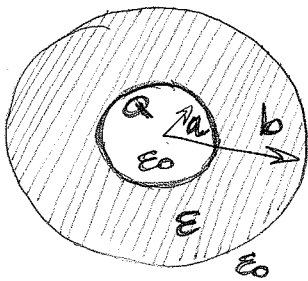


1



Diferencia de potencial entre $r=0$ y $r=\infty$

$$V = V(0) - V(\infty) = - \int_0^{\infty} \vec{E} \cdot d\vec{l}$$

Gauss:

$$\left. \begin{array}{l} r > a; \vec{D} = \frac{Q}{4\pi r^2} \hat{e}_r \\ r < a; \vec{D} = 0 \end{array} \right\} \begin{array}{l} \vec{E}(r) = \frac{Q}{4\pi \epsilon r^2} \hat{e}_r \\ \vec{E}(r) = 0 \end{array} \quad (a < r < b)$$

$$\Rightarrow V = - \int_{\infty}^b \left(\frac{Q}{4\pi \epsilon_0 r^2} \right) dr - \int_b^a \left(\frac{Q}{4\pi \epsilon r^2} \right) dr - \int_a^{\infty} (0) dr$$

$$= -\frac{Q}{4\pi} \left(-\frac{1}{r} \Big|_{\infty}^b \frac{1}{\epsilon_0} - \frac{1}{r} \Big|_b^a \frac{1}{\epsilon} \right) = \frac{Q}{4\pi} \left[\frac{1}{b\epsilon_0} + \frac{1}{\epsilon} \left(\frac{1}{a} - \frac{1}{b} \right) \right] =$$

$$V = \frac{Q}{4\pi} \left(\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b} \right)$$

2) Energía almacenada en la configuración:

$$U = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} (dv) = \frac{1}{2} \frac{Q^2}{(4\pi)^2} \left\{ \int_a^b \left(\frac{1}{r^2} \right)^2 r^2 dr + \frac{1}{\epsilon_0} \int_b^{\infty} \left(\frac{1}{r^2} \right)^2 r^2 dr \right\}$$

$$= \frac{1}{2} \frac{Q^2}{4\pi} \left\{ \frac{1}{\epsilon} \left(-\frac{1}{r} \Big|_a^b \right) + \frac{1}{\epsilon_0} \left(-\frac{1}{r} \Big|_b^{\infty} \right) \right\} = \frac{1}{2} \frac{Q^2}{4\pi} \left\{ \frac{1}{\epsilon} \left(\frac{1}{a} - \frac{1}{b} \right) + \frac{1}{\epsilon_0} \left(\frac{1}{b} \right) \right\}$$

$$U = \frac{Q^2}{8\pi} \left[\frac{1}{\epsilon a} + \frac{1}{b} \left(\frac{1}{\epsilon_0} - \frac{1}{\epsilon} \right) \right]$$

obs: $U = \frac{1}{2} QV$

La configuración se puede pensar como un capacitor cuya placa con carga $-Q$ se encuentra en el infinito.

SOLUCION EJERCICIO 3

Potencial entre placas: $\phi(\theta) = \left(\frac{V_0}{\theta_0}\right)\theta$

Campo eléctrico: $\vec{E} = -\left(\frac{\partial\phi}{\partial r} + \frac{1}{r} \frac{\partial\phi}{\partial\theta}\right) = -\left(0, \frac{1}{r} \frac{V_0}{\theta_0}\right)$

Carga sobre placa horizontal: $\epsilon_0 \vec{E} \cdot \hat{m} = \sigma$ donde $\hat{m} = (0, 1)$

$$\Rightarrow \boxed{\sigma = -\epsilon_0 \frac{V_0}{\theta_0}}$$

SOLUCION EJERCICIO 4

W (ENERGIA ELECTROST.) = $\frac{\epsilon_0}{2} \iiint |\vec{E}|^2 dV$ donde $|\vec{E}|^2 = \frac{1}{r^2} \left(\frac{V_0}{\theta_0}\right)^2$

$$= \frac{\epsilon_0 L}{2} \left(\frac{V_0}{\theta_0}\right)^2 \int_d^{L+d} \left[\int_0^{\theta_0} d\theta \right] \frac{r}{r^2} dr$$

$$= \frac{\epsilon_0 L}{2} \frac{V_0^2}{\theta_0^2} \ln\left(\frac{L+d}{d}\right)$$

$$Z \text{ (TORQUE)} = -\frac{\partial W}{\partial \theta_0} \Rightarrow \boxed{|Z| = \left(\frac{\epsilon_0 L}{2}\right) \left(\frac{V_0}{\theta_0}\right)^2 \ln\left(\frac{L+d}{d}\right)}$$

SOLUCION EJERCICIOS

Carga placa izquierda $Q_I(t) = Q_0 e^{-q_2 t/\epsilon_2}$

Carga placa derecha $Q_D(t) = -Q_0 e^{-q_1 t/\epsilon_1}$

Conservación de la carga;

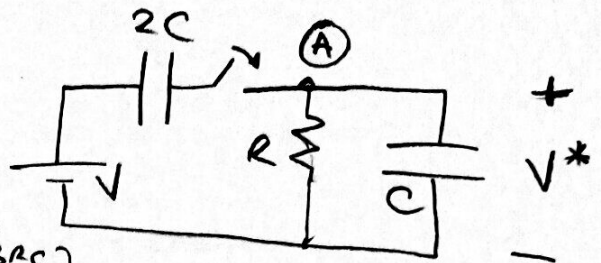
$$Q_I + Q_D + Q_{\text{INTERFASE}} = 0$$

$$\Rightarrow \boxed{Q_{\text{INTERFASE}} = Q_0 \left[e^{-q_2 t/\epsilon_2} - e^{-q_1 t/\epsilon_1} \right]}$$

* Solución ejercicio 6

- Nodo A: $2C \frac{d(V-V^*)}{dt} = \frac{V^*}{R} + C \frac{dV^*}{dt}$

$\Rightarrow -3C \frac{dV^*}{dt} = \frac{V^*}{R} \Rightarrow V^*(t) = A e^{-t/3RC}$
 $V^*(0) = \frac{q_0}{C}$



$\Rightarrow V^*(t) = \frac{q_0}{C} e^{-t/3RC}$

- $P = \frac{V^{*2}}{R} \Rightarrow E_{dis} = \int_0^{\infty} P dt = \int_0^{\infty} \frac{q_0^2}{RC^2} e^{-2t/3RC} dt = \frac{3RC}{2} \frac{q_0^2}{RC^2} = \frac{3q_0^2}{2C}$

* Solución ejercicio 7

Campos P_0, \vec{E} y \vec{D} radiales.

- Condiciones de borde: $\sigma_L = D = \epsilon_0 E + p_0 \Rightarrow \dot{\sigma}_L = -\frac{\partial \sigma_L}{\partial t} + \frac{\partial p_0}{\partial t}$

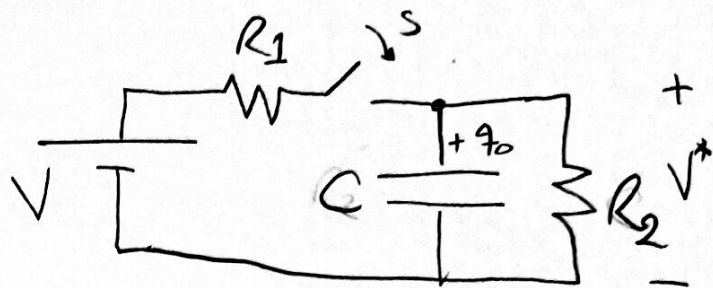
$-\dot{\sigma}_L = J = \gamma E$

$\Rightarrow \sigma_L(t) = A e^{-\frac{t}{\tau_0}} + p_0, A \in \mathbb{R}$

$\Rightarrow \sigma_L(0) = \frac{Q}{2\pi aL} = A + p_0 \Rightarrow \sigma_L(t) = \frac{Q}{2\pi aL} e^{-\frac{t}{\tau_0}} + p_0 (1 - e^{-\frac{t}{\tau_0}})$

* Solución ejercicio 8

- Clave S on: $\frac{V - V^*}{R_1} = C \dot{V}^* + \frac{V^*}{R_2}$

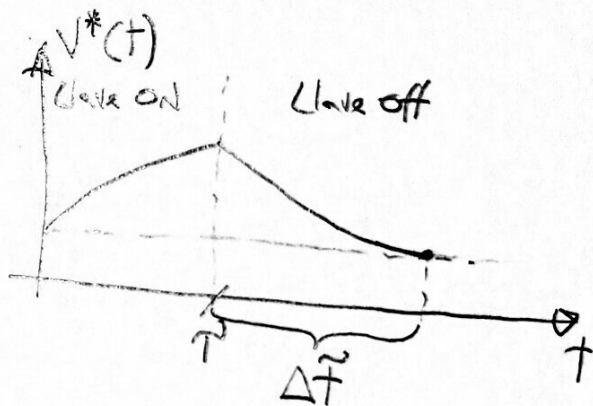


$$\Rightarrow \frac{dV^*}{dt} + \frac{R_1 + R_2}{R_1 R_2 C} V^* = \frac{V}{R_1 C}$$

$$\Rightarrow V^*(t) = A e^{-\frac{t}{\tau}} + \frac{R_2 V}{R_1 + R_2}, \text{ siendo } \tau = \frac{R_1 R_2 C}{R_1 + R_2}$$

$$V^*(0) = \frac{q_0}{C} = A + \frac{R_2 V}{R_1 + R_2} \Rightarrow \boxed{V^*(t) = \frac{q_0}{C} e^{-\frac{t}{\tau}} + \frac{R_2 V}{R_1 + R_2} (1 - e^{-\frac{t}{\tau}})} \quad \text{ON } t < \tau$$

$$V^*(\tau) = \frac{q_0}{C} e^{-1} + \frac{R_2 V}{R_1 + R_2} (1 - e^{-1})$$



- Clave S off:

$$t' = t - \tau \quad -\frac{t'}{R_2 C}$$

$$V^*(t') = B e^{-\frac{t'}{R_2 C}}$$

$$V^*(t' = 0) = \frac{q_0}{C} e^{-1} + \frac{R_2 V}{R_1 + R_2} (1 - e^{-1}) = B$$

$$\Rightarrow V^*(t' = \Delta \tilde{T}) = \frac{q_0}{C} = \left[\frac{q_0}{C} e^{-1} + \frac{R_2 V}{R_1 + R_2} (1 - e^{-1}) \right] e^{-\frac{\Delta \tilde{T}}{R_2 C}}$$

$$\Rightarrow \boxed{\Delta \tilde{T} = R_2 C \ln \left[e^{-1} + \frac{R_2 V}{R_1 + R_2} \cdot \frac{C}{q_0} (1 - e^{-1}) \right]}$$