec{D} = \varepsilon_0\vec{E} + \vec{P}$

En zona lineal: $\vec{P} = \dfrac{P_S}{E_0}\vec{E} \longrightarrow \varepsilon = \varepsilon_0 + \dfrac{P_S}{E_0} \longrightarrow \vec{D} = \varepsilon\vec{E}$

$(2a < r < 3a)$

$$\longrightarrow \vec{E} = \frac{Q}{4\pi\varepsilon r^2}\,\hat{e_r} \quad (2)$$

En zona saturada: $\vec{P} = P_S\,\hat{e_r} \longrightarrow \vec{E} = \dfrac{\vec{D}-\vec{P}}{\varepsilon_0} \overset{(1)}{=} \dfrac{1}{\varepsilon_0}\left[\dfrac{Q}{4\pi r^2} - P_S\right]\hat{e_r}$ (3)

$(a < r < 2a)$

$\cdot$ En $r = c\ (=2a) \longrightarrow \underline{E(2a) = E_0}$

Usando (2) $\longrightarrow \dfrac{Q}{4\pi\varepsilon(4a^2)} = E_0 \longrightarrow Q = 16\pi a^2\left(\varepsilon_0 + \dfrac{P_S}{E_0}\right)E_0$

$$Q = 16\pi a^2(\varepsilon_0 E_0 + P_S)$$

b)

$$\left\{\begin{array}{l} \vec{E} \overset{(2)}{=} \dfrac{16\pi a^2(\varepsilon_0E_0 + P_S)}{4\pi\left(\varepsilon_0 + \dfrac{P_S}{E_0}\right)r^2}\,\hat{e_r} = 4\left(\dfrac{a}{r}\right)^2 E_0\,\hat{e_r} \quad ; \quad 2a < r < 3a \\[2ex] \vec{E} \overset{(3)}{=} \dfrac{1}{\varepsilon_0}\left[\dfrac{16\pi a^2(\varepsilon_0E_0 + P_S)}{4\pi r^2} - P_S\right]\hat{e_r} = \left\{4\left(\dfrac{a}{r}\right)^2\left(E_0 + \dfrac{P_S}{\varepsilon_0}\right) - \dfrac{P_S}{\varepsilon_0}\right\}\hat{e_r} \end{array}\right.$$

$; \ a < r < 2a$

$$V_0 = \int_a^b \vec{E}(r)\cdot dr\,\hat{e_r} = \int_a^{2a}\left\{4\left(\frac{a}{r}\right)^2\left(E_0 + \frac{P_S}{\varepsilon_0}\right) - \frac{P_S}{\varepsilon_0}\right\}dr + \int_{2a}^{3a} 4\left(\frac{a}{r}\right)^2 E_0\,dr$$

$$= \int_a^{3a} 4\left(\frac{a}{r}\right)^2 E_0\,dr + \int_a^{2a}\left(4\left(\frac{a}{r}\right)^2 - 1\right)\frac{P_S}{\varepsilon_0}\,dr =$$

$$= 4E_0 . a^2 . \left(\underbrace{\frac{-1}{r}\Big|_a^{3a}}\right) + \frac{\rho_s}{\epsilon_0} . 4a^2 . \left(\underbrace{-\frac{1}{r}\Big|_a^{2a}}\right) - \frac{\rho_s}{\epsilon_0} . (\underbrace{2a - a}_{a})$$

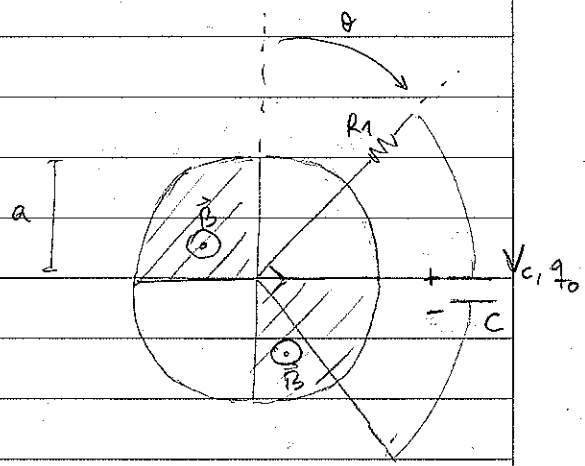
$$\frac{1}{a} - \frac{1}{3a} = \frac{2}{3a} \qquad \frac{1}{a} - \frac{1}{2a} = \frac{1}{2a}$$

$$V_0 = \frac{8}{3} E_0 . a + \frac{\rho_s}{\epsilon_0} . 2a - \frac{\rho_s}{\epsilon_0} . a \longrightarrow \boxed{V_0 = \left(\frac{8}{3} E_0 + \frac{\rho_s}{\epsilon_0}\right) a}$$

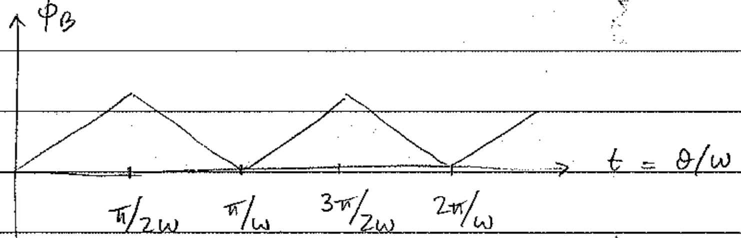
Papiros

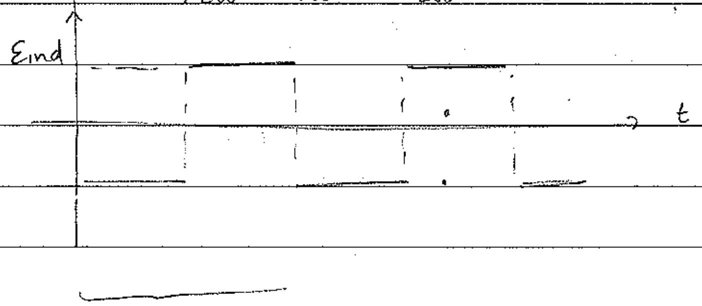
Ejercicio 2:

a) $\phi_B = \begin{cases} B\theta a^2/2 & , \theta \in [0, \pi/2] \\ B a^2 (\pi - \theta)/2 & , \theta \in [\pi/2, \pi] \end{cases}$

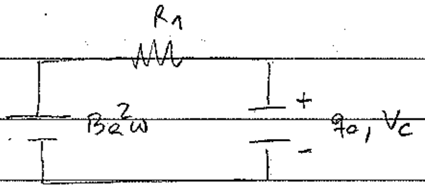


$\rightarrow \varepsilon_{ind} = -\dfrac{d\phi_B}{dt} = \begin{cases} -Ba^2\omega/2 & , \theta \in [0, \pi/2] \\ +Ba^2\omega/2 & , \theta \in (\pi/2, \pi] \end{cases}$





período del circuito



- $t \in [0, \pi/2\omega]$. Circuito equivalente:

$$\left.\begin{aligned} (Ba^2\omega/2) - V_c - R_1 . I &= 0 \\ I = +\frac{dq}{dt} &= +C\frac{dV_c}{dt} \end{aligned}\right\} \rightarrow \frac{Ba^2\omega}{2R_1C} = \frac{V_c}{R_1C} + \dot{V}_c \quad ; \quad \varepsilon := \frac{Ba^2\omega}{2}$$

$$\rightarrow V_c(t) = +\varepsilon + A . e^{-t/\tau} \quad ; \quad \tau = R_1 . C$$

$$V_c(0) = q_0/C \rightarrow A = \frac{q_0}{C} - \varepsilon \rightarrow \boxed{V_c(t) = +\varepsilon + \left(\frac{q_0}{C} - \varepsilon\right) . e^{-t/\tau}}$$

$$\text{En } t = \tilde{t} := \frac{\pi}{2\omega} \rightarrow V(\tilde{t}) = +\varepsilon + \left(\frac{q_0}{C} - \varepsilon\right) . e^{-\tilde{t}/\tau} := \frac{q_0'}{C}$$

$t \in [\pi/2, \pi]$ — Sea $t' = t - \tilde{t} \rightarrow t' \in [0, \pi/2\omega] = [0, \tilde{t}]$

$$\left.\begin{aligned} -\frac{Ba^2\omega}{2} - V_c - R_1 . I &= 0 \\ I &= -C.\dot{V}_c \end{aligned}\right\} \rightarrow -\frac{Ba^2\omega}{2R_1C} = \frac{V_c}{R_1C} + \dot{V}_c$$

$$\rightarrow V_c(t') = -\varepsilon + A.e^{-t'/\tau}$$

$$V_c(t'=0) = \frac{q_0'}{c} \rightarrow \boxed{V_c(t') = -\varepsilon + \left(\frac{q_0'}{c} + \varepsilon\right).e^{-t'/\tau}}$$

Para que sea periódico, $V_c(t' = \tilde{t}) = V_c(t=0) = q_0/c$

$$\rightarrow \frac{q_0}{c} = \frac{q_0'}{c}.e^{-\tilde{t}/\tau} + \varepsilon.\left(+e^{-\tilde{t}/\tau} - 1\right)$$

$$= \left[+\varepsilon + \left(\frac{q_0}{c} - \varepsilon\right).e^{-\tilde{t}/\tau}\right].e^{-\tilde{t}/\tau} + \varepsilon\left(+e^{-\tilde{t}/\tau} - 1\right)$$

$$= +2\varepsilon.e^{-\tilde{t}/\tau} + \left(\frac{q_0}{c} - \varepsilon\right).e^{-2\tilde{t}/\tau} - \varepsilon$$

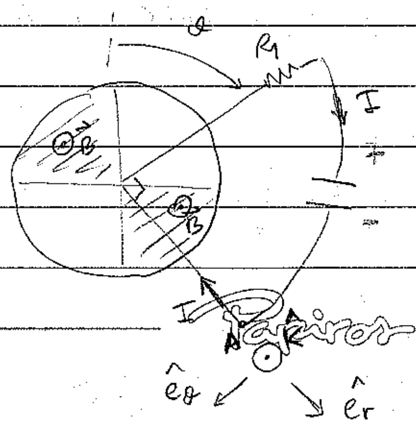
$$\frac{q_0}{c}\left(1 - e^{-2\tilde{t}/\tau}\right) = -\varepsilon + 2\varepsilon.e^{-\tilde{t}/\tau} - \varepsilon.e^{-2\tilde{t}/\tau}$$

$$q_0 = C\varepsilon.\left[\frac{-1 + 2e^{-\tilde{t}/\tau} - e^{-2\tilde{t}/\tau}}{1 - e^{-2\tilde{t}/\tau}}\right] = -C\varepsilon.\frac{\left(1 - e^{-\tilde{t}/\tau}\right)^2}{\left(1 - e^{-\tilde{t}/\tau}\right)\left(1 + e^{-\tilde{t}/\tau}\right)}$$

$$\boxed{q_0 = C\varepsilon.\left[\frac{-1 + e^{-\tilde{t}/\tau}}{1 + e^{-\tilde{t}/\tau}}\right]}$$

b) $V_c(t=0) = q_0/C \rightarrow I(t=0) = \frac{1}{R_1}.\left(-\frac{q_0}{c} + \frac{Ba^2\omega_0}{2}\right)$

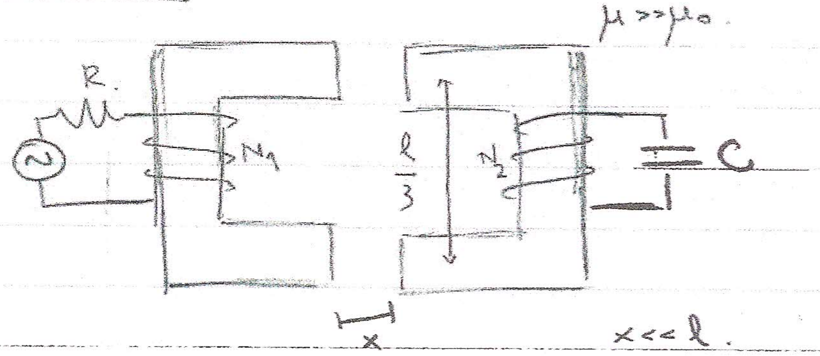
$$d\vec{f} = I\,d\vec{r} \times \vec{B} = -I\,dr\,\hat{e}_r \times B\hat{k}$$
$$= -IB\,dr\,\hat{e}_\theta$$



$$\vec{\tau}_0 = \int \vec{r} \wedge d\vec{f} = BI\hat{k}\int_0^a r\,dr$$

$$= \frac{BIa^2}{2}\hat{k} = \boxed{\frac{Ba^2}{2R_1}.\left(-\frac{q_0}{c} + \frac{Ba^2\omega_0}{2}\right)\hat{k}}$$

Ejercicio 3:



$\varepsilon(t) = \varepsilon_0 \cdot \cos(\omega t)$.

a) ¿L_1, L_2, M?

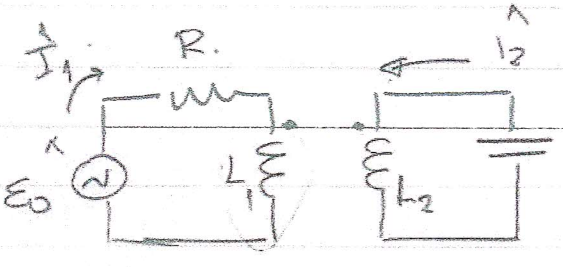
$$L_1 = \frac{N_1^2}{R_0} = \frac{N_1^2}{\frac{2l}{\mu S} + \frac{2x}{\mu_0 S}} = \boxed{\frac{N_1^2 S}{2\left(\frac{l}{\mu} + \frac{x}{\mu_0}\right)}}$$

x simetría, $\boxed{L_2 = \frac{N_2^2 S}{2\left(\frac{l}{\mu} + \frac{x}{\mu_0}\right)}}$

$$M = \sqrt{L_1 L_2} = \boxed{\frac{N_1 N_2 S}{2\left(\frac{l}{\mu} + \frac{x}{\mu_0}\right)}}$$

no hay pérdida de flujo.

b)



$\hat{I}_1 = 0 \iff \begin{cases} \hat{\varepsilon}_0 = j\omega M \hat{I}_2 & (1) \\ -j\omega L_2 \hat{I}_2 - \frac{1}{j\omega C}\hat{I}_2 = 0 \end{cases}$ $\rightarrow I_2 = 0$ Absurdo x (1)

$\downarrow$

$$\frac{1}{\omega C} = \omega L_2$$

$$L_2 = \frac{N_2^2 S}{2\left(\frac{l}{\mu} + \frac{x}{\mu_0}\right)} = \frac{1}{\omega^2 C}$$

$$\boxed{x = \mu_0 \left(\frac{N_2^2 S \omega^2 C}{2} - \frac{l}{\mu} \right)}$$