ELECTROMAGNETISMO (1128) SEGUNDO PARCIAL 1/12/22

a.i) $\nabla\times\vec{B} = \mu_0 \vec{J}_T = \mu_0(\vec{J}_L+\vec{J}_M)$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \to \vec{B} = \mu_0(\vec{H}+\vec{M})$

$\Rightarrow \cancel{\mu_0}[\nabla\times\vec{H}+\nabla\times\vec{M}] = \cancel{\mu_0}(\vec{J}_L+\vec{J}_M)$

donde $\boxed{\vec{J}_L = \nabla\times\vec{H}}$ y $\boxed{\vec{J}_M = \nabla\times\vec{M}}$

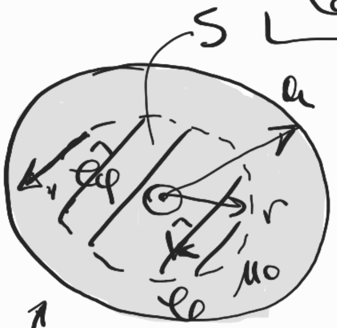
a.ii) $\int_V \underbrace{(\nabla\times\vec{M})}_{=\vec{J}_M} dv = \oint_S \underbrace{(\hat{n}\times\vec{M})}_{=-\vec{M}\times\hat{n}} da \Rightarrow \int_V \vec{J}_M dv + \oint_S \underbrace{(\vec{M}\times\hat{n})}_{=\vec{j}_M} da = 0$ ✓ $\boxed{\vec{j}_M = \vec{M}\times\hat{n}}$

b) b.i)

↳ dentro del conductor:

Ampère $\underbrace{\oint_{\mathcal{C}} \vec{H}\cdot d\vec{l}}_{= H(r)(2\pi r)} = I_S = \underbrace{\int_S \vec{J}_L\cdot\hat{n}\, da}_{= \frac{I}{\cancel{\pi} a^2}(\cancel{\pi} r^2)}$ con $\hat{n}$ normal a S (flujo de $\vec{J}_L$ a través de la sección S)

$J_L = \frac{I}{\pi a^2}$ (uniforme)



conductor cilíndrico

$\Rightarrow \boxed{\vec{H}(r) = \frac{I r}{2\pi a^2}\hat{e}_\varphi}$, $\boxed{\vec{M} = 0}$ y $\boxed{\vec{B} = \mu_0\vec{H} = \frac{\mu_0 I}{2\pi a^2} r\,\hat{e}_\varphi}$

↳ fuera del conductor:

$H(r)(2\pi r) = I \to \boxed{\vec{H}(r) = \frac{I}{2\pi r}\hat{e}_\varphi}$; $r>a$

$$\boxed{\vec{B}} = \begin{cases} \mu\vec{H} = \boxed{\frac{\mu I}{2\pi r}\hat{e}_\varphi} ; & a<r<b \\ \mu_0\vec{H} = \boxed{\frac{\mu_0 I}{2\pi r}\hat{e}_\varphi} ; & r>b \end{cases}$$

$$\boxed{\vec{M}} = \frac{\vec{B}}{\mu_0} - \vec{H} = \begin{cases} \left(\frac{\mu}{\mu_0}-1\right)\vec{H} = \boxed{\left(\frac{\mu-\mu_0}{\mu_0}\right)\frac{I}{2\pi r}\hat{e}_\varphi} ; & a<r<b \\ \left(\frac{\mu_0}{\mu_0}-1\right)\vec{H} = \boxed{0} ; & r>b \end{cases}$$

b.ii) $\vec M \neq 0$ sólo en el material magnetizado

$$\vec M = \underbrace{\left(\frac{\mu-\mu_0}{\mu_0}\right)\frac{I}{2\pi r}}_{= M_\varphi(r)} \hat e_\varphi$$

→ densidad de corriente volumétrica:

$$\boxed{\vec J_M = \nabla\times\vec M \overset{\text{(coord. cil.)}}{=} -\underbrace{\frac{\partial M_\varphi}{\partial z}}_{=0}\hat e_r + \frac{1}{r}\underbrace{\frac{\partial(rM_\varphi)}{\partial r}}_{=0}\hat k = 0}$$

→ densidad de corriente superficial:

$$\boxed{\vec j_M\Big|_{r=a} = M\times\underbrace{\hat n}_{-\hat e_r}\Big|_{r=a} = \frac{\mu-\mu_0}{\mu_0}\left(\frac{I}{2\pi a}\right)\underbrace{\hat e_\varphi\times(-\hat e_r)}_{=\hat k} = \frac{\mu-\mu_0}{\mu_0}\left(\frac{I}{2\pi a}\right)\hat k}$$

$-\hat e_r$ (normal saliente al material magnético en la cara lateral del cilindro de radio $r=a$)

$$\boxed{\vec j_M\Big|_{r=b} = \vec M\times\underbrace{\hat n}_{=\hat e_r}\Big|_{r=b} = \frac{\mu-\mu_0}{\mu_0}\left(\frac{I}{2\pi a}\right)\underbrace{\hat e_\varphi\times\hat e_r}_{=-\hat k} = \frac{\mu-\mu_0}{\mu_0}\left(\frac{I}{2\pi b}\right)(-\hat k)}$$

→ corriente superficial de magnetización:

flujo de $\vec j_M$ a través de $\mathcal{C}$ en la superficie cara lateral del cilindro de radio a con $\hat n$ tangente a la superficie y normal a $\mathcal{C}$

$$\boxed{I_M\Big|_{r=a} = \int_{\mathcal{C}} \vec j_M\Big|_{r=a}\cdot\underbrace{\hat n}_{=\hat k}\,dl = \frac{\mu-\mu_0}{\mu_0}\frac{I}{\cancel{2\pi a}}(\cancel{2\pi a}) = \left(\frac{\mu-\mu_0}{\mu_0}\right)I}$$

fluye según $\hat k$

$$\boxed{I_M\Big|_{r=b} = \int_{\mathcal{C}} \vec j_M\Big|_{r=b}\cdot\underbrace{\hat n}_{=\hat k}\,dl = -\left(\frac{\mu-\mu_0}{\mu_0}\right)\frac{I}{\cancel{2\pi b}}(\cancel{2\pi b}) = -\left(\frac{\mu-\mu_0}{\mu_0}\right)I}$$

fluye según $(-\hat k)$

— x —

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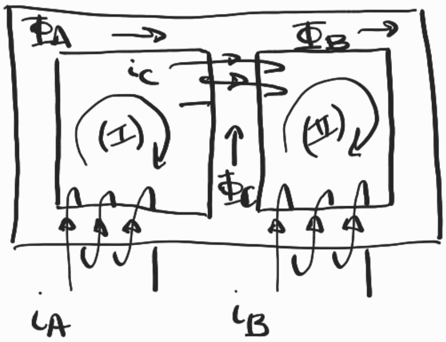


$$(I)\quad N i_A - N_c i_c = \mathcal{R}_{0A}\Phi_A - \mathcal{R}_{0c}\Phi_c$$

$$(II)\quad N_c i_c + N i_B = \mathcal{R}_{0c}\Phi_c + \mathcal{R}_{0B}\Phi_B$$

$$(III)\quad \Phi_B = \Phi_A + \Phi_c$$

a.i)

$$\mathcal{R}_{0A} = \mathcal{R}_{0B} = \frac{3l}{\mu S} \quad \text{y} \quad \mathcal{R}_{0c} = \frac{l}{\mu S}$$

$$(II)-(I):\ 2N_c i_c + N i_B - N i_A = 2\mathcal{R}_c\Phi_c + \mathcal{R}_{0A}\underbrace{(\Phi_B - \Phi_A)}_{\overset{(III)}{=}\ \Phi_c} = \underbrace{(2\mathcal{R}_{0c} + \mathcal{R}_{0A})}_{=\ (2+3)\frac{l}{\mu S}}\Phi_c = \frac{5l\,\Phi_c}{\mu S}$$

$$\Rightarrow \boxed{\Phi_c = \left[\frac{2N_c i_c + N i_B - N i_A}{5l}\right]\mu S} = \underbrace{\frac{2N_c i_c\,\mu S}{5l}}_{=\ \Phi_{cc}} + \underbrace{\frac{N i_B\,\mu S}{5l}}_{=\ \Phi_{cB}} - \underbrace{\frac{N i_A\,\mu S}{5l}}_{=\ \Phi_{cA}}$$

$$(I)\quad \mathcal{R}_{0A}\Phi_A = N i_A - N_c i_c + \mathcal{R}_{0c}\Phi_c$$

$$\frac{3l}{\mu S}\Phi_A = N i_A - N_c i_c + \cancel{\frac{l}{\mu S}}\,\cancel{\frac{\mu S}{l}}\,\frac{2N_c i_c + N i_B - N i_A}{5} = \frac{4N i_A - 3N_c i_c + N i_B}{5}$$

$$\Rightarrow \boxed{\Phi_A = \left[4N i_A + N i_B - 3N_c i_c\right]\frac{\mu S}{15l}} = \underbrace{\frac{4N i_A\,\mu S}{15l}}_{=\ \Phi_{AA}} + \underbrace{\frac{N i_B\,\mu S}{15l}}_{=\ \Phi_{AB}} - \underbrace{\frac{N_c i_c\,\mu S}{5l}}_{=\ \Phi_{Ac}}$$

$$\Phi_B = \Phi_A + \Phi_c = \left[6N_c i_c + 3N i_B - 3N i_A + 4N i_A + N i_B - 3N_c i_c\right]\frac{\mu S}{15l}$$

$$= \boxed{\left[3N_c i_c + 4N i_B + N i_A\right]\frac{\mu S}{15l}} = \underbrace{\frac{3N_c i_c\,\mu S}{15l}}_{=\ \Phi_{Bc}} + \underbrace{\frac{4N i_B\,\mu S}{15l}}_{=\ \Phi_{BB}} + \underbrace{\frac{N i_A\,\mu S}{15l}}_{=\ \Phi_{BA}}$$

a.ii)

$$\boxed{L_c = \frac{d(N_c\Phi_{cc})}{di_c} = \frac{2N_c^2\mu S}{5l}}$$

$$\boxed{L_A = \frac{d(N\Phi_{AA})}{di_A} = \frac{4N^2\mu S}{15l}}$$

$$\boxed{L_B = \frac{d(N\Phi_{BB})}{di_B} = \frac{4N^2\mu S}{15l}}$$

$$\boxed{M_{AC} = \frac{d(N\Phi_{AC})}{di_C} = -\frac{NN_C\mu S}{5l}} = M_{CA}$$

$$\boxed{M_{BC} = \frac{d(N\Phi_{BC})}{di_C} = 3NN_C\frac{\mu S}{15l} = \frac{NN_C\mu S}{5l}} = M_{CB}$$

$$\boxed{M_{AB} = \frac{d(N\Phi_{AB})}{di_B} = N^2\frac{\mu S}{15l}} = M_{BA}$$

Si $i_A = i_B$

b) $$V_C = L_C\frac{di_C}{dt} + \underbrace{M_{CB}\frac{di_B}{dt} + \overset{=-M_{CB}}{M_{CA}}\frac{di_A}{dt}}_{\text{contribución debido a la variación temporal de } i_A \text{ e } i_B} = L_C\frac{di_C}{dt}$$

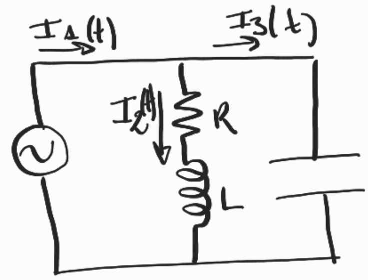
$$\parallel$$

$$M_{CB}\frac{d}{dt}\left(i_B(t) - i_A(t)\right) = 0 \checkmark$$

si $i_A = i_B$

—— X ——

③



$V(t) = V_0 \cos(\omega t)$

$\tilde{V}(t) = V_0\, e^{j\omega t}$

$\tilde{Z}_R = R\,,\ \tilde{Z}_L = j\omega L\,,\ \tilde{Z}_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$

a) $\tilde{Z}_{eq}^{-1} = (\tilde{Z}_R+\tilde{Z}_L)^{-1} + \tilde{Z}_C^{-1}$ ← $\tilde{Z}_R$ en serie con $\tilde{Z}_L$ y esa combinación en paralelo con $\tilde{Z}_C$

$$\tilde{Z}_{eq} = \frac{1}{\frac{1}{\tilde{Z}_R+\tilde{Z}_L} + \frac{1}{\tilde{Z}_C}} = \frac{\tilde{Z}_C(\tilde{Z}_R+\tilde{Z}_L)}{\tilde{Z}_C+\tilde{Z}_R+\tilde{Z}_L} = \frac{\frac{-j}{\omega C}(R+j\omega L)}{\frac{-j}{\omega C}+R+j\omega L}$$

$$= \frac{\omega L - jR}{R\omega C + j(\omega^2 LC-1)} = \frac{(\omega L - jR)\left[R\omega C - j(\omega^2LC-1)\right]}{(R\omega C)^2+(\omega^2LC-1)^2} =$$

$$= \frac{\cancel{R\omega^2LC} - \cancel{R\omega^2LC} + R}{(R\omega C)^2+(\omega^2LC-1)^2} + j\frac{\left[\omega L(1-\omega^2LC) - R^2\omega C\right]}{(R\omega C)^2+(\omega^2LC-1)^2}$$

$$\Rightarrow \boxed{Re(\tilde{Z}_{eq}) = \frac{R}{(R\omega C)^2+(\omega^2LC-1)^2}}\ ;\ \boxed{Im(\tilde{Z}_{eq}) = \frac{\omega L(1-\omega^2LC) - R^2\omega C}{(R\omega C)^2+(\omega^2LC-1)^2}}$$

$$\tilde{V} = \underbrace{\tilde{Z}_{eq}}_{=|\tilde{Z}_{eq}|e^{j\theta}}\tilde{I}_1 \rightarrow \boxed{\tilde{I}_1 = \frac{\tilde{V}}{|\tilde{Z}_{eq}|}e^{-j\theta} = \frac{V_0}{|\tilde{Z}_{eq}|}e^{j(\omega t-\theta)}} \rightarrow \boxed{i_1(t) = \frac{V_0}{|\tilde{Z}_{eq}|}\cos(\omega t-\theta)}$$

donde $\boxed{|\tilde{Z}_{eq}|} = \sqrt{Re^2(\tilde{Z}_{eq}) + Im^2(\tilde{Z}_{eq})} = \boxed{\sqrt{\dfrac{R^2+\left[\omega L(1-\omega^2LC) - R^2\omega C\right]^2}{\left[(R\omega C)^2+(\omega^2LC-1)^2\right]^2}}}$

y $\boxed{\theta = \operatorname{arctg}\left[\dfrac{Im(\tilde{Z}_{eq})}{Re(\tilde{Z}_{eq})}\right] = \operatorname{arctg}\left[\dfrac{\omega L(1-\omega^2LC) - R^2\omega C}{R}\right]}$

$$\tilde{V} = \underbrace{(\tilde{Z}_R+\tilde{Z}_L)}_{=|\tilde{Z}_R+\tilde{Z}_L|e^{j\theta_2}}\tilde{I}_2 \rightarrow \boxed{\tilde{I}_2 = \frac{V_0}{|\tilde{Z}_R+\tilde{Z}_L|}e^{j(\omega t-\theta_2)}}$$

donde $\boxed{|\tilde{Z}_R+\tilde{Z}_L| = |R+j\omega L| = \sqrt{R^2+(\omega L)^2}}$ y $\boxed{\theta_2 = \operatorname{arctg}\left(\dfrac{\omega L}{R}\right)}$

$$\boxed{i_2(t) = \frac{V_0}{\sqrt{R^2+(\omega L)^2}}\cos(\omega t-\theta_2)}$$

$$\tilde{V} = \tilde{Z}_C\tilde{I}_3 \rightarrow \boxed{\tilde{I}_3(t) = \frac{V_0\, e^{j\omega t}}{\left(\frac{1}{j\omega C}\right)} = (V_0\omega C)\underbrace{j}_{=e^{j\pi/2}}e^{j\omega t} = (V_0\omega C)e^{j(\omega t+\frac{\pi}{2})}}$$

$$\boxed{i_3(t) = (V_0\omega C)\cos(\omega t+\pi/2)}$$

b) buscamos ω_0 tal que $\theta = 0 \Rightarrow Im(\tilde{Z}_{eq}(\omega_0)) = 0$

$$\omega_0 L(1-\omega_0^2 LC) - R^2\omega_0 C = 0$$

$$\omega_0[L - \omega_0^2 L^2 C - R^2 C] = 0 \rightarrow \omega_0 = 0 \; X \text{ no sería corriente alterna}$$

$$\downarrow \; \omega_0^2 L^2 C = L - R^2 C$$

$$\rightarrow \boxed{\omega_0 = \sqrt{\frac{L-R^2C}{L^2C}} = \frac{1}{\sqrt{LC}}\sqrt{1-\frac{R^2C}{L}}}$$

(asumiendo $L > R^2C$)

$$\boxed{|I_1(\omega_0)|} = \frac{V_0}{|\tilde{Z}_{eq}(\omega_0)|} = \frac{V_0}{\sqrt{Re^2(\tilde{Z}_{eq}(\omega_0)) + \underbrace{Im^2(\tilde{Z}_{eq}(\omega_0))}_{0}}}$$

$$= \frac{V_0 \left[(R\omega_0 C)^2 + (\omega_0^2 LC - 1)^2\right]}{R}$$

$$= \frac{V_0}{R}\left\{ \underbrace{\left(\frac{L-R^2C}{L^2\cancel{C}}\right) R^2 C^{\cancel{2}}}_{\frac{R^2C}{L} - \cancel{\frac{R^4C^2}{L^2}}} + \underbrace{\left[\left(\frac{L-R^2C}{L\cancel{LC}}\right)\cancel{LC} - 1\right]^2}_{\left(\cancel{1} - \frac{R^2C}{L} - \cancel{1}\right)^2 = \cancel{\frac{R^4C^2}{L^2}}} \right\}$$

$$= \frac{V_0}{\cancel{R}} \frac{R^{\cancel{2}}C}{L} = \boxed{V_0\left(\frac{RC}{L}\right)}$$

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