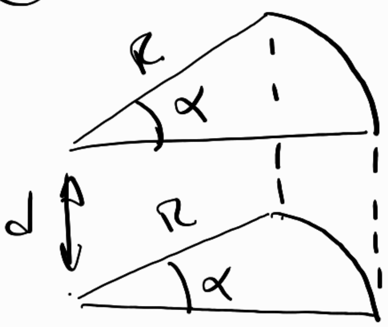
Solución Examen Electromagnetismo 23/07/22

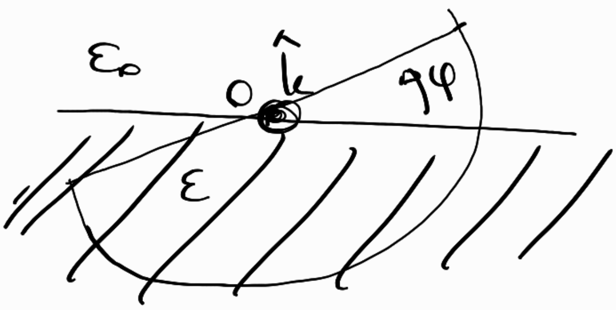
1) a)



$C = \frac{\varepsilon_0 A}{d}$ ← capacidad placas planas paralelas de área A y dist. d entre las placas

$$A = \int_0^R\int_0^\alpha r\,dr\,d\theta = \alpha\frac{1}{2}R^2 \Rightarrow \boxed{C_\alpha = \frac{\varepsilon_0 R^2 \alpha}{2d}}$$

b)



$$U = \frac{1}{2}C_{eq}V_0^2 = \frac{1}{2}\left(C_{\pi-\varphi} + C_\varphi\right)V_0^2$$

$$C_\varphi = \frac{\varepsilon_0 R^2 \varphi}{2d}$$

$$C_{(\pi-\varphi)} = \frac{\varepsilon R^2(\pi-\varphi)}{2d}$$

$$\Rightarrow C_{eq} = \frac{R^2\left[\pi\varepsilon + \varphi(\varepsilon_0-\varepsilon)\right]}{2d}$$

$$\boxed{\vec{\tau}_0 = \left.\frac{\partial U}{\partial \varphi}\right|_{V_0}\hat{k}} = \frac{1}{2}V_0^2\frac{\partial C_{eq}}{\partial\varphi}\hat{k} = \frac{1}{2}V_0^2\frac{R^2}{2d}(\varepsilon_0-\varepsilon)\hat{k}$$

$$= \frac{1}{4}\frac{(\varepsilon_0-\varepsilon)R^2V_0^2\hat{k}}{d} = \boxed{-\frac{1}{4}\frac{(\varepsilon-\varepsilon_0)R^2V_0^2\hat{k}}{d}}$$

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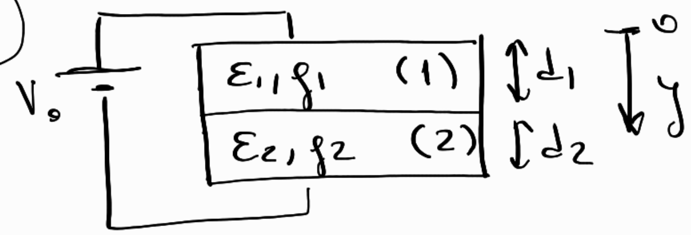

En situación estacionaria:

$$J_{1n}\big|_{d_1} = J_{2n}\big|_{d_2}$$

$$\rho_1 E_1 = \rho_2 E_2$$

$$V_0 = E_1 d_1 + E_2 d_2 = E_1 d_1 + \frac{\rho_1}{\rho_2} E_1 d_2 = E_1\left(d_1 + \frac{\rho_1}{\rho_2} d_2\right)$$

$$\Rightarrow \boxed{\vec{E}_1 = \frac{\rho_2 V_0}{d_1\rho_2 + \rho_1 d_2}\,\hat{y}} \quad \boxed{\vec{E}_2 = \frac{\rho_1 V_0}{d_1\rho_2 + \rho_1 d_2}\,\hat{y}}$$

$$\vec{D}_1 = \varepsilon_1 \vec{E}_1 \Rightarrow \boxed{\vec{D}_1 = \frac{\varepsilon_1 \rho_2 V_0}{d_1\rho_2 + \rho_1 d_2}\,\hat{y}} \; ; \; \boxed{\vec{D}_2 = \varepsilon_2 \vec{E}_2 = \frac{\varepsilon_2 \rho_1 V_0}{d_1\rho_2 + \rho_1 d_2}\,\hat{y}}$$

$$\vec{J}_1 = \rho_1 \vec{E}_1 \Rightarrow \boxed{\vec{J}_1 = \frac{\rho_1\rho_2 V_0}{d_1\rho_2 + \rho_1 d_2}\,\hat{y} = \vec{J}_2} \text{ " } \rho_2 \vec{E}_2$$

$$\boxed{R = \frac{V_0}{I} = \frac{V_0}{JA} = \frac{V_0\,(d_1\rho_2 + \rho_1 d_2)}{\rho_1\rho_2 V_0 A}}$$

b) $\boxed{\sigma(y=0) = D_1 = \dfrac{\varepsilon_1 \rho_2 V_0}{d_1\rho_2 + \rho_1 d_2}}$

$$\boxed{\sigma(y=d_1) = D_{2n}\big|_{d_1} - D_{1n}\big|_{d_1} = D_2 - D_1 = \frac{(\varepsilon_2\rho_1 - \varepsilon_1\rho_2)V_0}{d_1\rho_2 + \rho_1 d_2}}$$

$$\boxed{\sigma(y=d_1+d_2) = -D_2 = -\frac{\varepsilon_2\rho_1 V_0}{d_1\rho_2 + \rho_1 d_2}}$$

(Obs: $\sigma(y=0) + \sigma(y=d_1) + \sigma(y=d_1+d_2) = 0$)

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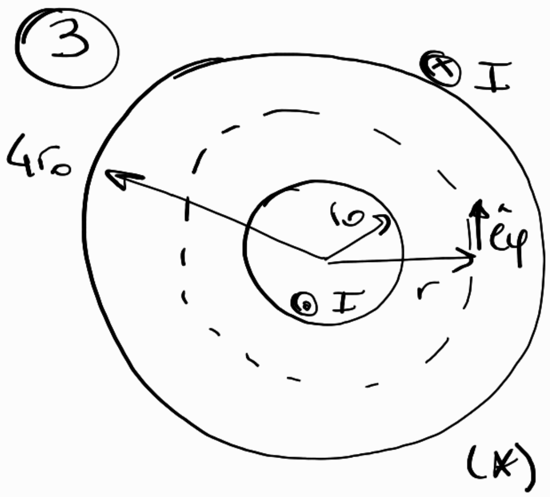
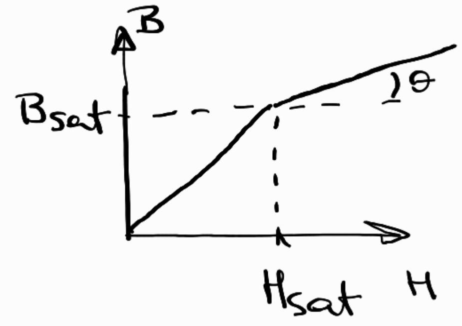


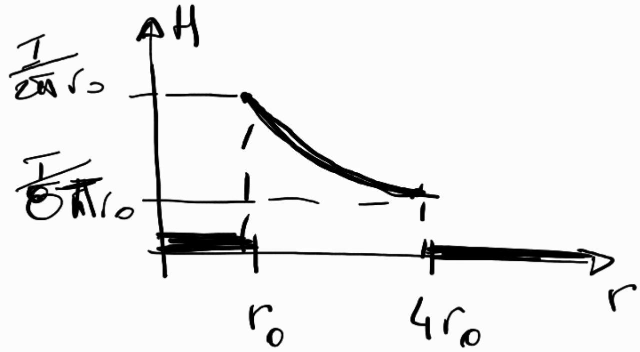


$\mu_0 = \operatorname{tg}\theta$

$$(*)\begin{cases} B(H) = \mu H \text{ si } H < H_{sat} \text{ con } \mu = \dfrac{B_{sat}}{H_{sat}} \\ B(H) = B_{sat} + \mu_0 (H - H_{sat}) \text{ si } H > H_{sat} \end{cases}$$

a) Por simetría de la configuración $\vec{H} = H(r)\,\hat{e}_\varphi$ (sentido antihorario) y aplicando Ampère;

$$\begin{cases} 0 < r < r_0 ;\ 2\pi H(r) = 0 \to \boxed{\vec{H}(r) = 0} \\ r_0 < r < 4r_0 ;\ 2\pi H(r) = I \to \boxed{\vec{H}(r) = \dfrac{I}{2\pi r}\,\hat{e}_\varphi} \\ 4r_0 < r ;\ 2\pi H(r) = (I - I) = 0 \to \boxed{\vec{H}(r) = 0} \end{cases}$$



b) el material satura $\forall r$ tal que $r_0 < r < 2r_0$

Para $r_0 < r < 2r_0 \to H(r) = \dfrac{I}{2\pi r} > H_{sat}$

de (*) $\Rightarrow B(H) = B_{sat} + \mu_0 (H - H_{sat})$ si $H > H_{sat}$

Entonces:

$$\vec{B}(r) = \begin{cases} \left(B_{sat} + \dfrac{\mu_0 I}{2\pi r} - \mu_0 H_{sat}\right)\hat{e}_\varphi ;\ \text{si } r_0 < r < 2r_0 \\ \dfrac{B_{sat}}{H_{sat}}\,\dfrac{I}{2\pi r}\,\hat{e}_\varphi ,\ \text{si } 2r_0 < r < 4r_0 \end{cases}$$

de (*) ($B = \mu H$ si $H < H_{sat}$)

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