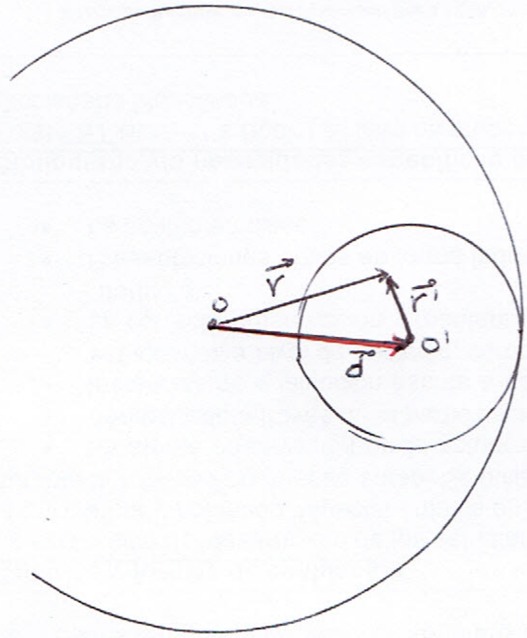
SOLUCION PROBLEMA 1

(a)



$$\vec{E}_{CAVIDAD} = \vec{E_1} + \vec{E_2}$$

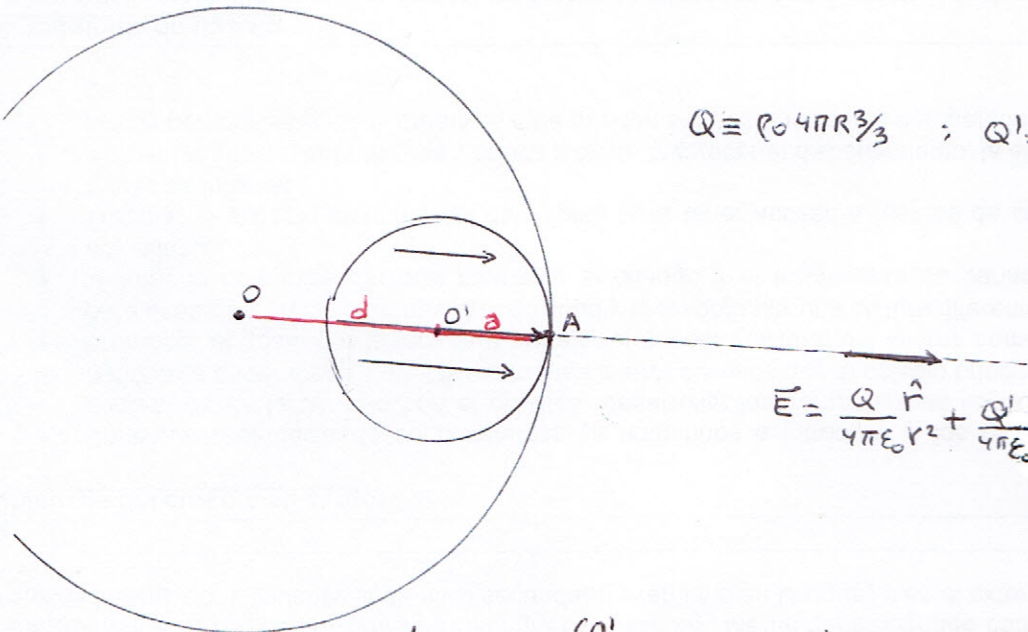
$\vec{E_1}$ campo de esfera uniform. cargada (ρ_0), radio R.

E_2 campo de esfera uniform. cargada ($-\rho_0$), radio a.

$$\vec{E_1} = \frac{\rho_0 \vec{r}}{3\varepsilon_0} \ ; \ \vec{E_2} = -\frac{\rho_0 \vec{r'}}{3\varepsilon_0} \ ; \ \vec{r'} = \vec{r} - \vec{d}$$

$$\Longrightarrow \vec{E}_{CAVIDAD} = \frac{\rho_0 \vec{d}}{3\varepsilon_0}$$

(b)



$$Q \equiv \rho_0 4\pi R^3/3 \ ; \ Q' = -\rho_0 4\pi a^3/3$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} + \frac{Q'}{4\pi\varepsilon_0 (r-d)^2}$$

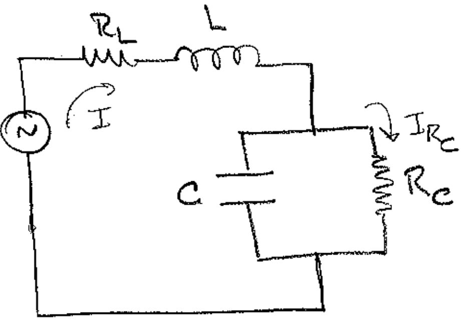
$$\phi(O') \equiv -\int_{\infty}^{O'} \vec{E}\cdot d\vec{l} = \underbrace{\underbrace{-\int_{\infty}^{A}}_{\phi(A)}}_{\frac{Q}{4\pi\varepsilon_0 R} + \frac{Q'}{4\pi\varepsilon_0 a}} + \underbrace{\underbrace{-\int_{A}^{O'}}_{\phi(O') - \phi(A)}}_{\underbrace{|E_{CAVIDAD}|a}_{\frac{\rho_0 d}{3\varepsilon_0}a}}$$

$$\boxed{\phi(O') = \frac{\rho_0 R^2}{3\varepsilon_0} - \frac{\rho_0 a^2}{3\varepsilon_0} + \frac{\rho_0 d}{3\varepsilon_0}a}$$

(2)

$V(t) = V_0 e^{j\omega t}$

$(j^2 = -1)$



$R_L = R_C = R$

$\omega RC = 1$

a) $tg\,\phi = \dfrac{Im(Z)}{Re(Z)} = 1 \quad (45°)$ $\qquad \dfrac{1}{j\omega C + \frac{1}{R}} = \dfrac{R}{1 + j\underbrace{\omega RC}_{=1}}$

$$Z = R + j\omega L + \frac{1}{\frac{1}{Z_C} + \frac{1}{R}}$$

$$\Rightarrow Z = R + j\omega L + \frac{R}{1+j} = \frac{(1-j)}{(1-j)}\left[\frac{(1+j)R + j(1+j)\omega L + R}{(1+j)}\right] = \frac{3R + j2\omega L - jR}{2}$$

$$\Rightarrow Z = \frac{3R + j(2\omega L - R)}{2}$$

$$\frac{Im(Z)}{Re(Z)} = 1 \Rightarrow \frac{2\omega L - R}{3R} = 1 \Longleftrightarrow 2\omega L - R = 3R \Longleftrightarrow \cancel{2}\omega L = \cancel{4}^{\,2}R \Longleftrightarrow \boxed{L = \frac{2R}{\omega}}$$

b) $Z = \dfrac{3R + j3R}{2} \rightarrow \boxed{|Z|^2 = Re^2(Z) + Im^2(Z) = \frac{1}{4}\left(9R^2 + 9R^2\right) = \frac{18R^2}{4} = \frac{9R^2}{2}}$

Pot. disipada por R_L:

$$P_{J_{R_L}}(t) = R\,Re[I(t)]^2 = R\frac{V_0^2}{|Z|^2}\cos^2(\omega t - \pi/4) = \boxed{\cancel{R}\,V_0^2\frac{2}{9R^{\cancel{2}}}\cos^2(\omega t - \pi/4)}$$

$$V_{R_C} = V_0 - \frac{V_0}{Z}\underbrace{(R + j\omega L)}_{Z_{R_L L}} = V_0\left[1 - \frac{R + j\overbrace{\omega L}^{=2R}}{\frac{3R(1+j)}{2}}\right] = V_0\left[1 - \frac{2\cancel{R}(1+2j)}{3\cancel{R}(1+j)}\right]$$

$$= V_0\left[\frac{3(1+j) - 2(1+2j)}{3(1+j)}\right] = V_0\left[\frac{1-j}{3(1+j)}\right] = \frac{(1-j)}{(1-j)}V_0\left[\frac{1-j}{3(1+j)}\right] = \frac{V_0(\cancel{1} - \cancel{1} - \cancel{2}j)}{3\times\cancel{2}}$$

$$= \boxed{-jV_0/3}$$

Pot. disipada por R_C: $P_{J_{R_C}}(t) = \dfrac{[Re(V_{R_C})]^2}{R} = \dfrac{V_0^2}{9R}\cos^2\left(\omega t - \frac{\pi}{2}\right) = \boxed{\dfrac{V_0^2}{9R}\operatorname{sen}^2(\omega t)}$

$$\Rightarrow \boxed{P_J(t) = \frac{V_0^2}{9R}\left[2\cos^2(\omega t - \pi/4) + \operatorname{sen}^2(\omega t)\right]}$$

Energía almacenada:

Capacitor:

$$E_C(t) = \frac{1}{2} C V_C^2(t) = \boxed{\frac{1}{2} \overbrace{C}^{=\frac{1}{\omega R}\ (\omega RC = 1)} \frac{V_0^2}{9} \text{sen}^2(\omega t)}$$

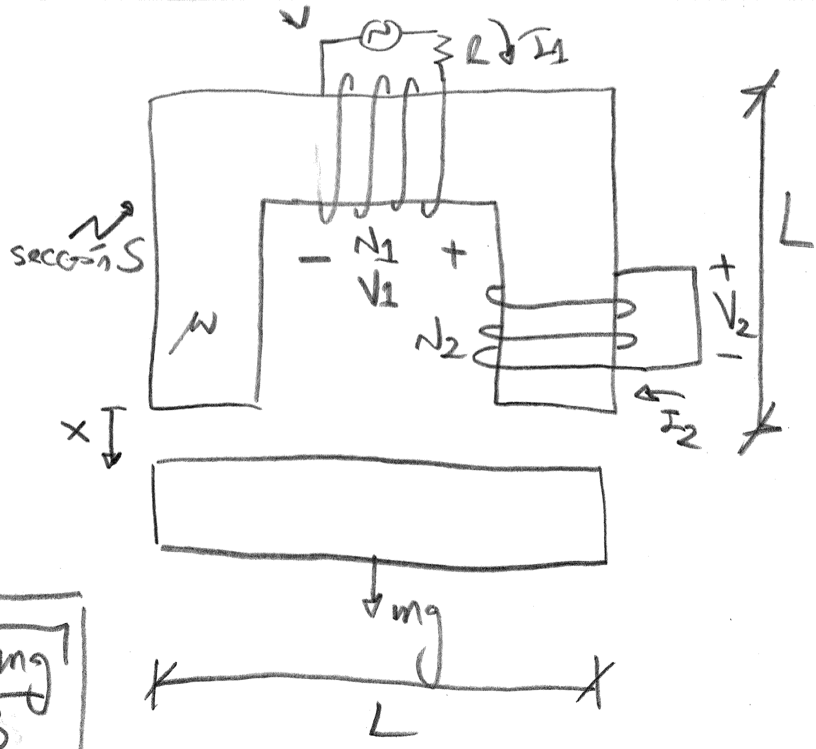
Inductor:

$$E_L(t) = \frac{1}{2} L I^2(t) = \frac{1}{\cancel{2}} \underbrace{L}_{=\frac{2\cancel{R}}{\omega}} \frac{V_0^2}{9R^{\cancel{2}}} \cancel{2} \cos^2(\omega t - \pi/4)$$

$$= \boxed{\frac{V_0^2}{9\omega R}\, 2\cos^2(\omega t - \pi/4)}$$

$$\Rightarrow \boxed{E(t) = \frac{V_0^2}{9\omega R}\left[\frac{1}{2}\, \text{sen}^2(\omega t) + 2\cos^2(\omega t - \pi/4)\right]}$$

— x —



3- a) $V_2=0 \Rightarrow \Phi_B$ - constante

$\Rightarrow B$ - constante

$$U = \frac{B^2}{2}\left(\frac{4LS}{\mu}+\frac{2xS}{\mu_0}\right)$$

$$\vec{F} = -\left.\frac{\partial U}{\partial x}\right|_{\Phi_B}\hat{x} = -\frac{B^2 S}{\mu_0}\hat{x}$$

$$-\frac{B^2 S}{\mu_0}+mg=0 \Rightarrow \boxed{B=\sqrt{\frac{\mu_0 mg}{S}}}$$

b) $$B\left(\frac{4L}{\mu}+\frac{2x}{\mu_0}\right)=N_1 I_1+N_2 I_2$$

Φ_B - constante $\Rightarrow V_1=0 \Rightarrow I_1=\frac{V_0\cos(\omega t)}{R}$

$$\Rightarrow \boxed{I_2 = \left[\sqrt{\frac{\mu_0 mg}{S}}\left(\frac{4L}{\mu}+\frac{2x}{\mu_0}\right)-\frac{N_1 V_0\cos(\omega t)}{R}\right]\frac{1}{N_2}}$$